Emerging Trends in Modern Business

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Examining Contemporary Management Practices of Select Indian Companies

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Abstract

There are various practices through which any organization can achieve and maintain its desired position in a particular industry. These practices need to undergo a continuous churning to make them specific and relevant in the present VUCA times. Adaptability and flexibility are the key levers that enable an organization to man oeuvre itself according to the customer needs and achieve its goal by remaining dynamic to the changes occurring in the turbulent business environment. Adoption of good business practices enables the organizations to fulfil the demands of its various stakeholders and build a good reputation in the industry. Since, human resources are the major source of competitive advantage as well as driving force towards success of any organizations, so the practices governing the people associated with the organizations are of prime importance and hold great relevance in understanding the critical success factors of any successful business entity. This paper primarily focuses on a qualitative study of select business practices which are being implemented by the Indian organizations in the present business scenario that have enabled these organizations to maintain its uniqueness and to help it to outgrow the competitors. It also highlights a operational business practices which are emerging in contemporary business scenario that the organizations deploy in response to the changes in business environment.

Keywords: Adaptation; Dynamic; VUCA; Business practices

Introduction

Every organization has a reason for its existence. It wants to achieve a set of objectives. The means or tactics are used by the organization to fulfil those objectives is referred as its business practices. Just as the end results or the objectives an organization wants to achieve keep on changing with time, likewise, the means to achieve them also need to be changed. Thus, business practices are dynamic in nature; they mostly keep on evolving with time. Contemporary business practices refer to the most recent business practices which are associated with the present time. A company tries to

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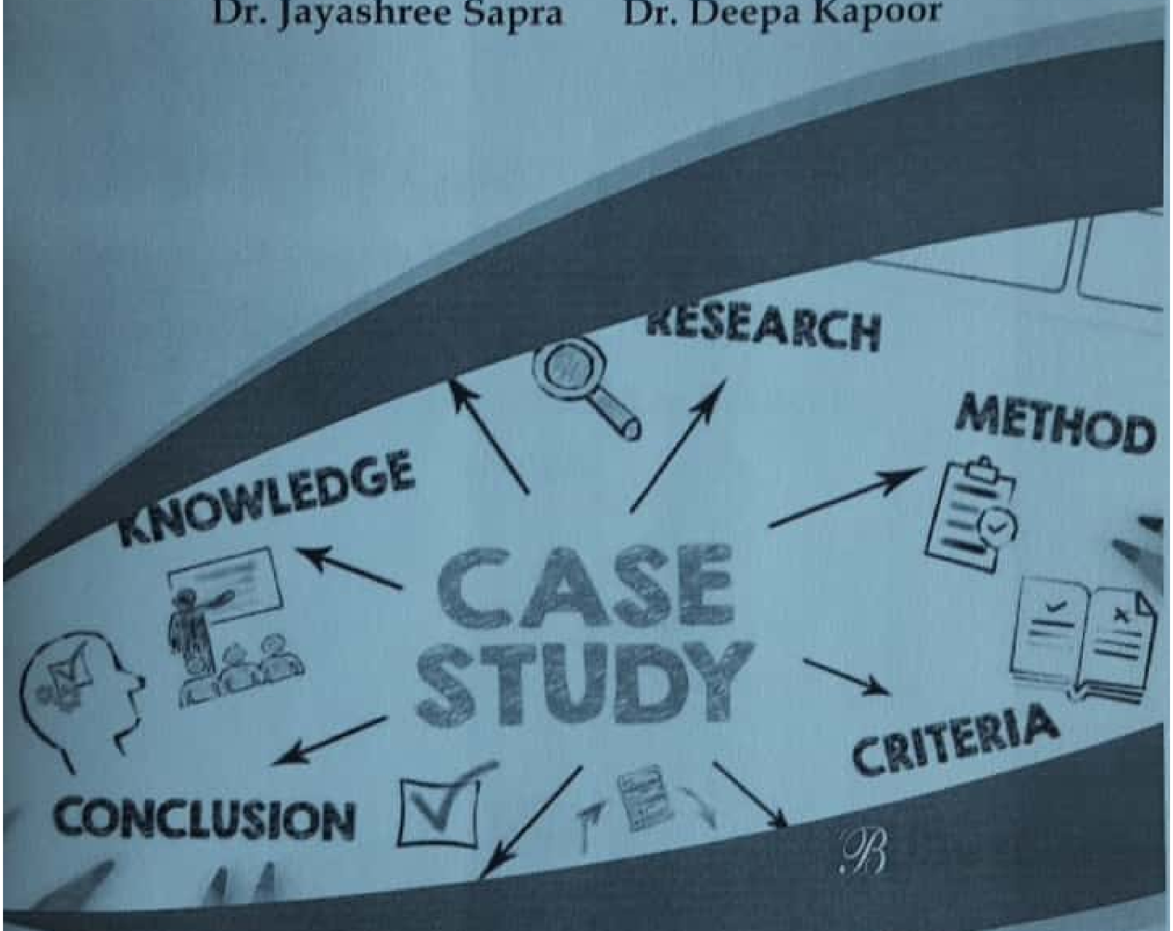

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ENTREPRENEURSHIP IN TIMES OF CRISES: THE MOUNTAINSIDE VIEW

Dr. Aditi Sharma* & Vikrant Chaudhary**

ABSTRACT

The first two decades of the twenty first centuries have been marked by uncertainties, crises, disruptions and turbulence. This all has led to slowing of economic growth in the emerging economies and has led to growth in unemployment rates across the major parts of world. The emerging economies like India too is undergoing transition from managerial economy to entrepreneurial economy and the (Drucker, 2014) current pandemic has accelerated the process as a significant chunk of population lost their jobs during the crises. Self-employment has become the "need of the hour" as people move back to their home towns. The study is based on the narratives shared by the entrepreneurs currently based in and around Dharamshala.

Keywords: Self Employment, Crisis, competencies, entrepreneurship, narrative.

INTRODUCTION AND FRAMEWORK

Entrepreneurs play a critical role in the growth and development of economy.The most important characteristic of an entrepreneur is self-confidence (Knight, 1921). Casson (1982) asserted that 'essence of entrepreneurship is being different'.

The present work examines the cases under study from three lens:

- Psychological Theories of the Entrepreneur
- Sociological Theories of Social Networking and Business Ownership
- An Eclectic social science model of Entrepreneurship

BACKGROUND INFORMATION

The present article centres around the stories of three entrepreneurs based in and around Dharamshala. The first story is of woman entrepreneur who runs a cafè in picturesque Bir

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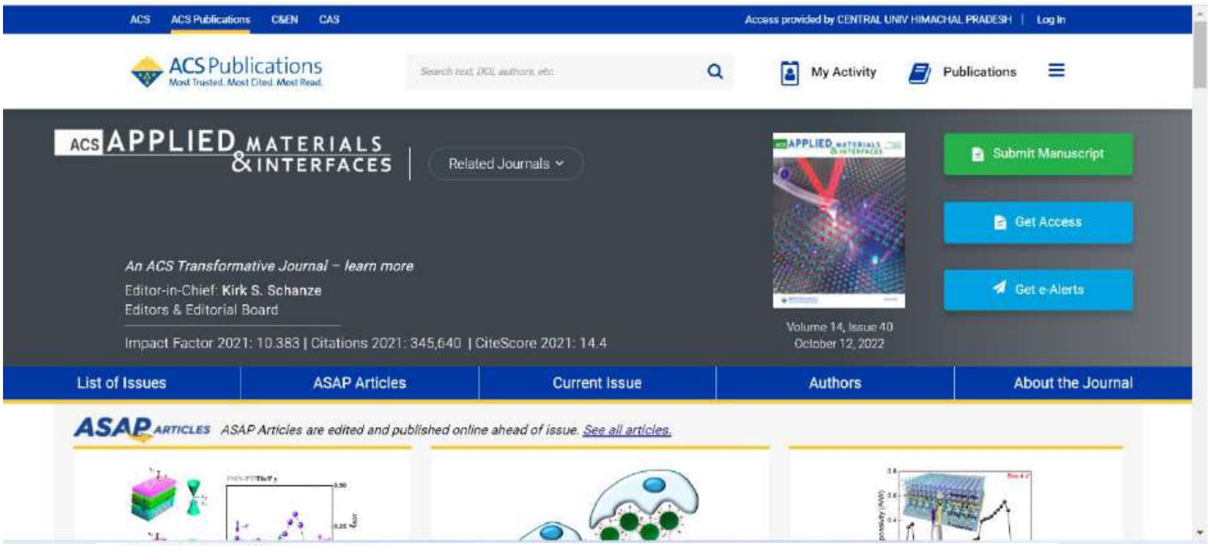

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Molecular Engineered Carbon-Based Sensor for Ultrafast and Specific Detection of Neurotransmitters

Jagadeesh Suriyaprakash, Kanchan Bala, Lianwei Shan, Lijun Wu,* and Neeraj Gupta*

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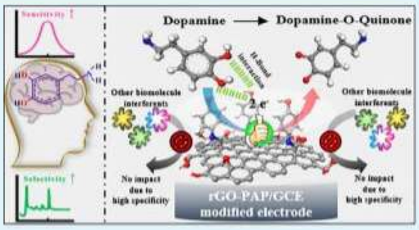
ABSTRACT: In the quest for designing affordable diagnostic devices with high performance, precisely functionalized carbon-based materials with high accuracy and selectivity are required. Every material has its own unique ability to interact with the analyte, and its performance can be enhanced by probing the interaction mechanism. Herein, *p*-aminophenol (PAP)-functionalized reduced graphene oxide (rGO) nanoscale material is developed by a one-step synthetic route as an all-organic-based sensor. As the PAP molecules are precisely covalently interacted with the rGO at the basal plane and form a wrinkled-paper-like structure, the functionalized material exhibits an outstanding sensing ability (7.5 nM neurotransmitter dopamine (DA) at a wide linear range, 0.01−100 μM) with fast electrical transduction (<3 s) and good recyclability (~10 cycles) in a real sample. Combining various analytical and density functional theory (DFT) calculation methods, physicochemical properties and the interaction mechanism of analyte−materials transduction are discussed exclusively. Besides, the potential application of the well-dispersed rGO-PAP gravure ink in flexible-printed electronics fields is explored. This study not only provides new insights into the surface/interface chemistry and working principle of this unique anchoring of PAP on rGO but also offers a new pathway for developing other forms of metal-free/organic functionalized biosensors with high efficiency.



KEYWORDS: *2D materials, electrocatalysis, bioelectronics, neurotransmitter, functionalization, dopamine, biosensor*

1. INTRODUCTION

Accurate diagnosis of neurological diseases to provide affordable health services to the user end is always required in the current health care system. Critical neurological diseases are mostly associated with the dysfunction of dopamine (DA) neurotransmitter in our nervous system.[1] This is found in human blood and serum in extremely tiny amounts [10^{-9}–10^{-5} mol L^{-1} (hereafter M)] along with other biomolecules.[2] Therefore, the advancement of noninvasive probing systems for the detection of DA is highly desirable. Such sensing probes must provide excellent specificity and sensitivity (tens of nM to hundreds of pM) while being inexpensive for commercial use. Over the last three decades, extensive sensing techniques have been employed for the detection of neurotransmitters.[3−13] Among them, the electrochemical sensing technique(s) excel in point-of-care (POC) tests, fast readout, accuracy, and real-time monitoring.[11,12] This technique further relies on the efficiency of the probing materials often designed with the help of metal-based,[14−18] metal-free,[19−21] and their hybrid[22−27] materials.

Since metal-free/organic-based sensors outperform metal-based/hybrid sensors in terms of their nontoxicity, sustainability, affordability, reproducibility, and flexibility in comparison to metal-based sensors, they are widely utilized in electrochemical biosensing techniques.[28,29] However, these sensors have limitations associated with their electronic nature, chemical stability, and fouling behavior. If the attached organic molecule does not have conjugated electrons in its structures, it is hard to control the electron movement in the resulting material. Also, the chemically tagged organic molecules are prone to hydrolysis during operational conditions. The different functional groups present on the surface of organic molecules can combine with different biomolecules and show a rapid fouling behavior, making their reproducibility challenging. Hence, finding a suitable organic molecule for tagging onto the surface of appropriate support is a challenging task, which makes exploration of this research field wide open to date. For instance, a carbon-based sensor was developed by combining the conjugated organic molecule polyaniline

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Catalysis

Metal-Free g-C_3N_4/Graphite Composite Based Carbocatalyst for Epoxidation of Styrene

Ashima Dogra,[a] Anil Kumar,[b] Mohit Kapoor,[c] and Neeraj Gupta*[a, d]

Tuning graphitic surface with nitrogen functionalities renders potential carbocatalysts for a wide range of reactions. Herein, g-C_3N_4/graphite composite material (g-C_3N_4@G) is reported for oxidation of styrene to styrene oxide. The detailed physiochemical properties of g-C_3N_4@G are investigated by X-ray photoelectron spectroscopy (XPS), Transmission electron microscopy (TEM), Scanning electron microscopy (SEM), X-ray diffraction (XRD) and Fourier transmission infrared spectroscopy (FTIR) analysis. The TEM and SEM micrographs indicate the formation of g-C_3N_4 sheets and N1s XPS results confirm the formation of g-C_3N_4 on the graphitic surface. The role of specific catalytic sites is experimentally studied by the use of model catalysts those mimic the functional groups on the surface of g-C_3N_4@G. It is suggested that the catalytic sites on the surface showcase a synergeistic effect, with sp^2 hybridized N site being most selective for styrene oxide.

Carbocatalysis has become a well celebrated area of research over the last few years.[1–3] For the purpose of sustainability and judicious use of resources, it is greatly recommended to employ metal-free carbon catalysts for mediating eclectic organic[4] /inorganic reactions[5] Besides sustainability, the other factor that leads to acceptance of carbon-catalysts as greener alternatives[6] to metallic materials, is economical viability, since feedstock of carbon materials is considerably cheaper than some precious metals.[7] The adjustable configuration, structural or topological defects and surface chemistry of the carbon materials play a crucial role in catalyzing reactions.[8] For instance, the electronic structure regulation of the carbon surface by introducing nitrogen functionalities might efficiently enhance its catalytic activity.[9] The modulation of carbon framework by N sites[10] develops a positive charge on the carbon next to nitrogen sites, thereby accelerating the reaction kinetics.[11] Until now, the nitrogen-functionalized carbon materials have been perfectly exemplified as metal free catalysts in C–H activation,[12] electroreduction of CO_2,[13] oxygen reduction reaction (ORR),[14] oxidation of alcohols,[15] hydrogenation of nitroarenes,[16] and dehydrogenation of ethylbezene.[17]

Researchers have clearly specified the role of nitrogen sites in oxidation reaction, which is one of the principal reactions in chemical industry.[18] Derived from the theoretical calculations, it is stated that carbon materials and their derivatives escalate oxidation reactions by stabilizing the peroxyl radicals.[19] In this catalytic reaction, charge transfer is expected between the pi conjugated structure of the carbon matrix and the radicals.[20] To quote a few metal free catalytic oxidative transformations, the selective oxidation of biomass feedstock, glycerol, to dihydroxyacetone,[21] is catalyzed by pyridinic N-sites in nitrogen containing carbon nanotubes (NCNTs). NCNTs have also shown remarkable activity in selective allylic oxidation of cyclohexene to 2-cyclohexen-1-one.[22] Carbon materials modified by nitrogen are known to have promoted aerobic oxidation of benzylic hydrocarbons and cycloctane.[23]

In this present report, g-C_3N_4/graphite composite carbon material (g-C_3N_4@G) has been synthesized by wet impregnation technique, using commercial graphite as the carbon precursor and urea as the nitrogen source. Subsequently, the material prepared was utilized as a metal-free catalyst in epoxidation of styrene.

The surface compositional features of the synthesized sample (g-C_3N_4@G) were verified by X-ray photoelectron spectroscopy (XPS). The different chemical regions in C1s and N1s spectra are shown in Figure 1. The intrinsic structure of graphite is displayed by the C1s spectrum in Figure 1a, along with a newly formed C–N bond at 284.7 eV due to g-C_3N_4 on the surface of graphite. The chemical identity of the nitrogen functionality present on the surface is revealed by the deconvolution of N1s spectrum (Figure 1b). The peaks at 398.8 eV, 400.2 eV and 401.5 eV are assigned to sp^2 hybridised N atoms, (N-(C_3) and quaternary N (graphitic N), respectively.[4,19]

The surface morphologic information of the g-C_3N_4@G was obtained by Transmission electron microscopy (TEM) and Scanning electron microscopy (SEM). The TEM micrographs in figure 2 (a, b) clearly show the crystalline layered structure of

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Catalysis

Pd-Au Supported Reduced Graphene Oxide Catalyst for Carbon- Hydrogen Bond Activation in Benzene

Deepika Sharma,[a] Kamal Kishor Thakur,[b] and Neeraj Gupta*[a, c]

An improved reduced graphene oxide supported bimetallic palladium and gold catalyst was synthesized for the carbon-hydrogen bond activation of un-activated aromatic organic substrate benzene. XPS analysis has revealed that catalyst contained Au(0) nanoparticles with Pd(II) ions on its surface. The surface of rGO was found to be rich in sp^2 carbon content with >C=O groups that most probably provided supporting sites for the gold and palladium metals. The XRD analysis further confirmed that Au(0) nanoparticles with palladium oxide and palladium nanoparticles were present on rGO surface. The catalytic activity of the material was tested towards activation of benzene to form biphenyl as an end product. Density function theory has revealed that Pd(II) ion was the active component that insert itself in between C–H bond of benzene and showed efficiency to interact with oxygen molecules. Simultaneously, Au(0) atoms have the tendency to adsorb on the π-bonded surface of the catalyst and extend its conjugation that reduced the HOMO-LUMO gap of the catalyst and provided the good catalytic activity.

Introduction

Metallic nanoparticles have attracted great attention due to their potential use in catalytic systems. Noble metal nanoparticles such as Pd, Au, Ru and Pt are very promising materials for different applications in C–C coupling,[1] alkynylation of cyclopropenes,[2] alkyne annulations[3] and C–H bond activation[4,5] reactions respectively. C–H bond activation is one of the most popular strategies for the synthesis of organic molecules and transition metal based catalysts are mostly involved in it. Especially, Pd nanoparticles have displayed great potential in this field.[6] We have synthesized biphenyl with 78% yield through C–H bond activation in benzene over Pd supported graphene oxide catalyst.[7] Biphenyl is an important intermediate, which is used in medicine, cosmetics, pharmacology and many fine chemicals.[8] Bimetallic nanoparticles based catalysts have received great interest to establish an important class of active catalysts.[9] They have been found to improve the efficiency of already existing catalysts for certain chemical reactions.

In this regard, bimetallic catalysts containing gold as one of the component have attracted great attention over the past decades.[2,10,11] The interaction of two transition metals for the C–H bond activation can open a great perspective for chemists in synthesis. The active Pd metal surrounded by Au metal can increase the overall selectivity and long-term stability. The ability of Au(III) to perform C–H bond activation was also established several decades ago.[12,13] This type of reactivity has authorized Au to take part in a range of bond-forming reactions such as cross-coupling reaction[14,15] and oxidation reactions.[16] Gold catalysts are generally robust, require mild reaction conditions and are tolerant to oxygen.[17] The direct functionalization or activation of the C–H bond by mean of gold catalysis has attracted a great deal of interest from both the industries and academia. Un-reactive C–H bond is directly converted into C–C, C-heteroatom bond through gold mediated oxidative cleavage of C–H bond.[13] A few reports are available making use of bimetallic catalysts for the C–H bond activation and biphenyl synthesis.[18] Au/Pd catalyst has been used in many other reactions such as CO oxidation, hydrogenation of hydrocarbons and synthesis of vinyl acetate.[19,20] All these reports highlight that Au is an ideal metal for making an efficient catalyst for the C–H bond activation reaction.

Graphene oxide is a unique support for designing catalysts due to its unique properties such as ease of modification, chemical inertness, tuneable surface functional groups and its strong interaction with metal clusters.[21] Gold nanoparticle decorated rGO (Au NPs-rGO) catalyst was developed for the Ullmann homo-coupling of iodobenzene to synthesize biphenyl.[22] The spontaneous chemical reduction of Au^+ by rGO regenerated the Au^0-rGO nanocomposite. The electron transfers from graphene sheets to Au nanoparticles caused an increase in the reactivity of the Au NPs-rGO nanocomposite.

In this study, we are reporting a bimetallic catalyst Pd-Au@rGO prepared by incorporating active Pd metal ion on Au

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pH controlled efficient conversion of extracted hemicellulose from agricultural waste to furfural using choline based BADES and NADES as homogenous acid catalysts

Ms. Shalini Arora[a], Dr. Neeraj Gupta[b] and Prof. Vasundhara Singh[a]*

Abstract: In the present work we report the valorization of hemicellulose isolated from lignocellulosic biomass (wheat straw, rice husk and bagasse) to furfural by pH controlled acid catalysed valorization using few choline based Bronsted acidic (BA) and natural acidic (NA) deep eutectic solvents (DES) serving both as catalyst and solvent. The effect of pH variation on the catalytic activity of various BADES and NADES prepared in 1:1 molar ratio was observed and ChCl:p-TSA among all the DESs was found to be the best with lower pH value of 1.0. The yield of furfural decreases from 85 to 51% with increase in the pH value from 1.0 to 3.0. The molar ratio of HBD (hydrogen bond donor) to HBA (hydrogen bond acceptor) components was varied from 1:1 to 1:9 to achieve the lowest possible pH values of the DESs and to increase the furfural yield. Further optimization of reaction conditions were also done in terms of DES loading, time of reaction and temperature using the model DES to achieve higher furfural yield. The best results were obtained using 5 mmol DES having pH 1.0 in 1.5 h at 120°C. ChCl:*p*-TSA (Choline Chloride:*p*-Toluene sulfonic acid), ChCl:oxalic acid (Choline chloride:oxalic acid) among BADES and ChCl:LevA (Choline chloride:Levulinic acid) among NADES investigated in this work yielding 85% furfural were found to be most efficient. The reported methodology is advantageous in terms of using bio-based green solvent, milder reaction conditions and efficient up scaling of reaction. The DESs were found to be efficiently recyclable up to five consecutive runs for the process.

Introduction

Global climate change and the sinking of fossil fuels have inspired the development of alternative sustainable resources to replace the current petroleum-based economy.[1-3] Lignocellulosic biomass (LCB), a renewable feedstock possesses the potential for serving as an enduring carbon source alternative to fossils, reducing the over-reliance on fossil resources for producing chemicals and fuel.[2,3] Among the major LCB components, hemicellulose, containing C5 and C6 sugars, could be valorized to fine chemicals and fuels.[4] Among the platform chemicals present in hemicellulose, furfural peculiarly fascinates the researchers, as it can be a precursor for large number of chemicals,[5] such as furfuryl alcohol, levulinic acid and γ-valerolactone (GVL) to mention a few. Additionally, furfural retains the potential to replace petroleum-based chemicals for the large scale production of plastics, adhesives, etc. and acts as a direct additive to fuel blends.

Processes for producing furfural on commercial basis possess multiple disadvantages of low yields; high energy–consumption, etc.[6] Obstacles co-related with the sustainable growth and betterment of chemical technology for furfural production is still a reasearch area of great importance.[7]

Typically, furfural is mostly produced from xylose or xylan (hemicellulose) using acid catalysts.[8,9] Recent studies target the designing of novel methodologies and developing efficient, stable and water–tolerant catalysts for biomass valorization and other applications such as fuel cell, feedstock, reforming in refineries, carbon dioxide valorization to mention a few.[10] Additionally, the development of various reaction systems for the synthesis of furfural has also been thoroughly investigated. Various approaches for the production of furfural have been reported in the literature, mainly focussing on the development of different catalytic systems including mineral acids,[11,12] metal salts,[13-15] zeolites,[16-18] Amberlyst,[19] heterogeneous solid acid catalysts either in monophasic or biphasic systems such as H_2SO_4 in water/THF,[20] metal halides in water/toluene[21] and SAPO–44 in water/toluene.[22] However, the complications with the reported methodologies is their tedious reaction protocols. Although, one-pot conversion of xylan has also been reported by using HUSY with Si/Al as catalyst, but with low yield of products.[23] The key shortcomings of above mentioned catalytic systems employed for the process are longer resistance times, higher temperature conditions and involvement of high-energy processes of distillation and additional treatments such as neutralization generating large waste streams, decreased furfural yield and selectivity. Recent literature also reports the use of various ionic liquids as catalyst for various applications in energy effective processes such as xylose and xylan hydrolysis[24] and fixation of industrial combustion CO_2 into high-value organic salts.[25] The use of acidic ionic liquids (ILs) has also been investigated as it is well known that the acidic strength of acid catalyst/ ionic liquid plays a primary role in the degradation of hemicellulose.[26-28] Various ionic liquids [BMIM]HSO_4,[29] H-ZSM-5,[30] sulphonted graphene oxide,[31] HSC-SO_3H,[32] [Ch-SO_4H][CF_3SO_3][33] have been reported for hydrolysis of xylan and xylose. Among them [Ch-SO_4H][CF_3SO_3][33] was found to show promising results for xylan hydrolysis to furfural in 82% yield. Integration of ionic liquids

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Brønsted acid functionalized carbon catalyst for synthesis of biologically active coumarin-substituted bis(indolyl)methanes

Vikrant Singh [a,1], Ashima Dogra [a,1], Joydeep Das [b], Prasenjit Manna [c], Neeraj Gupta [a,d,*]

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ABSTRACT

A novel carbon based catalyst has been prepared for the synthesis of potential anticancer compounds with low toxicity by an ionic liquid assisted doping process. Sulphonic acid functionalized nitrogen sulphur co-doped graphite (SO_3H-NSG) based catalyst was prepared by coating the carbon surface with the sulphonic acid bearing ionic liquid and annealing the resulting compound. X-ray photoelectron spectroscopy revealed the successful doping of nitrogen and sulphur atoms in the graphitic matrix and the functionalization of the surface with sulphonic acidic sites. The catalyst was used for the synthesis of biologically active coumarin-substituted bis (indolyl)methanes that were finally evaluated for their toxicity and anticancer properties. The cytotoxicity results unveiled that the iodo derivative of the coumarin-substituted bis(indolyl)methanes showed best anti-cancer activity among the four derivatives synthesized.

Introduction

Indole nucleus is a biologically celebrated pharmacophore in medicinal compounds, making it a multifaceted heterocyclic with gamut of biological applications [1]. Among the functionalized indoles, bis (indolyl)methanes (BIMs) exemplify privileged molecules holding a broad spectrum of pharmacological activities [2]. BIMs are known to serve as bio activators, such as antibacterial agents [3], anti-inflammatory agents [4], active regulators in colon cancer [5], and inhibitors against growth of human prostate cancer cells [6]. Considering that these organic molecules cover an extensive range of medicinal uses; various synthetic strategies have been investigated for their development. A couple of common approaches comprise are the Friedel-Crafts acylation of indoles with carbonyl compounds [7], dehydrative nucleophilic substitution of indolyl alcohols [8], cross-dehydrogenative coupling via methanol activation [9], and Friedel-Crafts alkylation of substituted indoles [10]. The structural accessibility of BIMs has led to exploration of diverse synthetic methods; nonetheless, these are somehow limited to tertiary substituted derivatives. The protocols for tetra-substituted are relatively scarce [10]. Thus, the first objective of our work was preparation of tetrasubstituted BIMs, with coumarin as the structural unit. The advancement in synthesis of coumarin skeleton and its derivatization has attracted many organic and medicinal chemists, due to its manifold industrial [11,12] and biological [13–15] utilities. Such molecules holds great potential in the discovery of new drugs and utilizing a metal free heterogeneous catalyst for the synthesis can help to accelerate their applicability especially during the clinical trials. If the drug is synthesized via a metal free pathway, it can avoid the lengthy purification procedures adopted to achieve the desirable specification limits for residual metals in the active pharmaceutical ingredients or drugs [16].

Hybridization of drugs to form a single molecule is emerging as an efficient tool in production of new drugs, fulfilling the purpose to diversify, intensify, or utilize the drug in two-fold action [17]. Therefore, the combination of these two set of molecules (BIMs and coumarin) with fascinating bioactivities would provide a better scope for construction of beneficial compounds [18,19]. Previously, Yuan et al [20] and Wang et al [21] synthesized coumarin- substituted bis(indolyl) methanes in a metal-Lewis acidic catalyzed reaction, carried out in a range of organic solvents (Fig. 1). Since, transition metal catalysis has several disadvantages, like toxicity, cost, air and moisture sensitivity, etc.; we aspired to conduct the study in a metal-free heterogeneous

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Development of metal free melamine modified graphene oxide for electrochemical sensing of epinephrine

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ABSTRACT

Metal free material melamine modified graphene oxide (MGO) was prepared by reacting melamine with graphene oxide followed by thermal annealing process. The prepared material was characterized by a range of microscopic and spectroscopic techniques. It was then applied for the detection of epinephrine at a biological pH. An MGO-modified GCE displayed improved electrocatalytic activity by shifting the anodic peak current negatively by 0.1193 V and the cathodic peak current positively by −0.2311 V, compared to those at the GCE. Density Functional Theory (TD-DFT method) was used to predict the electron transfer (ET) sites of MGO. The limit of detection of melamine at the MGO -modified glassy carbon electrode was estimated to be 0.13 μM which indicated the potential application of the electrode to the biosensing of the neurotransmitter epinephrine.

Introduction

Epinephrine is an inhibitory/catecholamine neurotransmitter [1] which exists in human blood serum as an organic cation in the quantity of 25–50 pg/mL in adults [2]. Its primary function is to increase the cardiac output by regulating blood pressure, raise the glucose level in blood and regulate the autonomic functions [3,4]. The fluctuating level of EP causes several diseases such as chronic active hepatitis, adrenal hyperplasia and hypoglycemia [5]. It is used to treat different allergic conditions, emphysema, glaucoma and bronchial infections [6]. Therefore, EP is an emergency healthcare medicine [7] and finding its trace level quantitatively is an important task. A number of analytical techniques have been developed for the determination of EP including high-performance liquid chromatography, fluorimetry, capillary electrophoresis, chemiluminescence and mass spectrometry [8,9]. However, these methods require complicated pretreatment of samples, skilled analysts, sophisticated instruments, and time-consuming procedures [10]. One of the notable drawbacks is their operation at acidic pH which is not favorable for the biological samples. As the pH value of human blood and urine is 7.35 to 7.45 and 4.5 to 8.0 respectively [3] and hence it is beneficial to develop the sensing material for application at in the pH range of 7–8.

In contrast to these techniques, the electrochemical methods have attracted much attention for the determination of EP [11]. The analytical performances of the voltammetric method were highlighted from previously developed chemical and biochemical sensors [12]. This molecule is electrochemically active because of its conjugated π-electrons and –OH groups that offer quantitative determination by a suitable electrochemical method [13]. This group of techniques is most appealing alternate because of its low cost, on-site monitoring, simplicity, selectivity, sensitivity, low detection limit and working efficiency at physiological pH. The EP detection using bare electrode requires high overpotential and show sluggish kinetics of electronic processes [14]. Also, adsorption of the oxidized product (epinephrinequinone) causes blocking of the electrode surface, which ultimately leads to the poor electrochemical response. Therefore, GCE [15], CPE [16] and gold electrodes [17,18] modified with different composites are used for the electrochemical detection of EP. There are numerous reports on the use of gold [5], iron [4], nickel [7], cobalt and palladium [19] based nanomaterials for the determination of EP. But, the use of metals should not be promoted in devising a sensor for biological system owing to their high cost and limited environmental reliability.

A wide range of carbon based materials are employed along with some modifiers for the electrochemical sensing of EP as a good

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Structural and optical amendment of PVDF into CQDs through high temperature calcination process

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ARTICLE INFO

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ABSTRACT

A controlled and successful high temperature calcination of PVDF was performed to synthesize high quality CQDs from polymeric membrane via electrospinning to control its shape and size. X-Ray Diffractograms show the formation of CQDs after thermal reduction of polymeric nanofibers with an average crystallite size of ~1.9 nm. Raman spectrum reveals the formation of intense G-band which confirms the presence of carbon nanomaterials in the form of CQDs with ratio of intensities I_G/I_D ~3.7. The variation of FTIR spectrum before and after calcination process supports the speculation of the suppression of oxygen containing functional groups. XPS and UV–Vis absorption spectrum confirms the highly stable and rich carbon content of C=C group. FE-SEM and TEM demonstrates the spherical distribution of quantum dots with an average size of ~2.6 nm.

1. Introduction

Carbon quantum dots (CQDs) have gained significant attention by various researchers due to its unique electrical properties caused by quantum confinement and edge effects. Conventional semiconductor QDs based on metallic elements such as CdS, CdSe, PbSe and Ag_2S suffer from various problems like toxicity, high cost and hydrophobicity which hinder their practical application. In this context, researchers have shifted their focus towards carbon-based quantum dots due to their high chemical stability, low toxicity, biocompatibility and easy functionalization [1,2]. There are a variety of approaches to synthesize CQDs including pyrolysis, electrochemical exfoliation, incomplete combustion oxidation, laser ablation, acidic oxidation, hydrothermal process, microwave passivation, plasma treatment. However, it is still desirable to synthesize high quality CQDs via a facile and feasible route using low-cost raw materials [3]. Compared to traditional semiconductor quantum dots and organic dyes, photoluminescent carbon-based quantum dots are superior in terms of high (aqueous) solubility, robust chemical inertness, facile modification and high resistance to photobleaching. In addition, superior biological properties such as low toxicity and good biocompatibility, makes them a good candidate for potential applications in bioimaging, biosensor and biomolecule/drug delivery. Many approaches have been used to synthesize carbon quantum dots like chemical ablation [4], electrochemical carbonization [5], laser ablation [6], microwave irradiation [7], solvothermal treatment [8]. It was first obtained in 2004 during the purification of single-walled carbon nanotubes through preparative electrophoresis and then by using laser ablation of graphite powder and cement in 2006 [9], however this method have several limitations like low quantum yield, and poor control over particle size. The chemical ablation process though is the most accessible and simple method to synthesize carbon quantum dots, however, is done under harsh conditions, yields poor control over size, and uses multiple steps and is therefore a tedious process to commercialize the production of CQDs. Both microwave irradiation and solvothermal treatment again yields poor control over shape, size and physical properties [8]. Therefore, controlling the size of carbon quantum dots is a critical step to reach stable properties for specific applications. Up to now, numerous investigations have been done to attain uniform and homogeneous CQDs through synthesis or post-treatment.

Herein, the main aim of this research work is to synthesize high quality CQDs via electrospinning followed by thermal reduction. Electrospinning is a feasible method to fabricate highly aligned nanofibers under the influence of large electric field [10]. To the best of our knowledge, no research work has been reported to synthesize CQDs using electrospun calcination method.

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A proof of concept for cooperation from the quinone groups adjacent to N sites during the metal-free oxidation of glycerol by nitrogen-rich graphene oxide†

Ashima Dogra,‡[a] Vinit Sharma,‡[a] Ilaria Barlocco,[b] Alberto Villa[b] and Neeraj Gupta *[ac]

Glycerol is a key by-product in biodiesel production and can be utilized in the synthesis of value-added chemicals. The low cost and fairly abundant availability of glycerol can be advantageous in producing a variety of pharmaceuticals and cosmetic products. Among the various catalytic transformations, selective oxidation is a promising pathway for the valorization of glycerol. In this present report, we deliver a first proof of concept for the involvement of quinone groups adjacent to N sites on the GO surface, for the selective oxidation of glycerol to dihydroxyacetone (DHA). Graphene oxide is covalently functionalized with 2,4-dihydroxypyridine (DHP), which resembles the identified active sites in the carbon clusters. As anticipated, the DHP-functionalized graphene oxide catalyst (DHP@GO) improved the conversion of glycerol to DHA, the main product, along with minor amounts of glyceric acid (GA) and fumaric acid (FA).

Introduction

The pertinent environmental indicators such as the over-consumption of fossil fuels and climate change make it imperative to substitute petroleum sources to produce fuels and chemicals. Biomass, an Earth-abundant resource is considered to be a significant replacement of fossil fuels for the synthesis of valuable chemicals, the production of clean energy and high-quality fuels.[1,2] Oxygenated biomass platform molecules have attracted keen interest in the catalysis community for exploring heterogeneous catalysts; specifically, heterogeneous carbon materials for the selective valorization of polyols, of which glycerol is seemingly the most well-known molecule.[3] Glycerol, a by-product formed in abundance during the mass-production of biodiesel by transesterification, has the potential to function as a feasible feedstock for synthesis of value-added chemicals.[4] The highly functionalized molecular constitution of glycerol allows its conversion into valuable compounds through various catalytic reactions, such as dehydrogenation, oxidation, esterification and so on. Among these reaction pathways, the selective catalytic oxidation of glycerol to dihydroxyacetone (DHA)[5] is conducive for the cosmetics, pharmaceuticals, food, chemical and many other industries.

The selective formation of DHA requires the specific cleavage of secondary (2°) C–H and O–H bonds. However, preferential activation of the secondary O–H over the primary O–H bond is not an easy task.[6] In addition, the difficulty of achieving C–C cleavage to form C_2 and C_1 side-products, as well as excessive oxidation to carboxylic acids is a matter of concern[7] with this reaction. Substantial attempts have been made in the field of metal-based heterogeneous carbon catalysis[8–10] for the selective oxidation of glycerol to DHA; however, not much has been explored in the area of metal-free carbon catalysis.[11,12] Previously, our group has assessed nitrogen-containing carbon nanotubes (NCNTs), which are rich in pyridinic N sites for the selective oxidation of glycerol to DHA.[13] This earlier work suggested the involvement of pyridine-N-oxide (then pyridine oxime) and cooperation from the quinone groups adjacent to the N sites during the selective oxidation of glycerol to DHA. In this report, we provide a proof of concept for the involvement of these active sites during the oxidation of glycerol. The active site in which the carbon atom adjacent to the N center is already oxidized (>C=O) was anchored to the carbon framework by covalent tagging (the catalyst design is given Fig. S1, ESI†). Therefore, graphene oxide (GO) sheets were covalently functionalized with 2,4-dihydroxypyridine, which resembles the proposed reactive sites, embedded in the carbon cluster.[13] The material DHP@GO was synthesized by refluxing 40 mg of 2,4-dihydroxypyridine with 200 mg of GO in H_2O at 50 °C for

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Immobilized Molecules' Impact on the Efficacy of Nanocarbon Organic Sensors for Ultralow Dopamine Detection in Biofluids

*Jagadeesh Suriyaprakash, Neeraj Gupta, Lianwei Shan, and Lijun Wu**

Dopamine (DA) is a key neurotransmitter that regulates many behaviors and physical functions in the human body. Therefore, there is a significant technological demand for an affordable point-of-care (POC) device to monitor low DA levels up to 10^{-12} M in human biofluids. This can be achieved if the underlying interaction mechanism of analyte– materials–transduction is tuned at the molecular level in the carbon-based organic sensors, and such knowledge is deficient in current literature. Herein, the one-step facile molecular engineering approach is adopted to develop three organic-based sensors consisting of different arylamines (hydroxyl/carboxyl/sulfonate) anchored to the surface of 2D-reduced graphene oxide (rGO). The impact of immobilized molecules on their efficacy is studied in detail by experimental and computational approaches. The carboxyl–arylamine attached to rGO surface outperforms others, resulting in the detection of DA up to 10×10^{-12} M (at a larger linear range of 10^{-10}–10^{-4} M) in the presence of interferents. As a proof of concept, a POC device is designed and evaluated in serum, artificial sweat, and urine samples, exhibiting exceptional results. The rational understanding and fundamental science behind the tunable features of carbon-based organic sensors presented in this work will enable the development of innovative probes for detecting neurotransmitters and related biomolecules.

1. Introduction

Neurological diseases are mostly associated with neurotransmitters[1] that are used by our nervous system to transmit messages between neurons. Dopamine (DA) is one such important chemical messenger that affects our behavior and physical functions such as learning, motivation, heart rate, functioning of blood vessels, pain processing, and movement.[2,3] The imbalance of DA is our body causes mental disorders and other crucial diseases such as Parkinson's disease, attention deficit hyperactivity disorder, and obesity.[4–8] Meanwhile, it is also an important biomarker for particular cancer strains like heochromocytoma and paraganglioma.[9,10] DA detection in real physiological conditions is very challenging because of its very low concentration of 195.8×10^{-12} and 274×10^{-9} M in blood and urine samples, respectively, which is further complicated by the presence of potential interfering agents.[11,12] Therefore, early diagnosis of this neurotransmitter with precision is required for providing effective treatment. Unfortunately, a fewer methods are available for the low-level detection of DA in the laboratory with almost negligible devices for the frontline healthcare of the affected persons.[13–19] Also these diagnostic approaches rely on the expensive equipment along with a tedious suitable sample preparation process, causing DA detection nearly impossible at the point-of-care (POC).

Dopamine has strong redox properties that make it suitable for electrochemical detection. The first commercial portable POC device was prepared for glucose detection, and it was also based on the electrochemical principle.[20,21] Therefore, numerous materials based on electrochemical sensing technologies have been developed so far.[22–33] Ultrasensitive electrochemical sensors with DA detection limit in femtomolar have recently been reported, but their use for developing the sensing probes (for POC units) remains limited due to the use of precious metals, aptamers, enzymes, composites, and highly processed carbon-based derivatives.[34–53] Since sensors based solely on 2D carbon derivatives perform better than metal-based/hybrid/polymer composite(s) in terms of compatibility, simplicity, and affordability, their use is generally promoted in electrochemical biosensing technologies.[54–62] Nevertheless, these sensors have drawbacks associated with processability, cross reactivity, and sensitivity. Some of the recent works have attempted to address these challenges, but they fail in terms of rapid data accessibility and antifouling characteristics.[63,64] Furthermore, probing the DA at the nanomolar level in undiluted waste biofluids such as sweat and urine is still in the pipeline.[12,29,65] The majority of previous studies have mainly

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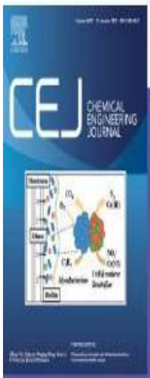

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Engineering of all solution/substrate processable biosensors for the detection of epinephrine as low as pM with rapid readout

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ABSTRACT

Designing next-generation affordable compact point-of-care (POC) epinephrine biosensors is a significant and challenging issue at the moment. In this context, all solution/substrate processable sensing material is developed by a simple one-step molecular engineering of 2D-reduced graphene oxide (rGO). As a proof-of-concept, a flexible POC device is fabricated which demonstrates a distinct and selective response to epinephrine down to 13/20 pM in the buffer/real sample solution in a wide linear range of 10^{-10}-10^{-4} M with rapid readout (2.2 s). Systematic experimental and density functional theoretical (DFT) studies are conducted to uncover the underlying reason for the sensor's remarkable performance. It is found that the precise link between the immobilized molecule [p-aminobenzoic acid (PAB)] and the 2D-rGO basal plane results in a beneficial change in the 2D-topological feature, charge mobility and interlayer chemistry. Besides, the sensing material functions as biomolecule selector, capturer and transducer via strong H-bond interaction and π-π electron coupling/resonance effect, which leads to enhanced sensitivity and specificity. The achievement of this simple yet efficient molecular engineering technique, which can successfully alter the electronic and chemical arrangement of the 2D matrix, opens up a new avenue for the development of various types of flexible and tunable biosensors.

1. Introduction

Epinephrine (EP) is one of the vital neurotransmitters and hormones in mammals and is essential for proper functioning in the central nervous system, hormonal system and numerous biological activities [1,2]. When EP is imbalanced in our body, neurodegenerative diseases and other chronic illnesses such as active hepatitis, adrenal hyperplasia, ischemic stroke, epilepsy and hypoglycemia will appear [3–5]. EP is also used in various allergic and emergency health condition treatments [6–9]. Moreover, the use of EP in sports was banned owing to its doping activity [10]. The quantity of EP in a biological sample is regarded as a significant diagnostic tool for monitoring behaviour, pharmacological action, or determining the inciting agent in possible poisoning targets [11,12]. Thus, developing an affordable POC device for the detection of EP in real physiological conditions at the trace level (0.01 to 10 nM) with rapid readout and high accuracy is a significant task in a variety of fields.

Since EP has good redox and electrochemically active characteristics, an integrated device based on electrochemistry is a promising methodology for its detection compared with others, such as magnetic resonance imaging (MRI), spectroscopy, chromatography and thermal wave transport [13–20]. Over the decade, extensive electrochemical biosensors (metal-based / metal-free / hybrid/ aptamers) have been employed for EP detection [21,22]. Among them, carbon-based organic sensors excel in biocompatibility, affordability, reproducibility and flexibility features [23–29]. However, the limitations of these sensors in terms of stability, selectivity, sensitivity, scalability and antifouling (against biofouling and electrochemical fouling) leave this research field wide open to date. Undoubtedly, one may construct a reasonable sensor by integrating the following strategies: (i) amplifying an electrochemical signal, (ii) anchoring suitable species at appropriate positions/proportions, (iii) diminishing the non-faradic component and proper band alignment between analyte and probe material. Which can be obtained via an add-on approach using individual physical/chemical moiety features. Most of the existing literatures followed this add-on approach, which makes sensor fabrication complicated and impractical; though they display exceptional EP detection capabilities.

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Janus 2D-carbon nanocomposite-based ascorbic acid sensing device: Experimental and theoretical approaches

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ABSTRACT

Precisely functionalized and structurally well-ordered carbon-based nanocomposites are in high demand for developing economical molecular-recognition-based devices with solid efficacy. In this context, a flexible point-of-care sensor is developed for the highly specific detection of ascorbic acid (AA) down to picomolar level using a Janus carbon nanocomposite (JCC) consisting of asymmetric molecular engineered porous/highly ordered carbon nanomaterial. JCC is constructed via a new strategy of simultaneous covalent and non-covalent molecular grafting techniques by exploiting the guest-host molecule's interface chemistry. As the guest molecules are precisely linked with the host at the basal plane, it exhibits better charge mobility, heterogeneous electron transfer, energy density, and stability. The alterable interlayer distance of this unique JCC is also advantageous for biomolecule binding and dissemination. An integrated flexible device that exhibits a specific and selective response to AA down to 47 pM in the buffer and similar excellent performance in serum/artificial sweat-urine samples, over a broad linear range of 10^{-12} M to 10^{-6} M. The integrated sensor has high selectivity, durability (88%/60 days), stability (95.3%/10 cycles) and reproducibility (RSD 1.7%/5 devices). The rationale behind this simple design technique and the science underpinning sensor operation are being systematically explored to the state of the art, which will aid in the development of high-performance Janus 2D-nanocomposite-based sensors.

1. Introduction

Ascorbic acid (AA) is one of the most important antioxidants and vitamins in humans, and it is required for proper functioning in a variety of life processes such as cell division, iron absorption, ligament/skin/blood vessel recovery, metabolism, and collagen synthesis acceleration [1–6]. It is commonly used to treat conditions such as scurvy, the common cold, anemia, infertility, and cancer [7]. Besides, it protects neurons/DNA from oxidative damage, glutamate toxicity, and aging-related degenerative diseases [8,9]. As a result of nutritional loss through our daily activities, AA is consumed in large amounts and can be probed in secreted biofluids such as urine, sweat, and blood. Since the human body is incapable of producing ascorbic acid, it must be received solely via diet. However, irregular intake of AA causes stomach upset, diarrhea, and calcium oxalate kidney stones. Thus, finding precise, quick, and simple techniques for assessing AA in real samples at a wide range from 10^{-12} mol L^{-1} (pM) to 10^{-3} mol L^{-1} (mM) is essential for prompt therapeutic response.

Since AA has good redox characteristics, an integrated device based on electrochemistry is a promising methodology [10–21]. Its matrix effect and device complexity are quite modest when compared to others such as chromatography [22], titrimetry [23], colorimetry [24], fluorimetry [25], and spectrophotometry [26]. Over the decade, extensive electrochemical biosensors such as metal-based [15,20]/metal-free [13,27]/hybrid [16,17,28]/aptamers [29] have been employed for AA detection. However, the interference of other biomolecules, such as neurotransmitters that coexisted with AA and had the same oxidation potential, made it difficult to apply these newly developed sensors to the direct electrochemical detection method of detecting AA in real sample analysis. This interference negatively impacted the accuracy of the detection. Further, devices of the metal-based, hybrid, and aptamer types failed to meet the requirements for biocompatibility, affordability,

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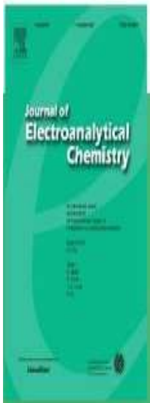

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Increasing the efficiency of reduced graphene oxide obtained via high temperature electrospun calcination process for the electrochemical detection of dopamine

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ABSTRACT

Reduced graphene oxide (rGO) has attracted significant attention for electrochemical sensing applications. In this work, rGO was obtained by thermal annealing of electrospun polymeric nanofiber membrane at 500 and 600 °C. The XRD patterns reveals the phase and crystalline formation of rGO. The interlayer spacing decreases at higher temperature that indicates the removal of oxygen containing moieties. FTIR spectrum shows the absence of epoxy, carboxyland hydroxylgroups for rGO-600 that resembles the surface feature of rGO. XPS further corroborates the XRD and FTIR results and quantifies the predominant functional groups in rGO-500 and rGO-600 °C. The synthesized materials were applied for the electrochemical sensing of dopamine (DA) by cyclic voltammetry. In the case of rGO-500/SPCE, a linear relationship for the DA concentration wasobserved in the range of 0.5 μM to 20 μM with a detection limit of 1.11 μM. Whereas, rGO-600/SPCEalso gave a linear relationship for the DA concentration in the range of 0.5 μM to 20 μM with 1.23 μM detection limit. These electrodes showed good electrocatalytic activity for the oxidation of DA with a minute variation in their detection limit. Therefore, the annealed material rGO can be efficiently used for the quantitative analysis of dopamine after carefully controlling the surface functional groups.

1. Introduction

Carbon materials have been tremendously used in electrochemical sensing applications due to theirvariablemorphologies and electronic conductions [1]. The surface functionality, structure and porosity of the carbon electrode have a direct impact on the analytical performance and electron transfer rate [2,3]. Graphene related materials exhibit unique and intriguing properties with a great potential in various applications such as preparation of supercapacitors, nano sensors, nanomedicines and nanoelectronics. In case of graphene oxide (GO), there is considerably high amount of carboxyl, hydroxyl and epoxy groups on its surface that decreases the overall electrical conductivity. Therefore, reduced graphene oxide (rGO) has gained enormous attention due to its robust electrical and mechanical properties. rGO exhibits properties between graphene and graphene oxide. There are various methods to synthesize rGO like micromechanical cleavage of natural graphite flakes using a Scotch tape [4], longitudinal unzipping of CNTs [5], hummers method [6], epitaxial growth of graphene on SiC [7], thermal or plasma enhanced chemical vapour deposition (CVD) of graphene [8], chemical exfoliation of graphitic materials [9], tour method [10] and electrochemical exfoliation of graphite [11].Generally, the chemical synthesis followed for the preparation of rGO involves three main steps i.e. oxidation, exfoliation, and reduction [12]. The first step involves the oxidation of graphite into graphite oxide, followed by exfoliation of graphite oxide into graphene oxide (GO) and finally the last step where GO undergoes reduction process to form reduced graphene oxide (rGO). Therefore, an approach to develop a simple and feasible synthesis route for rGO is required.

Dopamine (DA) is an important compound in neurotransmission and has been responsible for various neurological diseases [13]. The insufficient level of DA can lead to many neurological disorders such as hyperactivity, schizophrenia and Parkinson's disease [14]. DA is an electro-active molecule and therefore, the electrochemical process [15] for the simultaneous determination of these biomarkers has

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Graphitic sulfur functionalized carbon sheets as an efficient "turn-off" absorption probe for the optical sensing of mercury ions in aqueous solutions†

Vishal Bharati Jaryal,[a] Dilbag Singh[b] and Neeraj Gupta *[a]

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India is one of the world's largest producers of coconuts, with a turnover of 11706343 tonnes (11521459 long tons) in 2018 according to data reported by the Food and Agricultural Organization of the United States. The fleshy part (mesocarp) is either eaten or used to process oil, whereas the outer husk is usually removed and becomes waste. In this research work, a graphitic sulfur functionalized carbon sheet (g-SFCS) sample was prepared from *Cocos nucifera* (Coconut palm) by a chemical activation and thermal annealing process at 800 °C. X-Ray analysis confirmed the amorphous nature of the g-SFCS sample with increasing regularity of the crystalline structure. The BET specific surface area of the g-SFCS sample was found to be 659.41 m^2 g^{-1} with a mesoporous structure. The XPS analysis revealed that the material obtained has a 90.44% carbon content. The material was then analyzed for its selectivity towards Hg^{2+} ions, and the UV spectroscopy results indicate the presence of a strong absorption peak at 237 nm in the absence of Hg^{2+}. However, the presence of Hg^{2+} caused a significant reduction in the absorption intensity and a linear variation is seen with increasing concentration, with the LOD of 10.4 μM. The previous results clearly indicate that *C. nucifera* husks can be potentially applied as a precursor material for the preparation of highly efficient, low cost and eco-friendly g-SFCSs. This carbon material can act as a better adsorbent for adsorbing harmful and highly toxic heavy metals from water sources and make them suitable for use by humans and other living organisms.

Introduction

Heavy metal pollution has become widely prominent in different parts of the world, especially in developing countries with extremely high populations such as India and China. Even at low quantities, heavy metals are extremely toxic to aquatic plants and animals. The most prevalent hazardous metal found in natural ecosystems is mercury.[1] The electrical, paint, chlor-alkali, paper, pulp, and medical industries are just a few of the sectors that widely utilize mercury. However, the chlor-alkali industry is both the major consumer and producer of mercury pollution. When living organisms ingest mercury and its compounds, it leads to serious health implications. One of the special features of mercury is its strong attraction towards biological tissues. It is eliminated from biological systems very slowly. The potential health hazards caused due to the toxic effects of mercury are very common and are well known in Minamata, Japan and Iraq.[2] The harmful implications of mercury on living organisms depend on the nature of the compounds involved, its ingested amount and the mode of entry into the body of living organisms. Mercury and its salts found in industrial effluents are causing serious health hazards on a global scale.[3]

To reduce mercury pollution from various water resources, several environmental regulations have been imposed worldwide to limit its concentration in various liquid emissions.[4] Adsorption by activated carbon (AC) or several other adsorbents is one of the most efficient and cost-effective techniques for extracting mercury from waste water and other aqueous systems.[5] The most commonly used adsorbents for capturing mercury are powdered AC and granular AC.[6] The higher removal efficiency of AC is principally due to its well-developed internal pore structure and increased surface area, which are better suited for adsorption. The surface chemistry of AC (nature and number of functional groups) plays a crucial role in the elimination of mercury ions from water.[7]

Many studies have been targeting the enhancement of the selectivity of the AC by modifying their surface features for use

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RESEARCH ARTICLE

In-vitro and Bioimaging Studies of Mesoporous Silica Nanocomposites Encapsulated Iron-Oxide and Loaded Doxorubicin Drug (DOX/IO@Silica) as Magnetically Guided Drug Delivery System

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Abstract: ***Background***: In recent years, the delivery of drugs by nanocomposites has emerged as an exciting field of research for bio-imaging tools and targeted cancer treatment. The large surface area and porous volume of mesoporous silica nanocomposites (MSN's) have gained a lot of interest for their application in the delivery of drugs and the magnetic properties of iron oxide (IO) nanocomposites play a key role in the targeted delivery system.

Methods: In this study, mesoporous silica encapsulated IO nanocomposites loaded with doxorubicin (DOX) were synthesized for the magnetically guided delivery of anticancer drugs. The synthesis of IO nanocomposites was done through the precipitation method, and then silica encapsulation and drug loading were done by the StÖber method.

Results: The magnetically driven delivery of the drug is produced by the encapsulation of magnetically active IO in the mesoporous silica shell. The controlled release of DOX is possible because of the MSN's. TEM images show that the nanocomposites have a spherical morphology and average diameter in the range of 120 nm. Power-XRD data confirm the crystalline nature of nanocomposites. The strong absorption peak was observed in UV-Visible spectroscopy at 490 nm and quenching in fluorescence spectra confirms the encapsulation of DOX in the mesoporous silica shell. VSM data showed the magnetic nature of nanocomposites, with large magnetic susceptibility (74.88 emu/g). The use of DOX/IO@Silica nanocomposites as a sustainable drug release and targeted drug delivery vehicle has been reported here. The pH dependent release of DOX was studied and significant release was observed at lower pH. *In-vitro* cell viability assay and fluorescence imaging assay have demonstrated that these nanocomposites show significant dose-dependent toxicity to cancer cells in the presence of a magnetic field.

Conclusion: *In-vitro* studies via the MTT assay showed that these synthesized nanocomposites in culture are non-toxic to healthy cells compared to DOX-induced cytotoxicity due its controlled release and can be further strengthened by magnetic guidance. Therefore, due to its optical properties and potential for guided delivery of drug to the targeted site, these nanocomposites are ideal as an anticancer agent and bio-imaging prob.

Keywords: Mesoporous Silica Nanocomposites (MSN's), Core-Shell Iron Oxide/Silica Nanocomposites (IO@Silica), The Iron oxide (IO), Magnetically Guided Drug Delivery, (DOX), *in vivo*.

1. INTRODUCTION

The recent development in nanotechnology has drawn attention to nanoscale materials in the field of drug delivery, targeted drug delivery, diagnostic, medical imaging, and engineering [1-4]. There are non-existent platforms for the assembly of such multimodal agents due to size-dependent properties and the small morphology of the nanocomposites. Two types of nanocomposites were categorized in the view of applications: (a) structural nanocomposites, like biodegradable polymer, silica, etc.; and (b) functional nanocomposites with certain specific physical parameters, like metallic nanocomposites (iron oxide nanoparticle), quantum dots, etc., which provides a hosting network for potentially multi-

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Quantum dots-sensitized solar cells: a review on strategic developments

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Abstract. Quantum dots (QDs), the zero-dimensional semiconductor nanocrystals, due to their distinctive optoelectronic properties like size-tunable bandgap, broad absorption spectrum, size-dependent narrow emission profile, and better transport properties with the possibility of multiple exciton generation, have attracted wide attention as photosensitizers for developing QDs-sensitized solar cells (QDSSCs). Among all types of QDs, metal chalcogenide (MCh) QDs especially lead (Pb) and cadmium (Cd)-based chalcogenide QDs have been proven to be most suitable for the application as sensitizers in QDSSCs. This review paper presents a general overview of the QDSSC technology followed by the role of MCh QDs as the sensitizers in DSSCs. Despite their better sensitizing properties, lead (Pb)-based QDs have some serious issues such as lower stability and high toxicity associated with them. Therefore, this review is focused on Cd-based QDs (CdSe and CdTe) and presents a detailed prospective of recent trends in CdSe- and CdTe-based QDSSCs research. It tries to suggest the prospects of improvement strategies like co-sensitization of photoanode; deposition mechanisms, post-synthesis chemical treatments and doping/co-doping of sensitizers; optimization of synthesis parameters and interface modifications, etc. adopted for tailoring the bandgap of CdSe and CdTe QDs.

Keywords. Quantum dots; DSSC; QDSSC; photosensitizer; multiple exciton generation; metal chalcogenides.

1. Introduction

With the rapid development of human society and enormous demand for sustainable energy worldwide, research around the globe is mainly focused on developing state-of-the-art technologies linked to systematic energy generation and management patterns. One important example is that of photovoltaic (PV) technologies, which not only store energy from the sun but also are capable of releasing that energy on demand anytime and anywhere. The best representative of this kind of green energy is solar energy. However, the high cost of PV energy-based devices eclipses its benefits of abundant and clean availability. So, the solar energy conversion and storage systems need to be developed, for which the cost of fabrication of the device must come down along with an increase in the efficiency of the PV device. The Shockley-Queisser limit for single-junction PV devices ($\sim 33.7\%$) indicates that the maximum solar conversion efficiency occurs in a material with an energy bandgap $\sim$ 1.34 eV and hence silicon with a bandgap of 1.1 eV emerged as a good choice as a PV material for the first-generation solar cells [1]. Such silicon-based solar cells, though being highly efficient, are quite expensive because of the use of highly sophisticated crystal growing processes and purification techniques. Moreover, their efficiency is highly temperature-sensitive and is much lower at elevated temperatures. The quest for cost-effective solar-to-electrical energy conversion devices resulted in the evolution of thin-film-based solar cells, i.e., the second-generation solar cells. This generation of solar cells uses materials like amorphous silicon (a-Si), cadmium telluride/cadmium sulphide (CdTe/CdS), copper-indium-gallium-selenide (CIGS) and gallium arsenide (GaAs). The reduction in the cost of thin-film-based solar cells (except for GaAs-based cells) due to less requirement of material and cheaper manufacturing methods, is somewhat offset by the reduction in efficiency as compared to first-generation solar cells. Thin-film solar cells are flexible, have less temperature-dependent sensitivity for efficiency, and are found to be less space-efficient also. Different kinds of materials are used for enhancing the efficiency of the PV energy conversion devices even beyond the Shockley-Queisser limit by harnessing extended portions of the solar spectrum. The reduced per-unit costs of the devices and to

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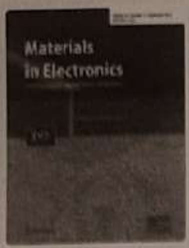

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Impact of ionic liquid incorporation on ionic transport and dielectric properties of PEO-lithium salt-based quasi-solid-state electrolytes: role of ion-pairing

Sujeet Kumar Chaurasia[1,*], Manish Pratap Singh[2], Manoj K. Singh[3], Pramod Kumar[4], and A. L. Saroj[5,*]

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ABSTRACT

The present paper reports the preparation and characterization of ionic liquid immobilized quasi-solid-state electrolytes (QS-SEs) "PEO-$LiPF_6$-ionic liquid (IL)" with polymer PEO, dopant salt ($LiPF_6$) and ionic liquid, $BMIMPF_6$ having common anion (PF_6^-). AC impedance and Raman spectroscopic measurements showed that ionic conductivity (σ) of QS-SEs increases with IL-content and partly controlled by number of free mobile ions and their mobility/polymeric chain flexibility. Differential scanning calorimetry (DSC) results showed that melting temperature (T_m), glass transition temperature (T_g) and degree of crystallinity (X_c) of QS-SEs decreases with IL-content due to the plasticization effect of added IL($BMIMPF_6$). Furthermore, the composition-dependent conductivity data with IL-concentration is discussed in terms of increasing amorphicity (or deceasing crystallinity) as well as ion-polymer and ion-ion interactions. Ion dynamics of QS-SEs is examined by converting the impedance data into different ionic transport formalisms like ionic mobility (μ), charge carrier density (N) using the loss tangent (tan δ) spectra and showing that amount of added IL/salt are the responsible factors for controlling overall ion transport mechanism.

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Investigation on Ionic Conductivity and Raman Spectroscopic Studies of Ionic Liquid Immobilized PEO-Based Polymer Electrolytes

Sujeet Kumar Chaurasia, Abhishek Kumar Gupta, Sarvesh Kumar Gupta, Shivani Gupta, Pramod Kumar, and Manish Pratap Singh

Abstract The ionic conductivity and Raman spectroscopic studies are reported for ionic liquid (IL)-based polymer electrolyte [PEO:$LiPF_6$ (as salt)] + $BMIMPF_6$ (as IL) in which the dopant salt and IL have common anion PF_6^-. These results are compared with another IL-based polymer electrolyte system with mixed anions (ClO_4^- & PF_6^-). X-ray diffraction (XRD) results showed that the structural modification in the polymer PEO matrix due to the change in its crystalline structure after the incorporation of salt and/or IL that gives reduced crystallinity (or enhanced amorphous content) of the polymer electrolyte films which, in turn, is accountable for enhancement in ionic conductivity. Raman spectroscopic analysis confirmed the occurrence of ion–polymer and ion–ion association/interaction phenomena in these polymer electrolyte membranes which is partly responsible for determining the number of mobile ions concentrations and hence ionic mobility. Furthermore, composition-dependent ionic conductivity results are discussed on the basis of changes in ion–polymer and ion–ion interactions as well as changes in the degree of crystallinity/amorphicity of the membranes.

Keywords Polymer electrolyte · Ionic liquid · Raman study · Ion–ion interaction

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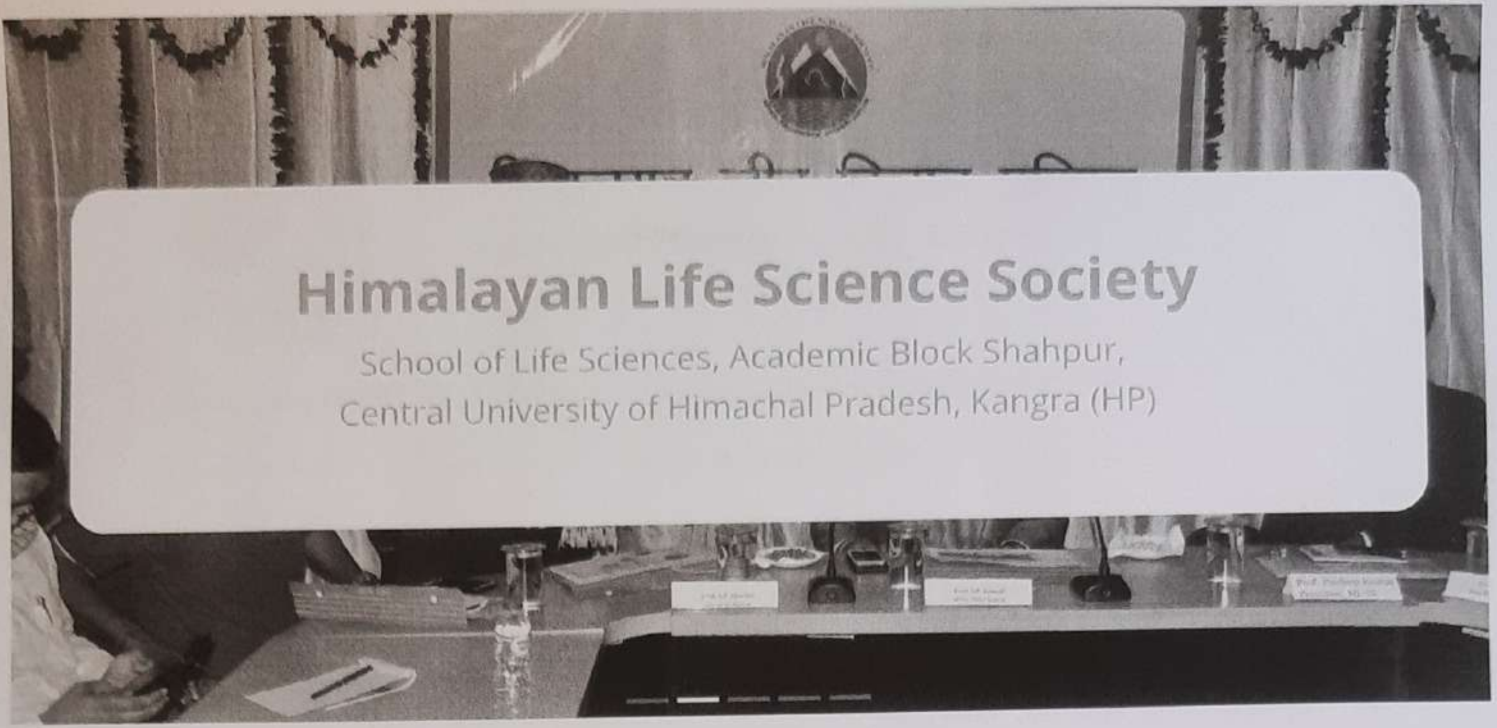

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Multimodal ORMOSIL Nanoparticles for biomedical application : A Review

Purnima Justa[1] , Hemant Kumar[2,3,4], Balaram Pani[4] and Pramod Kumar[1*]

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Abstract: Nanomedicine is a novel approach to conventional medicine in which problems are tackeled from bottom up rather than the top down, medical actions are carried out at the single cell level, tailored therapeutic prescriptions are carried out, and theronosis is promised. The synergistic effect of nanotechnology and biotechnology allow to develop multifunctional nanoprobe for combined therapt and diagnosis of diseases. This talk will highlight the use of multifunctional ORMOSIL NPs with combined imaging, diagnostic, and therapeutic functions for nanomedicine. ORMOSIL NPs serve as a new generation drug carrier for PDT of cancer, as well as for an efficient non viral gene delivery. The concern to use these multifunctional nanoprobes for clinical application lies within toxicity effects which need more investigation to carry out efficient clinical trial.

Keywords: Multifunctional, Drug carrier, PDT, Gene therapy.

Introduction

The biomedical applications of nanomaterials are covered by an interdisciplinary research area nanomedicine, that brings together nanotechnology, biology, chemistry, physics, material science, molecular biology and biotechnology for the enhancement of medical care. Quantitative superiority, multifunctionality, nano-size effect, additive/multiplier effect are just a few of the characteristics that make nanoparticles a desirable alternative for biomedical applications over conventional medications and imaging agents [1]. Nanotechnology provides a comprehensive technological platform for bioprocessing, molecular medicine, environmental and agricultural systems, as well as novel solutions for the transformation of biosystem. The size- dependent properties of nanoparticles contributes to their potential role in biomedical nanotechnology. Multimodal nanoprobe is a new generation of nanomaterials that possess the ability to resolve current issues related to medical diagnostics in terms of sensitivity and specificity and also lead to betterment of existing and developing therapies. However, the size of nanoprobes, their surface

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Research Article

In-vitro and Bioimaging Studies of Mesoporous Silica Nanocomposites Encapsulated Iron-Oxide and Loaded Doxorubicin Drug (DOX/IO@Silica) as Magnetically Guided Drug Delivery System

Author(s): Hemant Kumar, Jitender Kumar, Balaram Pani* and Pramod Kumar*

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Abstract

Background: In recent years, delivery of drugs by nanocomposites has emerged as an exciting field of research for bio-imaging tools and targeted cancer treatment. The large surface area and porous volume of mesoporous silica nanocomposites (MSN's) have gained a lot of interest for their application in the delivery of drugs and magnetic properties of Iron oxide (IO) nanocomposites play a key role in targeted delivery system.

Methods: In this study, mesoporous silica encapsulated IO nanocomposites loaded with doxorubicin (DOX) were synthesized for the magnetically guided delivery of anticancer drugs. The synthesis of IO nanocomposites was done through the precipitation method and then silica encapsulation and drug loading was done by StÖber method.

Results: The magnetically driven delivery of the drug is produced by the encapsulation of magnetically active IO in the mesoporous silica shell. The controlled release of DOX is possible because of the MSN's. TEM images shows that the nanocomposites have spherical morphology and average diameter in the rage of 120 nm. Power-XRD data confirms the crystalline nature of nanocomposites. The strong absorption peak was observed in UV-Visible spectroscopy at 490 nm and quenching in fluorescence spectra confirms the encapsulation of DOX in the mesoporous silica shell. VSM data showed magnetic nature of nanocomposites, with large magnetic susceptibility (74.88 emu/g). The use of DOX/IO@Silica nanocomposites as a sustainable drug release and targeted drug delivery vehicle has been reported here. The pH dependent release of DOX was studied and significant release was observed at lower pH. In-vitro cell viability assay and fluorescence imaging assay have demonstrated that these nanocomposites show significant dose-dependent toxicity to cancer cells in the presence magnetic field.

Conclusion: In-vitro studies via the MTT assay showed that these synthesized nanocomposites in culture are non-toxic to healthy cells compared DOX-induced cytotoxicity due its controlled release and can be further strengthened by magnetic guidance. Therefore, due to its optical properties and potential for guided delivery of drug to the targeted site, these nanocomposites are ideal as an anticancer agent and bio-imaging prob.

Keywords: Mesoporous Silica Nanocomposites (MSN's), Core-Shell Iron Oxide/Silica Nanocomposites (IO@Silica), The Iron oxide (IO), Magnetically Guided Drug Delivery, (DOX), in vivo

ACTA SCIENTIFIC CANCER BIOLOGY (ASCB) (ISSN: 2582-4473)

Review Article Volume 6 Issue 2

Synthesis and Characterization of Biogenic Iron Sulfide Nanoparticles in Cancer and Other Biomedical Applications: A Review

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Abstract

This review focuses on recent breakthroughs in the physicochemical approaches and biological approaches of metallic iron, iron sulfides, and iron oxide nanoparticles for a variety of applications. The range of kinds, topologies, and physicochemical features of nano-sized iron sulfides has piqued researchers' curiosity. Furthermore, this study examines the medicinal, environmental, and technical uses of biogenically synthesized NPs, as well as the hurdles that must be overcome in order to optimize the environmentally friendly production of these critical nanoparticles. FeS NPs have a good effect on biological activity due to their ease of production, magnetic properties, biocompatibility, and biodegradability. Also canvased are the FeS NPs nanoparticle-specific biomedical applications in cancer treatments. The goal of this review is to discuss the synthesis, characteristics, and uses of nano sized FeS NPs in biomedical domains, revealing that they have significant promise for enhancing human health and excellence of life.

Keywords: Iron Sulfides; Surface Modification; Biocompatibility; Cancer Therapy; Photothermal Therapy; Biosensors; Antifungal Agents; Enzyme-Like Catalysis

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Review Article

Synthesis and Characterization of Biogenic Iron Sulfide Nanoparticles in Cancer and Other Biomedical Applications: A Review

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Abstract

This review focuses on recent breakthroughs in the physicochemical approaches and biological approaches of metallic iron, iron sulfides, and iron oxide nanoparticles for a variety of applications. The range of kinds, topologies, and physicochemical features of nano-sized iron sulfides has piqued researchers' curiosity. Furthermore, this study examines the medicinal, environmental, and technical uses of biogenically synthesized NPs, as well as the hurdles that must be overcome in order to optimize the environmentally friendly production of these critical nanoparticles. FeS NPs have a good effect on biological activity due to their ease of production, magnetic properties, biocompatibility, and biodegradability. Also canvased are the FeS NPs nanoparticle-specific biomedical applications in cancer treatments. The goal of this review is to discuss the synthesis, characteristics, and uses of nano sized FeS NPs in biomedical domains, revealing that they have significant promise for enhancing human health and excellence of life.

Keywords: Iron Sulfides; Surface Modification; Biocompatibility; Cancer Therapy; Photothermal Therapy; Biosensors; Antifungal Agents; Enzyme-Like Catalysis

Introduction

Nanoscience can be explained as the study of the property of matter and structure on a nanoscale between 1 -100 nm. The several properties of the matter like interfacial and surface chemistry useful in nanotechnology's applications. The high surface-to-volume ratio useful in catalysis, it enhances the chemical properties of nanomaterials. Mechanical properties boost the strength hardness in lightweight nanomaterial and Nanocomposites [1]. It purely depends on the unique and size-dependent property of solid matter.

The biomedical nanotechnology is an interdisciplinary field of study that spans health, science and engineering. It also has a wide variety of molecular diagnostic, molecular imaging, and targeted treatment applications. The amalgamation of nanotechnology with molecular biology has begotten a new research area or sector that transforms biomedical research by providing specific imaging techniques, nano-robotics, and nano-devices, etc [2-3]. Nanomaterials with a spherical shape are very useful in the field of nanomedicine as they are similar in dimension to biological macromolecules, and smaller than cellular organelles. The nano-sized particles such as iron oxide and quantum dots have magnetically, optically, or structural properties that differ from the bulk materials. These biocompatible nanoparticles have a high affinity and selectivity for sick cells and organ-like malignant tumors. Nanopar-

2 Bio-Based Materials for Food Packaging Applications

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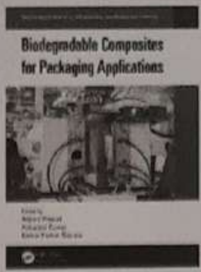

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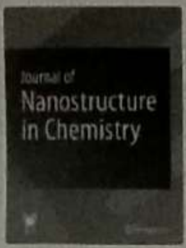

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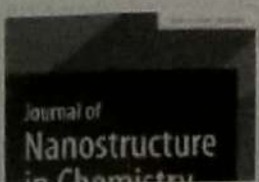

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REVIEW

Metal nanoparticles in cancer: from synthesis and metabolism to cellular interactions

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Abstract

Nanotechnology has encouraged new and amended materials (metal nanoparticles) for therapeutic applications with specific AQ1 prominence in healthcare. Metal nanoparticles (NPs) are versatile nanoscale entities, widely used to diagnose and treat cancer. Evidence suggested that metal NPs can modulate the expression of various intracellular and extra-cellular signaling molecules in the tumor microenvironment. Metal nanoparticles possess anti-cancer activities via apoptosis and cell cycle arrest. In AQ2 addition, metal NPs inhibit tumor angiogenesis, metastasis and inflammation to stop cancer proliferation. Synergistic applications of metal NPs with existing anti-cancer agents showed improvement in their bioactivity and bioavailability. This review explores the synthetic approaches, pharmacokinetics, and the cellular and molecular interactions of metal NPs in cancer. AQ3

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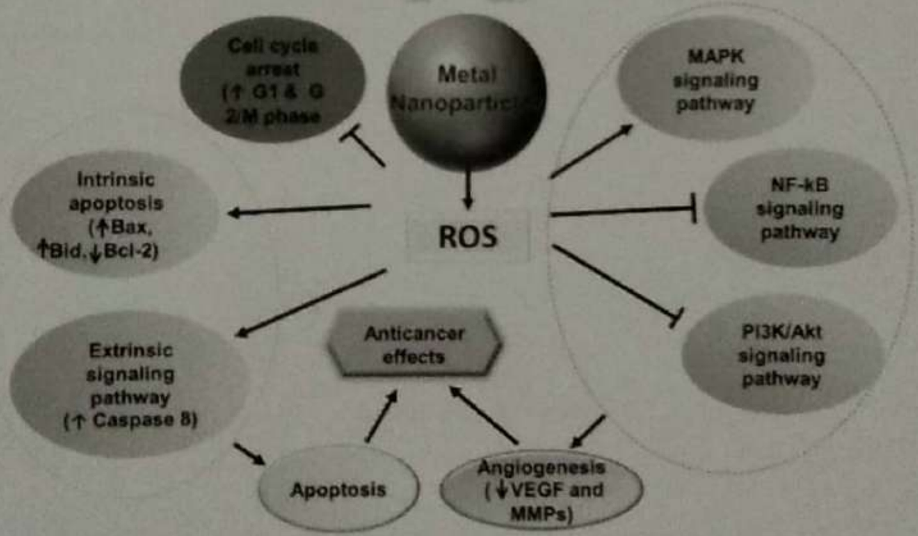

Keywords Nanoparticles · Apoptosis · Anti-angiogenesis · Anti-metastasis · Anti-inflammation · Synergistic effect

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Introduction

Cancer is a rapidly growing public health concern. While in 2008, an estimated 12.7 million new cancer cases with 7.6 million cancer-related deaths were detected globally

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Mechanistic and analytical understanding of biological immobilization of chromium metal ions from waste-sites

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ABSTRACT

In the proposed study, the isolation, identification and tolerance capacity of chromium (Cr(VI)) resistant fungi collected from different polluted sites of India has been studied. 12 fungal strains (FS1-FS12) have been isolated from the contaminated samples and cultured with different concentrations (100–500 ppm) of Cr(VI) to cumulate new biomasses. Screening and scrutinization divulges FS3 and FS5 isolates to be the most tolerant fungi against the higher concentrations of tested heavy metal with the deduction percentage and uptake of 72.5 ± 0.4%, 73.01 ± 0.12% and 4.30 ± 0.27, 4.39 ± 0.32 mg/g respectively. FS3 and FS5 were identified as *Aspergillus flavus* and *Aspergillus fumigatus* by grounding the data of internal transcribed spacer rDNA (ITS) region. The values of correlation coefficients (R^2) calculated for *A. flavus* and *A. fumigatus* supports the Langmuir isotherm more than that of Freundlich isotherm, thus putting forward the *multilayer adsorption* of Cr(VI) ions by the *mycelium* of the *fungal strains*. Characterization techniques such as Fourier transform infrared spectroscopy (FTIR) and Scanning electron microscopy (SEM) has rendered the comparative analysis of biosorbent before and after the removal of Cr(VI) ions, thus substantiating the results of adsorption isotherms. The obtained results have supported the detoxifying potential of both the isolated fungi towards Cr(VI), thus promoting the use of safe biotechnology as one of the sustained recuperative measures.

1. Introduction

The tainting of different spheres of earth with hazardous and toxic chemicals because of rapid industrialization has posed extortionate risks to its living beings, both directly and indirectly [1–3]. The most frequent anthropogenic activities which are a budding contributor of heavy metals toxification in water bodies are effluents released from industries, dumping of sewage sludge, mining and use of agrochemicals and landfill operations [4,5]. In general, heavy metals are resistant to natural degradation and sticks with the environment for an indefinite period of time [4]. One of the major non-essential and lethal constituent found in the industrial effluent is Cr(VI) [6,7]. In India, annually around 2000–3200 tons of elemental chromium escapes into the surroundings, out of which 2000 to 5000 ppm of share is contributed by the effluent discharge, which is far away from the suggested permissible limit of 2 ppm [8]. Generally, chromium is found in two oxidation forms viz. trivalent Cr(III) and hexavalent Cr(VI) [9]. Out of these two forms, Cr (VI) is of main concern and is categorized by the US Environmental Protection Agency as class A human carcinogen. Pronounced severity of Cr(VI) exposure is provoked by its easy diffusion through cell membranes of prokaryotes and eukaryotes [10–14]. It leads to the irritation of eyes and skin, ulcers, physical anxiety and lethal illness together with

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In-vitro cytotoxicity of nickel oxide nanoparticles against L-6 cell-lines: MMP, MTT and ROS studies

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ABSTRACT

In the present work we synthesize nickel oxide nanoparticles (NiO NPs) using *Rhododendron arboretum* (flower) (RNi), *Tinospora cordifolia* (stems) (GNi), *Corylus jacquemondii* (seeds) (CNi), and *Nardostachys jatamansi* (roots) (NNi) extracts by co-precipitation method. The synthesized NiO NPs were characterized in detail in terms of their morphological, crystalline nature, structural and antiproliferative activity against rat skeletal myoblast (L-6) cell lines. Morphological studies confirmed the formation of nanoparticles, while the structural and compositional characterization revealed the well-crystallinity and high purity of the synthesized nanoparticles. For biological applications and cytotoxicity examinations of the synthesized NPs, the rat skeletal myoblast (L-6) cell lines were subjected to study. By detailed cytotoxic investigations, it was observed that among the four kinds of NiO NPs prepared through different plant extracts, the *Tinospora cordifolia* (stems) showed strong antiproliferative activity against rat skeletal myoblast (L-6) cell lines and the calculated IC_{50} was 1.671 mg/mL. The observed anti-proliferative activity towards different NiO NPs were in the order of GNi > NNi > RNi > CNi. The present studies demonstrate that simply synthesized NiO can efficiently be used as antiproliferative agents.

1. Introduction

There has been an increasing interest in the synthesis of NiO NPs (Thema et al., 2016) due to their better characteristics and applications in various applications such as catalysis (Dooley et al., 1994), biomedical imaging, molecular diagnostics and bioengineering (Bakshi et al., 2010), battery cathodes (Sabouri et al., 2018)and magnetic materials (Jahromi et al., 2013), as well as their applications in the field of medicine, which involves the manufacture of cosmetics and hygiene products (Sabouri et al., 2019a), antibiotics (Savithramma et al., 2011),

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A sustainable approach to the degradation of dyes by fungal species isolated from industrial wastewaters: Performance, parametric optimization, kinetics and degradation mechanism

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ABSTRACT

Fungal abetted processes are among the finest approaches for the transformation or degradation and decolorization of dyes in effluents. In this piece of research; biodegradation and metabolic pathways of two toxic dyes Congo Red (CR) and Reactive black 5 (RB5) by two strains of Aspergillus sp. fungus in batch experiments has been investigated. Morphological characteristics of the isolates were observed with both light and electron microscopies. Based on molecular characterization the isolates were identified as *Aspergillus flavus* and *Aspergillus niger*. The degradation was also optimized *via* operational parameters such as pH, temperature, incubation time, inoculums size, dye concentration, carbon sources and nitrogen sources. Degradation measurements revealed that the isolates effectively degraded 90% and 96% of CR and RB5 respectively. Metabolites were identified with Liquid chromatography–mass spectrometry (LCMS) and degradation pathways of the dyes were proposed. Toxicity assay *Phaseolus mungo* seeds showed that pure CR and RB5 dyes exhibits significant toxicity whereas fungal treated dye solution resulted in an abatement of the toxicity and cell viability was increased. The results stipulated in this article clearly showed the effectiveness of the isolates on detoxification of CR and RB5 dyes.

1. Introduction

Surface water contamination from various sources have led the researchers to work intensively on the eco-friendly methods to combat and treat heavily loaded industrial wastewaters and real wastewaters (Alam et al., 2021; J et al., 2020). Dye pollution is among such recalcitrant which are not only aesthetically unacceptable but is also very toxic to the living beings (Kant, 2012; Lellis et al., 2019; Mani et al., 2019). Discharge of colored textile effluents into rivers and lakes results in reduced dissolved oxygen concentration and create toxic conditions in aquatic ecosystems (Berradi et al., 2019; Katal et al., 2014). For instance, congo red (CR) is an azo dye, it is toxic to many organisms and is a suspected carcinogen and mutagen (Asses et al., 2018; Berradi et al., 2019; Rani et al., 2017). It is a benzidine-based anionic di-azo dye and it is banned in many countries because of health concerns (Asses et al., 2018). But, it is still widely used in several countries such as India. Reactive dyes such as reactive blue 5 (RB5) are usually non-degradable under normal conditions and therefore contributes in strong color,

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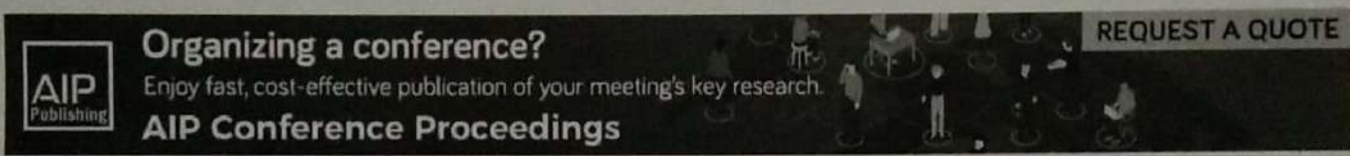

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Renu Bala, Ashutosh Dixit, Bhawna Pareek, et al.

A Synthesis, Characterization and Biological Function of Copper, Nickel and Iron Nano-Oxides with Various Plant Extract

Renu Bala[a)], Ashutosh Dixit, Bhawna Pareek, Vivek Sheel Jaswal, Ashun Chaudhary, Vikash Singh, Vivek Sharma

Department of Chemistry, MMDU, Mullana (Ambala), India.

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Abstract. Mixed metal oxides like copper, iron and nickel characterized by distinct processes in particular (sol-gel, co-precipitation and hydrothermal method etc.) using different plant abstract. The following literature study examined the biological activities of numerous metal and metal oxide nanoparticles such as copper, nickel and iron. The mean size, structure and the crystalline nature of the nanoparticles were examined by SEM, TEM, FTIR and SAED analysis. EDX analysis confirmed the presence of elements in the synthesized nanoparticles.

Keywords: Copper oxide, Nickel oxide and Iron Oxide Nan particles, Chemical Processes, Biological Activity and Characterizations

INTRODUCTION

The transition metals incarnation oxides of distinct stoichiometry due to their changeable oxidation states. These metal oxides have different commutative construction, out-turning in several chemical and electronic attributes. Due to their portable size, metal oxides nanoparticles have different attributes than those in the majority of a hug number of metal oxides in Mother Nature, like metal oxides are most convient in science and technology in line with their implementation to regular life [1]. The transition metals have the individual attributes of dyed compounds being structured and illustrate magnetic worldly goods. For several industrial implementations, metals from d-block elements are utilized. They perform as a catalyst, substance that are superconducting, laser, ceramic, sensors, phosphorous etc. They are also significant photoactive data and such as a photo-sensitizers. Due to their lofty top area and reactive plot mixed/metals oxides have broad implementation as a catalyst. In distinct organic reactions, numeral of researcher and academics used assorted/metal oxides as a catalyst. Assorted metal oxide nanoparticles distinct in chemicals, morphological and physical quality and are used in distinct implementation [2]. Metal oxide NPs are pliable data that could be used in implementation similar medical technology, environmental remediation, water treatment, electricity and individual care outcome [3]. Due to their restricted size and lofty density of part or edges top sites, NPs oxide may have individual chemical and physical properties [4]. Distinct metal oxides were characterized and more studied in several fields for their implementation [5-8] large research of this association leads to a superior mastery of the chemical bond in crystal. Due to their uncomplicated mode of emergence and multi-role behavior, metal oxides captivate special attention from researchers. Several scientists have reported different method of working of characterizing NPs of metal oxides by using different biological well spring such as fungi and bacteria [9]. Nickel oxide is a remarkable transition metal oxide with a cubic lattice construction. Due to its prospective use in a diversity of implementation such as catalysis [10], gas sensors [11], magnetic materials [12, 13], electro chromic films [14], battery cathodes [15, 16]. Now the another transition metal oxide (Iron Oxide) diameter are 1-100 nm as their excellent magnetic attributes and implementation in present day science i.e. modular, flexible, design, toxic [17] they have collected sizeable attention. In biomedical implementation magnetite and hematite are examples of iron oxides, where magnetite in opposition to oxidation is extra stable [18]. The most remarkable goods are biotechnology, optics, medicine, microbiology, mechanics and several field of material science [19]. NPs of biosynthesis in the newest years have been considered as a safer, greener and more researchable different to the synthesizing method of physical and chemical representatives. Plant mineral take out comprise a broad scale of anticancer and antioxidant activities [20-22]. Thermoelectric or electroluminescent material is used as a semiconductor [23]. For example, biomedicine, catalysis, optoelectronics systems, magnetic fluids and magnet recording media are used in a broad diversity of implementation [24-25]. A number of plan of attack are used in the characterize of NPs based on iron oxide: colloidal method [26], hydrothermal synthesis [27], sol-gel method [28], co-precipitation [29, 30] oxides of iron (III) are normally cheap, common and play an significant role in geological and biological procedures [31]. Fe_3O_4 particles have taken a great deal of negotiations because of the existence of Fe ions in the classification of non-position products and biological agreement [32]. The reducing representative plays a significant role in biosynthesis of nanoparticles like reductase, ascorbic acid, citric acid etc. [33]. Nanotechnology

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Green Synthesis of Cadmium Oxide Nanoparticles with Various Plant Extracts and their Use as an Anticancer Agent

Ashutosh Dixit [1], Renu Bala [1], Bhawna Pareek [1,*], Ashun Chaudhary [2], Saroj Arora [3], Davinder Singh [3], Anju Bala [4], Vivek Sheel Jaswal [5,*]

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Abstract: The co-precipitation method was used to produce cadmium oxide nanoparticles (CdO NPs) with different plant extracts such as *Tinospora Cardifolia* (stems), *Rhododendron arboretum* (flower), *Pichrorhiza Kurroa* (roots), *Nardostachys jatamansi* (roots), *Acorus Calamus* (roots), *Corylus Jacquemontii* (seeds), and *Emblica Officinalis* (fruit). To extract organic matter from the as-prepared sample, it was calcined at a temperature ranging from 500-600° C. X-ray diffraction (XRD), Scanning electron microscopy (SEM), Fourier transforms infrared spectroscopy (FTIR), and Transmission electron microscopy (TEM) were used to investigate the structure and morphology of the calcined oxide nanoparticles. The CdO NPs were well amorphous particle form and had an average particle size of 20-55 nm. The cytotoxicity of the *Pichrorhiza Kurroa* shows strong antiproliferative activity against rat skeletal myoblast cell lines (L-6).

Keywords: *Tinospora Cardifolia*; anticancer activity; FTIR; SEM; TEM; XRD.

1. Introduction

Cadmium (Cd^{2+}) is a rare genetically unregulated heavy metal, the most toxic to the living organism at relatively small levels and the most toxic environmental pollutant [1]. Different defense mechanism for overcoming Cd^{2+} toxicity has been identified in a living organism [2]. Cadmium oxide (CdO) is an n-type semiconducting material used as a transparent conducting film preparation. Nanomaterials are used to produce highly-developed structural ceramics as oxides and starting materials [3]. CdO has potential uses in photovoltaic cells [4], detectors [5], catalysts [6], and solar cells [7], etc., as a versatile semiconductor. Cadmium oxide is used for a transparent electrode, phototransistor, fluid crystal panels, photodiode, and IR. The design of quick, easy, and cost-efficient procedures to synthesize nanoparticles in the field of nanotechnology is worthwhile. Therefore the synthetisation of these nanoparticles of different shapes and sizes is very important [8]. These factors had a

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Chapter 2

Synthesis, Characterization, and Application of Metal Oxide Nanoparticles

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2.1 Introduction

This chapter presents the techniques used to characterize and investigate the inner construction and possessions of nanoparticles by the craved synthesis techniques [1]. The characterized NPs can be transferred out each of two by top-down talk to or by the extremity up approach. The act characterized implement is very climacteric in managing the possessions of evolved nanoparticles [2]. The most extensively used technique for characterizing the NPs is to cut a long story short reported in this chapter. The characterized technique used in our instance is the chemical coagulation process,

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Review

Metal coordinated macrocyclic complexes in different chemical transformations

Maheshwar S. Thakur [a,*], Neha Singh [b], Arti Sharma [c], Rohit Rana [d], A.R. Abdul Syukor [e], M. Naushad [f,g], Sunil Kumar [h], Manish Kumar [b], Lakhveer Singh [i,*]

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ABSTRACT

Macrocyclic ligand chemistry is seen by a growing number of scientists as a tool for designing new molecules with inherently selective properties. Metal coordinated macrocyclic rings are featured with exceptionally stable π-conjugated cyclic systems containing the metal ion in the central cavity. The cyclic tetra dentate framework of the four central nitrogen atoms makes these macrocyclic rings system unique chelating agents, in addition, these conjugated double bonds impart a vital effect in electrons transportation. Because of inimitable characteristics, these metal coordinated macrocyclic compounds have been effectively applied as catalysts in various important organic transformations. Hence, this review portrays the overview of metal coordinated macrocyclic molecules, classifications, and their catalytic application in redox reactions. This comprehensive and up-to-date review is particularly aligned toward the catalytic applications of four types of metal-coordinated macrocyclic rings, i.e., porphyrin, porphyrazine, corrole, and corrolazines, in different organic transformations. The different synthetic approaches for the preparation of metal coordinated macrocyclic rings and their UV spectrums are also outlined in this review.

Contents

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Limiting apparent molar volumes, limiting apparent molar isentropic compressions, transfer parameters and transition state theory for ternary mixtures of *Gly-Gly-Gly* in aqueous solutions of ascorbic acid at temperatures between (298.15 and 318.15) K

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ABSTRACT

Density, speed of sound, viscosity parameters were measured for glyclyl-glycyl-glycine in aqueous molal solutions of ascorbic acid were reported at condition of pressure (=0.1 MPa) and temperature (ranging 298.15–318.15) K. The results of thermophysical properties were obtained in the molality range (0.0499 to 0.3998) mol·kg^{-1} for glycyl-glycyl-glycine and of (0.1942, 0.3773, 0.5504, 0.7142) mol·kg^{-1} for ascorbic acid. The various thermophysical parameters like apparent molar volume, apparent molar isentropic compression, limiting apparent molar volumes, limiting apparent molar isentropic compression, empirical-interaction coefficients, transfer parameters (from volumetric and acoustic data), Falkenhagen coefficient, Jones-Dole coefficient, and *dB/dT* (first derivative of *B*-coefficient were computed with the help of experimental values. Also, the Gibbs free energy of activation of solvent and solute were calculated and explained with the help of transition state theory. The studied thermophysical parameters at different temperatures and molal concentrations were explained from the point of view of solute–solute and solute–solvent interactions and their structural effects.

1. Introduction

The various types of non-covalent interactions (like hydrogen-bonding, hydrophilic, hydrophobic interactions) are responsible for the stabilization of native confirmation of biological important molecules (like proteins) [1]. The mentioned non-covalent interaction depends on the surrounding and mainly on the behaviour of solute, solvent molecules constituting the surrounding atmosphere [2]. Thus, the study of thermodynamic properties of certain solute–solvent molecular interactions becomes a curious field of research. These molecular (solute–solvent) interactions contribute a lot in predicting the stability of proteins in presence of solutes [3]. The poly-ols compound helps in preventing the dehydration of protein by other compounds/substances. Thus, the idea of thermophysical properties in aqueous solution of poly-ols are of great importance and detailed study of results so obtained enhance our knowledge about protiens and their behaviour to great extent [4-6]. Glyclyl-glycyl-glycine (*Gly-Gly-Gly*) utilized in reproducible serum protein measurements, taken as substrate in the evaluation of aminotripeptidases [7-9]. The poly-ol compound taken in the present work is ascorbic acid. Ascorbic acid (*AA*), a conjugate acid of L-ascorbate, widely available as vitamin C, is from organic acid family with molecular formula $C_6H_8O_6$. Ascorbic acid (*AA*), as water soluble vitamin, important for steadiness of bones, connective tissues, muscles, blood vessels and as agent in absorbtion of iron, important for body in producing red blood cell. Ascorbic acid (*AA*) exists as radicals in and around the physiological pH in conjunction with the unpaired electron present in the C (4) region. Since a distinction can be made between both types of radicals, the electron spin resonance technique can be used for discrimination between the epimers of vitamin C. The radical has a cyclic side chain structure which is formed by the hydrogen bond C(3)-O------- HO – C(6) and which engulfs Na^+ (or) K^+ in the case of ascorbyl (isoascorbyl) radial, respectively. As radicals Na – ASC and K – ISO – ASC present in act as electroneutral, therefore mobile carriers Na – ASC and K – ISO – ASC helps in transmission of Na^+ and K^+ across membranes. Thereby, red glutathione impacts the both types of radicals thus reinstating the original electronic configuration at C (4) without modifying the electroneutral bicyclic

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Vikas Bharti, Deepika Kaushal, Sunil Kumar, Abhishek Thakur, Dilbag Singh Rana, Manish Kumar* and Shashi Kant

Molecular interaction studies on the binding ability of hydrated zinc sulphate with aqueous solution of ascorbic acid at different temperatures

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Abstract: The ternary systems containing Water, Ascorbic acid (AA) and $ZnSO_4\cdot 7H_2O$ were investigated using three approaches namely volumetric studies, viscosity studies and conductance studies. The solvent systems used were 2, 4 and 6% (by weight) of AA in water. The studies were conducted at four temperatures (303.15–318.15 K with an interval of 5 K) and pressure 0.1 MPa with concentration o $ZnSO_4\cdot 7H_2O$ in the solution ranging from 0.01 to 0.12 m. Various parameters lik partial molar volume (ϕ_v), apparent molar volume (ϕ_v^o), Hepler's constan ($(d^2\phi_v^o/dT^2)_p$), partial molar expansibility (ϕ_E^o) and transfer volume ($\Delta_{tr}\phi_v^o$) have been evaluated from volumetric studies. The viscosity studies have yielded Jones-Dole parameters (A and B) and free energy of activation per mole for solvent ($\Delta\mu_1^{0\ddagger}$) and solute ($\Delta\mu_2^{0\ddagger}$). The conductance data has been used to calculate molar conductance (Λ_m), limiting molar conductance (Λ_m^o) and Walden product ($\Lambda_m^o\eta_o$). The results of these studies agree with each other and have concluded the structure breaker behavior of $ZnSO_4\cdot 7H_2O$ in the solvent system containing AA and water.

Keywords: apparent molar volume; partial molar expansibility; structure breaker behavior; transfer parameters; Walden product.

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Topics

Manish Kumar*, Shashi Kant and Deepika Kaushal

Molecular interaction investigation of some alkaline earth metal salts in aqueous citric acid at various temperatures by physiochemical studies

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Abstract: Densities, ultrasonic velocity, conductance and viscosity of some alkaline earth metal chlorides such as magnesium chloride ($MgCl_2$) and calcium chloride ($CaCl_2$) were calculated in the concentration range (0.01–0.12 mol kg^{-1}) in 0.01 mol kg^{-1} aqueous solution of citric acid (CA + H_2O) at four varying temperatures $T_1 = 303.15$ K, $T_2 = 308.15$ K, $T_3 = 313.15$ K and $T_4 = 318.15$ K. The parameters like apparent molar volume (ϕ_v), limiting apparent molar volume (ϕ_v^o) and transfer volume ($\Delta_{tr}\phi_v^o$) were calculated from density data. Viscosity data have been employed to calculate Falkenhagen coefficient (A), Jone–Dole's coefficient (B), relative viscosity (η_r), and relaxation time (τ) whereas limiting molar conductance (Λ_m^o) has been evaluated from conductance studies. Using these parameters, various type of interactions occurred in the molecules have been discussed. Values of Hepler's constant $(d^2\phi_v^o/dT^2)_p$, (dB/dT) and $d(\Lambda_m^o\eta_o)/dT$ suggests that both $MgCl_2$ and $CaCl_2$ behave as structure breaker in (CA + H_2O) system. The positive value of transfer volume exclusively tells about solute–solvent interactions which further indicate that both metal chlorides distort the structure of water and act as structure breaker. These studies are helpful in understanding the nature of interactions occurs in biological systems as CA and metal salts are essential for normal functioning of body.

Keywords: apparent molar volume; citric acid; molecular interactions; transfer volume.

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A sustainable approach to the degradation of dyes by fungal species isolated from industrial wastewaters: Performance, parametric optimization, kinetics and degradation mechanism

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ABSTRACT

Fungal abetted processes are among the finest approaches for the transformation or degradation and decolorization of dyes in effluents. In this piece of research; biodegradation and metabolic pathways of two toxic dyes Congo Red (CR) and Reactive black 5 (RB5) by two strains of Aspergillus sp. fungus in batch experiments has been investigated. Morphological characteristics of the isolates were observed with both light and electron microscopies. Based on molecular characterization the isolates were identified as *Aspergillus flavus* and *Aspergillus niger*. The degradation was also optimized *via.* operational parameters such as pH, temperature, incubation time, inoculums size, dye concentration, carbon sources and nitrogen sources. Degradation measurements revealed that the isolates effectively degraded 90% and 96% of CR and RB5 respectively. Metabolites were identified with Liquid chromatography–mass spectrometry (LCMS) and degradation pathways of the dyes were proposed. Toxicity assay *Phaseolus mungo* seeds showed that pure CR and RB5 dyes exhibits significant toxicity whereas fungal treated dye solution resulted in an abatement of the toxicity and cell viability was increased. The results stipulated in this article clearly showed the effectiveness of the isolates on detoxification of CR and RB5 dyes.

1. Introduction

Surface water contamination from various sources have led the researchers to work intensively on the eco-friendly methods to combat and treat heavily loaded industrial wastewaters and real wastewaters (Alam et al., 2021; J et al., 2020). Dye pollution is among such recalcitrant which are not only aesthetically unacceptable but is also very toxic to the living beings (Kant, 2012; Lellis et al., 2019; Mani et al., 2019). Discharge of colored textile effluents into rivers and lakes results in reduced dissolved oxygen concentration and create toxic conditions in aquatic ecosystems (Berradi et al., 2019; Katal et al., 2014). For instance, congo red (CR) is an azo dye, it is toxic to many organisms and is a suspected carcinogen and mutagen (Asses et al., 2018; Berradi et al., 2019; Rani et al., 2017). It is a benzidine-based anionic di-azo dye and it is banned in many countries because of health concerns (Asses et al., 2018). But, it is still widely used in several countries such as India. Reactive dyes such as reactive blue 5 (RB5) are usually non-degradable under normal conditions and therefore contributes in strong color.

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Functional assignment to hypothetical proteins in *Orientia tsutsugamushi* strain Ikeda

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Synthesis, characterizations and antifungal activities of copper oxide and differentially doped copper oxide nanostructures

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ABSTRACT

Nanoparticles of Co, Ni, and Fe doped copper oxide along with nanoparticles of bare copper oxide were prepared using a co-precipitation reaction. The XRD pattern suggested the crystalline nature of the nanoparticles with a crystallite size of 28 nm and these size calculations were complimented by TEM results where grain shape nanostructures were visible with a size around 20–40 nm. The EDAX results showed the presence of Co, Ni, and Fe in doped copper oxide nanoparticles thus confirming doping in the nanoparticles. The FTIR and Uv–Visible spectra for the nanoparticles were recorded to complement the chemical structure of the nanoparticles as established by XRD and TEM. The optical band gap for these nanoparticles was also calculated and the nanoparticles were tested against some clinical fungal species to record their antimicrobial activities.

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1. Introduction

With the evolution of nanotechnology, the field of chemical sciences has flourished a lot as the materials at the nanoscale exhibit different physical properties without changing their chemical nature. The physical properties of the material have sorted more attention at the nanoscale as most of the applications of materials are due to their exciting physical parameters like improved surface area, bandgap, magnetic, dielectric, and electrical properties which make these materials suitable for a wide range of applications in various fields [1–4].

To harvest the nanoscale properties of materials, there is a need to synthesize materials at the nanoscale. A variety of synthetic methods are available for the synthesis of materials at the nanoscale including sol-gel, co-precipitation, solid-state reaction, ball milling, alkoxide-based synthesis, sonochemical preparation, microwave irradiation, and thermal decomposition [5]. Among these methods, chemical methods like sol-gel, and co-precipitation are widely applied to be cost-effective and eco-friendly [6].

A number of nanomaterials are being prepared yearly among which metal and metal oxide comprise the maximum number. From pure metal machinery to trace metal ions in biological systems, metals exhibit an important role in our life. Among the various compounds of metals, metal oxides are equally significant as metals. Metal oxide nanomaterials are being produced on large scale for scientific and industrial applications and have been used in sensors, optical and electronic devices, paints, batteries, etc.[7–9]. The most widely used nanoscale oxides are nano TiO_2 and nano ZnO due to their fascinating properties and wide applications [10–12], but nano copper oxides are comparatively less explored although it also has the potential in the fields of catalysis, sensing, ceramics, batteries, magnetic storage media, solar energy, superconductors, energy and environment etc. [13–16]. The shape and morphology of nanostructures is also an important parameter to decide their application spectrum. Copper oxide has been prepared and studied in the form of nanobelts, nanowires [17], nanorods, nanoneedles, nanoflowers, and nanoparticles [18,19].

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Vancomycin Grafted Polydopamine Coated Silver Nanoparticles for Enhanced Antibacterial Action Against Vancomycin-Resistant Bacteria *E. Faecalis.*

Khushbu Patel[a] and Rajender Kumar*[a, b]

A wide variety of pathogens causing nosocomial infections have emerged, showing resistance towards commonly used antimicrobial agents. Nosocomial infections are rapidly emerging as a severe threat leading to high morbidity and mortality. In this study, we have introduced Vancomycin grafted polydopamine(pDA) coated silver nanoparticles (AgNPs) to combat vancomycin-resistant bacterial species *Enterococci faecalis (E.faecalis)*. The prepared nanoparticles were characterized by High Resolution -Transmission Electron Microscopy (HR-TEM), Energy Dispersive X-ray Spectroscopy (EDS), Ultraviolet (UV) Visible spectroscopy and Attenuated Total Reflection-Fourier Transform Infrared spectroscopy (ATR-FTIR). The nanoparticles were 33 nm in size as revealed by HR-TEM. Antimicrobial assays were conducted to investigate the antimicrobial activity of vancomycin-pDA@AgNPs against E. *faecalis* by a zone of inhibition method. The vancomycin-pDA@AgNPs exhibited good bactericidal activity against vancomycin-resistant *E. faecalis*. Additionally, pDA@AgNPs also show strong antibacterial activity against resistant bacteria, even in the absence of Vancomycin. A small concentration of Vancomycin on pDA@AgNPS led to significant antibacterial action reducing the need for a higher level of the drug. Thus, the one-step synthesis of pDA@AgNPs grafted with Vancomycin serves as a potent antimicrobial agent against antibiotic-resistant *E. faecalis.*

1 Introduction

Antibiotic-resistant and multidrug-resistant bacteria are growing at an alarming rate despite advances in the clinical treatment of bacterial-caused diseases. The large scale use of the antibiotics and blindly following the antibiotics regime even for minor healthcare issues is one of the leading causes for the increase in the strains of antibiotic-resistant bacteria leading to serious mortality rates and nosocomial infections.[1] Among the various types of bacteria, *Enterococci*, which initially were considered harmless to the human population, have very rapidly developed into the highly virulent strains and resistant towards generally prescribed antibiotics.[2] Due to their resistance developing tendency, the *Enterococci* bacteria are often treated either with a more massive dose of antibiotics or requires a regime of more than two antibiotics for effective treatment. The most commonly used antibiotics to treat *Enterococci* infections are ciprofloxacin,[3] ampicillin,[4] Teicoplanin,[5] cephalosporins,[6] gentamicin,[7] and Vancomycin, etc. Vancomycin is one of the most effective antibiotics prescribed worldwide against several different types of bacterial infections.[8]

Vancomycin, have been effective against methicillin-resistant *staphylococci* and other gram-positive bacteria.[9] For several years there is no significant resistance was reported against this antibiotic.[10] However, recently vancomycin-resistant *Enterococci* have been reported, which is an alarming situation considering the use of Vancomycin in treating a host of bacterial diseases. Uttley *et al.*[11] described *Enterococcus faecalis* (*E. faecalis*) as vancomycin-resistant. Thus, the emergence of entirely vancomycin-resistant Enterococci strain and lack of alternative treatment regimes can lead to catastrophic consequences to humans. The way forward is to minimize the use of all antibiotics in general and Vancomycin in particular and to develop other therapeutic regimes free of antibiotic use.

The antibiotic-free regime has seen some success with the use of silver NPs as active antibacterial agents.[12] Most of the bacterial strains fail to develop the resistance against AgNPs, thus making them the right candidate for an antibiotic-free alternate treatment regime.[13] Also, AgNPs based materials have been successively studied as potent broad-spectrum antimicrobial agents.[14]

However, for *enterococci* bacteria, it has been reported that even the use of AgNPs alone is not effective alternate, and treatment still depends on the use of the NPs in conjunction with the antibiotics. AgNPs functionalized with antibiotics have gained a great interest in their enhanced antibacterial activity against antibiotic-resistant bacteria.[15]

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Hemolysis tendency of anticancer nanoparticles changes with type of blood group antigen: An insight into blood nanoparticle interactions

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ABSTRACT

Different blood groups of ABO system have specific antigen which bestows them with different biochemical properties and hence they can show different hemolytic activity. In this report, hemolytic activity of thiol-functionalized Fe_3O_4-Au nanoparticles were studied in presence and absence of doxorubicin and the effect of various thiol coatings were correlated towards their hemolysis tendency. The nanoparticles were functionalized with four different amino thiols, cysteamine (CEA), cystamine (CA), cysteine (Cys) and cystine (Cyt) to form Fe_3O_4-Au CEA, Fe_3O_4-Au CA, Fe_3O_4-Au Cys and Fe_3O_4-Au Cyt nanoparticles which were loaded with anticancer drug, doxorubicin. The functionalization was characterized using ATR-FTIR, HR-TEM, XPS and other spectroscopic methods. Maximum drug encapsulation efficiency of 83% was observed with Fe_3O_4-Au CA nanoparticles. *In-vitro* experiments were performed on HeLa cells to check the cellular uptake and cytotoxicity using MTT assay. Hemolytic activity was then analyzed with all the blood groups (positive and negative). The amino acid functionalized, Fe_3O_4-Au Cys and Fe_3O_4-Au Cyt nanoparticles, shows lesser hemolysis compared to amino thiol functionalized Fe_3O_4-Au CEA, and Fe_3O_4-Au CA nanoparticles. In positive blood groups, the Fe_3O_4-Au CA nanoparticles shows the highest rate of hemolysis followed by Fe_3O_4-Au CEA, while the lowest hemolysis rate was observed for Fe_3O_4-Au Cyt nanoparticles. For negative blood groups, the thiol coated nanoparticles show more abrupt hemolysis rate depending upon the type of antigen.

1. Introduction

Nanomaterials are known to possess unique optical and magnetic properties and have the potential to store therapeutic and imaging agents for effective cancer theranostics. Several nanoparticles have been employed as targeted drug delivery vehicles that can improve the current cancer detection and therapies [1,2]. Liposome – Doxil, loaded with doxorubicin (DOX) is one of the examples that have been successfully shown to increase the blood half-life and reduces the cardiotoxicity when compared to the standard DOX treatment [3]. Thus, it is very important to advance the study of cancer nanomedicine to expand its role as intended and overcome its limited therapeutic efficacy. One of the major areas which remain largely unexplored, is a study of the biotoxicity and biodistribution of the nanoparticles. It has been reported that only 10% of the nanocarrier accumulates in the tumor cells [4]. Consequently, the number of nanoparticles to actually hit the target cells is even smaller. Thus, to achieve the maximum biodistribution, it is important to study the actual interaction of intravenously injected nanoparticles and blood. Such interactions may prove pivotal in improving the transport and targeting ability of the nanoparticles besides reducing the biotoxicity [5,6].

Nanoparticles when entered into the body, formation of protein corona takes place due to the adsorption of blood protein on the surface of nanoparticles [7]. This results into the change in size and surface charge of the nanoparticles [8,9] causing the unexpected alteration in cellular interaction, biodistribution, and cellular uptake [10,11].

Also, intravenous administration of the nanoparticles can results into producing high local concentration in the circulating blood, which makes it mandatory to evaluate its biological safety [12]. Several hemolytic studies were carried out on the impact of nanoparticles (silica nanoparticles [13], carbon nanotubes [14], silver nanoparticles [15], iron oxide [16] and Au nanoparticles [17]) on erythrocytes in the blood. It is also known that hemolytic activity differs for bare and coated nanoparticles as its interaction depends on surface, size or nature of the particles too [18].

Thus, hemolysis falls among one of the fundamental tests in

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Taurine-Conjugated Mussel-Inspired Iron Oxide Nanoparticles with an Elongated Shape for Effective Delivery of Doxorubicin into the Tumor Cells

Nimisha Singh, Nadine Millot,* Lionel Maurizi, Gérard Lizard, and Rajender Kumar*

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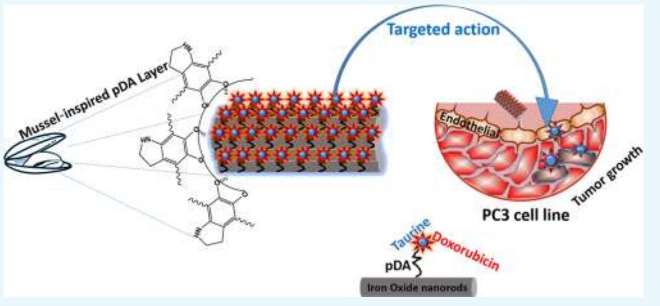
ABSTRACT: Multifunctional iron oxide magnetic nanoparticles, among them nanorods, were prepared with a mussel-inspired polydopamine (pDA) surface coating agent for cancer therapeutics. Taurine, a free sulfur-containing ß amino acid, was grafted on the pDA at the iron oxide nanoparticle surface to enhance its biocompatibility and targeted delivery action. Doxorubicin (DOX), an anticancer drug, was loaded on the prepared nanovehicles with an entrapment efficiency of 70.1%. Drug release kinetics were then analyzed using UV−vis and fluorescence spectroscopies, suggesting the pH-responsive behavior of the developed nanovehicle. The developed system was then tested on PC-3 cell lines to check its cellular response. Confocal microscopy observations and (3-(4,5-dimethylthiazol-2-yl)-5-(3-carboxymethoxyphenyl)-2-(4-sulfophenyl)-2*H*-tetrazolium) and Annexin V-FITC assays used to evaluate cell toxicity and apoptosis reveal a dose-dependent nature of nanorods and can overcome the side effects of using free DOX with a targeted action.



1. INTRODUCTION

Cancer remains the leading cause of fatality in the developed world because of several inducing factors such as environmental pollution and unhealthy living habits.[1,2] Current therapies require intrusive processes such as surgery to remove a tumor, followed by chemotherapy and/or radiation therapy. These treatments often kill healthy cells and lead to acute toxicity in the patients.[3] Scientists have made several efforts to improve chemotherapy over 25 years, but an effective regime for cancer treatment is still a distant dream.[4] One of the essential requirements for the improvement in cancer treatment is drug targeting either by molecular or by physical targeting.[5] Molecular drug targeting although promising suffers from inherent high cost, immune system activation, and reduced blood residence time. The high blood residence time requires conjugating the drug carrier with polymers, chemical agents, and so forth, which have strong antibiofouling properties, thus preventing the protein carrier interactions and autoimmune response.[6,7] The physical targeting treatment involves the use of magnetic nanocarriers which increase permeability and provide magnetic targeting.[8] Several magnetic drug carriers based on iron oxide nanoparticles (IONPs) are in use for the cancer drug carriers.[9] These drug carriers are biocompatible in nature, induce magnetic hyperthermia, and thus reduce the drug resistance of tumor cells.[10,11] The IONPs thus provide several properties such as magnetic targeting, high blood-residence time, and effective payload of the drug.[12] The ideal drug carrier which encompasses the above properties besides providing strong anti-biofouling properties and delivers the required payload of the drug at tumor tissue remains a challenge.[13]

Along with surface chemistry, the shapes of these nanostructures play a vital role in deciding the applicability of the developed nanohybrids.[14−16] A nanostructure dimension within a decided geometrical frame carries a strong determinant of the total cellular uptake. Among different shapes, the rod-shaped nanostructure offers larger uptake than spherical-shaped particles,[16,17] as it offers two different orientations (long and short axes) that result in providing a different level of control in presenting the drug to targeted sites.[18] Even the blood half-life also depends on the shape and size of the nanostructure, as shown by rod-shaped micelles having 10 times longer circulation time than that of the spherical micelles.[19] Research efforts have always been made to achieve small, uniform, and highly dispersed particles. However, recent studies have realized the importance of shape, and thus, one-dimensional structures such as nanorods and nanotubes have come into the picture.[20,21] Because of their unique properties, we attempt to develop iron oxide

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Study of Anticancer Drugs Interaction with Hemoglobin by Electrochemical Methods and Molecular Docking: Implications towards Anticancer Treatment

Jyotsnamayee Nayak,[b] Suban K. Sahoo,[b] and Rajender Kumar*[a]

Chemotherapeutic drugs tend to cause hemolysis; however, hemoglobin's mode of action is still less explored. In the present study, we used electrochemical methods and molecular docking approach to study anticancer drugs and hemoglobin interaction. For this purpose, we prepared self-assembled monolayers (SAMs) of thiolated hemoglobin on the gold substrate. Electrochemically active chemo drugs, i.e., doxorubicin, withaferin-a, and 5-fluorouracil, are selected and allowed to interact with SAMs of hemoglobin at the physiological condition to study the redox activity and impedimetric behavior. Doxorubicin and withaferin-a show similar behavior in cyclic voltammetry and impedance spectroscopic, i.e., redox process due to hydroquinone/quinone/ketone ring, and faster interfacial electro-transfer kinetics and decreasing in electron-transfer resistance with increasing concentration. In contrast, 5-fluorouracil shows only the oxidation reaction. The molecular docking studies were in good agreement with the electrochemical study. All selected anticancer drugs are found to bind hemoglobin, with doxorubicin showing maximum binding efficiency and 5-fluorouracil showing the least. Doxorubicin's highest binding sites contain Tyr 145, which may be involved in the electron transport pathway between doxorubicin and hemoglobin, while Tyr35 in withaferin-a, but absent in case of 5-fluorouracil.

Introduction

Cancer is one of the deadliest disease worldwide, resulting in approximately 10 million deaths every year.[1] With the advancement of science and technology, several methods to treat cancer, like surgical removal of cancerous tissue, radiotherapy, and chemotherapy, have been developed, resulting in significant improvement in patient outcomes. Chemotherapy is regarded as the most accepted method of early cancer treatment as it interferes quite differently with cellular and molecular processes.[2] However, chemotherapy's major drawback is an unspecific mode of action towards healthy and tumor cells, limiting its use.[3] Chemotherapeutic agent attacks the cancerous cells and unspecific targets and destroys the healthy cells. However, the advent of nanotechnology and the discovery of more and more target-specific nano-drug delivery vehicles[4] have been successful to a small extent in reducing the side effects of chemotherapy. Reports shows, decrease in hemoglobin (Hb) while chemotherapy leading to anemia. There are several reasons for anemia as bone marrow suppression due to chemotherapy, erythropoietin deficiency nephrotoxicity induced by chemotherapy, hemolysis, etc.[5]

Nevertheless, completely eradicating the side-effects of chemotherapy requires a multi-angle approach, including but not restricted to the development of target-specific drug delivery vehicles. One of the significant areas in eliminating chemotherapy's side effects is the effect of anticancer drugs on the Hb. Since most of the chemotherapeutic drugs are given intravenously, their first site of interaction is Hb leading to numerous interactions with the protein Hb.[6] There are a few reports which have studied the detailed interaction of anticancer drugs with the blood Hb. Although the anticancer drugs' hemolysis tendency is being reported, drug-Hb interaction's mechanism and fate are still poorly understood.[6b,7] In our recent report, we have found that hemolysis tendency varies mostly with the type of blood groups when treated with anticancer nanoparticles carrying doxorubicin.[8]

Hb is a supreme tetrameric protein component in the erythrocyte. It can reversibly bind several endogenous and exogenous agents like flavonoids,[9] alkaloids,[10] herbicides,,[11] and different food colorants.[12] Several reported works explain the interaction of the various compounds with Hb. Such as oxali-palladium,[13] Pt(II) complex,[14] etc., anion exchange inhibitor, e.g., DIDS & antiseptic drug, e.g., DNP.[5a] Nevertheless, there are few reports which study the interaction between Hb and anticancer drug directly.[7c,d]

Herein, we studied Hb interaction with three different anticancer drugs (doxorubicin, 5-fluorouracil, and withaferin-a) through electrochemical impedance spectroscopy and cyclic voltammetry supplemented with molecular docking. For the electrochemical approach, we prepared SAMs of Hb on the

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Lysine and α-Aminoisobutyric Acid Conjugated Bioinspired Polydopamine Surfaces for the Enhanced Antibacterial Performance of the Foley Catheter

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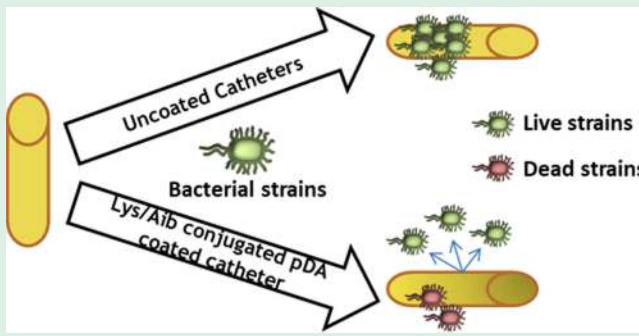
ABSTRACT: Microbial adhesion onto implanted devices was reduced by the immobilization of amino acid lysine and α-aminoisobutyric acid to polydopamine functionalized PET films and Foley catheters. The polydopamine functionalized film was prepared by a dip coating method followed by incorporation of biocompatible amino acids to prepared films. The purpose of development of the modified pDA film is to improve the anti-biofouling and antibacterial activity of the film which can be successfully applied for medical devices. The characterization of modification was done using different techniques such as contact angle measurement, ATR-FTIR, FE-SEM, AFM, and XPS analysis. ATR-FTIR spectroscopy and XPS confirmed the successful amino modification of film. The anti-biofouling and antimicrobial behavior of the prepared surfaces were evaluated using the bacterial attachment and death assay. The resulting coatings repelled bacterial cell attachment and killed clinically applicable Gram-negative and Gram-positive strains. The developed coatings were applied to the Foley catheters to study the antibacterial activity by the log reduction method. The results demonstrate that tested amino acid-modified film increases the antibacterial activity of the catheters and can significantly help in reduction of nosocomial infections.



KEYWORDS: *anti-biofouling, antibacterial coating, polydopamine, amino acids, bacterial attachment and death assay, Foley catheters, log reduction method*

■ INTRODUCTION

The revolution in the state of art in medical devices has tremendously improved life expectancy and saved numerous people from untimely deaths and health issues.[1] However, despite development in the fields of medical devices such as biodegradable sutures, contact lenses, implant devices, and catheters, there remains the serious problem of biofouling and microbial infestation in to humans through the route of these devices.[2] Besides causing microbial device infection and diseases, the biofouling also denigrates the devices and increases the cost of medical devices. Thus, the design of such biocompatible surfaces which resist the microbial adhesion and biofouling is a hot topic of research. The devices required should be able to prevent the secondary microbial infection to humans by preventing the initial microbial adhesions or by killing microorganisms as they come in close contact with the devices.[3,4] Many studies have focused on the development of antibacterial surfaces to inhibit biofilm formation by functionalizing the surface with different antimicrobial agents like silver, quaternary ammonium groups, and chitosan which are considered good candidates for reducing microbial adhesion and colonization.[5−7] Several anti-biotic-based surfaces have also been developed. However, the excessive use of anti-biotics leads to the development of anti-biotic-resistant bacterial strains[8] thus posing the even more serious problem of anti-biotic-tolerant bacteria.

The antibacterial biomolecules such as proteins, natural polymers, and biomolecules have strong potential to prevent or inhibit microbial adhesion, besides also controlling bacterial resistance toward them.[9] In a recent report, Kandiyote et al. have generated an excellent antifouling surface by grafting antimicrobial peptide magainin-2 to the surface.[10] The immobilization of biomolecules onto the surfaces of medical devices is an important research area in many biotechnological applications such as medical diagnostics, tissue engineering, and bioprocess engineering.[11] The immobilization of biomolecules to surfaces is done by either covalent or noncovalent approaches.[12] However, despite all advances, there are several gaps in terms of bonding efficiency, conjugation, the resulting activity of the functionalized surface, and so forth. The efficiency of surface bioconjugation decreases, as the reactions

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Mimicking biological process to detect alkaline phosphatase activity using the vitamin B_6 cofactor conjugated bovine serum albumin capped CdS quantum dots

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ABSTRACT

This manuscript presents a novel bioanalytical approach for the selective ratiometric fluorescent sensing of enzymatic activity of the alkaline phosphatase (ALP) in the biological samples. The probe was designed by conjugating the pyridoxal 5′-phosphate (PLP) over the surface of bovine serum albumin (BSA) stabilized CdS quantum dots (QDs) through the interaction of free amine present in BSA with the aldehyde group of PLP. The conjugation of PLP quenched the emission of QDs. Upon addition of the ALP, the emission of QDs was restored due to the dephosphorylation and the conversion of the functionalized PLP in to pyridoxal. With this probe, the ALP activity can be detected down to 0.05 U/L and also successfully applied for the detection of ALP activity in biological samples such as human serum and plasma.

1. Introduction

Alkaline phosphatase (ALP) is one of the vastly assayed enzyme due to its ubiquitous roles in numerous biological processes in human health [1]. ALP is found mainly in the intestine, liver, bone, kidney and placenta of human body. ALP mainly catalyzes the hydrolysis and transphosphorylation of phosphomonoesters [2]. The detection of ALP as biomarker had provided inexhaustible opportunities to diagnose a number of diseases associated with the human body, such as osteoblastic bone cancer, breast cancer, ovarian cancer, prostate cancers and liver tumors. Also, the ALP quantification is done for some other diseases like diabetes, hepatitis, osteomalacia, Paget's disease, heart failure, obstructive jaundice and rickets [3–5]. Therefore, there is burgeoning interest among the chemists to develop convenient and reliable assay to monitor the ALP activity in biological samples for the disease diagnosis, biomedical research and other relevant applications [6,7].

Analytical techniques such as chromatography, isotope label, colorimetric, chemiluminescence, fluorometric, surface-enhanced Raman scattering (SERS) and electrochemical etc. have been utilized for the monitoring of ALP activity [8–12]. Except the fluorometric methods, most of the other analytical approaches require complicated pre-treatment, sophisticated instruments and also lack high selectivity, specificity and sensitivity. Therefore, recent research was devoted to developing ALP selective fluorometric assays using the suitably derived organic dyes, functionalized nanoparticles, quantum dots and noble metal nanoclusters with the added analytical advantages of high sensitivity, selectivity, simplicity, less sample consumption and cost-effectiveness [13]. Most of the recent reports on ALP selective fluorescent assays were designed by using non-natural monophosphates as substrate which upon catalytic hydrolysis with ALP generates analytically useful optical response for the ALP detection and quantification. However, very less effort is made to use the naturally occurring monophosphates like pyridoxal 5′-phosphate (PLP) to develop an ALP selective assay [14]. The vitamin B_6 PLP serves as a cofactor for more than 140 enzymes in our body and participates in various kinds of enzymatic reactions like transamination, racemization and decarboxylation. Importantly, there is a unique relation between the PLP and ALP in the biological processes. The albumin-bound PLP present in the blood can't cross the cell membranes directly; instead, this complex acts as a reservoir of pyridoxal. Before entering into cells, ALP converts PLP into pyridoxal which can cross the cell membranes. Therefore, PLP can be used as a natural substrate for the designing of chemosensors for the sensing of ALP.

There is potential applications of water-soluble semiconductor quantum dots (QDs) in the field of sensing and biosensing due to their attractive optical properties of high Stokes shifts, high fluorescence quantum yields, stability against photobleaching and longer

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Research Article

Developing a Cost-Effective Bioassay to Detect Alkaline Phosphatase Activity and Generating White Light Emission from a Single Nano-Assembly by Conjugating Vitamin B_6 Cofactors with Lysozyme-Stabilized Fluorescent Gold Nanoclusters

Yachana Upadhyay, Rajender Kumar, and Suban K. Sahoo*

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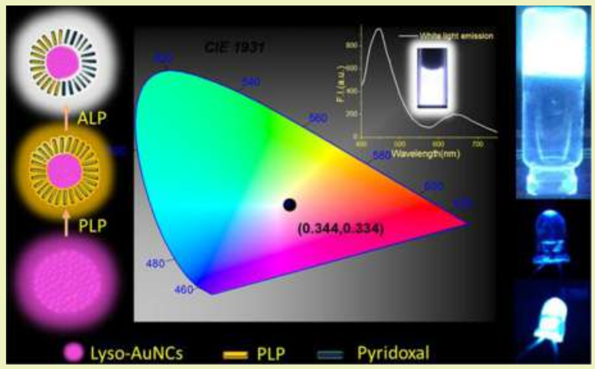
ABSTRACT: In this work, lysozyme-stabilized fluorescent gold nanoclusters (Lyso-AuNCs) and pyridoxal 5′-phosphate (PLP) as a monophosphate ester substrate were used to develop a highly selective and cost-effective bioassay for the detection of alkaline phosphatase (ALP) activity. The vitamin B_6 cofactor, PLP, was conjugated with the red-emitting nanoclusters to obtain a probe, PLP_Lyso-AuNCs, *via* forming a Schiff base linkage between the free amino group of lysozyme and the aldehyde group of PLP. At pH = 10.08, addition of ALP to the yellow-emitting PLP_Lyso-AuNCs solution catalyzed the hydrolysis of PLP and converted it into pyridoxal, which produced a distinct ratiometric fluorescence response and the fluorescent color turned pale white. Using the probe, PLP_Lyso-AuNCs, the ALP activity could be detected down to 0.002 U/L. Further, the changes in the fluorescent color intensity (red, blue, and green) of PLP_Lyso-AuNCs were recorded with the back camera of a smartphone to quantify the ALP activity. Both the fluorimetric and smartphone approaches gave satisfactory recovery percentage, when the practical utility of PLP_Lyso-AuNCs was applied to quantify the ALP activity in environmental and biological samples, such as river and lab tap water, blood plasma, and serum. Finally, a pure white light-emitting nano-assembly was developed by conjugating optimized amounts of both PLP and pyridoxal over the surface of Lyso-AuNCs.



KEYWORDS: *fluorescent gold nanoclusters, vitamin B_6 cofactors, ALP activity detection, smartphone, white light emission*

INTRODUCTION

The enzyme alkaline phosphatase (ALP) catalyzes the dephosphorylation of a variety of monophosphate ester substrates, such as ascorbic acid-2-phosphate, phenyl phosphate, *p*-nitrophenyl phosphate, adenosine triphosphate (ATP), amifostine, 5-bromo-4-chloro-3-indolyl-phosphate, etc., and also promotes the transphosphorylation of many phosphoric acid monoesters in various metabolic processes.[1,2] ALP is found throughout the body with higher concentrations in the cells of bones and liver. ALP has been extensively studied as a biomarker in clinical practice for the diagnosis of diseases such as diabetes, prostate cancer, Paget's disease, bone cancer, osteomalacia and osteoblastic damage, liver dysfunction, and hepatic and coronary artery diseases.[3−7] It is often utilized in the enzyme-linked immunosorbent assay (ELISA) and histochemical assay.[8] Also, the human placental ALP is used as an important labeling enzyme and a genetic marker to track cartilage and bone cells and western blotting in clinical experiments.[9−11] Therefore, development of facile analytical approaches with high selectivity and sensitivity to quantify levels of serum ALP is a scientific goal of current research interest in the fields of drug screening and clinical diagnostics.

Different analytical approaches, such as isotopic labeling, fluorescence, chemiluminescence, chromatography, colorimetry, electrochemistry, surface-enhanced resonance Raman scattering, etc., were developed to assay the ALP activity.[12−19] However, there is burgeoning interest in the designing of fluorescence-based sensors and biosensors because of their simplicity, low cost, high selectivity, sensitivity, and real-time detection.[20] A survey of recent literature revealed that most of the reported ALP-selective fluorescent sensors are mainly based on organic fluorescent dyes, nanomaterials, semiconductor quantum dots (QDs), and conjugated polyelectrolytes.[21−27] The reported approaches can detect ALP activity

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Vitamin B_6 cofactors conjugated ovalbumin-stabilized gold nanoclusters: Application in alkaline phosphatase activity detection and generating white-light emission

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ALP detection
Smartphone
White-light emission

ABSTRACT

The red-emitting ovalbumin-stabilized gold nanoclusters (OVA@AuNCs) was prepared and applied for the selective detection of vitamin B_6 cofactors. The addition of pyridoxal 5′-phosphate (PLP) and pyridoxal perturbed the fluorescence profile of OVA@AuNCs, and showed new emission band respectively at 532 nm and 437 nm due to the formation of Schiff base linkage between the aldehyde group of the cofactors and the free amine groups present in the OVA. Using OVA@AuNCs, the cofactors PLP and pyridoxal can be detected down to 6.14×10^{-7} M and 5.47×10^{-7} M, respectively. Also, the OVA@AuNCs can be mixed with optimized amount of pyridoxal to generate pure white-light emission. Further, considering the catalytic role of the alkaline phosphatase (ALP) in dephosphorylation, the PLP conjugated OVA@AuNCs was applied as a probe to detect ALP activity. The yellow-emitting PLP_OVA@AuNCs turned to pale-white emitting nanoclusters due to the ALP directed catalytic conversion of PLP into pyridoxal. With PLP_OVA@AuNCs, the ALP activity can be detected down to 3.9 U/L. In addition, the fluorescent color changes of PLP_OVA@AuNCs in the presence of varying amounts of the ALP were integrated with smartphone to develop a simple and cost-effective approach for the on-site detection of ALP activity. Finally, the practical utilities of both the fluorimetric and smartphone approaches were validated by quantifying ALP in various environmental and biological samples.

1. Introduction

Alkaline phosphatase (ALP), a crucial enzyme that catalyzes the dephosphorylation of many molecules like proteins, nucleotides, alkaloids etc. in the biological processes [1]. In the human body, the ALP exists predominantly in the intestine, bone, liver, placenta and kidney with the normal serum ALP ranges from 20 U/L to 140 U/L [2]. In clinical assay, the quantification of ALP activity in the serum has been of significant interest, and conceived as a diagnostic biomarker for various bone diseases (Paget's disease, osteomalacia and osteoblastic bone cancer), dysfunction of liver, prostatic cancer, bile duct blockage [3–7]. Moreover, ALP serves as a useful tool in the molecular biology for keeping the DNA molecules linear for various genetic modifications [8]. Therefore, there is a growing interest for the designing of highly selective and sensitive, cost-effective and facile bioassay for the instantaneous detection of ALP activity in the real biological samples.

The literature survey revealed that several approaches based on chemiluminescence, surface enhanced Raman spectroscopy, electrochemical, colorimetric, fluorimetric etc. were reported for the quantification of ALP activity [9–14]. Among the reported methods, the fluorescence based ALP detection gained considerable attention due to their high sensitivity, low-cost, easy operation and real-time detection without the need of highly skilled analyst. The reported ALP selective fluorescent probes are mainly used either the organic dyes or the functionalized nanomaterials as the light-emitting units, and the monophosphates like *p*-nitrophenylphosphate, 5-bromo-4-chloro-3-indolyl-phosphate, adenosine triphosphate and ascorbic acid-2-phosphate as substrate that can undergo catalytic hydrolysis in the presence of ALP to produce analytically useful optical responses for the monitoring of ALP activity [15–20]. However, with some analytical advantages in detecting ALP activity, most of the reported fluorescent sensing approaches have their own disadvantages of poor photo-stability, insolubility in aqueous medium, small Stokes shift, complicated synthesis process and lack of biocompatibility [21]. Therefore, there is need of

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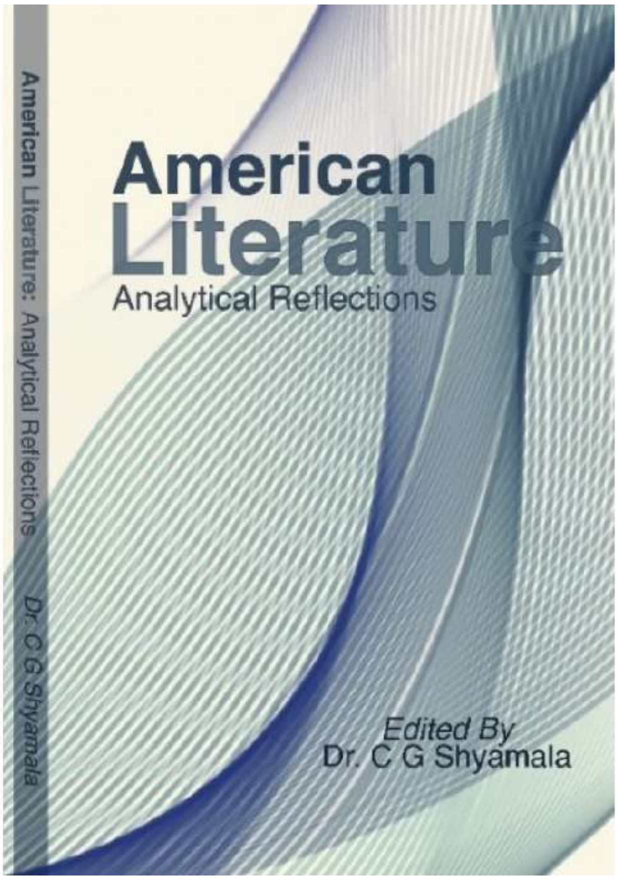

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Chapter- 4

A REVIEW ON PHYTOREMEDIATION - AN ECOFRIENDLY TECHNOLOGY

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CHAPTER - 4

A review on Phytoremediation - an eco-friendly technology

Neha Thakur, Munish Sharma and *Munish Sharma

Abstract

Phytoremediation a technology is used from last decade or two to solve the problem of eradication of harmful metal ions in sustainable, environment friendly way. Rapid increase of industrialization, modernization and various other factors such as agricultural activities and due to man's greed, the excessive use of fertilizers, untreated waste, untreated laboratory effluents lead to degradation of soil as well as environment. Therefore, it become our foremost duty towards sustainable development goals to eradicate the toxic harmful metal ions. Certain physical and chemical technologies are used to eradicate such toxins but due to certain limitations natural method is preferred which is use of plants for eradication of toxins from soil. Phytoremediation is a technique which help in eradication of heavy metal ions through plants metabolic pathway. The plants which are used as phytoremediator are generally hyperaccumulators, that can accumulate metal ions in concentration of more than 1000ppm. Plant that act as phytoremediator must have certain properties such as branched root system, less biomass, easy harvestable, and many more. Besides the benefits there are certain limitations of this technique. This review will focus on heavy metal stress, its sources and various technologies to eradicates heavy metal or metal toxin from soil through various processes and public and regulatory acceptance.

Keywords

Phytoremediation, toxic effects, heavy metals, hyperaccumulators, metallophytes

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Abstract and Figures

Marketing Communication and Importance in 21st Century.

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Critically Exploring Well-being and Resilience Strategies during and after Covid-19 Pandemic

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Authors:

Manpreet Arora

Roshan Lal Sharma
Central University of Himachal Pradesh, Dharamshala, India

Abstract

The outbreak of Covid-19 pandemic forced us not only to think but also to pause, and objectively reflect how to deal with the situation that has arisen and how to develop well-being and resilience strategies for the welfare of humanity at large. It has also forced us to see the various issues of life concerning education, poor medical facilities, unemployment, and conservation of wildlife along with environmental degradation. The pandemic brought a global economic shock of enormous magnitude in most of the countries as health systems failed, and education came to a grinding halt initially but was later put back on the track through online modes of teaching and counseling. Policymakers across the globe today have been facing alarming situations and unpredictable challenges as they try to find solutions to problems in health sector, at macroeconomic levels, and in socio�cultural arena. People in developing countries like India are concerned more with earning livelihood and supporting their families and therefore social distancing is impossible to achieve as people have to travel through public transportation system. At times, they do not have the money to buy tickets and it results in their movement in groups from one place to another. This paper deals with how in such a scenario, developing well-being and resilience strategies may really help us overcome challenges of diverse sorts. Key words: Well-being, Resilience, Covid-19, Strategies of well�being / resilience

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Post-Pandemic Psycho-Social Wellbeing in India: Challenges and the Way Ahead

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Abstract

The COVID-19 has affected physical and psychological well-being adversely. Besides causing large-scale damage to peoples' livelihoods, the pandemic has caused grievous injury to people's psyche as well as psychosocial well-being. Sheer magnitude and intensity of devastation caused by the pandemic forces us to reflect seriously on how crucial psychosocial well-being from the viewpoint of coming to terms with the havoc caused by COVID-19. This chapter proposes to unravel varied facets of psychosocial well-being by first developing a conceptual understanding of it, and then analyze issues and challenges that people have been facing in India and elsewhere in the world during post-pandemic times. In common parlance, psychosocial well-being implies a blend of mental and social health. Whereas the former signifies psychological wellness, the latter points toward how well and meaningfully we can relate to people in the society at large. If we share a good rapport with them, we feel happy, nice and cared for; in case we cannot get along well with people in our community or society, we may remain peeved, anxious and irritated. Psychosocial wellness therefore depends on diverse range of real-life experiences, interpersonal behavior, happiness and satisfaction. COVID-19 pandemic has caused so much damage that it has caused deep harm to our physical, emotional, social, spiritual and psychological health. Being conceptual in nature, this chapter employs qualitative method of research by analyzing data from secondary sources such as websites, reports, and other sources to build our perspective. The insights arrived at conceptually shall be discussed in the last part of the chapter where we have suggested a way ahead via suggesting strategies that may have meaningful implications if visualized, planned and executed properly.

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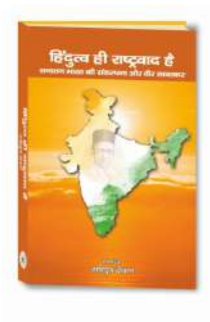

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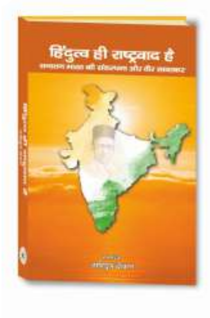

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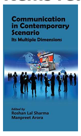

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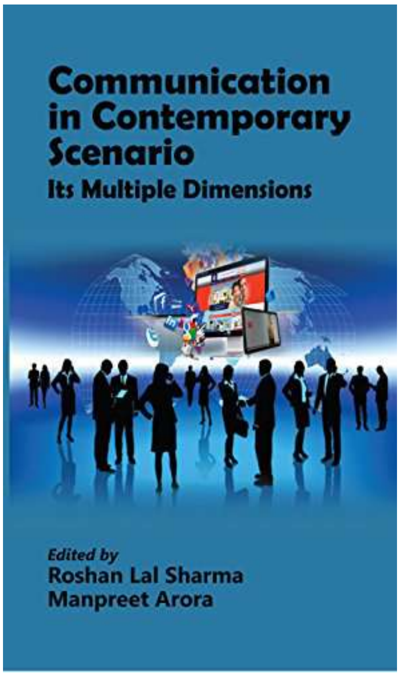

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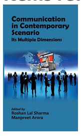

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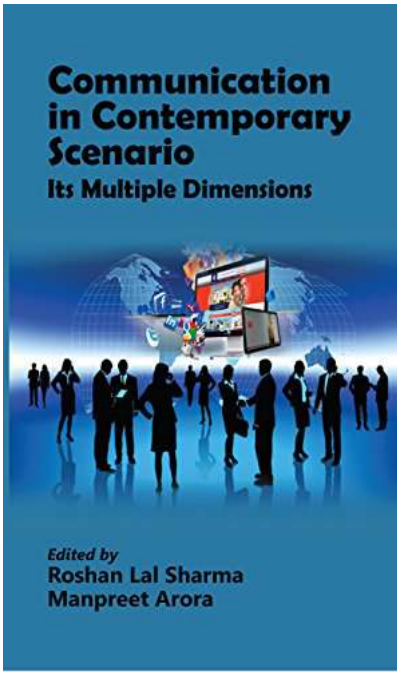

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18. राष्ट्रवाद की अवधारणा : एक चिन्तन

प्रीती सिंह

राष्ट्रवाद एक ऐसी भावना है जो देश के प्रति प्रेम, समर्पण, त्याग और बलिदान प्रकट करती है। जब कोई व्यक्ति देश के सभी नागरिकों के विकास और भलाई के विषय में समान रूप से बिना किसी धार्मिक, जातिगत, भेदभाव के बात करे। देश के सम्पूर्ण विकास के विषय की बात करे तथा राष्ट्रहित की भावना को सबसे ऊपर रखकर कार्य करे। समाज के हर वर्ग के लोगों की रक्षा करना उनकी मदद करना ही 'राष्ट्रवाद' है

राष्ट्रवाद राष्ट्र की आत्मा और जीवनी शक्ति को पुनर्जागृत करने का सशक्त माध्यम है। वह राष्ट्रीय जीवन की समस्त गतिविधियों, अनुभवों, अनुभूति, जागृति और पतन का दर्पण होती है इसलिए उसकी गहरी सहानुभूति और आस्था अपने देश की जनता के साथ होती है न कि शासकों के प्रति। राष्ट्रवाद का प्रधान लक्षण अतीत की गहराईयों का अनुसंधान है। प्रत्येक राष्ट्र की जीवन धारा में ज्वार भाटे की तरह उत्थान पतन पुनरूत्थान का अनवरत क्रम चलता रहता है।

मार्गरीटा बर्न्स के शब्दों में :–"वास्तव में भारतीय राष्ट्रवाद के विकास की कहानी भारतीय राष्ट्रीयता के विकास की कहानी है। सुशासन से स्वशासन, स्वराज्य से पूर्ण स्वतन्त्रता स्वतंत्र की यात्रा क्रमशः विकसित राजनीतिक चेतना के प्रस्फुटन का इतिहास है।"[1]

राष्ट्रवाद के विषय में यह जान लेना आवश्यक है कि राष्ट्रीयता की भावना भारत में प्रारम्भ से ही रही है। भारत को एक राष्ट्र तथा एक पृथक भौगोलिक अस्तित्व की गरिमा प्राप्त की। भारत में राष्ट्रवाद का आधुनिक स्वरूप अंग्रेजी शासन के अन्तर्गत बौद्धिक, पुनर्जागरण एवं दासता से मुक्ति के प्रयास में परिलक्षित होता है। भारतीय राष्ट्रवाद का उदय बौद्धिक पुनर्जागरण से माना जा सकता है।

डॉ० वी० पी० वर्मा लिखते है –"भारत का बौद्धिक पुनर्जागरण आधुनिक भारतीय राष्ट्रवाद के उदय का महत्वपूर्ण कारण था। जिस प्रकार इटली के पुनर्जागरण तथा जर्मनी के धर्मसुधार आन्दोलन ने यूरोपीय राष्ट्रवाद के उदय के लिए बौद्धिक आधार पर काम किया था इसी प्रकार भारत के सुधारकों तथा धार्मिक नेताओं के उपदेशों ने देशवासियों में स्वतंत्र तथा आत्मनिर्णय पर आधारित राजनीतिक जीवन का निर्माण करने की इच्छा उत्पन्न की। भारतीय आत्मा के जागरण की सर्जनात्मक अभिव्यक्ति

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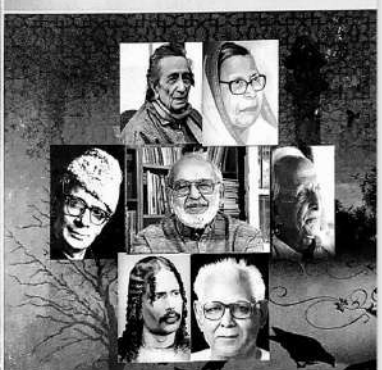

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अनुक्रमणिका

दलित और आदिवासी साहित्य : एक तुलनात्मक अध्ययन

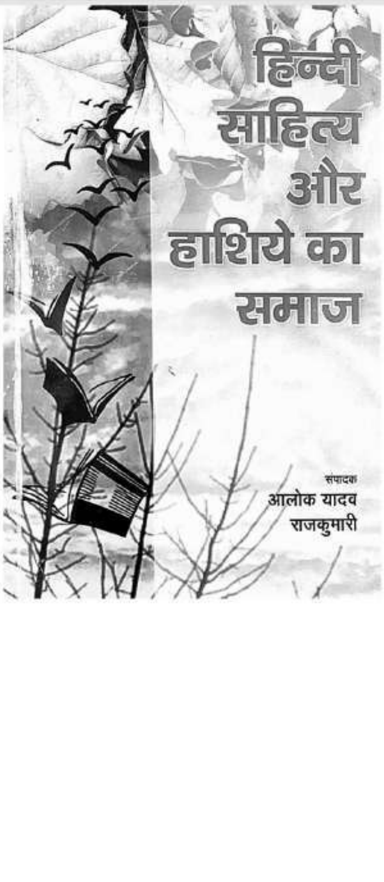
–डॉ० प्रीति सिंह*

दलित और आदिवासी विमर्श हाशिये का विमर्श है। ये दोनों समाज हाशिये का समाज कहलाते हैं। इसलिये इन दोनो विमर्शों के एक होने का भ्रम होने लगता है। इनके साहित्य एक दूसरे में गड्डमगड्ड होने लगते हैं। तब यह जरूरी हो जाता है कि हम साफ कर लें कि दोनों में पर्याप्त अन्तर है। इसमें अलग–अलग समाजों का अन्तर है। आदिवासी सम्पूर्णतः हिन्दू समाज के बाहर खड़े हैं। उनकी भाषा संस्कृति बिल्कुल उनकी अपनी भाषा और संस्कृति है जबकि दलितों की कोई अपनी अलग भाषा नहीं है. वे वही भाषा बोलते है जो गाँव के बाकी हिन्दू बोलते हैं। उनकी संस्कृति पर भी हिन्दुओं की संस्कृति का काफी असर दिखायी देता है। मसलन नारी की स्थिति या जाति प्रथा को लेकर जबकि आदिवासियों के अन्दर जाति प्रथा का प्रभाव नहीं मिलता है।

ऐतिहासिक रूप से आदिवासी एक स्वतन्त्र समाज रहे हैं। जबकि दलितों की स्थिति हिन्दू समाज में 'गुलामों' जैसी रही है। आदिवासी भौगोलिक दृष्टि से भी स्वतन्त्र रहे है। यानी वह बाकी मुख्य समाज से अलग दूर जंगलों पहाड़ों पर रहते आये हैं, जबकि दलितों का कोई अलग स्वतन्त्र प्रदेश नहीं रहा है। वे हर हिन्दू गाँव में उसके एक अंग के रूप में, उस गाँव से बंधे हुए रहे हैं। आदिवासी एक दलित की तुलना में ज्यादा सीमित और बंद दुनिया में रहता है, जबकि दलित आदिवासियों जैसी सीमित व बंद दुनिया में नहीं रहता है।

दलितों की मुख्य समस्या सामाजिक अपमान की रही है। उनकी लड़ाई सामाजिक न्याय की है। एक समाज के भीतर अपने समस्त श्रम का योगदान करके समस्त मूल अधिकारों से वंचित होने की रही है। आदिवासियों की समस्या पहचान की है। मुख्यधारा का सभ्य समाज उनकी पहचान ही मिटाने पर तुला हुआ है। उनकी स्वायत्त अर्थव्यवस्था को

*पी०डी०एफ०, हिन्दी विभाग, लखनऊ विश्वविद्यालय ।



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डॉ० प्रीती सिंह
पी.डी.एफ., हिन्दी विभाग, लखनऊ विश्वविद्यालय

"संसार भर के आदिवासी, उपनिवेशवाद तथा तथाकथित विकास के सिद्धान्तों के शिकार हैं।" – भुजंग मेश्राम

वैश्वीकरण का शाब्दिक अर्थ स्थानीय या क्षेत्रीय वस्तुओं या घटनाओं के विश्व स्तर पर रूपांतरण की प्रक्रिया है। इसे एक ऐसी प्रक्रिया का वर्णन करने के लिए भी प्रयुक्त किया जा सकता है जिसके द्वारा पूरे विश्व के लोग मिलकर एक समाज बनाते हैं तथा एक साथ कार्य करते हैं। यह प्रक्रिया आर्थिक, तकनीकी, सामाजिक और राजनीतिक ताकतों का एक संयोजन है।

प्रसिद्ध भाषा वैज्ञानिक नोम चॉमस्की का मानना है कि 'वैश्वीकरण का अर्थ अन्तर्राष्ट्रीय एकीकरण है। इस एकीकरण में भाषा की अहम् भूमिका होगी और जो भाषा व्यापक रूप में प्रयोग में रहेगी, उसी का स्थान विश्व में सुनिश्चित होगा।'

इसमें कोई शक नहीं कि वैश्वीकरण ने शहरी जीवन स्तर में सुधार किया और उसे ऊँचा उठाया है, लेकिन हाल फिलहाल में भूमण्डलीकरण के विरोध में आवाज उठाई जा रही है जो समाज के ऐसे वर्ग से आती है जो तथाकथित जंगली हैं, असभ्य हैं, बर्बर हैं और मूल समाज का हिस्सा नहीं हैं। इनकी आवाज में ऐसी समस्या की पीड़ा है जिसने समाज के शिक्षितों को इनकी आवाज में आवाज मिलने को विवश किया है। सवाल उठता है कि ऐसी क्या वजह है जिसने इस समाज को भूमण्डलीकरण के विरोध में खड़ा किया है जिस वैश्वीकरण से सम्पूर्ण विश्व का लाभ हो रहा है, तरक्की हो रही है, उनके जीवन स्तर में सुधार हो रहा है, आदिवासी समाज उसका विरोध क्यों कर रहा है।

भगवान ग्याहड़े की पुस्तक "आदिवासी मोर्चा" में कविता "वैश्वीकरण की असली शक्ल" शीर्षक से ज्ञात हो जाता है कि आदिवासी समाज के लिए वैश्वीकरण की छवि नकारात्मक है। आदिवासी प्रकृति से जुड़े हुए हैं, उनकी आत्मा, उनके पूर्वज पेड़ों में बसते हैं, उनका जीवन, उनकी सम्पूर्ण दिनचर्या

बदलता परिवेश साहित्य और मीडिया

मिराण्डा हाउस, हिन्दी विभाग

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भीमदी उप महानगरपालिका-15, नया रतनपुर,
गोपाल चौक, दाङ, नेपाल (पिन : 22415)
फोन : 9857826417, 9815465315

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अनुक्रम

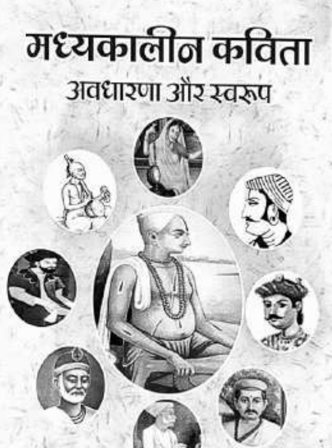
आदिवासी साहित्यिक विमर्श के प्रतिमान, समस्याएँ व सम्भावनाएँ

डॉ. प्रीति सिंह

आदिवासी साहित्य का अध्ययन करते समय अनेक समस्याओं का सामना करना पड़ता है। इनमें आदिवासी की पहचान का संकट, संपर्क का अभाव, लिपि विहीन मौखिक साहित्यिक परंपरा, मूलपाठ की समस्या, राज्य द्वारा सहयोग और संरक्षण का अभाव, बेरोजगारी और विस्थापन, आदिवासियों के अस्तित्व का संकट शैक्षणिक क्षेत्र में आदिवासी साहित्य को पाठ्य पुस्तक के रूप में स्वीकृति प्राप्त नहीं होना, आदिवासी साहित्य से संबंधित कोशों का अभाव, कुछ क्षेत्रों में बाहरी समाज की राजनीतिक और वर्चस्ववादी मनोवृत्ति प्रमुख है।

समस्या यहाँ भी आती है कि लिखित मुख्यधारा के साहित्य समाज में आदिवासियों की अभिव्यक्ति को अल्प स्तर पर रखा गया है, तो यहाँ स्वानुभूति बनाम सहानुभूति की बहस से इतर अनुभव की आधिकारिता का प्रश्न उठता है। इस संदर्भ में हरिराम मीणा जी का कथन उल्लेखनीय है-"कोई लेखक जन्मना आदिवासी है कि नहीं यह महत्त्वपूर्ण है, लेकिन यदि कोई गैर-आदिवासी लेखक अपने आदिवासी जीवन के आधिकारिक अनुभव के आधार पर साहित्य रच रहा है तो ऐसी साहित्यिक अभिव्यक्ति आदिवासी साहित्य की श्रेणी में आयेगी, इसलिए हमारा यह आग्रह नहीं है कि जो जन्मना आदिवासी नहीं है वो आदिवासी साहित्य नहीं रच सकता। सवाल आधिकारिक अनुभव का है, अधिकारिक अनुभव का मतलब है उसके भौतिक जीवन, सांस्कृतिक, सामाजिक, आर्थिक और राजनीतिक पहलू की अभिव्यक्ति क्या है? उसकी मानसिकता, भौगोलिक अंचल, परिवेश किस तरह के हैं? उसका जमीन, आसमान, हवा, पानी, जंगल, पहाड़, नदियों संपूर्ण प्रकृति के साथ संबंध क्या है? तब उस रचनाकार को आदिवासी जीवन का आधिकारिक अनुभव होगा।"[1]

मध्यकालीन कविता
अवधारणा और स्वरूप



संपादक
डॉ. प्रीति सिंह
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अनुक्रम

6.

भक्ति काव्य : प्रकृति एवं स्वरूप

डॉ. प्रीति सिंह

मध्यकालीन भक्ति आंदोलन को प्रायः एक स्वर से उत्तर भारत की एक महान सांस्कृतिक घटना के रूप में स्वीकार किया गया है। हिंदी साहित्य के इतिहास में संवत् 1375 से लेकर संवत् 1700 विक्रमी का समय भक्ति काल कहा जाता है। भक्ति धारा का पर्यवसान शृंगार या रीति काव्यधारा में होता है। यह धारा संवत् 1700 से लेकर 1900 विक्रमी तक प्रवाहित होती रही है। भक्ति-काल में रीति काल को सम्मिलित करके इसे हिंदी साहित्य का 'मध्ययुग' कहा गया है। भक्तिकाल को पूर्व-मध्यकाल के नाम से भी जाना जाता है। साहित्य की कसौटी मनुष्य है एवं भक्ति आंदोलन का साहित्य मनुष्य की आकांक्षा का उद्घोष है। साहित्य किसी भी युग का हो, उसके निर्माण में तत्कालीन समाज के धार्मिक विश्वास एवं परिस्थितियाँ सदैव अपना महत्व रखती हैं। शुक्ल जी कहते भी हैं कि प्रत्येक देश का साहित्य वहाँ की जनता की चित्तवृत्ति का संचित प्रतिबिम्ब होता है।

भक्तिकाल के विषय में डॉ. नगेन्द्र के विचार है- "हिंदी साहित्य के संदर्भ में भक्तिकाल से तात्पर्य उस काल से है, जिसमें मुख्यतः भागवत धर्म के प्रचार तथा प्रसार के परिणामस्वरूप भक्ति आंदोलन का सूत्रपात हुआ था और उसकी लोकोन्मुखी प्रवृत्ति के कारण धीरे धीरे लोक प्रचलित भाषाएँ भक्ति भावना की अभिव्यक्ति का माध्यम बनती गयी और कालांतर में भक्ति विषयक विपुल साहित्य की बाढ़-सी आ गयी। परंतु यह भावना वैष्णव धर्म तक ही सीमित ना थी अपितु शैव, शाक्त, आदि धर्मों के अतिरिक्त बौद्ध और जैन सम्प्रदाय तक इस प्रवाह से प्रभावित हुए बिना न रह सके।"[1]

माना जाता है कि यह पहला भारतीय नवजागरण था, जो कश्मीर से कन्याकुमारी एवं गुजरात से असम तक फैला हुआ था। जहाँ तक मध्यकालीन भक्ति आंदोलन का प्रश्न उठता है, यह मूलतः धार्मिक सांस्कृतिक आंदोलन के रूप में हमारे समक्ष आता है। भक्ति काव्य के आंदोलन व स्वरूप पर चर्चा करने से पूर्व उस समय की पृष्ठभूमि के विषय में जान लेना अति आवश्यक है।

पर्यावरणीय विमर्श
सैद्धान्तिकी एवं अनुप्रयोग

संपादक
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अनुक्रम

16.

समकालीन हिंदी उपन्यासों में पर्यावरणीय चिंतन

डॉ. प्रीती सिंह

पर्यावरण शब्द अनादिकाल से संपूर्ण ब्रह्मांड से जुड़ा हुआ है। पर्यावरण 'परि' व 'आवरण' दो शब्दों से बना है परि का अर्थ है– चारों तरफ तथा आवरण का अर्थ है घेरा। अर्थात कह सकते है कि प्रकृति में जो भी चारों ओर परिलक्षित है यथा– वायु, जल, मृदा, पेड़–पौधे तथा प्राणी आदि सभी पर्यावरण के अंग है पर्यावरण अनेक प्राकृतिक तत्वों का समूह है। मनुष्य हो या जीव–जंतु सभी पर्यावरण की ही उपज है। आज मानवीय क्रियाकलापों के द्वारा पर्यावरण के संतुलन में विशेष अंतर होता जा रहा है। वनों के कट जाने से पर्यावरण के संतुलन पर सबसे अधिक प्रभाव पड़ रहा है इस पारिस्थितिक संकट को दूर करने के लिए पूरा विश्व प्रयत्नशील है। विश्व भर के साहित्यकारों ने भी इस भयंकर समस्या को अपने साहित्य का विषय बनाया है। समकालीन हिंदी साहित्यकारों ने भी इस पारिस्थितिक संकट को चुना है और अपने साहित्य के माध्यम से इस भीषण समस्या से सभी को अवगत कराने का प्रयास कर रहे हैं। वर्तमान समय विमर्शों का कहा जा सकता है। जहाँ भी अस्मिता अर्थात पहचान का खतरा है वहीं पर विमर्श हथियार के रूप में हाशिए के लोगों का साथ दे रहा है। पर्यावरण चिंतन की जब हम बात करते है तो साहित्य के आदिकाल से ही हमें प्रकृति के रूप में पर्यावरण दिखाई देता है। जब से साहित्य की रचना शुरू हुई है प्रकृति किसी ना किसी रूप में साहित्य में सदैव विद्यमान रही है। आदिकाल के साहित्य में प्रकृति का चित्र हमें विभिन्न रूपों में देखने को मिलता है बसंत विलास, बीसलदेव रासो, राउलवेल आदि कई ऐसे काव्य है जहां पर प्रकृति विभिन्न रूपों में दिखाई देती है विद्यापति का एक गीत दृष्टिगत है–

"डम डम डम्फ दिमिक द्रीमी मादल, रूनु झुनु मंजीर बोल
...घटिता घटिता धुनि मर्दंग गरजनि, चंचल स्वर मण्डल करूँ राय
सम्य बसंत रास रस वर्णन, विद्यापति मति छोभित होथि।"

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जिल्दसाज : रहमत अली, कानपुर

विषय सूची

3

एस.आर. हरनोट का 'हिडिम्ब' उपन्यास : दलित जीवन संघर्ष व चेतना का यथार्थ दस्तावेज

डॉ. प्रीती सिंह

बदलते समय के साथ साथ सारी दुनिया वैश्विक ग्राम बनने जा रही है। यह विश्व ग्राम उत्तर आधुनिक और भूमंडलीय संस्कृति में उपनिवेशवादी प्रवृत्ति की ओर खींचा जाना अनेक खतरों के इशारा है। जल, जंगल, पहाड़, जमीन, पर्यावरण के अंधाधुंध उद्योग के कारण फैक्ट्रियों, कारखानों, सड़कों, शहरों के लिए नष्ट किए जा रहे हैं, जिनसे अनेक समस्याएँ उत्पन्न हुई हैं। प्रकृति का दोहन तेजी से हो रहा है। अब औद्योगिकीकरण की लहर शहरों से ग्रामीण क्षेत्र की ओर भी आ रही है। ग्रामीण समाज भी इससे विक्षिप्त हुआ है। वैश्विक संकट आज उनके सामने चुनौती बन कर उभर रहा है। ग्रामीण जीवन इससे चिंतित भी है। औद्योगिकीकरण के लिए किसानों की जमीनें छीनी जा रही हैं। देश की सीमा पर ही नहीं गांव की भूमि पर भी हड़प नीति का घना खतरा मंडरा रहा है। वैश्वीकरण और उदारीकरण के जमाने में पहाड़ों की जमीनें और खेत बेभाव बढ़ कर बिक रहे हैं। इक्कीसवीं सदी में जल, जंगल, जमीन, पहाड़ को उजाड़ कर उद्योगीकरण स्थापित किया जा रहा है, उसे रोकने की आवश्यकता है। जिसके कारण वहां रहने वाले ग्रामीण वासियों का निरंतर शोषण हो रहा है।

इस उत्तर-आधुनिकता समय में जब जल, जंगल, पहाड़, संस्कृति एवं परम्पराओं की चिंताएं हाशिये पर चली जा रही हैं। उस समय एस. आर.हरनोट द्वारा रचित हिडिम्ब' उपन्यास को पढ़ने की आवश्यकता है। ग्रामीण पहाड़ी क्षेत्र के विशृंखलित होते जन-जीवन की त्रासदी 'हिडिम्ब' उपन्यास की केंद्रीय संकल्पना है। "वैश्वीकरण के और उदारीकरण की बयार में भारतीय सभ्यता और संस्कृति संकट के नए दौर में है। यह संकट वैश्विक ग्राम में अपने गांव के खो जाने का है।"[1]

लेकिन एस. आर.हरनोट ने हिमाचल के दुर्गम पहाड़ी ग्रामीण क्षेत्र की व्यथा को अपने हिडिम्ब उपन्यास में समेटकर वैश्विक सरोकार को पकड़ने की कोशिश की है। हरनोट हिमाचल प्रदेश शिमला के निवासी हैं। इनके साहित्य में हिमाचली परिवेश व संस्कृति की अभिव्यक्ति परिलक्षित होती है। हरनोट अपने जीवन के

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An Identity-Based Authentication Framework for Big Data Security

Vinod Kumar, Musheer Ahmad & Pankaj Kumar

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Abstract

Big data raises a robust need for a network structure with the ability to support information retrieval and sharing. Many companies start to supply of big data services for Internet users and at the same time, these services also bring sever all security issues. Presently, the great part of big data set provides digital identity for users to use their services. At the moment, most of these systems use asymmetric and conventional public key cryptography (PKC) to give mutual authentication data security. However, existing authentication schemes trust commonly on the centralized servers to offer record and facilitation facilities for information retrieval. Pairing-free identity-based cryptography has some pull characteristics that gives the idea to healthy requirements in that scenario. The presented scheme is carried out in three phases and are as follows: initialization phase, registration phase and mutual authentication, and session key agreement phase. Detailed security analyzes have been made to authenticate big data server and user. Further, the paper has the resistance to possible attacks in this environment.

Keywords

Big data **Elliptic curve cryptography** **MapReduce**

Authentication and security

Anamika Prakashan

Proceedings of the XXIV DAE-BRNS High Energy Physics Symposium, Jatni, India pp 931–934

Formation of Marginally Trapped Surfaces in Gravitational Collapse

Suresh C. Jaryal & Ayan Chatterjee

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218 Accesses

Part of the Springer Proceedings in Physics book series (SPPHY,volume 277)

Abstract

Using a combination of analytical and numerical techniques, we study the formation and time evolution of collapsing shells, spherically symmetric marginally trapped tubes, as well as the event horizon. Depending on the mass function, density, and the velocity profile, there can be situations where these marginally trapped surfaces becomes space-like, time-like, or null.

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Orientation effects in the fusion of ^{28}Si+^{28}Si system using SEDF

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Introduction

The study of orientation effects is significant as it gives the most favourable configuration for the nuclei to fuse. For this, we have extended the semiclassical extended Thomas-Fermi approach of Skryme energy density formalism (SEDF) for deformed and oriented nuclei, although quadrupole deformation only. The orientation and deformations dependence in SEDF is included through the nuclear density. Here, two parameter Fermi density is used as nuclear density and is extended to deformed nuclei by using deformed nuclear radius and surface diffuseness of ref. [1]. These parameters for spherical nuclei are taken from the earlier work of one of us [2]. The nuclear proximity potential is obtained in slab approximation and overlap of density distribution is considered in sudden approximation. The total interaction potential is obtained by adding deformed Coulomb and centrifugal terms explicitly to the proximity part. We fixed the orientation of the one of the interacting nucleus and varied the orientation of the other in steps of 15° so as to obtain the maximum barrier height and the minimum interaction radius, a configuration favoured for hot fusion [3]. The characteristic properties of the total interaction potential obtained for this configuration is used in Hill-Wheeler approximation, an alternate to the one dimensional solution to the Schrödinger wave equation with appropriate boundary conditions, for a given partial wave. The transmission probability is obtained in parabolic approximation and the total fusion cross-section is calculated by adding partial waves up to certain maximum value ℓ_{max}.

For comparing our results, we have chosen ^{28}Si+^{28}Si system for which fusion evaporation residues were measured in a recent experiment [4] over center of mass-energy (E_{cm}) range $\simeq$ 31-39 MeV and coupled channel (CC) calculations were implemented to reproduce the observed fusion data. In an another experiment [5] of same group (a year ago) over $E_{cm} \simeq 22-30$ MeV, the CC calculations were done by adjusting the parameters of the interaction potential. Similarly, for some other older experiments [6–8] over E_{cm} range ($\simeq 30-220$ MeV), Esbensen, et al. [9] has calibrated the potential parameters within the CC calculation. Now, with these above mentioned experiments the fusion cross-section data for ^{28}Si+^{28}Si system is available for over a wide range of energy both well below and quite above the barrier but there is no unique choice of interaction potential available in literature for the calculations of fusion cross-section. So, here we have achieved suitable interaction potential for the fusion process by varying nuclear orientations.

Methodology

The nuclear potential, $V_N(R)$ in semiclassical extended Thomas-Fermi approach of SEDF using slab approximation (see Ref. [2]) is,

$$V_N(R) = 2\pi\bar{R}\int_{s_0}^{\infty}\left[H(\rho,\tau,\vec{J}) - \sum_{i=1}^{2} H_i(\rho_i,\tau_i,\vec{J}_i)\right]dz$$

where $\bar{R}$ is the mean curvature radius for deformed nuclei, $H(\rho,\tau,\vec{J})$ is Skyrme Hamiltonian density and $\rho(=\sum_i \rho_i)$, $\tau(=\sum_i \tau_i)$, $\vec{J}(=\sum_i \vec{J}_i)$, are nuclear, kinetic energy and spin-orbit densities respectively for composite system, $i = 1,2$ for the two interacting nuclei. The two parameter Fermi density distribution in slab approximation for axially symmetric deformed and oriented nuclei is

$$\rho_i(z_i,\alpha_i,T) = \rho_{0i}(T)\left[1+\exp\left(\frac{z_i - R_i(\alpha_i,T)}{a_i(\alpha_i,T)}\right)\right]^{-1} \quad (1)$$

where $R_i(\alpha_i,T)$ and $a_i(\alpha_i,T)$ are nuclear radii and surface diffuseness parameters for deformed, oriented and coplanar nuclei of [1], respectively. The angle α_i is the angle between nuclear symmetry axis and radius vector, and are uniquely defined for each nuclei after satisfying the minimisation condition [3] for the separation between the interacting surfaces, for a fixed orientation. The nuclear radii and surface diffuseness parameters $R_{0i}(T)$ and a_{i0}(T) are taken from the earlier work of one of us

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[2] and $\lambda(=2)$ for axially symmetric quadrupole deformation. The total interaction potential $V_T(R)$ is obtained by adding Coulomb $V_C(R)$ and centrifugal terms $V_\ell(R)(=\hbar^2\ell(\ell+1)/2\mu R^2)$ to nuclear proximity $V_N(R)$. The characteristic properties of this potential is used in Hill-Wheeler approximation [10] to calculate the fusion cross-section, as

$$\sigma_{HW} = = \frac{\pi\hbar^2}{2\mu E_{cm}} \sum_{\ell=0}^{\ell_{max}} (2\ell+1)\frac{e^x}{1+e^x} \quad (2)$$

where $x = 2\pi\{V_B(\ell) - E_{cm}\}/\hbar\omega_\ell$, $V_{B\ell}$ is barrier height, $R_{B\ell}$ is barrier position and μ is the reduced mass of interacting nuclei and $\ell_{max} = 38\hbar$ [9].

Calculations and results

The total interaction potential is obtained by taking various orientation of target and projectile at $\ell = 0$. The orientations of projectile and target are varied in steps of 15° to obtained an orientation which gives maximum barrier height at $\ell = 0$. Fig. 1 shows this variation of V_B with projectile orientation θ_1 for fixed orientation of target θ_2, at different target orientations θ_2. This figure clearly shows that the V_B first increase, become maximum and then decreases when $\theta_1 \to 90°$. Out of all possible orientations, the orientation (θ_1, θ_2 =30°, 30°) gives maximum V_B and minimum value of R_B, a favourable configuration for the fusion reaction. This is also clear from the Table I, where the orientation effect on other potential characteristics ($V_B, R_B, \hbar\omega_0$) is given. The characteristic properties of the potential for (30°, 30°) configuration are used to calculate fusion cross-section with HW approximation and partial waves are considered up to 38$\hbar$ as per ref. [9]. Fig. 2, shows the calculated fusion excitation function for ^{28}Si+^{28}Si system over $E_{cm} \simeq 22-229$ MeV at (30°, 30°) orientation and is compared with the observed data [4–8]. It clear from the figure that (30°, 30°) configuration is the favoured configuration for the fusion process and the number of partial waves required to be included is 38$\hbar$.

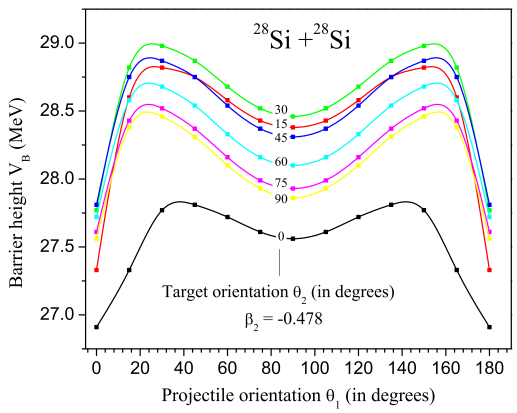

FIG. 1: The interaction potential barrier V_B as a function projectile orientation angle θ_1 at different target orientations angle θ_2.

Orientations:(θ_1, θ_2)	V_B(MeV)	R_B(fm)	$\hbar\omega_0$(MeV)
Spherical	28.23	9.05	2.87
(0°, 30°), (180°, 30°)	27.77	9.14	2.90
(15°, 30°), (165°, 30°)	28.82	8.76	2.85
(30°, 30°), (150°, 30°)	28.98	8.73	2.81
(45°, 30°), (135°, 30°)	28.87	8.81	2.78
(60°, 30°), (120°, 30°)	28.68	8.90	2.75
(75°, 30°), (105°, 30°)	28.52	8.98	2.75
(90°, 30°),(90°, 30°)	28.46	9.01	2.72

TABLE I: The characteristic properties of the total interaction potential for various orientations of projectile at fixed target $\theta_2 = 30°$ for $\ell = 0$.

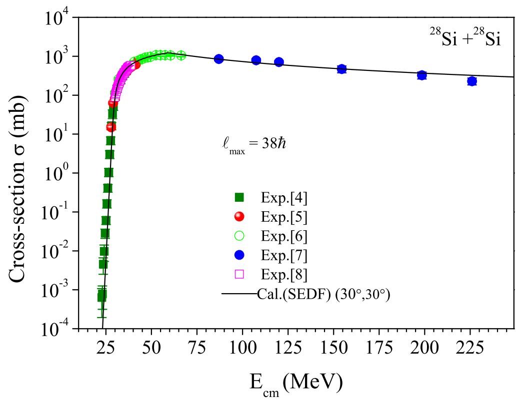

FIG. 2: Fusion excitation function for ^{28}Si+^{28}Si system for most favour configuration for hot fusion compared with experimental data [4–8] over a wide E_{cm} range.

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Isospin effects on the decay of $^{118,122,134}Ba^{*}$ isotopes

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Introduction

One of the importance of studying the nuclear reaction is to study the effects of isospin degree of freedom on reaction mechanism that is the effect of the incoming channel N/Z ratio on the outgoing fragments in a given channel. The effect of N/Z ratio of the compound nucleus is studied here by considering $^{118,122,134}Ba^{*}$ isotopes formed in the reaction of $^{78,82,86}Kr +^{40,40,48}Ca$ at centre of mass energies, E_{cm} =264.41, 268.85 and 308.06 MeV, which corresponds to 10 MeV/nucleon energy [1]. The study is done by using dynamical cluster decay model, DCM ([2] and references there in), where $\overline{\Delta R}$ is the only parameter of the model and is fixed arbitrarily at 1.01 fm for all isotopes of barium. Out of all possible fragments combination, the outgoing channel contains only the energetically favoured combination. In the calculations of decay cross-sections, the number of partial waves to be included are obtained by plotting the preformation probability P_0 or σ as a function of angular momentum ℓ and from the plot it is found that at a particular value of ℓ, below critical angular momentum ℓ_c, the $P_0 \to 0$ or $\sigma \to 0$. This angular momentum is called ℓ_{max} and gives the number of partial waves to be included in the decay cross-section. It is well established that the compound nucleus formed carries large angular momentum and is in excited state. The de-excitation of the CN takes place with the emission of both light particles (LPs: Z$\leq$ 2) and intermediate mass fragments (IMFs: $2 < Z \leq 15$, here). Here, in this work we have calculated the decay cross-section for both LPs and IMFs and a comparison is made between the three isotopes of Ba under study.

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Formalism

The decay cross-section defined by DCM ([2] and references there in) is,

$$\sigma(E_{cm},\ell) = \frac{\pi\hbar^2}{2\mu E_{cm}}\sum_{\ell=0}^{\ell_c}(2\ell+1)P_0P \qquad (1)$$

where, P_0 is the preformation probability related to η-motion and is obtained from the solution of stationary Schrödinger equation in η-coordinate with temperature dependent collective fragmentation potentials (which include sum of binding energies, proximity [3], Coulomb and centrifugal potential terms) at a fixed $R = R_a = C_t(\eta,T) + \overline{\Delta R}(T)$, given as

$$P_0(A_i) = |\psi(\eta(A_i))|^2\sqrt{B_{\eta\eta}}\left(\frac{2}{A}\right) \qquad (2)$$

where ($C_t = \sum C_i$, the Süssman central radii) $i = 1, 2$ for two out going fragments, the penetrability P, referring to R-motion, is WKB integral solved analytically with R_a and R_b as the first and second turning point, satisfying $V(R_a) = V(R_b) = Q_{eff}$, the effective Q-value for outgoing fragments, given as

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_b}\{2\mu[V(R)-Q_{eff}]\}^{1/2}dR\right] \qquad (3)$$

The critical angular momentum ℓ_c in terms of incident energy E_{cm}, reduced mass μ and the first turning point R_a of the entrance channel η_{in} is given by

$$\ell_c = \frac{R_a}{\hbar}\sqrt{2\mu(E_{cm}-V(R_a,\eta_{in},\ell=0))} \qquad (4)$$

with μ as the reduced mass and η_{in} as the entrance channel mass asymmetry.

Calculations and results

Barium isotopes considered for the study of isospin effects are $^{118,122,134}Ba^*$ andP_0, P and decay cross-section are the quantities on effects have been observed. The calculations have been done at fixed $\overline{\Delta R}$ =1.01 fm and the ℓ_c-values to the corresponding center of mass energies E_{cm} =264.41, 268.85 and 308.06 MeV are 153, 158 and 191$\hbar$. But, the cross-sections have been added up to the respective ℓ_{max} values i.e up to 76, 90 and 168 $\hbar$, because beyond ℓ_{max} the decay cross-section drops to zero. The Q_{in}-values are shown in Fig.2.

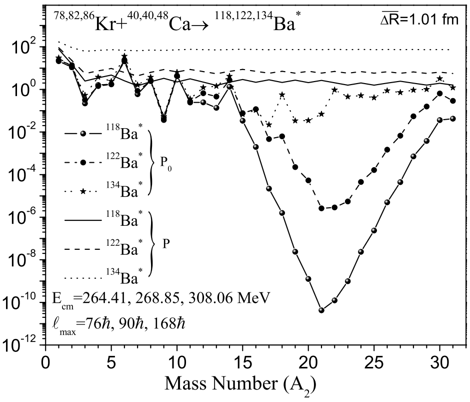

FIG. 1: The P_0 (shown by line plus symbol type) and P (shown by line types) are calculated by using DCM for the energetically favoured LPs and IMFs emitted from the decay of hot and rotating compound systems $^{118,122,134}Ba^*$ at centre of mass energies, E_{cm} =264.41, 268.85 and 308.06 MeV at $\overline{\Delta R}$ =1.01 fm.

Fig. 1, shows the P_0 and P for particles of mass A_2 = 1 - 31, which correspond to Z up to 15, emitted from the isotope of Ba^* compound system. From Fig. 1, we observe the following: (i) the P_0 for outgoing fragments from $^{118,122}Ba^*$ below mass $A_2 \leq 14$ is large compared to fragments of $A_2 > 14$ and (ii) both the P_0 and P increases with increase in N/Z ratio. (iii) the relative change for fragments of $A_2 > 14$ is more then for the fragments of $A_2 \leq 14$. As N/Z→ 1.39286, the P_0 for all the fragments comes almost with in an order of 2. In other words, the two windows of ($A_2 = 1 - 14$ and $A_2 = 15 - 31$) observed in case of $^{118,122}Ba^*$ almost disappears for $^{134}Ba^*$ (iv) the magnitude of P for $A_2 = 1 - 2$ is large by one order compare to the fragments of $A_2 > 2$ and hence the decay cross-section for $A_2 = 1 - 2$ is comparatively large (see Fig. 2) and for other observations for the decays cross-sections remains same as for P_0, see Fig. 2. Also, Figs.1 and 2,

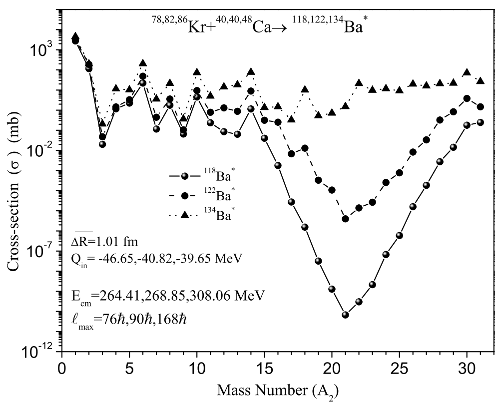

FIG. 2: Same as Fig.1, but for cross section σ.

shows that the magnitude as well as structure of $P_0(A_2)$ and $\sigma(A_2)$ does not change much up to $A_2 = 14$, while for fragments $A_2 > 14$ there is considerable change with increase in N/Z ratio. So, we conclude that the two windows observed for $^{118,122}Ba^*$ disappear for $^{134}Ba*$ which is equivalent to the fact that with increase in N/Z-ratio there is comparatively large increase in the decay probabilities of mass range $A_2 = 15 - 31$ than for masses $A_2 \leq 15$.

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Chapter

Bioremediation of Metals from Lithium-Ion Battery (LIB) Waste

January 2018
DOI:10.1007/978-981-10-7413-4_14
In book: Waste Bioremediation (pp.265-278)

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Abstract

Technological advancement has greatly increased the demand for newer lithium-ion batteries (LIBs) due to the more use of advanced energy storage devices like electric vehicles, consumer electronics, renewable energy storage, backup power, medical devices. The existing methods for metal recovery from LIB recycling involved: (i) aqueous stream-based limited recycling using the liquid stream mixed with soda ash with the hammer mill and shaker table (ii) supercritical CO2-based recycling of cathode and anode (iii) pyro- and hydrometallurgical processes. Microbial participation for the recovery of metals from waste (LIBs) was found to be an attractive method due to its environmental-friendly approaches. Lysinibacillus, Micrococcus, Sporosarcina, Empedobacter, Barrientosiimonas, Lysinibacillus, Paenibacillus, Bacillus, Acidithiobacillus are among the species involved in the recycling of metal form LIB.

Chapter

Bioremediation Techniques for E-waste Management

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Abstract

Bioremediation or microbial cooperation can improve the e-waste management process in a greener way. Every management strategy is concentrated upon the organic and inorganic portion of the e-waste. Organic part consists of variety of thermo and thermosetting plastic with the presence of halogenated material. Microbes are involved in the process of dehalogenation in many ways. Microbes can manage the leaching of inorganic portion of e-waste which consists of both metallic and nonmetallic components.

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Economics and Policy of Energy and Environmental Sustainability

Editors: Narendra N. Dalei, Anshuman Gupta

Investigates the economics and policy facets of both energy and environmental sustainability

Analyses the interaction and relationship among economy energy and environment

Offers a course of action to assist policymakers in designing efficacious policies for their respective countries

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Keywords
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Sustainable Energy Challenges in Realizing SDG 7

Narendra N. Dalei, Pramod Kumar Painuly, Atul Rawat & Githa S. Heggde

Living reference work entry | First Online: 14 September 2021

Part of the Encyclopedia of the UN Sustainable Development Goals book series (ENUNSDG)

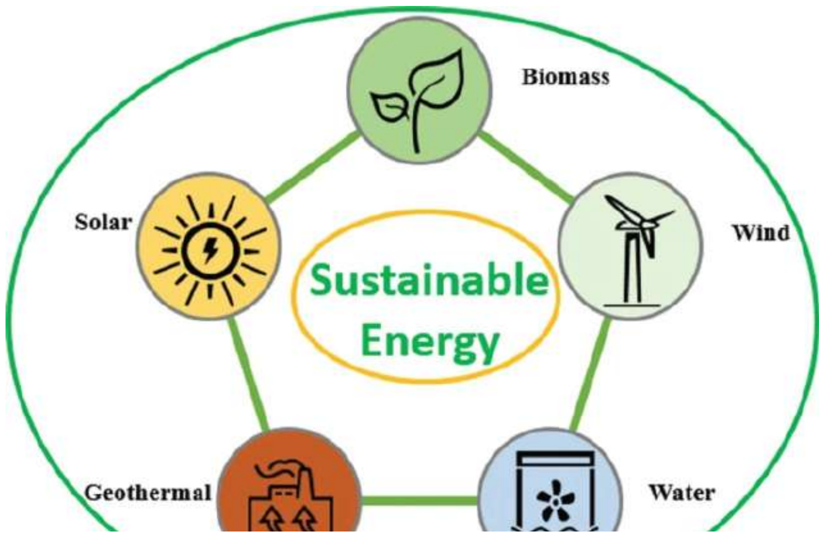
Synonyms

Barriers; Clean energy; Modern energy; Obstacles; Renewable energy

Definitions

The energy that can meet the demand of all sections of society and that can be available for present and future generations can be considered as sustainable energy. This energy is thus to fulfill our present demand without harming our environment, and the same will continue for the future without being depleted while contributing to the survival of all species (Jain and Jain 2020; Rinkesh 2020; Lund 2010). This is the energy which is very much aligned with SDG 7 and basically comes from biomass, solar, wind, water, and geothermal (Kutscher et al. 2018) (see Fig. 1). However, there are numerous challenges to face in order to transit from dirty energy to sustainable energy. This entry discusses all such challenges in realizing Sustainable Development Goal 7 (SDG7), which ensures sustainable energy that is affordable, reliable, modern, and clean for all (UN 2018).

Fig. 1



Economics and Policy of Energy and Environmental Sustainability pp 1–9

Sustainable Energy and Environmental Sustainability in an Economics and Policy Prospective

Narendra N. Dalei & Anshuman Gupta

Chapter | First Online: 24 November 2022

Abstract

Transitioning to renewable and sustainable energy has been considered as one of the major drivers of keeping the global mean temperature rise well below 2 °C above the pre-industrial level as per the Paris agreement. Thus, energy policies of most of the countries at large have been aligned to sustainable energy with the objective of providing affordable and clean energy to all citizens along with decarbonizing the energy systems. Despite lots of effort put by global communities for cost-effective and environmentally friendly generation, distribution, and access to sustainable energy, there are many challenges to the realization of environmental sustainability and green energy. Therefore, analysis of economics and policies of energy and environment is of utmost importance before bringing any reforms to policies of sustainable development. In view of this, the book investigates the economics and policy aspects of both energy and environmental sustainability and interactions between them through many channels and suggests course of actions to help policymakers design effective policies for their respective countries. The current chapter is the snapshot of what the effort has been made in this book for the global policymakers and concerned stakeholders.

Economics and Policy of Energy and Environmental Sustainability pp 13–31

Renewable Energy Policy Responses to Observed Impact of Climate Change: Evidence from DPSIR Framework

Narendra N. Dalei

Chapter | First Online: 24 November 2022

Abstract

Renewable energy has already been proved to be the most reliable, affordable and green energy so far as its evolvement over the past decades is. Most of the countries in the world have realized its importance in addressing and mitigating the climate change impact and thus formulated policy framework to harvest sustainable energy. Over the past decades, there was a tremendous growth in renewable energy, which has given the hope to provide a sustainable energy supply to the entire globe. However, the sustainable supply of renewable energy depends upon policy effectiveness. Therefore, using the Driving Forces–Pressure–State–Impact–Response (DPSIR) framework, this study investigates the drivers, pressures, state, impact and policy responses that will help policymakers to not only introduce new policies but also help them to bring reforms in the existing policies for global sustainable energy security. Identifying the drivers of climate change, studying its observed impact and investigating the policy responses to the observed impact of climate change the study found that renewable energy policy brings sustainable energy security for the entire globe.

Notes

Book | Feb 2023

Financial Inclusion in Circular Economy

A Bumpy Road Towards Sustainable Development

Authors: Vinay Kandpal , Deep Chandra , Narendra N. Dalei , Jatinder Handoo

An indispensable source for the Students of PG and UG programs, researchers and practitioners

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About this book

This book presents an assessment of endeavors towards Financial Inclusion and its role in Sustainable development. An attractive feature is that it deals with almost all the contemporary issues essential for reaching UN Sustainable Development Goals. This book would be an exclusive and authentic source to the students of undergraduates, postgraduates and professional courses in Commerce and Management. This manuscript is divided into nine chapters.

The book looks at various salient topics, including financial inclusion measurement, the impact of various financial inclusion indicators on development outcomes and macroeconomic volatility using aggregate data, and the effects of financial inclusion on poverty and development outcomes using microdata. Using the recently adopted Sustainable Development Goals as an overall framing of the issues, it exhibits how poor and disadvantaged women and men can be bankable if the adequate facilitation for maximizing opportunities and addressing constraints. This book attempts to cover different dimensions of Financial Inclusion towards attaining Sustainability and Circular Economy through financing instruments and investments. This book highlights different goals of UN SDG as an Initiative towards Inclusive Growth and Circular Economy, which is also influenced by Micro Finance Institutions and NBFCs.

This book will be an indispensable source for the Students of PG and UG programs, Researchers and practitioners from areas of Commerce, Economics and Management and the faculty members and professionals like bankers and financial consultants. We hope this book will meet the requirement of all the categories of readers.

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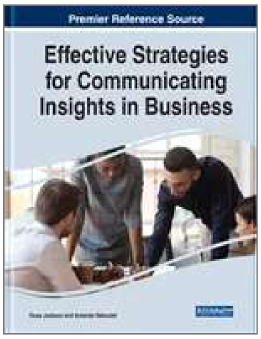
Keywords

Sustainability and circular economy **Financial inclusion and capability**



Neutralizing Maleficent Effects of COVID-19 Through Entrepreneurship: Peeping Through the Lens of Communication

Manpreet Arora (Central University of Himachal Pradesh, Dharamshala, India) and Roshan Lal Sharma (Central University of Himachal Pradesh, Dharamshala, India)

Source Title: Effective Strategies for Communicating Insights in Business (/book/effective-strategies-communicating-insights-business/244299)

Pages: 20
DOI: 10.4018/978-1-7998-3964-4.ch005

Abstract

Entrepreneurs are regarded as the leaders of socio-economic, industrial, and business development. They are important pillars of any economy. They are the employment generators and contribute to economic growth. In the current scenario caused by the COVID-19 pandemic, we witnessed people across the world losing their jobs. An inclination towards entrepreneurial activities has been observed in 2020. People have started communicating how entrepreneurial activities are the only solution for economies in crises. The role of social media platforms cannot be under-emphasized, and therefore, an entrepreneur today has to be a master of various media of communication. This chapter aims at exploring the concept of entrepreneurship with special reference to communication. It highlights challenges and issues in the post-pandemic world with reference to entrepreneurship and the role effective communication can play in handling them. It also attempts to examine the use of the term 'entrepreneurship' by general public on social media such as Twitter in particular with a view to work out its implications.

Chapter Preview

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Introduction

In today's fast changing business environment, key developmental strategies and planning done by the policy makers require a combination of vision, mission, value creation, resource organization and sustainable development of various businesses. Such businesses/ business ventures are initiated based on the risk-taking abilities of entrepreneurs. According to Joseph Alios Schumpter, it is the entrepreneur who shackles and disturbs the equilibrium in business cycles and contributes toward economic development. Schumpter (2000) further observes that the entrepreneurial spirit of entrepreneurs contributes directly towards creativity and innovation. All the technological changes which are brought in the economy are the result of innovation only which leads to prosperity and welfare of the state (Audretsch, 2007). Entrepreneurs are regarded as the leaders of industrial development (Davis, 1968). They are important pillars of any economy. They are basically the employment generators and primarily contribute to the economic growth (Mitra, 2019). Entrepreneurship is often explained as the process of starting and running one's own business. Entrepreneurs are the key drivers of fostering production process in the economy by setting up small, medium or large-scale enterprises (Vogel et al., 2014).

Entrepreneurship also contributes toward structural changes in the society (Gries & Naudé 2010). The structural transformation of the society helps in moving towards equitable distribution of income and greater opportunities leading to prosperity. Entrepreneurial activities foster the development of new projects and ventures by encouraging creativity and innovation (Autio et al., 2014). When new capital is invested in the economy it helps create additional capacities in existing and new product lines. This creates new demand and helps in the growth of the economy due to contribution of consumers who ultimately spend and buy these products. Entrepreneurs take risks and try to grab opportunities (Brockhaus 1980). They help discover new ideas, create knowledge by thinking creatively, innovate and solve various problems pertaining to capital formation, resource utilization, and resource allocation for starting new ventures. Entrepreneurs are often visionary (Allen 1997). They basically carry the trait of "having the ability to see the unseen". Quite often it has been seen that they dream big. Most of the entrepreneurs carry the intrinsic ability of visualizing the unseen things. They spend a lot of time in interpersonal communication with various stakeholders.

Various dimensions of communication play crucial part in the successful realization of entrepreneurial vision. Entrepreneurs can undoubtedly be charismatic but most of the things that they do and envision can be learnt through experience. Considerable degree of persistence towards working in a single direction to actualize a dream/vision is their passion. We all dream and most of the times visualize several ventures/endeavors, but we lack that risk-taking ability to make our dreams come true. This trait of converting dreams into reality is inherent in entrepreneurs and they have the capacity to work passionately and with sincerity (Murmann & Sardana). The ability of an entrepreneur and his inner drive to chase his passion, to achieve something unseen is what makes

Soft Computing: Theories and Applications pp 715–722

On the Security of Certificateless Aggregate Signature Scheme in Vehicular Ad Hoc Networks

Pankaj Kumar & Vishnu Sharma

Conference paper | First Online: 25 November 2017

Part of the Advances in Intelligent Systems and Computing book series (AISC,volume 583)

Abstract

Certificateless aggregate signature scheme is a very effective technique for ad hoc networks such as vehicular ad hoc network. An aggregate scheme aggregates the individual signature, which reduces the computational overhead and useful in the areas, where bandwidth limitation is a major issue. Recently, Malhi and Batra proposed a certificateless aggregate signature scheme for vehicular ad hoc networks and proved the scheme mathematically secure against possible types of security attacks. In this paper, we present the cryptanalysis of the scheme while applying some concrete attack "honest but curious". The additional contribution to this paper is an improvement of the certificateless aggregated signature scheme.

Keywords

Cryptography **CLAS** **CLS** **Digital signature**

References

Chapter

Bio-Based Materials for Food Packaging Applications

By Purnima Justa, Hemant Kumar, Sujeet Kumar Chaurasia, Adesh Kumar, Balaram Pani, Pramod Kumar

Book Biodegradable Composites for Packaging Applications (https://www.taylorfrancis.com/books/mono/10.1201/9781003227908/biodegradable-composites-packaging-applications?refId=7862f767-9ce5-47f6-a9cf-8f6091886965&context=ubx)

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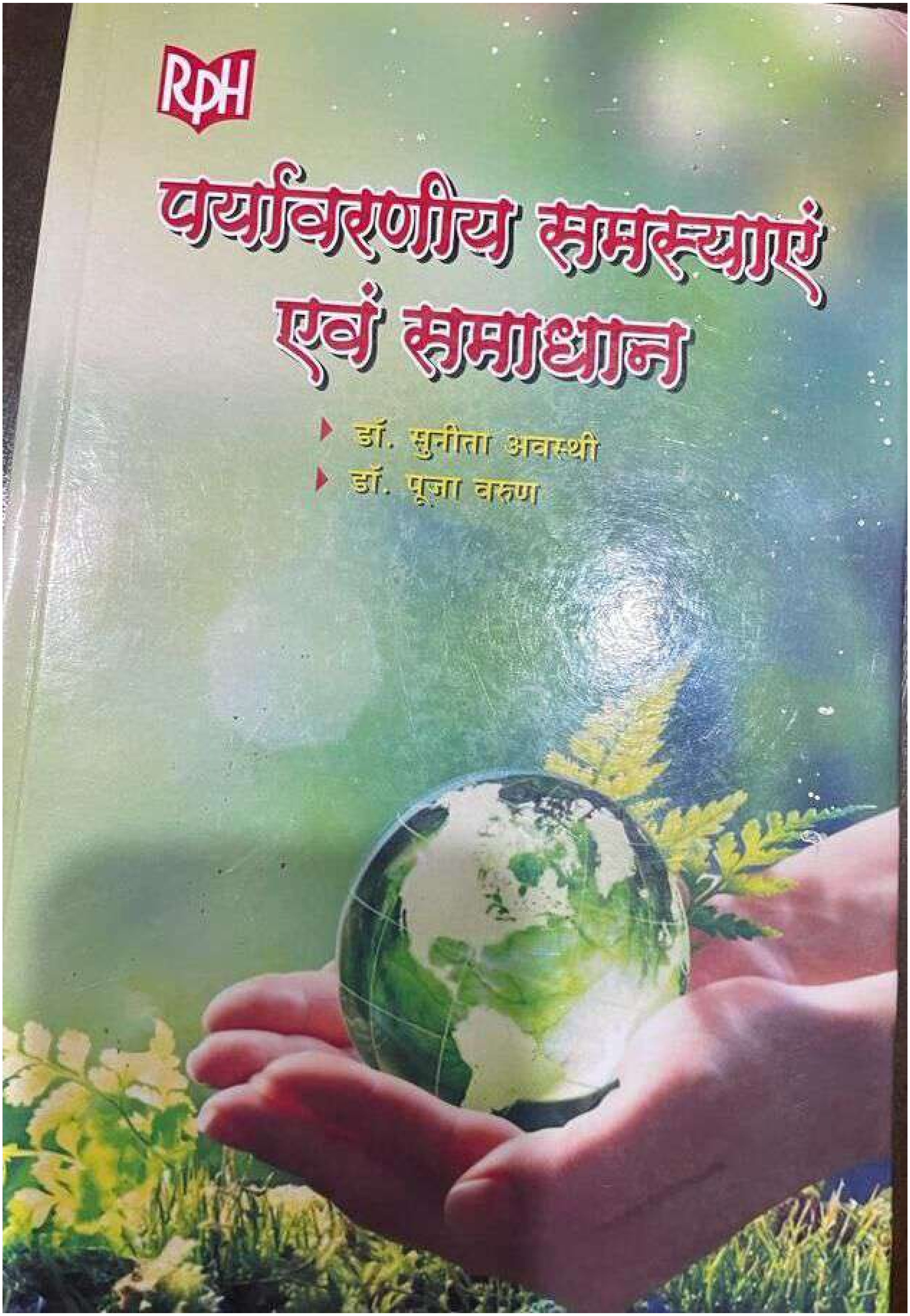
ABSTRACT

< Previous Chapter (chapters/edit/10.1201/9781003227908-1/introduction-biodegradable-polymers-arbind-prasad-gourhari-chakraborty-ashwani-kumar-kishor-kumar?context=ubx)
Next Chapter > (chapters/edit/10.1201/9781003227908-3/processing-biodegradable-composites-gourhari-chakraborty-arbind-prasad-ashwani-kumar?context=ubx)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नानकवाणी में मानस पर्यावरण

डॉ. प्रिया शर्मा*

पर्यावरण और मनुष्य का अटूट संबंध है। वायु, जल, अग्नि, आकाश, थल, इन पांच तत्वों पर मानव जीवन आधारित है। आज ग्लोबल वार्मिंग की समस्या विश्व भर में फैल रही है, किंतु इसका पक्का समाधान नानकवाणी के जपुजी साहिब में दर्ज है–

पवणु गुरु पाणी पिता माता धरति महतु।
दिवसु राति दुइ दाई दाइआ खेलै सगल जगतु।।

अर्थात पवन गुरु के समान है, पानी पिता का प्यार देता है, धरती माता है, जीवन देने वाली है। दिन और रात दाई–दाया हैं, अर्थात् सबके अपने नियम है। दिन कर्म करने के लिए है, परिश्रम करने के लिए है, रात विश्राम करने के लिए है। अगर इतने सरल शब्द भी मनुष्य की समझ में नहीं आए हैं, फिर प्रकृति के साथ हमारा संबंध कैसे कायम होगा? देश के लोगों को पर्यावरण के महत्व को समझाने के लिए 5 जून से 16 जून के बीच 'पर्यावरण दिवस' मनाने के लिए प्रेरित किया है। मानवता पर आज जो दुःख, निराशा, आक्रोश फैला हुआ है, उसके मूल में मानव की असीम आकांक्षाएं हैं। मनुष्य अपने स्वार्थवश पेड़–पौधों की कटाई करके प्राकृतिक संसाधनों से खिलवाड़ कर रहा है, परिणामस्वरूप करोना महामारी हमारे समक्ष चुनौती बन कर खड़ी है। अगर मनन किया जाए तो स्पष्ट है कि मनुष्य को किसी का भय ही नहीं रहा। प्रकृति रूपी संपदा का हनन करते हुए उसे लेशमात्र भी दुःख नहीं हुआ। गुरु नानक देव ने मनुष्य को समझाते हुए कहा है कि प्राकृतिक वस्तुएं भी परमात्मा के भय से वंचित नहीं।

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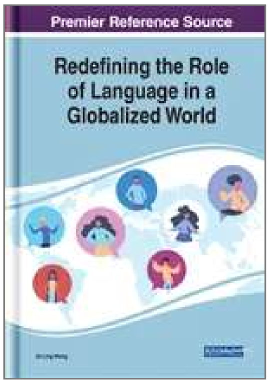

Post-Truth (Mis)communication as Enigmatic Mystification and Misuse of Language

Manpreet Arora (School of Commerce & Management Studies, Central University of Himachal Pradesh, Dharamshala, India) and Roshan Lal Sharma (Department of English, Central University of Himachal Pradesh, Dharamshala, India)

Source Title: Redefining the Role of Language in a Globalized World (/book/redefining-role-language-globalized-world/238472)

Pages: 11
DOI: 10.4018/978-1-7998-2831-0.ch003

Abstract

It would be erroneous to assume that language can only be used in positive sense and that it cannot be misused to cause instability and unrest in our professional as well as personal lives. On the contrary, language can be weaponised to cause irreparable harm/damage to vulnerable communities, ethnicities, marginalized populations, and deprived chunks of humanity. Language has innumerable forms such as genuine and authentic language, truthful and honest language, ingenuine and false language, and fake and deceptive language. Be that negative or positive aspect of language, it can be tilted either way as per one's will and choice; nevertheless, inauthentic and manipulative language can seldom have acceptability among the masses. The term 'post-truth' stems from fabrication of truth and it has a deep connection with politics. Post-truth may be understood as a situation wherein the objective facts are less influential in shaping opinion, and emotions and personal beliefs are important to connect people. In this chapter, the authors have analyzed language in the age of post-truth.

Chapter Preview

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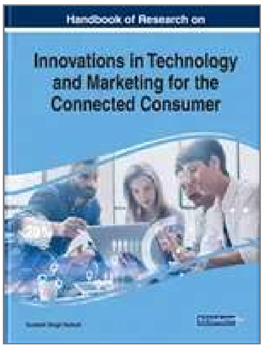
Introduction

The origin of language, a great extent, could be attributed to the survival needs of Homo sapiens as they had to struggle hard to get food and stay safe from the harsh surroundings. Ever since man could use language, he had an advantage over other species not only from survival perspective but also from the viewpoint of his need to interact with others (Bryant). Not that communication has been impossible without language; we have ample evidence to the contrary as birds and animals communicate nonverbally without language. Nevertheless, humans are different from members of other species majorly because of their ability to use words to articulate and vocalize our thoughts. In common parlance, language is a means of communication in a non-verbal, verbal or written manner. One tends to make use of words while s/he uses language be that in a spoken or written manner. The term 'language' also implies a 'system of communication' practiced by a particular community. Language can also be viewed as "a system of conventional spoken, manual (signed), or written symbols by means of which human beings, as members of a social group and participants in its culture, express themselves" (Robins and Crystal). Besides communication, it is also a means to express our identity, imagination, ideas, emotions and feelings. Since time immemorial, man has been using language in the form of signs, signals and gestures. Prior to the advent of the word, man has been using various nonverbal signs to express his desires and needs. Language as we see it today is an evolved form of self-expression and exchange of ideas.

Language as Communication

As the story of civilization developed, human's narration skills also improved considerably. Language as communication in its highly evolved form has been a serious cause of concern across humanities and social sciences. Language as means to communicate one's inner-self has been core area of concern for linguists and writers. Interestingly, language can be used, misused, abused, over-used and under-used as per volition and choice of the user. In this sense, language can be viewed as a weapon to achieve desired results.



Post-Truth and Marketing Communication in Technological Age

Manpreet Arora (Department of Commerce and Management, Himachal Pradesh Kendriya Vishvidyalye Business School, Central University of Himachal Pradesh, Dharamshala, India)

Source Title: Handbook of Research on Innovations in Technology and Marketing for the Connected Consumer (/book/handbook-research-innovations-technology-marketing/226306)

Pages: 15
DOI: 10.4018/978-1-7998-0131-3.ch005

Abstract

The way by which the communication is done depends upon the purpose of the communication. The complex technology-driven environment is affected by a syndrome called post-truth. Post-truth scenario is marred with a situation where there are spread of lies, rumors, propaganda, and deceit. Human perception is distorted by the spread of lies and fake news. We struggle hard to decide whether any communication which we read, or listen to, or share is true or untrue. The strategic advancements aspired by any company are based more or less on the marketing tactics of the product or service. Many strategies of the organisations are based on the communicative interactions of the corporate world with the consumers. The era of post-truth is based on emotions, opinions, and distorted facts. False advertising tactics are hitting the emotions and sentiments of the public at large. Many social media players in the move to curb the menace of false news, misinformation, and false advertisements have opted for a voluntary code of ethics.This chapter analyses the marketing communication in the era of post-truth.

Chapter Preview

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Communication, Technology, And Post- Truth

Communication: a process of creating, exchanging and receiving information, messages, and ideas, etc in order to influence or get a response from someone. Communication Technologies include all equipment and programs that are used to process and communicate information Every communication channel or each communication we do in our daily lives has its own dimensions and on different occasions, we require different types of communication. The way by which communication is done depends upon the purpose of the communication. It can be done verbally or non-verbally also. Firstly, we need to decide what will we communicate? We can communicate through various languages, expressions of languages, a mix of languages, of course, smiley's these days, jargons, local dialects, and buzzwords but how well the words are used plays an important role in it. Therefore, the choice and use of appropriate words in a particular situation determine the efficiency of the individual and the effectiveness of the communication. Gone are the days when sender and receiver were identified, and few types of identified noise played a role in distorting the communication.

Key Terms in this Chapter

Fake News (/dictionary/fake-news/51885): Refers to nontruthful information, false news, hoax news, false information, or propaganda published under the guise of being authentic news and mostly spread out using a range of media including social media with some ulterior motives and sometimes for the sake of fun, or influencing others opinions.

Digital Platforms (/dictionary/digital-platforms/78668): Are internet and web-based virtual spaces that are based on some kind of unique business models in order to exchange, information, knowledge, goods, ideas, or services, etc, either with financial or nonfinancial returns, Eg Google search engine, Facebook, Spotify, Youtube, Apple store, Amazon market place, Uber, Apple/Google Pay, etc.

Advertisements (/dictionary/advertisements/78718): A paid form of communication that employees a range of media channels to send out an open, non-personal message to inform, persuade, and influence users of a product, idea, service, or place or person.

Post-Truth (/dictionary/post-truth/63377): It relates or demonstrates those circumstances where the objective facts are not important or worth enough for shaping public opinion and appeals to emotion or personal beliefs play an important role than the objective analysis.

Human Resource Development and MSME Development

Editors :
D M Pestonjee
Sanjay Pal

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"Human is a possibility and not a resource…" This was the guiding spirit behind the Conference which led to chalking out agenda for the Conferenece themes. Similarly the unexplored potential of Micro, Small and Medium Enterprises and its connect to Human Resource Development led to combining the two themes to resurge as Human Resource Development and MSME Development.

The Third International Conference on 'Human Resource Development and MSME Development' (January, 24-25, 2019) thus was organised jointly in collaboration with Entrepreneurship Development Institute of India and Academy of Human Resources Development. Twenty seven papers were presented during the two-day conference.

Organizing and hosting an International Conference was a toilsome task especially in the intriguing domains of Human Resource Development & MSME Development, which could not have seen light of the day without moral and whole-hearted support of many contributors. We take this opportunity to thank each and everyone who has made this possible.

Firstly we thank Dr Sunil Shukla, Director General, Entrepreneurship Development Institute of India, for his exceptional guidance, advice and suggestion for ideating and conducting the conference and providing the facilities in their sprawling campus. We are truly thankful to the significant contribution of Dr. Rajeshwari Narendran, Honorary Director, AHRD for her vision, execution excellence and untiring efforts for making this conference possible. We would like to express our special thanks and gratitude to Dr. T. V. Rao for iluminating us with his rich experience under his umparalleled umbrella of support and guidance.

We acknowledge the contribution of Conference participants and researchers from India and abroad for their research paper, case studies and

scholarly work. We express our gratitude for the active participation of presenters and guests from different countries.

We extend our gratitude to all the Board members of Academy of Human Resources Development-India, without their able support the Conference would not have been successful.

We extend our gesture of appreciation to all the Reviewers, Jury, Session Chairs, Discussants for the Conference papers and help in finalising award winners in both HRD and MSME segments.

Finally we would like to acknowledge the efforts and commendable contribution of Team EDII: Dr. Ishwar Kumar and Mr. Ganapathi Batthini and Team AHRD: Merlin George, Aparna M., Savita Gopalan, Jyothu Francis and Arpit Patel for excellent facilitation and working day and night despite many challenges.

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Dr. D M Pestonjee
Dr. Sanjay Pal

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Editors :
D M Pestonjee
Sanjay Pal

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Entrepreneurship Development Institute of India

Human Resource Development and MSME Development

International Conference 2019

Entrepreneurship Development Institute of India, Ahmedabad (24-25 January 2019)

Editors :

D M Pestonjee
Sanjay Pal

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"Human is a possibility and not a resource..." This was the guiding spirit behind the Conference which led to chalking out agenda for the Conferenece themes. Similarly the unexplored potential of Micro, Small and Medium Enterprises and its connect to Human Resource Development led to combining the two themes to resurge as Human Resource Development and MSME Development.

The Third International Conference on 'Human Resource Development and MSME Development' (January, 24-25, 2019) thus was organised jointly in collaboration with Entrepreneurship Development Institute of India and Academy of Human Resources Development. Twenty seven papers were presented during the two-day conference.

Organizing and hosting an International Conference was a toilsome task especially in the intriguing domains of Human Resource Development & MSME Development, which could not have seen light of the day without moral and whole-hearted support of many contributors. We take this opportunity to thank each and everyone who has made this possible.

Firstly we thank Dr Sunil Shukla, Director General, Entrepreneurship Development Institute of India, for his exceptional guidance, advice and suggestion for ideating and conducting the conference and providing the facilities in their sprawling campus. We are truly thankful to the significant contribution of Dr. Rajeshwari Narendran, Honorary Director, AHRD for her vision, execution excellence and untiring efforts for making this conference possible. We would like to express our special thanks and gratitude to Dr. T. V. Rao for iluminating us with his rich experience under his umparalleled umbrella of support and guidance.

We acknowledge the contribution of Conference participants and researchers from India and abroad for their research paper, case studies and

scholarly work. We express our gratitude for the active participation of presenters and guests from different countries.

We extend our gratitude to all the Board members of Academy of Human Resources Development-India, without their able support the Conference would not have been successful.

We extend our gesture of appreciation to all the Reviewers, Jury, Session Chairs, Discussants for the Conference papers and help in finalising award winners in both HRD and MSME segments.

Finally we would like to acknowledge the efforts and commendable contribution of Team EDII: Dr. Ishwar Kumar and Mr. Ganapathi Batthini and Team AHRD: Merlin George, Aparna M., Savita Gopalan, Jyothu Francis and Arpit Patel for excellent facilitation and working day and night despite many challenges.

Our gratitude and thanks to all who have continued to shower their undaunted support, inspiration and zeal for making the conference a great success.

Dr. D M Pestonjee
Dr. Sanjay Pal

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Description

डॉ- प्रिया शर्मा की ये कविताएँ बच्चों के सरल चित्त पर समय की चेतावनियाँ लाल-हरे रंग में झिलमिलाती चलने वाली हँसती-बोलती सी कविताएँ हैं। 'फास्ट फूड' या 'ब्यूटी पार्लर' या 'कविताएँ न पढ़ने की प्रवृत्ति' बाल जगत् में भी प्रवेश पा गए हैं। एक से एक रोचक विज्ञापन मायानगर के ऐयार की तरह टीवी पर आते हैं और फास्ट फूड, श्रृंगार प्रसाधन आदि के प्रति बाल-मन में भी ऐसा आकर्षण भर देते हैं कि घर का पौष्टिक और संतुलित आहार उन्हें अखाद्य लगने लगता है और नहाया-धोया, सादा-सा चेहरा भी
'ब्यूटी पार्लर' का प्रत्याशी! गुड़ियानुमा वे ही मम्मियाँ अच्छी लगने लगती हैं जो 'सुपर बायर्स'
होंμमहाखरीदारμरंगीन पैकेटों में जहर खरीदकर घर लाने वाले। परीकथा के जंगलों में भी उन्हें 'ब्यूटी पार्लर' चाहिए।
आंतरिक सौंदर्य, शांति, सौहार्द और प्रेम से भरे सहकारितामूलक जीवन की प्रेरणा सहज ही मन में जगाने वाले ये बालगीत इसलिए भी महत्त्वपूर्ण हैं कि पर्यावरण की चुनौतियों का सामना करते हुए पशु-पक्षी और वनस्पति जगत् से तादात्म्य रखने की उमंग ये मन में भरते हैं। पशु-पक्षी, वनस्पति और बच्चेμये ही युद्ध और आतंक की छाया में पल रहे इस अतिशय भौतिक युद्ध का सच्चा वैकल्पिक प्रतिपक्ष रचेंगे इन कविताओं के दम से।
-अनामिका

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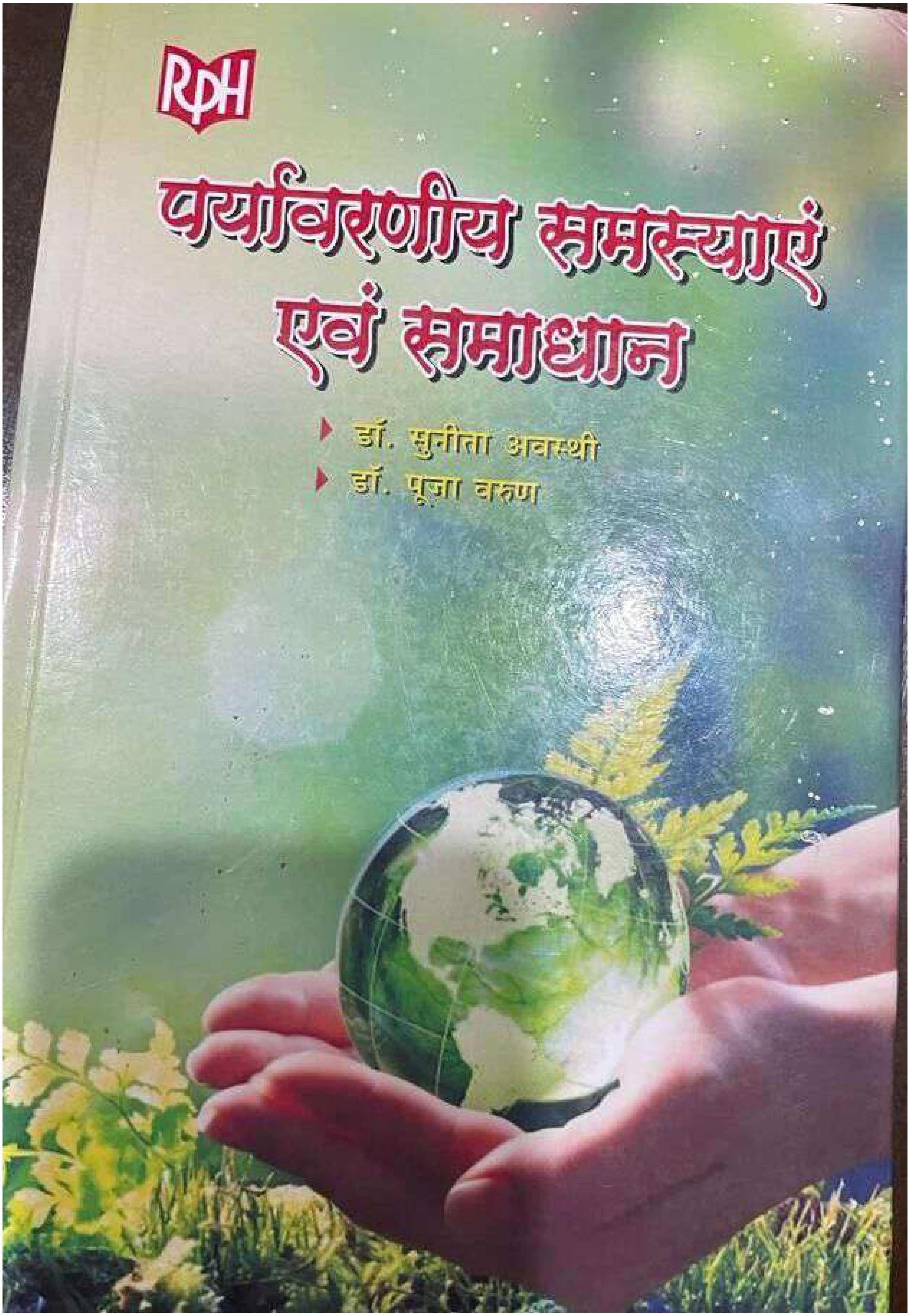
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पुस्तक प्रकाशन में सम्पूर्ण सावधानी बरती गई है। फिर भी किसी त्रुटि, कमी अथवा लोप का रह जाना मानवीय भूल के कारण संभव हो सकता है। पुस्तक में प्रकाशित लेख के विचारों हेतु सम्पादक एवं प्रकाशक की कोई जिम्मेदारी नहीं है।

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नानकवाणी में मानस पर्यावरण

डॉ. प्रिया शर्मा*

पर्यावरण और मनुष्य का अटूट संबंध है। वायु, जल, अग्नि, आकाश, थल, इन पांच तत्वों पर मानव जीवन आधारित है। आज ग्लोबल वार्मिंग की समस्या विश्व भर में फैल रही है, किंतु इसका पक्का समाधान नानकवाणी के जपुजी साहिब में दर्ज है–

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दिवसु राति दुइ दाइ दाइआ खेलै सगल जगतु।।

अर्थात पवन गुरु के समान है, पानी पिता का प्यार देता है, धरती माता है, जीवन देने वाली है। दिन और रात दाई–दाया हैं, अर्थात् सबके अपने नियम है। दिन कर्म करने के लिए है, परिश्रम करने के लिए है, रात विश्राम करने के लिए है। अगर इतने सरल शब्द भी मनुष्य की समझ में नहीं आए हैं, फिर प्रकृति के साथ हमारा संबंध कैसे कायम होगा? देश के लोगों को पर्यावरण के महत्व को समझाने के लिए 5 जून से 16 जून के बीच 'पर्यावरण दिवस' मनाने के लिए प्रेरित किया है। मानवता पर आज जो दुःख, निराशा, आक्रोश फैला हुआ है, उसके मूल में मानव की असीम आकांक्षाएं हैं। मनुष्य अपने स्वार्थवश पेड़–पौधों की कटाई करके प्राकृतिक संसाधनों से खिलवाड़ कर रहा है, परिणामस्वरूप करोना महामारी हमारे समक्ष चुनौती बन कर खड़ी है। अगर मनन किया जाए तो स्पष्ट है कि मनुष्य को किसी का भय ही नहीं रहा। प्रकृति रूपी संपदा का हनन करते हुए उसे लेशमात्र भी दुःख नहीं हुआ। गुरु नानक देव ने मनुष्य को समझाते हुए कहा है कि प्राकृतिक वस्तुएं भी परमात्मा के भय से वंचित नहीं।

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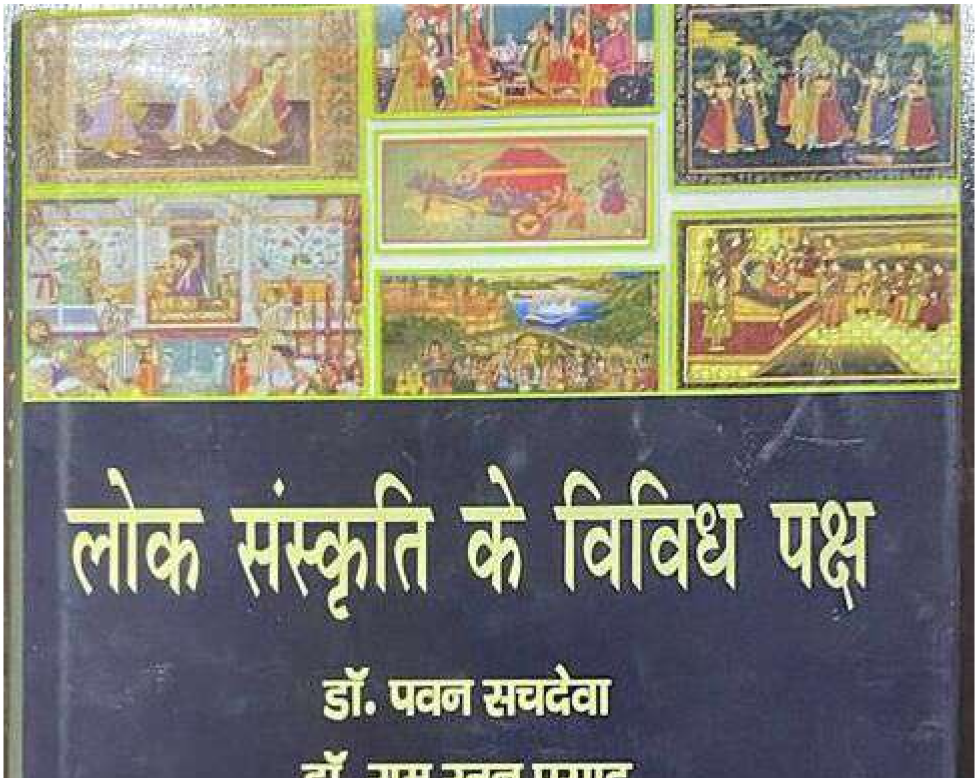
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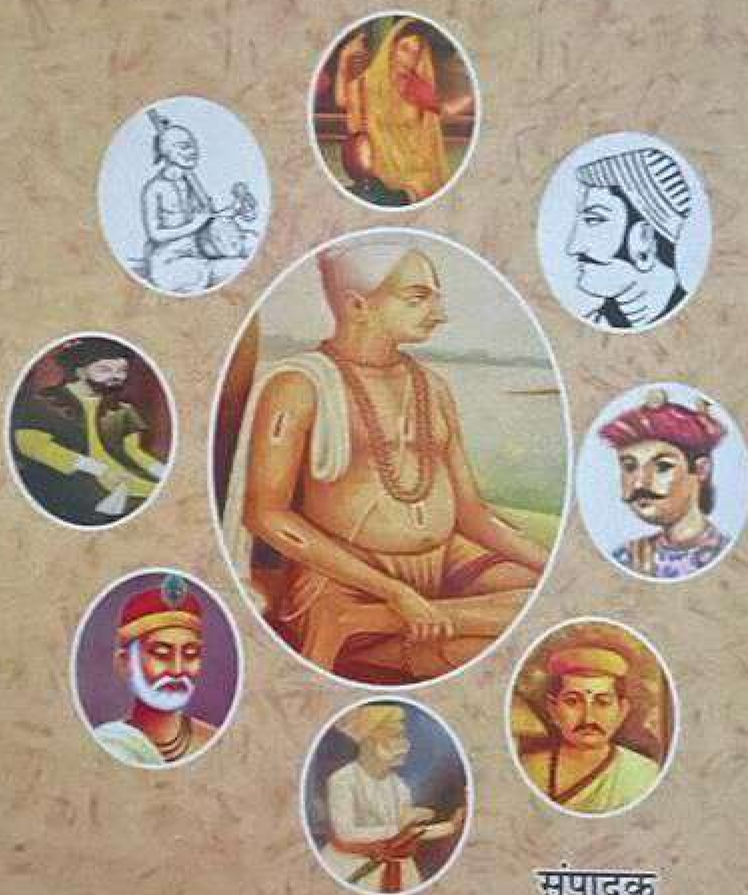
लोक में नानक

– डॉ. प्रिया शर्मा,

'लोक' शब्द अत्यंत व्यापक और सम है, यह ब्रह्म की ही तरह अनंत अक्षर और असीम है, जीवन का प्रतीक और जन का पर्याय है। 'लोक' की सीमा केवल ग्राम या साधारण जनता तक ही सीमित नहीं है, बल्कि समस्त चराचर मात्र में 'लोक' की समीचीन अलंकृति ही परम उपादेय और मांगलिक है। 'लोक' मनुष्य के हज़ारों विश्वासों, रीतियों, रिवाज़ों, रूढ़ियों, व्यवहारों, परंपराओं और संकल्पों से बनता है। लोक अनंत है, असीम है। लोक धरती से आसमान तक फैला है। जहां–जहां तक मनुष्य की बुद्धि और कल्पना पहुंचती है, वहां–वहां तक लोक की सीमा मानी जा सकती है। सृष्टि की उत्पत्ति से लगाकर प्रलय तक लोक रहेगा। जब तक मनुष्य रहेगा, तब तक लोक रहेगा। 'लोक' की व्याख्या बहुत विशद है, वह सदैव चिरंतन है। 'लोक' काल का अनुगामी है। काल निरंतर है, नित्य है, लोक भी निरंतर और नित्य है। आदमी की मृत्यु के पूर्व और पश्चात, काल की तरह लोक भी विद्यमान है। काल की गणना संभव है, लोक की गणना नहीं हो सकती लेकिन वह सर्वत्र होता है। काल को खंडों में विभक्त किया जा सकता है, लेकिन लोक को विभाजित करना संभव नहीं। लोक की शक्ति अपार है।[1] लोक शक्ति साहित्य में वेदों से लगाकर उपनिषद्, आरण्यक, पुराण, तांत्रिकी से होते हुए संस्कृत, अपभ्रंश वाङ्गमय में पूरी ताकत के साथ अभिव्यक्त हुई। गुप्त, राजपूत और भक्तिकाल के भारतीय समाज की कला साहित्य संस्कृति में लोक शक्ति संपूर्ण ओजस्विता के साथ प्रकट हुई। मुगलकाल में लोक शक्ति शिवाजी,

मध्यकालीन कविता

अवधारणा और स्वरूप



संपादक

डॉ. प्रीति सिंह

नितेश उपाध्याय

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अनुक्रम

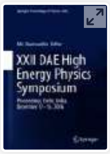
10.

गुरुनानक के काव्य में धर्म और आचरण की प्रासंगिकता

डॉ. प्रिया शर्मा

गुरुनानक देव सिक्ख धर्म के जन्मदाता थे। उन्होंने आदर्श जीवन के जो सिद्धांत मनुष्य को दिए, उन्हें हम सिक्ख धर्म के नाम से जानते हैं। यह धर्म सबसे अधिक बल ईश्वर की भक्ति अर्थात् नाम–स्मरण पर देता है और मनुष्य को निरर्थक विश्वासों तथा कर्मकाण्डों का त्याग कर उच्च, निर्मल, धार्मिक तथा सदाचारिक गुण धारण करके आचरण को पवित्र रखने की शिक्षा देता है। इसका कार्य–क्षेत्र यहीं तक सीमित नहीं बल्कि इसका मनोरथ तो मनुष्य को जीवन के सभी क्षेत्रों धार्मिक, सामाजिक, आर्थिक तथा राजनीतिक आदि में उचित मार्गदर्शन करवाना है। अपने इस मौलिक तथा स्वतंत्र धर्म को प्रचारित करने के लिए गुरुनानक जी को उस समय के प्रचलित धर्मों का विरोध भी सहना पड़ा, क्योंकि उन्होंने उनके अनुचित कार्यों का बड़े साहस तथा निर्भयता से खंडन किया था। प्रत्येक मनुष्य चाहे वह किसी भी धर्म को मानने वाला क्यों न हो, उसे उस धर्म की असलियत का भी अहसास कराया। ऐसे गुरुनानक देव का धर्म मज़हबों, सम्प्रदायों से ऊपर उठकर मानवतावादी भावना तक आ पहुँचता है।

वास्तव में धर्म की कसौटी मनुष्य ही है। मनुष्यों की विशिष्टता दिखाने वाली यदि कोई वस्तु है तो वह धर्म है। 'धर्म वह है, जो व्यक्ति और समाज को धारण करे, उसका पोषण और संवर्धन करे। गर्नारी महतो ने धर्म को कर्म का पर्याय माना है– 'वह कर्म जिसका संपादन, किसी संबंध या गुणविशेष के विचार से उचित और आवश्यक हो, धर्म है।' औचित्य की दृष्टि से धर्म सार्वकालिक, सार्वदेशिक और सार्वभौमिक है। समाज में रहते हुए व्यक्ति जिस धर्म का पालन करता है, वही मानव धर्म है। धर्म व्यक्ति को दूसरों पर श्रद्धा एवं विश्वास करना सिखाता है। हजारी प्रसाद द्विवेदी 'धर्म को अन्य व्यक्ति में श्रद्धा उद्रिक्त करने वाला' मानते हैं। धर्म मानव का उदार कर्तव्य है। 'यह कर्तव्य मानवीय संबंधों और परिस्थितियों के विविध रूपों में चरितार्थ होता है। धर्म



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Probing *CP* Violation in Neutrino Oscillation Experiments and Leptonic Unitarity Quadrangle

Surender Verma, Shankita Bhardwaj, B. C. Chauhan & Gazal Sharma

Conference paper | First Online: 24 May 2018

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Abstract

We have constructed Leptonic Unitarity Quadrangle (LUQ) in order to investigate *CP* violation in the leptonic sector. We have obtained, probability distribution plots for independent geometric parameters of LUQ consistent with the current experimental data on neutrino mixing parameters. We have, also, obtained *CP* asymmetry $P_{\nu_f \rightarrow \nu_{f'}} - P_{\bar{\nu}_f \rightarrow \bar{\nu}_{f'}}$ for both long baseline (LBL) and short baseline (SBL) experiments, in term of independent parameters of the LUQ.

Keywords

Neutrinos oscillation **Non-unitarity** ***CP* Violation** **Neutrino mixing**

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CONTENT

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EFFECT OF INTERNAL HEAT SOURCE ON THE ONSET OF CONVECTION IN A LAYER OF OLDROYDIAN VISCO-ELASTIC NANOFLUID IN A POROUS MEDIUM

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ABSTRACT:
Analytical investigation for study of the thermal instability of Oldroydian visco-elastic nanofluid in a porous medium with realistic boundary condition is presented. For porous medium the Brinkman model is considered. The used model incorporates the effect of Brownian diffusion and thermophoresis. The eigen value problem is solved by employing the Galerkin weighted residuals method. The influence of the Lewis number, Brinkman-Darcy number, nanoparticle Rayleigh number, modified diffusivity ratio and porosity parameter on the stationary convection studied analytically and found that Lewis number, modified diffusivity ratio and nanoparticles Rayleigh number destabilizes while Brinkman-Darcy number and porosity parameter stabilize the stationary convection.

1. INTRODUCTION

Thermal instability of nanofluids in a porous medium is an important phenomenon because of its importance in many fields of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. The natural convection for flow in a porous medium based upon Buongiorno's model [Buongiorno (2006)] was studied by Nield and Kuznetsov (2009) using the Darcy model. The extension to the Brinkman model was made by Kuznetsov and Nield (2010, 2011) and Chand and Rana (2012a). In studying these convective instability problems, the volume fraction of nanoparticles was prescribed at the boundaries. Nield and Kuznetsov (2014) suggested that the value of the temperature can be imposed on the boundaries, but the nanoparticle fraction adjusts so that the nanoparticle flux is zero on the boundaries. With these new boundary conditions, Chand and Rana (2014, 2015a, 2015b) and Yadav et al. (2016) studied thermal instability in a layer of nanofluid based upon Buongiorno's model. The above literature deals with the study of nanofluids as Newtonian fluids. The onset of convection in a horizontal layer of nanofluid as Newtonian fluids uniformly heated from below (Bénard convection) has been extensively investigated but a little attention has been made to study the thermal convection of non-Newtonian fluids. With the growing importance of non-Newtonian fluids in technology and industries, the investigations of such fluids are desirable. In the category of non- Newtonian fluids Oldroydian nanofluids have distinct features and well represented by the Oldroydian constitutive model as given by Oldroyd (1958). The Oldroydian constitutive model is adopted widely to examine the influence of elasticity on thermal convective instability. Thermal convection in a layer of Oldroydian visco-elastic fluid saturated by Brinkman–

Darcy porous medium was investigated by Chand (2015) and found that Brinkman stabilize the fluid layer. Thermal instability problems in non-Newtonian nanofluid problems were investigated theoretically by Sheu (2011), Shivakumara et al. (2015), Rana and Chand (2015), Chand and Rana (2012b) by taking different type of non-Newtonian fluid as base fluid.

Due to importance of Oldroydian visco-elastic nanofluids in porous medium an attempt has been made to study the thermal instability of a horizontal layer of Oldroydian visco-elastic nanofluids for more realistic boundary conditions in Brinkman porous medium.

2. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of Oldroydian visco-elastic nanofluid of thickness ‘d’ bounded by horizontal boundaries z = 0 and z = d. Fluid layer is acted upon by a gravity force **g**(0,0,-g) and is heated from below in such a way that horizontal boundaries z = 0 and z = d respectively maintained at a uniform temperature T_0 and T_1 ($T_0 > T_1$) as shown is Fig.1. The normal component of the nanoparticles flux has to vanish at an impermeable boundaries and the reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively.

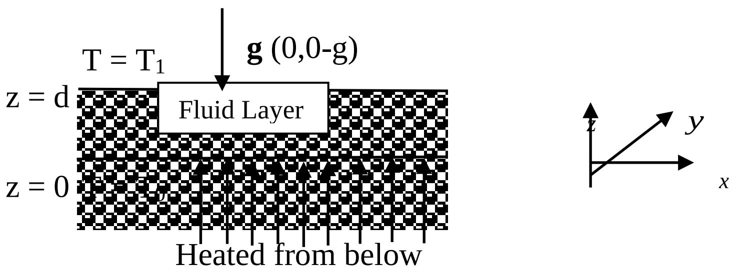

Fig.1 Physical configuration of the problem

The equation of continuity and motion for Oldroydian visco-elastic nanofluid in Brinkman porous medium under the Boussinesq approximation are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{d\mathbf{q}}{dt} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p + \left(\varphi\rho_p + (1-\varphi)\{\rho_f(1-\alpha(T-T_0))\}\right)\mathbf{g}\right) - \left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\tilde{\mu}\nabla^2 - \frac{\mu}{k_1}\right)\mathbf{q}, \tag{2}$$

where **q(u, v, w)** is the Darcy velocity vector, p is the hydrostatic pressure, μ is viscosity, $\tilde{\mu}$ effective viscosity, α is the coefficient of thermal expansion, λ the relaxation time, λ_0 the retardation time, T the temperature of the nanofluid, φ is the volume fraction of the nanoparticles, ρ_p density of nanoparticles, ρ_f density of base fluid and $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)$ is stands for convection derivative.

The equation of energy for Oldroydian visco-elastic nanofluid in porous medium is

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f\, \mathbf{q}\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p\left(D_B \nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right) + Q_0, \tag{3}$$

where $(\rho c)_m$ is effective heat capacity of fluid, $(\rho c)_p$ is heat capacity of nanoparticles, Q_0 is the overall uniformly distributed effective volumetric internal heat source, T_1 is the temperature of the fluid layer at z = d and k_m is effective thermal conductivity of the porous medium.

The continuity equation for the nanoparticles is

$$\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_1}\nabla^2 T\,, \tag{4}$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation and D_T is the thermoporetic diffusion coefficient of the nanoparticles.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus boundary conditions [Chandrasekhar (1981), Nield and Kuznetsov (2014)] are

$$w=0, T=T_0,\ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_0}\frac{\partial T}{\partial z}=0 \text{ at } z=0 \text{ and } w=0, T=T_1,\ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \text{ at } z=d. \tag{5}$$

Introducing non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right), v'(u',v',w')=v\left(\frac{u,v,w}{\kappa}\right)d,\ t'=\frac{t\kappa}{\sigma d^2},\ p'=\frac{pk_1}{\mu\kappa},\ \varphi'=\frac{(\varphi-\varphi_0)}{\varphi_0},$$

$T'=\dfrac{(T-T_1)}{(T_0-T_1)}$, where $\kappa=\dfrac{k_m}{(\rho c_p)_f}$ is thermal diffusivity of the fluid.

Equations (1) - (5) in non-dimensional form can be written as

$$\nabla'\cdot\mathbf{q}'=0, \tag{6}$$

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t'}\right)\frac{\partial\mathbf{q}'}{\partial t'}=\left(1+F\frac{\partial}{\partial t'}\right)\left(-\nabla'p'-Rm\hat{e}_z+RaT'\hat{e}_z-Rn\varphi'\hat{e}_z\right)-\left(1+F_0\frac{\partial}{\partial t'}\right)\left(\tilde{D}a\nabla'^2\mathbf{q}'-\mathbf{q}'\right),$$

(7)

$$\frac{\partial T'}{\partial t'}+\mathbf{q}'\cdot\nabla'T'=\nabla'^2T'+\frac{N_B}{Le}\nabla'\varphi'\cdot\nabla'T'+\frac{N_AN_B}{Le}\nabla'T'\cdot\nabla'T'+Hs\,, \tag{8}$$

$$\frac{1}{\sigma}\frac{\partial\varphi'}{\partial t'}+\frac{1}{\varepsilon}\mathbf{q}'\cdot\nabla'\varphi'=\frac{1}{Le}\nabla'^2\varphi'+\frac{N_A}{Le}\nabla'^2T', \tag{9}$$

Here the non-dimensional parameters are given as

$Le=\dfrac{\kappa}{D_B}$ is the Lewis number,

$Pr=\dfrac{\mu}{\rho\kappa}$ is the Prandtl number,

$Da=\dfrac{k_1}{d^2}$ is the Darcy number,

$\tilde{D}a=\dfrac{\tilde{\mu}k_1}{\mu d^2}$ is the Brinkman-Darcy number,

$\mathrm{Hs} = \dfrac{Q_0 d^2}{k_m (T_0 - T_1)}$ is the dimensionless constant of heat source strength,

$\mathrm{Va} = \dfrac{\varepsilon \Pr}{\mathrm{Da}}$ is the Prandtl-Darcy number,

$F = \dfrac{\lambda\kappa}{d^2}$ is the stress relaxation parameter,

$F_0 = \dfrac{\lambda_0 \kappa}{d^2}$ is the strain retardation parameter,

$\mathrm{Ra} = \dfrac{\rho\alpha d k_1 g (T_0 - T_1)}{\mu\kappa}$ is the Rayleigh Darcy number,

$\mathrm{Rm} = \dfrac{(\rho_p \varphi_0 + \rho(1 - \varphi_0)) g d k_1}{\mu\kappa}$ is the density Rayleigh Darcy number,

$\mathrm{Rn} = \dfrac{(\rho_p - \rho)\varphi_0 g d k_1}{\mu\kappa}$ is the nanoparticles Rayleigh Darcy number,

$N_A = \dfrac{D_T (T_0 - T_1)}{D_B T_1 \varphi_0}$ is the modified diffusivity ratio,

$N_B = \dfrac{(\rho c)_p \varphi_0}{(\rho c)_f}$ is the modified particle-density increment,

$\mathbf{e}_z$ is the unit vector along z-axis.

In spirit of Oberbeck-Boussinesq approximation, equation (7) has been linearized by the neglect of a term proportional to the product of φ_0 and T. This approximation is valid in the case of small temperature gradients in a dilute suspension of nanoparticles.

The dimensionless boundary conditions are

$$w' = 0, \quad T' = 1, \quad \frac{\partial \varphi'}{\partial z'} + N_A \frac{\partial T'}{\partial z'} = 0 \ \text{at} \quad z' = 0 \ \text{and} \ w' = 0,\ T' = 0,\ \frac{\partial \varphi'}{\partial z'} + N_A \frac{\partial T'}{\partial z'} = 0 \ \text{at} \ z' = 1. \tag{10}$$

2.3 THE BASIC STATE AND ITS SOLUTIONS

The basic state was assumed to be quiescent and is given by

$$\mathbf{q}'(u', v', w') = 0, \quad p' = p_b(z),\ T' = T_b(z),\ \varphi' = \varphi_b(z)\,, \tag{11}$$

Equations (6) – (9) reduce to

$$0 = -\frac{dp_b}{dz'} - \mathrm{Rm} + \mathrm{Ra} T_b - \mathrm{Rn}\varphi_b, \tag{12}$$

$$\frac{d^2 T_b}{dz'^2} + \frac{N_B}{\mathrm{Le}} \frac{d\varphi_b}{dz'} \frac{dT_b}{dz'} + \frac{N_A N_B}{\mathrm{Le}} \left(\frac{dT_b}{dz'} \right)^2 + \mathrm{Hs} = 0, \tag{13}$$

$$\frac{d^2 \varphi_b}{dz'^2} + N_A \frac{d^2 T_b}{dz'^2} = 0\,. \tag{14}$$

Using boundary conditions in (10), equation (14) gives

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$$\varphi_b = -N_A T_b + \varphi_0 + N_A. \tag{15}$$

On substituting the value of the φ_b from equation (15) in equation (13), we get

$$\frac{d^2 T_b}{dz^2} + \frac{(1 - N_A)N_B}{Le}\frac{dT_b}{dz^2} + Hs = 0. \tag{16}$$

On integrating equation (16) with respect to z and using boundary conditions (10), we get

$$T_b = \frac{e^{\frac{-(1-N_A)N_B}{Le}z}\left[-N_B(-1+N_A) - LeHs\right] - LeHs(-1+z) + e^{\frac{-(1-N_A)N_B}{Le}\left[(-1+N_A)N_B + LeHsz\right]}}{-1 + e^{\frac{-(1-N_A)N_B}{Le}(-1+N_A)N_B}}. \tag{17}$$

$$\varphi_b = z + \frac{N_A\left[(-1+N_A)N_B - LeHs\right]\left(-1 + e^{\frac{-(1-N_A)N_B}{Le}z} + z - ze^{\frac{-(1-N_A)N_B}{Le}}\right)}{-1 + e^{\frac{-(1-N_A)N_B}{Le}(-1+N_A)N_B}}. \tag{18}$$

According to Buongiorno (2006), for most nanofluid investigated so far Le is large, is of order 10^2-10^3, while N_A is no greater than about 10, N_B is of order 10^{-4} to 10^{-2}. Then, the exponents $\frac{-(1-N_A)N_B}{Le}$ in equations (17)–(18) are very small. By expanding the exponential function into the power series and retaining up to the first order is negligible and so to a good approximation for the solution of basic state is given by

$$T_b = \frac{1}{2}\left(2 - 2z + Hsz - Hsz^2\right)$$

and

$$\varphi_b = \varphi_0 + N_A\left(z - \frac{Hsz}{2} + \frac{Hsz^2}{2}\right).$$

In the absence of internal heat generation i.e. Hs = 0, then basic flow distributions for temperature and nanoparticles volume fraction are:

$T_b = 1 - z$ and

$$\varphi_b = \varphi_0 + N_A z. \tag{19}$$

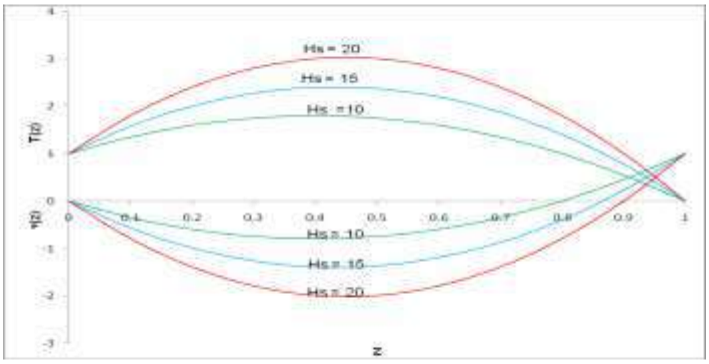

Fig.10.2 Basic state temperature and basic state nanoparticles volumetric distributions for different values of internal heat source strength parameter Hs

To see the effect of internal heat source strength Hs on the criterion for the onset of thermal convection in nanofluids, the distributions of dimensionless basic temperature and basic nanoparticles volumetric fraction are drawn in the Fig. 10.2 for different values of Hs. The discrete values of Hs are purposely taken to see the behavior of both distributions. This plot shows the behavior of basic temperature distribution which is parabolic in positive direction and same behavior in negative direction for the basic nanoparticles distribution as internal heat source strength Hs increases. That is increase in the internal heat source strength Hs amounts to increase in energy supply to the system. This gives large deviations in these distributions which in turn improve the disturbances in the layer and thus system is more unstable.

2.3. PERTURBATION EQUATIONS

Let the initial basic state described by (11) is slightly perturbed so that perturbed state is given by

$$\mathbf{q}'(u',v',w') = 0 + \mathbf{q}''(u'',v'',w''),\ T' = T_b + T'',\ \varphi' = \varphi_b + \varphi'',\ p' = p_b + p'', \tag{20}$$

where $T_b = 1 - z'$, $\varphi_b = \varphi_0 + N_A z'$ and (u'',v'',w''), T'', φ'' and p'' respectively the perturbations in initial velocity, temperature, volume fraction of the nanoparticles and pressure.

By substituting (20) in equations (6) – (9) and linearize by neglecting the product of the prime quantities, we obtained following equations

$$\nabla\cdot\mathbf{q} = 0, \tag{21}$$

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t}\right)\frac{\partial\mathbf{q}}{\partial t} = \left(1+F\frac{\partial}{\partial t}\right)\left(-\nabla p + RaT\hat{e}_z - Rn\varphi\hat{e}_z\right) - \left(1+F_0\frac{\partial}{\partial t}\right)\left(\tilde{D}a\nabla^2\mathbf{q} - \mathbf{q}\right), \tag{22}$$

$$\begin{aligned}\frac{\partial T}{\partial t} - w(-2+Hs-2Hsz) &= \nabla^2 T + \frac{2N_A N_B}{Le}(-2+Hs-2Hsz)\frac{\partial T}{\partial z}\\ &+ \frac{N_B N_A}{Le}\left(1-\frac{Hs}{2}+2Hsz\right)\frac{\partial T}{\partial z} + \frac{2N_B}{Le}(-2+Hs-2Hsz)\frac{\partial\varphi}{\partial z},\end{aligned} \tag{23}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}wN_A = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T. \tag{24}$$

Boundary conditions for the infinitesimal perturbation are given by

$$w = 0, T = 0,\ \frac{\partial\varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 0,1. \tag{25}$$

[Dashes (") have been suppressed for convenience]

Applying Curl operator twice to the equation (22) under the assumption of linear theory, the resulting equations are given by

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 w = \left(1+F\frac{\partial}{\partial t}\right)\left(Ra\nabla_H^2 T - Rn\nabla_H^2\varphi\right) + \left(1+F_0\frac{\partial}{\partial t}\right)\left(\tilde{D}a\nabla^4 w - \nabla^2 w\right), \tag{26}$$

where ∇_H^2 is two-dimensional Laplacian operator.

4. NORMAL MODES ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)]\exp\left(ik_x x + ik_y y + nt\right), \tag{27}$$

where k_x, k_y are wave numbers in x and y direction and n is growth rate of disturbances.
Using equation (27), equations (26), (23) and (24) become

$$\left[\left(\frac{n(1+Fn)}{Va}\left(D^2-a^2\right)-\left(1+F_0 n\right)\left(\tilde{D}a\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right)\right)D^2\right]W-(1+nF)\left(a^2 Ra\Theta - a^2 Rn\Phi\right)=0, \tag{28}$$

$$-\frac{1}{2}(-2+Hs-2Hsz)W+\begin{pmatrix} D^2-a^2-n+\dfrac{N_A N_B}{Le}(-2+Hs-2Hsz)D \\ -\dfrac{N_A N_B}{Le}\left(1-\dfrac{Hs}{2}+Hsz\right)D \end{pmatrix}\Theta \tag{29}$$

$$-\frac{N_B}{2Le}(-2+Hs-2Hsz)D\Phi = 0,$$

$$\frac{1}{\varepsilon}N_A W-\frac{N_A}{Le}\left(D^2-a^2\right)\Theta-\left(\frac{1}{Le}\left(D^2-a^2\right)-\frac{n}{\sigma}\right)\Phi=0, \tag{30}$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the dimensionless resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W = 0, D^2W = 0, \Theta = 0, D\Phi + N_A D\Theta = 0 \text{ at } z = 0,1. \tag{31}$$

5. METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (28) – (30) with boundary conditions (31). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{N} A_p W_p,\ \Theta = \sum_{p=1}^{N} B_p \Theta_p,\ \Phi = \sum_{p=1}^{N} C_p \Phi_p, \tag{32}$$

where $W_p = \Theta_p = \sin p\pi z, \Phi_p = -N_A \sin p\pi z$, A_p, B_p and C_p are unknown coefficients, p = 1, 2, 3, ..., N and the base functions W_p, Θ_p, and Φ_p satisfying the boundary conditions (31). Using expression for W, Θ and Φ in equations (28) – (30) and multiplying the first equation by W_p the second equation by Θ_p and third equation by Φ_p and then integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations with 3N unknown A_p, B_p and C_p; p = 1, 2, 3, ..., N. For existing of nontrivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number Ra.

6. LINEAR STABILITY ANALYSIS

For the present formulation, we have considered the which system of equations (28) – (30) together with the boundary conditions (31) constitute a linear eigen value problem with variable coefficient for the growth rate of disturbance n of the system. Substituting equation (32 into the system of equations (28) – (30) and multiplying the first equation by

W_p the second equation by Θ_p and third equation by Φ_p and then integrating in the limits from zero to unity and performing some integration by parts, one obtains the following matrix equation

$$\begin{bmatrix} \frac{(1+nF_0)\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)}{(1+nF)}+\frac{n}{Va}(\pi^2+a^2) & -a^2Ra & -a^2N_ARn \\ 1-\frac{Hs}{2}+Hsz & -(\pi^2+a^2+n) & 0 \\ \frac{N_A}{\varepsilon} & \frac{N_A}{Le}(\pi^2+a^2) & -N_A\left(\frac{1}{Le}(\pi^2+a^2)+\frac{n}{\sigma}\right) \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The non-trivial solution of the above matrix requires that

$$Ra=\frac{1}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{(1+nF_0)}{(1+nF)}\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)+\frac{n}{Va}(\pi^2+a^2)\right)(\pi^2+a^2+n)$$
$$-\frac{\frac{(\pi^2+a^2)}{Le}+\frac{(\pi^2+a^2+n)}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}}{\frac{(\pi^2+a^2)}{Le}+\frac{n}{\sigma}}N_ARn. \tag{26}$$

The growth rate n is in general a complex quantity such that $n = \omega_r + i\omega$, the system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability, the real part of n is zero, thus on putting $n = i\omega$, (where ω is real and is a dimensionless frequency) in equation (26), we have

$$Ra=\Delta_1+i\omega\Delta_2, \tag{27}$$

where

$$\Delta_1=\frac{(\pi^2+a^2)}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{(1+\omega^2FF_0)\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)-\omega^2\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)(F_0-F)-\frac{(\pi^2+a^2)}{Va}}{1+\omega^2F^2}\right)$$
$$-\frac{\left(\frac{(\pi^2+a^2)}{Le}\right)^2\left(1+\frac{Le}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)+\frac{\omega^2}{\varepsilon\sigma\left(1-\frac{Hs}{2}+Hsz\right)}}{\left(\frac{(\pi^2+a^2)}{Le}\right)^2+\left(\frac{\omega}{\sigma}\right)^2}N_ARn \tag{28}$$

and

$$\Delta_2 = \frac{\left(\tilde{D}a\left(\pi^2+a^2\right)^2+\left(\pi^2+a^2\right)\right)}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{\left(1+\omega^2 FF_0\right)+\left(\pi^2+a^2\right)\left(F_0-F\right)}{1+\omega^2F^2}\right)+\frac{\left(\pi^2+a^2\right)^2}{Va}$$
$$-\frac{\frac{1}{Le\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}-\frac{1}{\sigma}\left(\frac{\left(\pi^2+a^2\right)}{Le}+\frac{\left(\pi^2+a^2\right)}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)}{\left(\frac{\left(\pi^2+a^2\right)}{Le}\right)^2+\left(\frac{\omega}{\sigma}\right)^2}N_A Rn \tag{29}$$

Since Ra is a physical quantity, so it must be real. Hence, it follows from the equation (27) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset).

7. STATIONARY CONVECTION

For the case of stationary convection $\omega = 0$, equation (27) reduces to

$$(Ra)_s = \frac{\tilde{D}a\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}-\left(1+\frac{Le}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)N_A Rn. \tag{30}$$

We find that for the stationary convection, the stress relaxation time parameter F and strain retardation time parameter F_0 vanishes with n and the Oldroydian visco-elastic nanofluid behaves like an ordinary Newtonian nanofluid.

It is observed that stationary Rayleigh number Ra is function of the Lewis number, the modified diffusivity ratio, Brinkman-Darcy number, nanoparticles Rayleigh and porosity parameter but independent of the stress relaxation time parameter F and strain retardation time parameter F_0, Vadasz number Va and modified particle- density increment N_B. Thus Oldroydian nanofluid behaves like an ordinary Newtonian nanofluid and instability is purely a phenomenon due to buoyancy coupled with the conservation of nanoparticles.

$$(Ra)_s = \frac{\tilde{D}a\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2}{a^2}-\left(1+\frac{Le}{\varepsilon}\right)N_A Rn. \tag{30}$$

If $\tilde{D}a = 0$ then the critical value of the wave number is attained at $a_c = \pi$ and corresponding critical Rayleigh number given by

$$(Ra)_c = 4\pi^2-\left(1+\frac{Le}{\varepsilon}\right)N_A Rn\,. \tag{31}$$

In the absence of nanoparticles ($Rn = Le = N_A = 0$) i.e. for ordinary Newtonian fluid, one recovers the well-known results that the critical Rayleigh-Darcy number is equal to $(Ra)_c = 4\pi^2$.

This is good agreement of the result obtained by Nield and Kuznetsov (2009).

If $\tilde{D}a$ is very large as compared to the unity, then critical value of wave number is attained at $a = \pi/\sqrt{2}$, critical value of the Rayleigh Darcy number is

$$(\mathrm{Ra})_c = \frac{27\pi^4}{4} - \left(1 + \frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{N_A Rn}\,. \tag{32}$$

In the absence of nanoparticles (Rn = Le = N_A = 0), one recovers the well- known results that the critical Rayleigh-Darcy number is equal to $(\mathrm{Ra})_c = \frac{27\pi^4}{4}$.

8. RESULTS AND DISCUSSION

To study the effect of Lewis number Le, Brinkman-Darcy number $\tilde{\mathrm{D}}\mathrm{a}$, nanoparticles Rayleigh Rn, modified diffusivity ratio N_A and porosity parameter ε on stationary convection, we examine the behavior of $\frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Le}}, \frac{\partial(\mathrm{Ra})_s}{\partial \tilde{\mathrm{D}}\mathrm{a}}, \frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Rn}}, \frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{N_A}}$ and $\frac{\partial(\mathrm{Ra})_s}{\partial \varepsilon}$ analytically.

$$(\mathrm{Ra})_s = \frac{\tilde{\mathrm{D}}\mathrm{a}\left(\pi^2 + a^2\right)^3 + \left(\pi^2 + a^2\right)^2}{a^2\left(1 - \frac{\mathrm{Hs}}{2} + \mathrm{Hsz}\right)} - \left(1 + \frac{\mathrm{Le}}{\varepsilon\left(1 - \frac{\mathrm{Hs}}{2} + \mathrm{Hsz}\right)}\right)\mathrm{N_A Rn}.$$

From equation (30), we have

(i) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{Le}} < 0$,

(ii) $\frac{(\partial \mathrm{Ra})_s}{\partial \tilde{\mathrm{D}}\mathrm{a}} > 0$

(iii) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{Rn}} < 0$,

(iv) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{N_A}} < 0$,

(v) $\frac{(\partial \mathrm{Ra})_s}{\partial \varepsilon} > 0$.

These inequalities shows that Lewis number Le, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn have destabilizing effect while Brinkman-Darcy number $\tilde{\mathrm{D}}\mathrm{a}$ and porosity parameter ε have stabilizing effect on the stationary convection.

9. CONCLUSIONS

Thermal instability in a horizontal layer of Oldroydian visco- elastic nanofluid in a porous medium is investigated theoretically. Brinkman model is used for porous medium. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries and the eigen value problem is solved using the Galerkin residual method. The results have been obtained analytically.

The main conclusions derived from the present analysis are as follows:

(i) The instability purely phenomenon due to buoyancy coupled with the conservation of nanoparticle and is independent of the contribution of Brownian motion and thermophoresis.

(ii) For stationary convection Oldroydian visco- elastic nanofluid behaves like an ordinary Newtonian nanofluid.

(iii) Lewis number Le, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn destabilizes while Brinkman-Darcy number $\tilde{D}a$ and porosity parameter ε stabilizes the stationary convection.

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Propagation of Rayleigh Waves in Semi-infinite Thermoelastic Porous Material under an Inviscid Liquid Half-space

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Abstract
In the present paper, an attempt is made to investigate the propagation of Rayleigh waves in a semi-infinite homogeneous isotropic porous generalized thermoelastic medium underlying a homogeneous inviscid liquid half-space in the context of Lord Shulman theory. Helmholtz decomposition technique is employed to simplify the problem. The governing equations are derived and solved to show the existence of four waves in a generalized thermoelastic porous solid half-space and one wave in the liquid half-space. By invoking appropriate boundary conditions at the interface, secular equations are derived for stress-free isothermal $(H \to 0)$ and stress-free insulated $(H \to \infty)$ porous thermoelastic medium. The secular equation is also obtained in the context of uncoupled thermoelasticity. The frequency equation is found to agree with the standard result. Some special cases are also discussed in the paper. The study may find various applications in the field of acoustics and seismology.

INTRODUCTION

The theory of thermoelasticity has become very important due to its practical and theoretical relevance. Beneath the earth's surface are lying many valuable materials like oils, minerals, water, hydrocarbons, etc. To know the exact nature of the layers under the Earth's surface, we need to consider many appropriate models for the theoretical investigation. These models help in the exploration of valuable materials as well as provides better information regarding the internal composition of the earth. Surface waves provide a lot of information about the crust of the earth and dispersion analysis of surface waves is concerning with the phase velocity and wave number.

Rayleigh waves are a type of surface acoustic wave that travels along the surface of solids. They include both longitudinal and transverse motions that decrease exponentially in amplitude as the distance from the surface increases. In isotropic solids, these waves cause the surface particles to move in ellipses. Surfaces wave propagation in an elastic medium is of great practical importance. To detect deposits of oil and other valuable organic and inorganic material under the earth's surface, the most economic technique is wave propagation as it is a difficult and tedious process to drill randomly.

For the present paper, the previous works of Cowin and Nunziato [4] is found helpful. They developed a theory of linear elastic material with voids and discussed several applications like response to homogeneous deformation. Puri and Cowin [8] studied the behavior of plane harmonic waves in linear elastic material with voids. They derived two dilatational waves in the paper. Both waves are found to attenuate in the direction of their propagation, to be dissipative and dispersive. The nature of surface waves in an elastic half-space with voids is studied by Chandrasekharaiah [3]. Iesan [6] gave a theory of thermoelastic material with

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voids. He studied acceleration waves and some problems of equilibrium. Rayleigh surface waves problem in linear thermoviscoelasticity with voids is studied by Bucur [2]. In his paper the dissipative character of the porous thermoviscoelastic model was taken into consideration and dampness in time wave solutions was studied.

Biot [1] studied the interaction of Stoneley and Rayleigh waves in the ocean bottom. They developed a theory for the two-dimensional unattenuated waves. Kumar and Kumar [7] studied the wave motion in transversely isotropic elastic material with voids under an inviscid liquid layer. The paper found useful in studying geophysical problems. Thermoelastic Lamb waves in a homogeneous isotropic plate bordered with layers of inviscid liquid discussed by Sharma and Pathania [9]. Also, Rayleigh waves in rotating thermoelastic solids with voids were studied by Sharma and Kaur [10]. They observed a significant effect of voids, thermal relaxation, and rotation in certain ranges of frequency.

In the present investigation, we consider a semi-infinite isotropic thermoelastic material with voids in contact with inviscid liquid half-space which helps in exploring geophysical problems like explosions and earthquakes. The secular equations in mathematical form for Rayleigh wave propagation are derived for stress-free insulated and isothermal boundaries, for the thermoelastic solid half-space, elastic half-space with voids in contact with inviscid liquid.

FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, thermoelastic solid half-space with voids lying under a homogeneous inviscid liquid half-space. The surface is assumed to be stress-free, thermally insulated boundaries with no fractional change across the boundary. The z-axis is taken vertically downwards and the direction of the propagation of the wave is along the x-axis so that all the particles on a line parallel to the y-axis are equally displaced. Therefore all the field quantities will be independent of the y-axis. Further, the disturbance is assumed to be confined to the neighborhood of the free surface $z = 0$ and hence vanishes as $z \to \infty$.

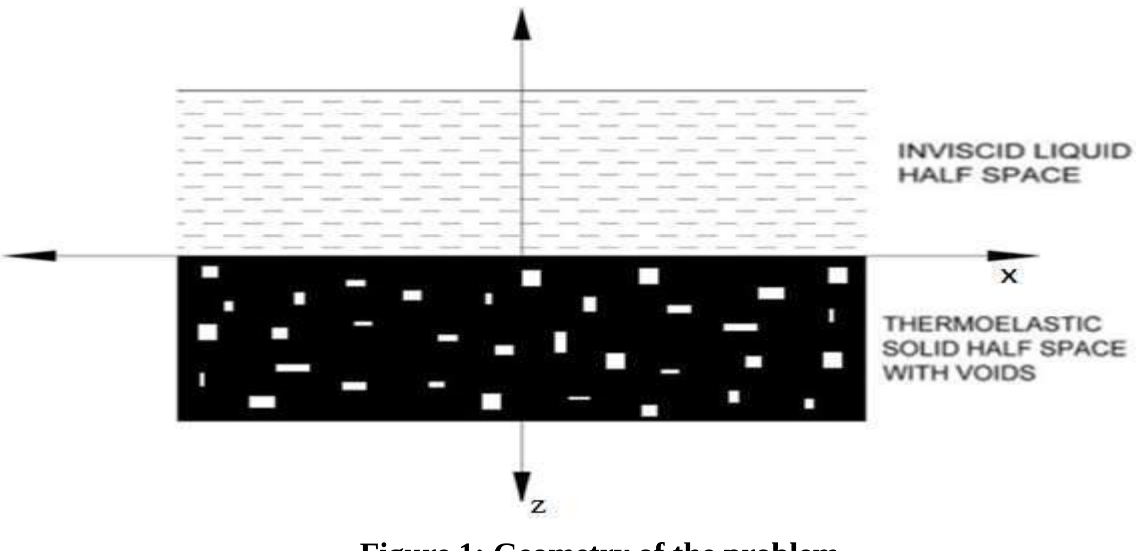

Figure 1: Geometry of the problem

The basic governing equations for stress-free thermoelastic solid with voids in contact with inviscid liquid half-space in the absence of body forces, equilibrated forces and heat sources following Iesan [6] and Sharma and Pathania [9] are given as:

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla.\vec{u} - \beta\nabla T + b\nabla\phi = \rho\ddot{\vec{u}} \tag{1}$$

$$K\nabla^2 T - \rho C_e\left(\dot{T} + t_0\ddot{T}\right) = \beta T_0\nabla.\left(\dot{\vec{u}} + t_0\ddot{\vec{u}}\right) + mT_0\left(\dot{\phi} + t_0\ddot{\phi}\right) \tag{2}$$

$$\alpha\nabla^2\phi - \xi_1\phi - \xi_2\dot{\phi} - b\nabla.\vec{u} + mT = \rho\chi\ddot{\phi} \tag{3}$$

$$\nabla\nabla.\vec{u}_L = \frac{\rho_L}{\lambda_L}\ddot{\vec{u}}_L \tag{4}$$

where $\vec{u}(x,z,t) = (u,0,w)$ and $\vec{u}_L = (u_L, 0, w_L)$ are the displacement vectors in the solid half-space and liquid half-space respectively, $\phi(x,z,t)$ is the volume fractional change, $T(x,z,t)$ is the temperature change in the medium, λ and μ are Lame's parameter, ρ and ρ_L are densities for solid and liquid half-space, C_e is the specific heat at constant strain, K is the thermal conductivity, $\alpha\ b\ m\ \xi_1\ \xi_2\ \chi$ are material parameters due to the presence of voids, t_0 is thermal relaxation time and $\beta = \left(3\lambda + 2\mu\right)\alpha_T$, α_T being coefficient of linear thermal expansion. Defining the dimensionless quantities as

$$x' = \frac{\omega^* x}{c_1},\ z' = \frac{\omega^* z}{c_1},\ u' = \frac{\rho\omega^* c_1 u}{\beta T_0},\ w' = \frac{\rho\omega^* c_1 w}{\beta T_0},\ \ \phi' = \frac{\omega^{*^2}\chi}{c_1^2}\phi,\ u'_L = \frac{\rho\omega^* c_1 u_L}{\beta T_0},$$

$$w'_L = \frac{\rho\omega^* c_1 w_L}{\beta T_0},$$

$$t' = \omega^* t,\ t_0' = \omega^* t_0,\ T' = \frac{T}{T_0}, \sigma_{ij}' = \frac{\sigma_{ij}}{\beta T_0},\ \varepsilon_T = \frac{\beta^2 T_0}{\rho c_e\left(\lambda + 2\mu\right)}, a_1 = \frac{c_1{}^2 b}{\beta T_0\chi\omega^{*^2}},\ a_2 = \frac{\beta T_0\chi b}{\alpha\rho c_1{}^2}, a_3 = \frac{\xi_1 c_1{}^2}{\alpha\omega^{*^2}}$$

$$a_4 = \frac{mT_0\chi}{\alpha},\ a_5 = \frac{mc_1{}^4}{K\chi\omega^{*^3}},\ \ \omega^* = \frac{C_e\left(\lambda+2\mu\right)}{K}, \tag{5}$$

Introducing equation (5) and suppressing the primes, the equations (1)-(4) are obtained as:

$$u_{,xx} + \left(1-\delta^2\right)w_{,xz} + \delta^2 u_{,zz} - T_{,x} + a_1\phi_{,x} = \ddot{u} \tag{6}$$

$$\left(1-\delta^2\right)u_{,xz} + \delta^2 w_{,xx} + w_{,zz} - T_{,z} + a_1\phi_{,z} = \ddot{w} \tag{7}$$

$$\left(T_{,xx} + T_{,zz}\right) - \left(\dot{T} + t_0\ddot{T}\right) - \varepsilon_T\left[\left(\dot{u}_{,x} + \dot{w}_{,z}\right) + t_0\left(\ddot{u}_{,x} + \ddot{w}_{,z}\right)\right] - a_5\left(\dot{\phi} + t_0\ddot{\phi}\right) = 0 \tag{8}$$

$$\left(\phi_{,xx} + \phi_{,zz}\right) - a_2\left(u_{,x} + w_{,z}\right) - a_3(\phi + \bar{\xi}\dot{\phi}) + a_4 T = \frac{\ddot{\phi}}{\delta_1^2} \tag{9}$$

$$u_{L,_{xx}} + w_{L,_{xz}} = \frac{1}{\delta_L^2}\ddot{u}_L \tag{10}$$

where $\delta^2 = \dfrac{c_2^2}{c_1^2},\ \delta_1^2 = \dfrac{c_3^2}{c_1^2},\ \bar{\xi} = \dfrac{\xi_2}{\xi_1}\omega^*,\ c_1{}^2 = \dfrac{\left(\lambda+2\mu\right)}{\rho}, c_2{}^2 = \dfrac{\mu}{\rho},\ c_3{}^2 = \dfrac{\alpha}{\rho\chi}, \delta_L^2 = \dfrac{c_L^2}{c_1^2}, c_L^2 = \dfrac{\lambda_L}{\rho_L}.$

Here ω^* is the characteristic frequency of solid half-space, ε_T is the thermoelastic coupling parameter, c_L is the velocity of sound in the liquid, λ_L is the bulk modulus, c_1, c_2, and c_3 are velocities of longitudinal, transverse and volume fractional fields respectively.The coefficient ξ and equilibrated inertia χ must be positive and satisfy the dissipation inequality resulting from the second law of thermodynamics. The comma denotes the spatial derivatives and superposed dot is used for time differentiation.

FORMAL SOLUTION OF THE PROBLEM

Introducing the scalar point function G, G_L and vector point function ψ through the relations:

$$u = G_{,x} + \psi_{,z}, \ \ , w = G_{,z} - \psi_{,x}, \ \ u_L = G_{L,x}, \ w_L = G_{L,z} \tag{11}$$

where G , G_L and ψ represent the displacement potentials of longitudinal and transverse waves respectively for solid and liquid half-spaces. Substituting the values from equation (11) in equations (6)-(10), the equations become

$$G_{,xx} + G_{,zz} - T + a_1\phi = \ddot{G} \tag{12}$$

$$\delta^2\psi_{,xx} + \delta^2\psi_{,zz} = \ddot{\psi} \tag{13}$$

$$\left(T_{,xx} + T_{,zz}\right) - \left(\dot{T} + t_0\ddot{T}\right) - \varepsilon_T\left[\left(\dot{G}_{,xx} + \dot{G}_{,zz}\right) + t_0\left(\ddot{G}_{,xx} + \ddot{G}_{,zz}\right)\right] - a_5\left(\dot{\phi} + t_0\ddot{\phi}\right) = 0 \tag{14}$$

$$-a_2\left(G_{,xx} + G_{,zz}\right) + \left[\left(\phi_{,xx} + \phi_{,zz}\right) - a_3\left(\phi + \bar{\xi}\dot{\phi}\right)\right] + a_4T = \frac{\ddot{\phi}}{\delta_1^2} \tag{15}$$

$$\mathrm{G}_{L,xx} + \mathrm{G}_{L,zz} - \frac{1}{\delta_L^2}\ddot{\mathrm{G}}_L = 0 \tag{16}$$

It can be seen that equations (12), (14) and (15) are coupled through the quantities G, ϕ and T while (13) and (16) are uncoupled in potentials ψ and G_L respectively.

BOUNDARY CONDITIONS

The boundary conditions at the solid–liquid interface $z = 0$ to be satisfied are as follows:

(i) $\sigma_{zz} = \left(\sigma_{zz}\right)_L$.

(ii). $\sigma_{xz} = \left(\sigma_{xz}\right)_L$.

(iii) $w = w_L$.

(iv) $\phi_{,z} = 0$.

(v) $T_{,z} + HT = 0$, where H is the Biot's heat transfer coefficient.

SOLUTION OF THE PROBLEM

To discuss the wave propagation in a linear homogeneous thermoelastic material with voids of infinite extent underlying over an inviscid liquid, consider the form of plane waves traveling in the positive direction of x-axis, which are given by

$$\left(G, \psi, \phi, T, G_L\right) = \left[1, V, W, S, R\right] U \exp\left\{ik\left(x - ct\right) - rz\right\} \tag{17}$$

where k is the wavenumber, ω is the angular frequency of the plane wave propagating in $x-z$ plane. The phase velocity c is connected with angular frequency ω and wavenumber k through the relation $c=\dfrac{\omega}{k}$. U is the scalar constant and V,W,S,R are amplitude ratios. Invoking equation (17) in equations (12)-(16), we have

$$(r^2-\alpha_1^2)+a_1W-S=0 \tag{18}$$

$$\left(r^2-\beta_1^2\right)V=0 \tag{19}$$

$$\varepsilon_T\tau_0\omega^2\left(r^2-k^2\right)+a_5\tau_0\omega^2W+\left[r^2-k^2\left(1-\tau_0c^2\right)\right]S=0 \tag{20}$$

$$-a_2\left(r^2-k^2\right)+\left(r^2-\gamma^2\right)W+a_4S=0 \tag{21}$$

$$(r^2-\beta_2^2)R=0 \tag{22}$$

where

$$\alpha_1^2=k^2\left(1-c^2\right),\quad \beta_1^2=k^2\left(1-\frac{c^2}{\delta^2}\right),\ \beta_2^2=k^2\left(1-\frac{c^2}{\delta_L^2}\right),\ \tau_0=t_0+i\omega^{-1},\ \bar{\xi}_0=1-i\bar{\xi}\omega,$$

$$\gamma^2=k^2\left[1-c^2\left(\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_0}{\omega^2}\right)\right]. \tag{23}$$

For the requirement of non-trivial solution of the system of equations (18)-(22) the determinant of their coefficients vanish. On simplifying the determinant, after some algebraic manipulations which leads to the determination of characteristics roots as

$$r_j^2=k^2\left(1-\lambda_j^2c^2\right);\quad j=1,2,3,4,5 \tag{24}$$

where, $\lambda_j^2;\ j=1,2,3,4,5$ are the roots of the polynomial.

$$\sum\lambda_1^2=1+\tau_0\left(1+\varepsilon_T\right)+\frac{1}{\delta_1^2}-\frac{a_3}{\omega^2}\left(\bar{\xi}_o-\frac{\varepsilon_\phi\varepsilon_b^2}{\varepsilon_T}\right),$$

$$\sum\lambda_1^2\lambda_2^2=\tau_0+\frac{1+\tau_0\left(1+\varepsilon_T\right)}{\delta_1^2}-\frac{a_3}{\omega^2}\left[\bar{\xi}_o\left\{1+\tau_0\left(1+\varepsilon_T\right)\right\}+\frac{\tau_0\varepsilon_\phi}{\varepsilon_T}\left\{\varepsilon_T-\varepsilon_b\left(2\varepsilon_T+\varepsilon_b\right)\right\}\right],$$

$$\lambda_1^2\lambda_2^2\lambda_3^2=\tau_0\left[\frac{1}{\delta_1^2}-\frac{a_3}{\omega^2}\left(\bar{\xi}_o+\varepsilon_\phi\right)\right],\ \lambda_4^2=\frac{1}{\delta^2},\ \lambda_5^2=\frac{1}{\delta_L^2}. \tag{25}$$

where $\varepsilon_\phi=\dfrac{a_4a_5}{a_3}$, $\varepsilon_b=\dfrac{a_2}{a_4}$, $\dfrac{\varepsilon_\phi\varepsilon_b}{\varepsilon_T}=\dfrac{a_4a_1}{a_3}$ (26)

Dealing with surface wave motion must be confined to the free surface $z=0$ of the half-space. Consequently, the formal solutions which satisfy the radiations condition $\mathrm{Re}\left(r_j\right)\ge 0$ is given by

$$\left(G,\psi,\phi,T,G_L\right)=\sum_{k=1}^{5}\left(1,V_k,W_k,S_k,R_k\right)U_k\exp\left\{ik\left(x-ct\right)-r_kz\right\} \tag{27}$$

where the amplitude ratios are given as

$$V_k = \begin{cases} 0 \ , \ k = 1,2,3,5 \\ \infty \ , \ k = 4 \end{cases} , \quad S_k = \begin{cases} \omega^2 \left(1-\lambda_k^2\right) + a_1 W_k \ , \ k = 1,2,3 \\ 0 \ , \ k = 4,5 \end{cases}$$

$$W_k = \begin{cases} \dfrac{a_4\left[\left(\varepsilon_b - 1\right)\lambda_k^2 + 1\right]}{\lambda_k^2 - \dfrac{1}{\delta_1^2} + \dfrac{a_3}{\omega^2}\left(\bar{\xi}_o - \dfrac{\varepsilon_b \varepsilon_\phi}{\varepsilon_T}\right)} \ , \ k = 1,2,3 \\ 0 \ , \ k = 4,5 \end{cases} , \quad \mathrm{R}_k = \begin{cases} 0, k = 1,2,3,4 \\ \infty, k = 5 \end{cases} \tag{28}$$

PARTICULAR CASES OF FORMAL SOLUTIONS

The particular cases in the absence of inviscid liquid half-space are discussed below.

(i) Thermoelasticity without voids

In the absence of voids $b = 0$ which leads to $\varepsilon_\phi = \varepsilon_b = 0$ one can obtain the characteristic roots (25) and amplitude ratios (28) as

$$\lambda_1^2 + \lambda_3^2 = 1 + \tau_0 \left(1 + \varepsilon_T\right) , \ \lambda_2^2 = \frac{1}{\delta_1^2} - \frac{a_3 \bar{\xi}_o}{\omega^2} , \ \lambda_1^2 \lambda_3^2 = \tau_0 , \ \lambda_4^2 = \frac{1}{\delta^2} \tag{29}$$

$$W_k = \begin{cases} 1 \ , \ k = 2 \\ 0 \ , \ k = 1,3 \end{cases} , \quad S_k = \begin{cases} \omega^2 \left(1-\lambda_k^2\right) \ , \ k = 1,3 \\ 0 \ , \ k = 2 \end{cases} \tag{30}$$

(ii) Elasticity with voids

If we neglect the thermal effect, then the elastic field and thermal field are independent of each other $m = \beta = 0$ which implies that $\varepsilon_T = \varepsilon_\phi = a_4 = a_5 = 0$. The characteristic roots (25) and amplitude ratios (28) are thus reduced to

$$\lambda_1^2 + \lambda_2^2 = 1 + \frac{1}{\delta_1^2} - \frac{a_3 \bar{\xi}_0}{\omega^2} + \frac{a_1 a_2}{\omega^2} , \ \lambda_1^2 \lambda_2^2 = \frac{1}{\delta_1^2} - \frac{a_3 \bar{\xi}_0}{\omega^2} , \ \lambda_3^2 = \tau_0 , \ \lambda_4^2 = \frac{1}{\delta^2} \tag{31}$$

$$W_k = \begin{cases} \dfrac{a_2 \lambda_k^2}{\lambda_k^2 - \dfrac{1}{\delta_1^2} + \dfrac{a_3 \bar{\xi}_o}{\omega^2}} \ , \ k = 1,2 \\ 0 \ , \ k = 3 \end{cases} , \quad S_k = \begin{cases} 0 \ , \ k = 1,2 \\ 1 \ , \ k = 3 \end{cases} \tag{32}$$

(iii) Elasticity without voids

The characteristic roots (25) and the amplitude ratios (28), in the absence of voids and thermomechanical coupling parameters, $m = b = 0 = \beta$ lead to $\varepsilon_T = \varepsilon_\phi = \varepsilon_b = 0$ and are obtained as

$$\lambda_1^2 = 1, \ \lambda_2^2 = \frac{1}{\delta_1^2} - \frac{a_3 \bar{\xi}_o}{\omega^2}, \quad \lambda_3^2 = \tau_0 , \ \lambda_4^2 = \frac{1}{\delta^2} \tag{33}$$

$$W_k = \begin{cases} 1 \ , \ k = 2 \\ 0 \ , \ k = 1,3 \end{cases} , \quad S_k = \begin{cases} 0 \ , \ k = 1,2 \\ 1 \ , \ k = 3 \end{cases} \tag{34}$$

The above results are verified and same as obtained by Sharma and Kaur [10].

DERIVATION OF THE SECULAR EQUATIONS

Invoking the boundary conditions and equation (27) at the stress-free surface i.e. $z=0,$ the equations of Rayleigh waves for insulated and isothermal surface of solid half-space in contact with inviscid liquid half-space are obtained as

$$1+QV_k r_k = \mathrm{MR}_k, \quad V_k - Qr_k = 0, \quad W_k r_k = 0, \quad r_k + ikV_k = R_k r_k, \quad S_k = 0, \quad S_k r_k = 0. \tag{35}$$

where

$$Q=\frac{2ik}{P}, \quad P=k^2\left(2-\frac{c^2}{\delta^2}\right), \quad M=\frac{1}{P}\frac{\rho_L\omega^2}{\rho\delta^2} \tag{36}$$

The system ofequations (35) have a non-trivial solution if the determinant of the coefficients vanishes and it leads to following secular equations for insulated and isothermal surface respectively.

$$r_1S_1F_1 - r_2S_2F_2 + r_3S_3F_3 - r_4S_4F_4 - r_5S_5F_5 = 0, \; S_1F_1 - S_2F_2 + S_3F_3 - S_4F_4 + S_5F_5 = 0. \tag{37}$$

where

$$F_1 = \begin{vmatrix} 1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_4-MR_4 & 1+QV_5-MR_5 \\ V_2-Qr_2 & V_3-Qr_3 & V_4-Qr_4 & V_5-Qr_5 \\ W_2r_2 & W_3r_3 & W_4r_4 & W_5r_5 \\ r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5 \end{vmatrix}$$

$$F_2 = \begin{vmatrix} 1+QV_1-MR_1 & 1+QV_3-MR_3 & 1+QV_4-MR_4 & 1+QV_5-MR_5 \\ V_1-Qr_1 & V_3-Qr_3 & V_4-Qr_4 & V_5-Qr_5 \\ W_1r_1 & W_3r_3 & W_4r_4 & W_5r_5 \\ r_1+ikV_1-R_1r_1 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5 \end{vmatrix}$$

$$F_3 = \begin{vmatrix} 1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_4-MR_4 & 1+QV_5-MR_5 \\ V_1-Qr_1 & V_2-Qr_2 & V_4-Qr_4 & V_5-Qr_5 \\ W_1r_1 & W_2r_2 & W_4r_4 & W_5r_5 \\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5 \end{vmatrix}$$

$$F_4 = \begin{vmatrix} 1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_5-MR_5 \\ V_1-Qr_1 & V_2-Qr_2 & V_3-Qr_3 & V_5-Qr_5 \\ W_1r_1 & W_2r_2 & W_3r_3 & W_5r_5 \\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_5+ikV_5-R_5r_5 \end{vmatrix}$$

$$F_5 = \begin{vmatrix} 1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_4-MR_4 \\ V_1-Qr_1 & V_2-Qr_2 & V_3-Qr_3 & V_4-Qr_4 \\ W_1r_1 & W_2r_2 & W_3r_3 & W_4r_4 \\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4 \end{vmatrix} \tag{38}$$

The complete information regarding wave number, phase velocity and attenuation coefficient of Rayleigh waves in the considered medium in case of thermally insulated and stress-free, isothermal boundaries, can be obtained from equations (37). In the absence of voids, inviscid liquid and thermomechanical coupling,both the secular equations (37) with the help of equation (33) lead to

$$\left(2-\frac{c^2}{\delta^2}\right)^2 = 4\sqrt{1-c^2}\sqrt{1-\frac{c^2}{\delta^2}} \tag{39}$$

Equation (39) represents the classical Rayleigh wave velocity equation in an elastic solid half-space which is the same as given by Graff [5], Kumar and Kumar [7], Sharma and Pathania [9] and Sharma and Kaur [10].

CONCLUSIONS

The propagation of thermoelastic waves in a semi-infinite homogeneous isotropic thermoelastic material with voids in contact with inviscid liquid half-space is investigated and the results concluded from the above analysis can be summarized as:

1. There exist four waves in a generalized thermoelastic porous solid and one wave in inviscid liquid half-space. In the solid half-space, one of the waves is a transverse wave and the remaining three are set of coupled longitudinal waves.
2. Each set of longitudinal waves is found to be influenced by void volume fraction and the thermal properties.
3. The transverse wave remains independent of the presence of voids and thermal properties of the medium.
4. The Rayleigh wave propagation has importance in geophysical problems as this field studies the propagation of progressive elastic waves. The surface wave propagation in a thermoelastic material containing voids finds many applications in the industrial world in the manufacturing of resonators, filters, and sensors, detectors, electronic components, etc.
5. The study may find applications in the development and designs of surface acoustic wave devices.

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FINITE DIFFERENCE TECHNIQUE FOR THREE DIMENSIONAL MHD FREE CONVECTIVE FLOWS PAST A POROUS MEDIUM WITH VARIABLE SUCTION AND PERIODIC PERMEABILITY

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ABSTRACT:
The free convection three dimensional steady viscous incompressible flow in porous medium is investigated. The effect of transverse magnetic on three dimensional flow and mass transfer in the viscous dissipative heat when porous plate is subjected to a slightly sinusoidal transverse suction velocity distribution. The aim of this study is to know the effect of periodic permeability on the flow in the presence of magnetic field and heat transfer. The effect of magnetic field, Schmidt and Grashof number and Reynolds number on velocity, concentration and temperature explained with the help of graphs. The governing equations are solved numerically with finite difference method.

KEYWORDS: Free Convection, Periodic Permeability, Grashof number, magnetic field porous medium.

1. INTRODUCTION:

Magneto-Hydrodynamics (MHD) is the branch of continuum mechanics deals with the study of magnetic properties and behaviour of electrically conducting fluids. Many engineering problems and natural phenomenon are based upon the concept of magneto-hydrodynamics. MHD is related to engineering problems such as plasma confinement, liquid metal cooling into nuclear reactors and electromagnetic casting. The concept of MHD is used in Geo-physics to study the flow pattern in the core of the earth. Some examples of magneto fluids are plasma, salt water, electrolytes and liquid metals. Hannes Alfen in 1942 introduced the concept of Magneto-hydrodynamics for which he received the Nobel prize in 1970. The word Magneto-Hydrodynamics is derived from the word '*magne*t' which means magnetic field , '*hydro*' means liquid and '*dynamics'* means in motion.

The movement of conducting fluid in magnetic field produces an induced electric current in fluid. The magnetic field exerts a force known as Lorentz force. The set of equations for MHD are combination of Navier-Stokes equations for fluid dynamics and Maxwell's equations for electromagnetism. An ideal MHD, assumes that the fluid has very low resistivity that it can be treated as perfect conductor. The problems of MHD laminar flow through a porous medium have become very important in recent years due to its applications in many branches of Science and Technology.

Free convection flows have number of industrial applications such as fibre, granular insulation and geo-thermal energy recover, oil extraction and flow through filtering devices. Soundalgekar (1981) studied MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field. Bejan & Khaair (1985) discussed heat and mass transfer by natural convection. Hossain & Mandal (1985) have investigated the effects of magnetic field on natural convection flow past a vertical surface. Hydromagnetic free convective flow past a porous plate studied by Singh (1991). Singh, Chand & Rana(1993) studied the effect of heat transfer in three dimensional flow past a porous plate. Later on Gupta & Johari (2000) have investigated the MHD three dimensional flow past a porous plate. Singh & Sharma (2002) have discussed three dimensional free convective flow and heat transfer in rotating elastic-viscous fluid past an infinite vertical porous flat plate with constant suction. Cookey et.al. (2003) have studied the problems which includes thermal radiation, heat source, thermal diffusion on unsteady MHD free convection flows past an infinite heated vertical plate in porous medium with time-dependent suction. A numerical solution of unsteady free convection and mass transfer flow is presented by Alam & Rahman (2005). Sharma & Yadav (2006) have discussed steady MHD boundary layer flow and heat transfer between two long vertical wavy walls. R.C.Choudhary & Arpita Jain (2010) investigated an exact solution of MHD convection flow past an accelerated surface embedded in porous. S. Siviach et.al. (2012) have studied finite element analysis of chemical reaction and radiation effects on isothermal vertical oscillating plate with variable mass diffusion. Ravi kumar & Raju (2012) have discussed MHD three dimensional couette flow past a porous plate with heat transfer.

Ahmed, Batin & Chamkha (2013) have investigated finite difference approach in porous media transport modelling for MHD unsteady flow over a vertical plate using Darcian Model. Finite difference solutions of MHD free convective flow with constant suction and variable thermal conductivity in a Darcy- Forchheimer porous medium studied by Uwanta & Usman (2015). Reddy & Raju (2018) have studied MHD free convective flow past a porous plate.

In this paper, we have investigated the MHD free convection flow through a highly porous medium. Due to three dimensional flow the permeability of the porous medium is considered to be periodic. The flow is moving sinusoidal with variable suction velocity. The governing partial differential equations are transformed into a set of ordinary equations and solved numerically by finite difference method. We discussed the effect of parameter involved on the flow; computed skin friction and rate of heat transfer.

2. MATHEMATICAL FORMULATION:

Consider the flow of electrical conducting and incompressible viscous fluid with heat and mass transfer along an infinite vertical non-conducting porous plate with variable suction velocity. The plate is lying vertically on x' -z' plane with x'-axis is taken along the plate in the upward direction. The y'-axis is taken normal to the plate and directed along the fluid with free stream velocity U . All physical quantities will be independent of x', flow remains three dimensional due to variable suction. The sinusoidal suction velocity at plate is considered to be of the form

$$v'(z') = -V\left[1+\varepsilon\cos\frac{\pi z'}{L}\right] \quad (1)$$

which consists of basic steady distribution V > 0 superimposed with a very weak transversally direction $\varepsilon\cos\frac{\pi z'}{L}$. Here L is half wave length of periodic suction velocity. The negative sign in (1) indicates that suction is towards the plate. The magnetic field B_0 is applied along y'-axis. The permeability of the porous medium is assumed to be of the form

$$k'(z') = \frac{K_0'}{\left[1+\varepsilon\cos\frac{\pi z'}{L}\right]} \tag{2}$$

where K'$_0$ is the mean permeability of the porous medium. The ε (<<1) is amplitude of permeability variation. For the flow problem under consideration the following assumptions are made:

(i) The flow is steady, laminar and applied magnetic field B_0 perpendicular to the plate.
(ii) Molecular transport properties are constant.
(iii)The fluid under consideration is incompressible and viscous.
(iv)The density variation due to temperature and concentration are approximated by Boussinesq approximation.
(v) Hall effect, polarization and chemical reactions are neglected.

Denoting velocities components u', v', w' in the direction of x', y', z' respectively and temperature T' and concentration C', the flow through highly porous medium is governed by the following equations:

Continuity Equation

$$\frac{\partial v'}{\partial y'}+\frac{\partial w'}{\partial z'}=0 \tag{3}$$

Momentum Equations

$$v'\frac{\partial u'}{\partial y'}+w'\frac{\partial u'}{\partial z'}=g\beta(T'-T'_\infty)+g\beta'(C'-C'_\infty)+\upsilon\left(\frac{\partial^2 u'}{\partial y'^2}+\frac{\partial^2 u'}{\partial z'^2}\right)-\frac{\upsilon}{k'}(u'-U)-\frac{\sigma B_0^2(u'-U)}{\rho} \tag{4}$$

$$v'\frac{\partial v'}{\partial y'}+w'\frac{\partial v'}{\partial z'}=-\frac{1}{\rho}\frac{\partial p'}{\partial y'}+\upsilon\left(\frac{\partial^2 v'}{\partial y'^2}+\frac{\partial^2 v'}{\partial z'^2}\right)-\frac{\upsilon}{k'}(v') \tag{5}$$

$$v'\frac{\partial w'}{\partial y'}+w'\frac{\partial w'}{\partial z'}=-\frac{1}{\rho}\frac{\partial p'}{\partial z'}+\upsilon\left(\frac{\partial^2 w'}{\partial y'^2}+\frac{\partial^2 w'}{\partial z'^2}\right)-\frac{\upsilon}{k'}(w')-\frac{\sigma B_0^2 w'}{\rho} \tag{6}$$

Energy Equation

$$\rho c_p\left(v'\frac{\partial T'}{\partial y'}+w'\frac{\partial T'}{\partial z'}\right)=k\left(\frac{\partial T'}{\partial y'^2}+\frac{\partial^2 T'}{\partial z'^2}\right) \tag{7}$$

Concentration Equation

$$\left(v'\frac{\partial C'}{\partial y'}+w'\frac{\partial C'}{\partial z'}\right)=D\left(\frac{\partial^2 C'}{\partial y'^2}+\frac{\partial^2 C'}{\partial z'^2}\right) \tag{8}$$

where g- acceleration due to gravity, β- coefficient of volume expansion, β'- coefficient of mass expansion, p'- pressure, ρ- density, ν- kinematic viscosity, μ- dynamic viscosity, k- thermal conductivity, c_p- specific heat at constant pressure, σ- electrical conductivity and D- concentration diffusivity.

The boundary conditions are

y' = 0 ; u' = 0 , w' = 0 ,T' = T'_w , C' = C'_w v' = -V $\left[1+\varepsilon\cos\frac{\pi z'}{L}\right]$

y'→ ∞ ; u' = U , w' = 0 , T' = T'_∞ , p' = p^*_∞ , v' = -V, C' = C'_∞ (9)

The subscripts ' w' and '∞' denote the physical quantities at the wall and in free stream respectively.
Introducing the following non-dimensional quantities

$$y=\frac{y'}{L},z=\frac{z'}{L},u=\frac{u'}{U},v=\frac{v'}{U},w=\frac{w'}{U},p=\frac{p'}{\rho U^2},T=\left(\frac{T'-T'_\infty}{T'_w-T'_\infty}\right),C=\left(\frac{C'-C'_\infty}{C'_w-C'_\infty}\right) \tag{10}$$

With the help of above (10) non-dimensional variables equations (3) to (8) reduces:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{11}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=GrRT+GmRC+\frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{(u-1)(1+\varepsilon\cos\pi z)}{R\ K'_0}-\frac{M^2(u-1)}{R} \tag{12}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{(1+\varepsilon\cos\pi z)v}{R\ K'_0} \tag{13}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{(1+\varepsilon\cos\pi z)w}{R\ K'_0}-\frac{M^2 w}{R} \tag{14}$$

$$v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{R\Pr}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{15}$$

$$v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{RSc}\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right) \tag{16}$$

Where

$Gr=\frac{vg\beta(T'_w-T'_\infty)}{U\ V^2}$ - Grashof number , $Gm=\frac{vg\beta(C'_w-C'_\infty)}{U\ V^2}$ - Modified Grashof number

$R=\frac{UL}{\upsilon}$ - Reynolds number , $\Pr=\frac{\mu c_p}{k}$ - Prandlt number

$K=\frac{K'_0}{L^2}$ - Permeability parameter, $Sc=\frac{\nu}{D}$ - Schmidt number

$M=\left(\frac{\sigma}{\mu}\right)^{1/2}B_0L$ - Hartmann number or magnetic parameter and $\alpha=\frac{V}{U}$ -Suction parameter

The corresponding boundary conditions become:
y = 0; u = 0, v = - α(1 + cos πz), w = 0, T = 1 and C = 1
y→∞: u = 1, w = 0 , p = p_∞, T = 0 and C = 0. (17)

3. Method of Solution

We assume that the solutions of following form because ε(<<1) amplitude permeability variation is very small.

u (y, z) = $u_0(y)+\varepsilon u_1(y,z)+\varepsilon^2 u_2(y,z)+$ ....

$v(y, z) = v_0(y) + \varepsilon v_1(y, z) + \varepsilon^2 v_2(y, z) + \ldots$
$w(y, z) = w_0(y) + \varepsilon w_1(y, z) + \varepsilon^2 w_2(y, z) + \ldots$
$p(y, z) = p_0(y) + \varepsilon p_1(y, z) + \varepsilon^2 p_2(y, z) + \ldots$

$T(y, z) = T_0(y) + \varepsilon T_1(y, z) + \varepsilon^2 T_2(y, z) + \ldots$
$C(y, z) = C_0(y) + \varepsilon C_1(y, z) + \varepsilon^2 C_2(y, z) +$ (18)

When ε = 0, the using equations (18) into equations (11) – (16), we get the following equations.

$$\frac{dv_0}{dy} = 0 \tag{19}$$

$$\frac{d^2u_0}{d\ y^2} - v_0 R\frac{du_0}{dy} - \frac{1}{K}u_0 - M^2u_0 = -GrR^2T_0 - GmR^2C_0 - \left(\frac{1}{K} + M^2\right) \tag{20}$$

$$\frac{d^2T_0}{dy^2} - v_0 R \Pr\frac{dT_0}{dy} = 0 \tag{21}$$

$$\frac{dC_0}{dy^2} - v_0 RSc\frac{dC_0}{dy} = 0 \tag{22}$$

The corresponding boundary conditions are:

$y = 0$; $u_0 = 0$, $v_0 = -\alpha$, $w_0 = 0$, $T_0 = 1$ and $C_0 = 1$
$y \to \infty$: $u_0 = 1$, $w_0 = 0$, $p_0 = p_\infty$, $T_0 = 0$ and $C_0 = 0$. (23)

The solutions of equations (19) – (22) with boundary conditions (23) are

$$v_0 = -\alpha \tag{24}$$

$$u_0 = 1 + (Gr\lambda_0 + Gm\lambda_1 - 1)e^{\alpha\lambda y} - Gr\lambda_0 e^{\alpha R \Pr y} - Gm\lambda_1 e^{\alpha RScy} \tag{25}$$

$$T_0 = e^{-\alpha R \Pr y} \tag{26}$$

$$C_0 = e^{-\alpha RScy} \tag{27}$$

with $v_0 = -\alpha$, $w_0 = 0$ and $p_0 = p_\infty$, (28)

where $\lambda = \frac{\alpha R}{2} + \sqrt{\frac{\alpha^2R^2}{4} + \frac{1}{K} + M^2}$ and $\lambda_0 = \frac{R^2}{\alpha^2R^2 \Pr(\Pr - 1) - \left(\frac{1}{K} + M^2\right)}$

$$\lambda_1 = \frac{R^2}{\alpha^2R^2Sc(Sc-1) - \left(\frac{1}{K} + M^2\right)}$$

When ϵ ≠ 0, the periodic permeability enters the equations (11) – (16) and comparing the coefficients of like power of ϵ and neglecting higher power of ϵ^2, we get the following with equations using equation (28):

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{29}$$

$$v_1\frac{\partial u_0}{\partial y} - \alpha\frac{\partial u_1}{\partial y} = GrRT_1 + GmRC_1 + \frac{1}{R}\left(\frac{\partial^2u_1}{\partial y^2} + \frac{\partial^2u_1}{\partial\ z^2}\right) - \frac{(u_0 - 1)\cos\pi z + u_1}{R\ K} - \frac{M^2u_1}{R} \tag{30}$$

$$-\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial\ z^2}\right)-\frac{(v_1-\cos\pi z)}{R\ K} \tag{31}$$

$$-\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial\ z^2}\right)-\frac{w_1}{R\ K}-\frac{M^2 w_1}{R} \tag{32}$$

$$v_1\frac{\partial T_0}{\partial y}-\alpha\frac{\partial T_1}{\partial y}=\frac{1}{R\Pr}\left(\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial\ z^2}\right) \tag{33}$$

$$v_1\frac{\partial C_0}{\partial y}-\alpha\frac{\partial C_1}{\partial y}=\frac{1}{RSc}\left(\frac{\partial^2 C_1}{\partial y^2}+\frac{\partial^2 C_1}{\partial\ z^2}\right) \tag{34}$$

The corresponding boundary conditions are:
$y = 0$; $u_1 = 0$, $v_1 = -\alpha\cos\pi z$, $w_1 = 0$, $T_1 = 0$ and $C_1 = 0$
$y\to\infty$: $u_1 = 0$, $w_1 = 0$, $p_1 = 0$, $T_1 = 0$ and $C_1 = 0$. (35)

The equations (29) – (34) are the linear partial differential equations, which describes free convective, three dimensional flow. We assume the solution of equations (29) – (34) of the form:

$v_1(y, z) = v_{11}(y)\cos\pi z$ (36)

$w_1(y, z) = -\frac{1}{\pi}v'_{11}(y)\sin\pi z$ (37)

$p_1(y, z) = p_{11}(y)\cos\pi z$ (38)

$u_1(y, z) = u_{11}(y)\cos\pi z$ (39)

$T_1(y, z) = T_{11}(y)\cos\pi z$ (40)

$C_1(y, z) = C_{11}(y)\cos\pi z$ (41)

Where prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expressions for $v_1(y, z)$ and $w_1(y, z)$ are chosen in such a way that equation of continuity (29) is satisfied. Substituting the expressions (36)-(38) into equations (31)-(32) we have

$$v''_{11}+\alpha Rv'_{11}-\left(\pi^2+\frac{1}{K}\right)v_{11}=Rp'_{11}-\frac{1}{K} \tag{42}$$

$$v'''_{11}+\alpha Rv''_{11}-\left(\pi^2+M^2+\frac{1}{K}\right)v'_{11}=R\pi^2 p_{11} \tag{43}$$

Eliminating the terms p_{11} and p'_{11} in equations (42) and (43), we get the following,

$$v^{iv}_{11}+\alpha Rv'''_{11}-\left(M^2+\frac{1}{K}+2\pi^2\right)v''_{11}-R\pi^2 v'_{11}+\left(\pi^4+\frac{\pi^2}{K}\right)v_{11}+\frac{\pi^2}{K}=0 \tag{44}$$

The corresponding boundary conditions are:
$y = 0$; $v_{11} = -\alpha$, $v'_{11} = 0$
$y\to\infty$; $v_{11} = 0$ (45)

Substituting the equations (39), (40) and (41) in equations (30), (33) and (34), we get following:

$$u''_{11}+\alpha Ru'_{11}-\left(M^2+\frac{1}{K}+\pi^2\right)u_{11}=Rv_{11}u'_0-GrR^2T_{11}-GmR^2C_{11}+\frac{u_0-1}{K} \tag{46}$$

$$T_{11}'' + \alpha RprT_{11}' - \pi^2 T_{11} = -R\Pr v_{11}T_0' \tag{47}$$

$$C_{11}'' + \alpha RScC_{11}' - \pi^2 C_{11} = -RScv_{11}C_0' \tag{48}$$

The corresponding boundary conditions are:

y = 0 ; $u_{11} = 0$, $T_{11} = 0$, $C_{11} = 0$

$y \to \infty$; $u_{11} \to 0$, $T_{11} \to 0$, $C_{11} \to 0$ (49)

Using Finite difference formulae

$$v_{11}'(i) = \frac{v_{11}(i+1) - v_{11}(i-1)}{2h}$$

$$v_{11}''(i) = \frac{v_{11}(i+1) - 2v_{11}(i) + v_{11}(i-1)}{h^2}$$

$$v_{11}'''(i) = \frac{v_{11}(i+2) - 2v_{11}(i+1) + 2v_{11}(i-1) - v_{11}(i-2)}{2h^3}$$

$$v_{11}''''(i) = \frac{v_{11}(i+2) - 4v_{11}(i+1) + 6v_{11}(i) - 4v_{11}(i-1) + v_{11}(i-2)}{h^4} \tag{50}$$

Equation (44) using finite difference form (50) becomes

$$D_1 v_{11}(i+2) - D_2 v_{11}(i+1) + D_3 v_{11}(i) - D_4 v_{11}(i-1) - D_5 v_{11}(i-2) + \frac{2h^4\pi^2}{K} = 0 \tag{51}$$

$D_1 = 2 + \alpha Rh$

,

$$D_2 = 8 + 2\alpha Rh + 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + Rh^3\pi^2$$

$$D_3 = 12 + 4h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + 2h^4\left(\pi^4 + \frac{\pi^2}{K}\right)$$

$$D_4 = 8 - 2\alpha Rh + 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) - R\pi^2 h^3$$

$D_5 = 2 - \alpha Rh$

Similarly solving equations (46), (47) and (48) using (50), we get the following:

$D_1 u_{11}(i+1) - B_1 u_{11}(i) + D_5 u_{11}(i-1) = B(i)$(52)

$K_1 T_{11}(i+1) - K_2 T_{11}(i) + K_3 T_{11}(i-1) = D(i)$(53)

$F_1 C_{11}(i+1) - K_2 C_{11}(i) + F_2 C_{11}(i-1) = E(i)$(54)

Where $B_1 = 4 + 2h^2\left(M^2 + \frac{1}{K} + \pi^2\right)$

$$B(i) = -2h^2 Rv_{11}(i)B_2(i) - 2h^2R^2(GrT_{11}(i) - GmC_{11}(i) + \frac{2h^2}{K}B_3(i)$$

$$B_2(i) = [\alpha\lambda(Gr\lambda_0 + Gm\lambda_1 - 1)]e^{\alpha\lambda ih} - \alpha R\Pr Gr\lambda_0 e^{\alpha R\Pr ih} - \alpha RScGm\lambda_1 e^{\alpha RScih}$$

$$B_3(i) = (Gr\lambda_0 + Gm\lambda_1 - 1)e^{\alpha\lambda ih} - Gr\lambda_0 e^{\alpha R\Pr ih} - Gm\lambda_1 e^{\alpha RScih}$$

$$K_1 = 2 + \alpha R \Pr h$$
$$K_2 = 4 + 2\pi^2 h^2 \qquad K_3 = 2 - \alpha R \Pr h$$
$$D(i) = 2\alpha R^2 \Pr^2 h^2 v_{11}(i) e^{\alpha R \Pr ih}$$
$$F_1 = 2 + \alpha RSch$$
$$F_2 = 2 - \alpha RSch$$
$$E(i) = 2\alpha R^2 Sc^2 h^2 v_{11}(i) e^{\alpha RScih}$$

Equations (51) – (54) have been solved by Gauss-Seidal iteration method for velocity and temperature. To prove the convergence of finite difference scheme, the calculations are carried out slightly changed value of h .Smaller change is observed in the value of velocity and temperture.Thus, it is concluded that finite difference scheme is convergent and stable.

5.4 Skin –friction coefficient:

We know discuss the skin friction components in the x' – direction in the non- dimensional form.

$$\tau_x = \frac{\tau'_x}{\rho UV} = \frac{\nu}{VL}\left(\frac{du_0}{dy}\right)_{y=0} \quad \tau_x = \frac{1}{R}\left(\frac{du_0}{dy}\right)_{y=0} + \frac{1}{R}\varepsilon\left(\frac{du_{11}}{dy}\cos\pi z\right)_{y=0} \tag{55}$$

5.5 Nusselt number

The rate of heat transfer i.e heat flux at the surface in terms of Nusselt number(Nu) is given by

$$Nu = \frac{q_w}{\rho V c_p (T_w - T_\infty)} = \frac{k}{\rho V c_p L}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{1}{Rpr}\left[\frac{dT}{dy} + \varepsilon\frac{dT_{11}}{dy}\cos\pi z\right]_{y=0} \tag{56}$$

5.6 Results and discussion

In view of physical nature of the problem, numerical calculations are carried out for different parameters such as Grashof number(Gr), Modified Grashof number(Gm), Hartmann number(M) , Suction velocity(α), Permeability(K) , Reynolds number(R), Prandtl number (Pr) and Schmidt number(Sc) are studied.

Figure 1 depicts the effect of suction parameter(α) on velocity field when R=10, K=1, M=2,ϵ=0.1 and z=0. As the suction parameter increases there is increase in velocity of the fluid.

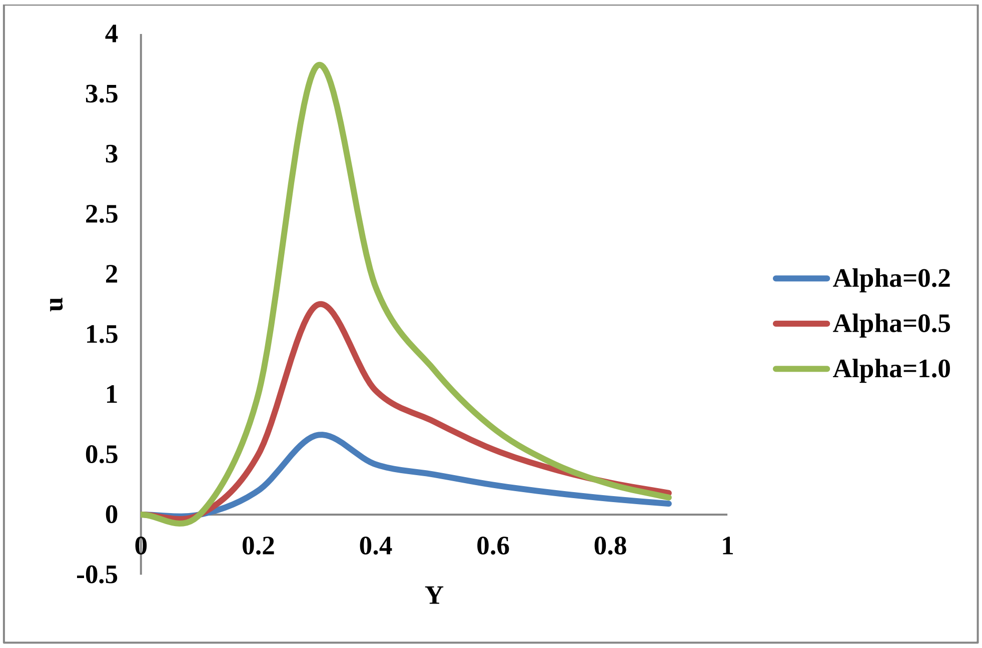

Figure 1. Effect of suction parameter (α) on velocity field when R = 10, K=1, M=2,ϵ=0.1 and z=0.

Figure 2 shows the effect of Hartmann number(M) on velocity field when α = 0.2, R= 10, K=1, ϵ= 0.1 and z=0. This indicates that as magnetic parameter increases there is decrease in velocity field.

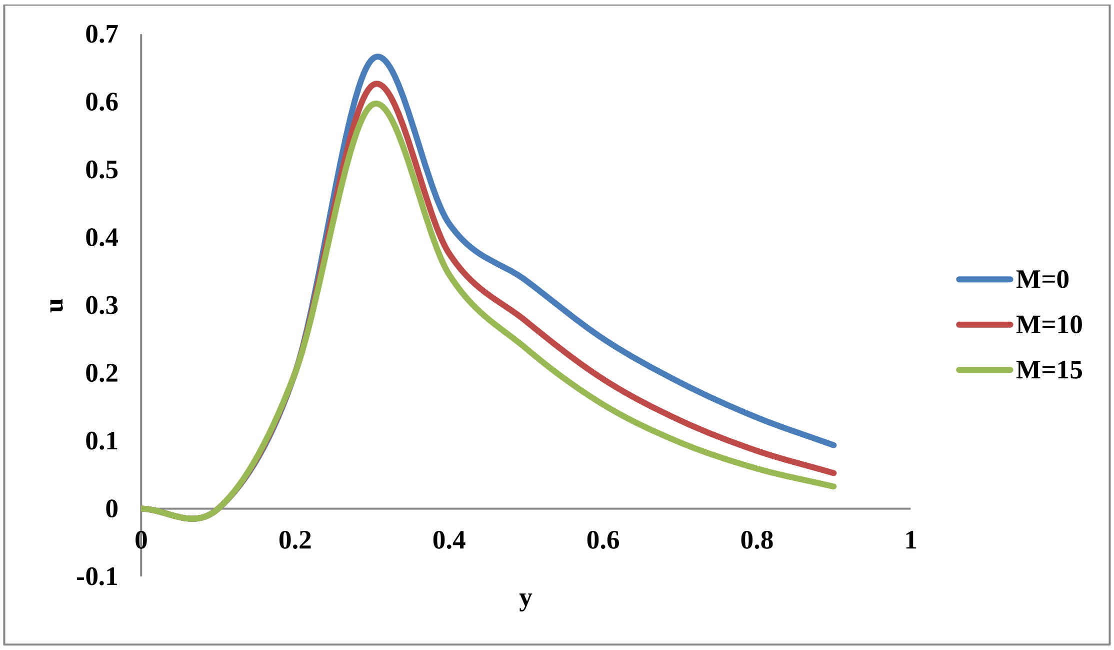

Figure 2. Effect of magnetic parameter (M) on velocity field when α =0.2,R = 10, K=1,,ϵ=0.1 and z=0.

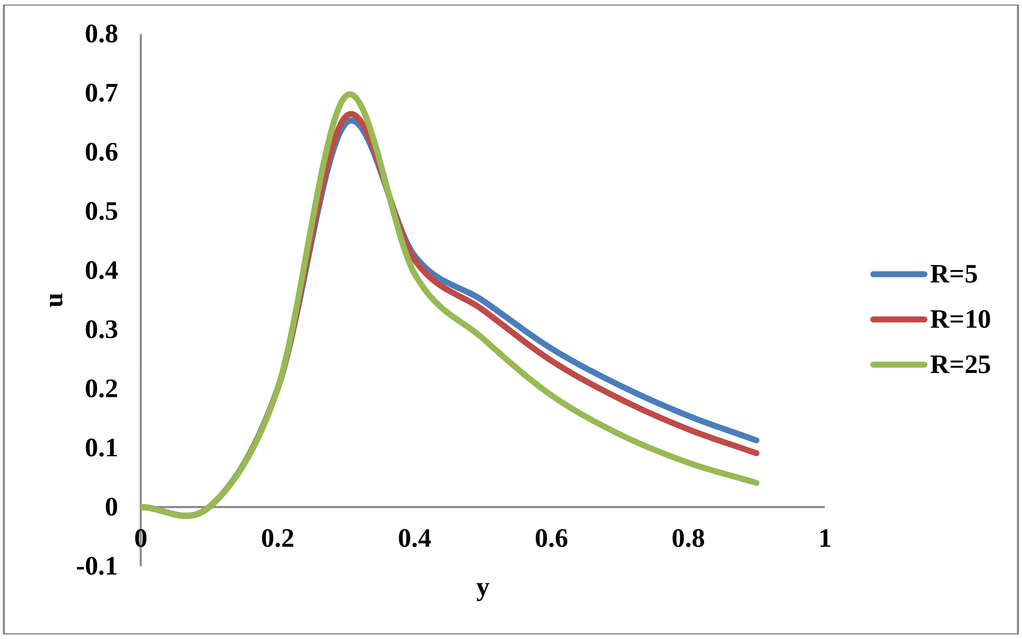

Figure 3. Effect of Reynolds number (R) on velocity field when α=0.2, K=1, M=2,ϵ=0.1 and z=0.

Figures 3 and 4 depicts the Reynolds number and permeability effect on velocity field.

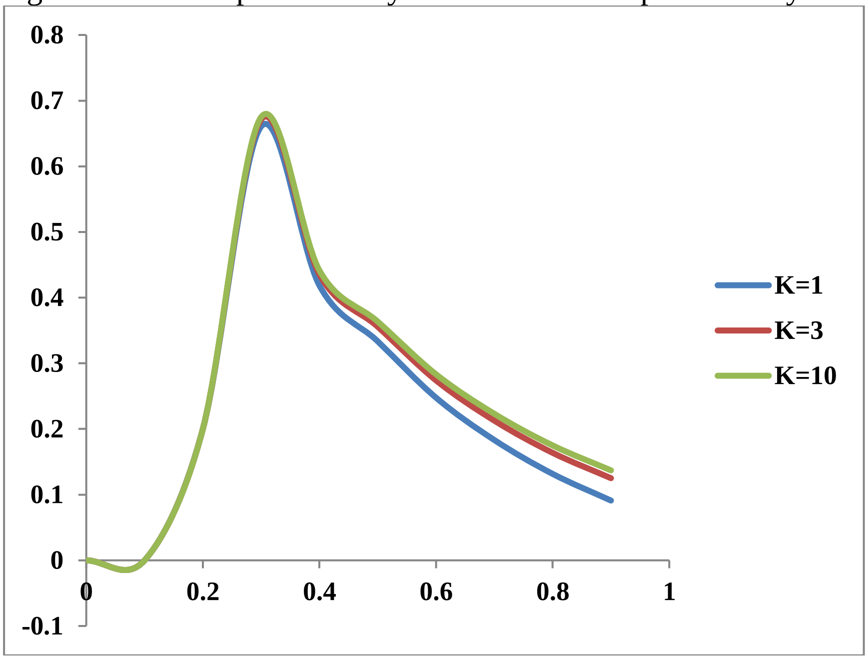

Figure 4. Effect of Permeability parameter (K) on velocity field when α=0.2, R=1, M=2,ϵ=0.1 and z=0.

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STABILITY OF STRATIFIED ELASTICO-VISCOUS WALTERS' (MODEL B') FLUID IN THE PRESENCE OF VARIABLE MAGNETIC FIELD AND ROTATION SATURATING POROUS MEDIUM

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ABSTRACT:
The influence of viscosity, viscoelasticity, medium permeability and medium porosity on the stability of a stratified elastic-viscous fluid is examined for viscoelastic polymeric solutions in the simultaneous presence of a variable horizontal magnetic field $H(H_0(z),0,0)$ and uniform horizontal rotation $\Omega(\Omega,0,0)$ in porous medium. These solutions are known as Walters' (model B') fluid and their rheology is approximated by the Walters' (model B') fluid constitutive relations, proposed by Walters'. The effects of coriolis force on the stability are chosen along the direction of the magnetic field and transverse to that of the gravitational field $g(0, 0, -g)$. Assuming the exponential stratifications in density, viscosity and viscoelasticity, the appropriate solution for the case of free boundaries is obtained using a linearized stability theory and normal mode analysis method. The dispersion relation is obtained and the behaviour of growth rates with respect to kinematic viscosity, kinematic viscoelasticity, medium permeability and medium porosity is examined numerically using Newton-Raphson method through the software Fortran-90 and Mathcad. In contrast to the Newtonian fluids, the system is found to be unstable, for stable stratifications, for small wavelength perturbations. It is found that the magnetic field stabilizes the certain wave number band, for unstable stratification in the presence of rotation and this wave number range increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity implying thereby the stabilizing effect of magnetic field and kinematic viscoelasticity and the kinematic viscosity has a stabilizing effect on the system for the low wave number range. The medium permeability has enhancing effect on the growth rates with its increase for a fixed wave number. These results are shown graphically.

KEY WORDS: Walters' (model B') fluid; magnetic field; rotation; viscosity; viscoelasticity, medium permeability, medium porosity.

1. INTRODUCTION:

The flow through porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamics. When we consider flow in porous medium, some additional complexities arise which are due to the interaction between the fluids and the porous medium. Here we consider those fluid flows for which Darcy's law is applicable. This law is empirical in nature and is usually considered valid for creeping flows where the Reynolds's number as defined for a porous medium is less than one. Darcy's law states that the gross effect, as the fluid slowly percolated through the pores of rock, is that usual viscous term in the equation of elastic-viscous fluid motion will be replaced by the resistance

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terms $\left[-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t} \right) q \right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity, of Walters' (model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity (seepage) of the fluid. The stability of flow of a single component fluid through porous medium taking into account the Darcy's resistance has been studied by Lapwood [1] and Wooding [2]. The effect of the Earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The physical properties of comets and meteorites strongly suggest importance of porosity in astrophysical context (McDonnell [3]).

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid) was investigated by Rayleigh [4]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases everywhere. An experimental demonstration of the development of the Rayleigh–Taylor instability was performed by Taylor [5]. The effect of a vertical magnetic field on the development of Rayleigh–Taylor instability was considered by Hide[6]. Reid [7] studied the effect of surface tension and viscosity on the stability of two superposed fluids. The Rayleigh–Taylor instability of a Newtonian fluid has been studied by several authors accepting varying assumptions of hydrodynamics and hydromagnetics and Chandrasekhar [8] in his celebrated monograph has given a detailed account of these investigations. Bellman and Pennington [9] further investigated in detail illustrating the combined effects of viscosity and surface tension. Gupta [10] again studied the stability of a horizontal layer of a perfectly conducting fluid with continuous density and viscosity stratifications in the presence of a horizontal magnetic field. The Rayleigh–Taylor instability problems arise in oceanography, limnology and engineering.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [11] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role on the stability of the system. In all the above studies the fluid has been assumed to be Newtonian.

With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. Fredricksen [12] has given a good review of non–Newtonian fluids whereas Joseph [13] has also considered the stability of viscoelastic fluids. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One of such viscoelastic fluids is Walters' (model B') fluid. Walters' [14] has proposed a constitutive equation for such type of elastico–viscous fluids. Many other research workers have paid their attention towards the study of Walters' (model B') fluid. The mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 grams of polymers per litre behaves very nearly as the Walters' (model B') viscoelastic fluid and which is proposed by Walters'[15]. This class of fluids is used in the manufacture of parts of space cafts, aeroplane, tyres, beltconveyors, rops, cushions, seats, foams, plastics, engineering equipments etc. Sharma and Kumar [16] have studied the steady flow and heat transfer of Walters' fluid (model B') through a porous pipe of uniform circular–cross section with small suction. Sharma and Kumar [17] have studied the stability of two superposed Walters' (model B') viscoelastic fluid. The magnetic field stabilizes the system. The viscoelasticity of the medium has damping effects on the growth rates but has enhancing effects for certain ranges of the wave-numbers. Sharma *et al.* [18] have studied the stability of stratified Walters' (model B') fluid in the presence of horizontal magnetic field and rotation in porous medium. Yadav and Sharma [19] have studied the effects of porous medium on MHD fluid flow along a stretching cylinder. Sharma and Gupta[20] have also studied the stability of stratified elastico–viscous Walters' (model B') fluid in the presence of variable magnetic field and rotation.

Keeping in mind the importance of non–Newtonian fluids, medium permeability in modern technology and their various applications mentioned above, the present paper is devoted to consider the stability of rotating stratified elastico–viscous Walters' (model B') fluid in the presence of variable magnetic field and rotation in porous medium.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:

The initial stationary state whose stability we wish to examine is that of an incompressible, heterogeneous infinitely extending and conducting $(\sigma \to \propto)$ elastico–viscous Walters' (model B') fluid of thickness d bounded by the planes $z = 0, d$ and of variable density, kinematic viscosity and viscoelasticity, arranged in horizontal strata in a porous medium of variable porosity and medium permeability so that the free surface is almost horizontal and the electrical conductivity $\eta = \frac{1}{4\pi\mu_e\sigma}$ is zero. The fluid is acted on by gravity force $\boldsymbol{g}(0,0,-g)$, a uniform horizontal rotation $\boldsymbol{\Omega}(\Omega,0,0)$ and a variable horizontal magnetic field $\boldsymbol{H}(H_0(z),0,0)$.The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution..

The equations expressing conservation of momentum, mass, incompressibility and Maxwell's equations for the elastico–viscous Walters' (model B') fluid are

$$\rho\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\in}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right] = -\nabla p + \rho\,\boldsymbol{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{q} + \frac{2\rho}{\in}(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\mu_e}{4\pi}[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}], \tag{1}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{2}$$

$$\in\frac{\partial\rho}{\partial t} + (\boldsymbol{q}\cdot\nabla)\rho = 0, \tag{3}$$

$$\nabla\cdot\boldsymbol{H} = 0, \tag{4}$$

$$\in\frac{\partial\boldsymbol{H}}{\partial t} = \nabla\times(\boldsymbol{q}\times\boldsymbol{H}), \tag{5}$$

where μ_e, the medium permeability, is assumed to be constant. Equation (3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.

Let $\delta\rho$, δp, $\boldsymbol{q}(u,v,w)$ and $\boldsymbol{h}(h_x,h_y,h_z)$ denote, respectively, the perturbations in density $\rho(z)$, pressure $p(z)$, velocity $\boldsymbol{v}(0,0,0)$ and horizontal magnetic field $\boldsymbol{H}(H,0,0)$. Then the equations (1)–(5) after perturbations in the cartesian form become

$$\frac{\rho}{\in}\frac{\partial u}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial}{\partial x}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 u+\frac{\mu_e}{4\pi}\left(h_z\frac{\partial}{\partial z}H_0\right)+\frac{2}{\in}\rho\, v\,\Omega, \tag{6}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial}{\partial y}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 v+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y-\frac{\partial}{\partial y}h_x\right)-\frac{2}{\in}\rho\, u\,\Omega, \tag{7}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial}{\partial z}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_z-\frac{\partial}{\partial z}h_x-\frac{h_z}{H_0}\frac{\partial}{\partial z}H_0\right)-g\delta\rho, \tag{8}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{9}$$

$$\in\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial\rho}{\partial z}=0, \tag{10}$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \tag{11}$$

$$\in\frac{\partial}{\partial t}h_x=\frac{\partial}{\partial x}\left\{u\left(H_0+h_x\right)-uh_y\right\}-\frac{\partial}{\partial z}\left\{w\left(H_0+h_x\right)-uh_z\right\}, \tag{12}$$

$$\in\frac{\partial}{\partial t}h_y=\frac{\partial}{\partial z}\left\{vh_z-wh_y\right\}-\frac{\partial}{\partial x}\left\{uh_y-v\left(H_0+h_x\right)\right\}, \tag{13}$$

$$\in\frac{\partial}{\partial t}h_z=\frac{\partial}{\partial x}\left\{w\left(H_0+h_x\right)-uh_z\right\}-\frac{\partial}{\partial y}\left\{vh_z-wh_y\right\}, \tag{14}$$

$$\frac{\rho}{\in}\frac{\partial u}{\partial t}=-\frac{\partial}{\partial x}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 u+\frac{\mu_e}{4\pi}h_z\frac{\partial}{\partial z}H_0+\frac{2}{\in}\rho\, v\,\Omega, \tag{15}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t}=-\frac{\partial}{\partial y}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 v+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y-\frac{\partial}{\partial y}h_x\right)-\frac{2}{\in}\rho\, u\Omega, \tag{16}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t}=-\frac{\partial}{\partial z}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}-\frac{h_x}{H_0}\frac{\partial H_0}{\partial z}\right)-g\delta\rho, \tag{17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{18}$$

$$\in\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial\rho}{\partial z}=0, \tag{19}$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \tag{20}$$

$$\in\frac{\partial}{\partial t}h_x=H_0\frac{\partial}{\partial x}u-w\frac{\partial}{\partial z}H_0, \tag{21}$$

$$\in\frac{\partial}{\partial t}h_y=H_0\frac{\partial}{\partial x}v, \tag{22}$$

$$\in\frac{\partial}{\partial t}h_z=H_0\frac{\partial}{\partial x}w\cdot \tag{23}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, z and time t is given by

$$f(z)\exp\left(ik_x x+ik_y y+nt\right), \tag{24}$$

where $f(z)$ is the some function of z–only; k_x, k_y are the wave-numbers in the x– and y–directions, respectively, $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave-number and n is the growth rate of the disturbance which is, in general, a complex constant.

Equations (15)–(23) using expression (24) become

$$\frac{\rho}{\in} n\, u = -ik_x \delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)u + \frac{\mu_e}{4\pi} h_z\, DH_0 + \frac{2}{\in}\rho\, v\, \Omega, \tag{25}$$

$$\frac{\rho}{\in}\, n\, v = -ik_y \delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)v + \frac{\mu_e H_0}{4\pi}\left(ik_x h_y - ik_y h_x\right) + \frac{2}{\in}\rho u \Omega, \tag{26}$$

$$\frac{\rho}{\in}\, n\, w = -D\,\delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)w + \frac{\mu_e H_0}{4\pi}\left(ik_x h_z - Dh_x - \frac{h_x DH_0}{H_0}\right) - g\delta\rho, \tag{27}$$

$$ik_x\, u + ik_y v + Dw = 0, \tag{28}$$

$$\in n\delta\rho + wD\rho = 0, \tag{29}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \tag{30}$$

$$\in nh_x = ik_x H_0\, u - w\, DH_0, \tag{31}$$

$$\in nh_y = ik_x H_0 v, \tag{32}$$

$$\in nh_z = ik_x H_0 w, \tag{33}$$

Now substituting the values of h_x, h_y and h_z from equations (31)–(33) in equations (25)–(27), we get

$$\frac{\rho}{\in} n\, u = -ik_x \delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)\, u + \frac{\mu_e}{4\pi}\left(\frac{ik_x H_0 w}{n}\right) DH_0 + \frac{2}{\in}\rho\, v\Omega, \tag{34}$$

$$\frac{\rho}{\in} n\, v = -ik_y \delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)\, v + \frac{\mu_e}{4\pi} H_0\left(\frac{ik_x H_0 \zeta_z}{n} + \frac{ik_y\, w\, DH_0}{n}\right) - \frac{2}{\in}\rho\, u\Omega, \tag{35}$$

$$\frac{\rho}{\in} n\, w = -D\,\delta p - \frac{1}{k_1}(\mu - \mu' n)(D^2 - k^2)w + \frac{\mu_e H_0}{4\pi \in}\left[-\frac{k_x^2 H_0 w}{n} - D\left(\frac{ik_x H_0\, u}{n} - \frac{w\, DH_0}{n}\right) - \left(\frac{ik_x H_0 u}{n} - \frac{w\, DH_0}{n}\right)\frac{DH_0}{H_0}\right] + \frac{g\,(D\rho)\, w}{\in}, \tag{36}$$

where $\zeta_z = ik_x v - ik_y u$, is the z–component of vorticity.

Multiplying equations (34) and (35) by $-ik_y$ and ik_x, respectively, and then adding we get

$$\frac{\rho\, n}{\in}\zeta_z = -\frac{\rho}{k_1}(\upsilon + \upsilon' n)(D^2 - k^2)\zeta_z - \frac{\mu_e\, k_x^2 H_0^2}{\in 4\pi\, n}\zeta_z + \frac{2}{\in}\Omega\, Dw,$$

or
$$\zeta_z = \frac{2\, n\, \Omega\, Dw}{n^2 - \frac{\in n}{k_1}(\upsilon - \upsilon' n)(D^2 - k^2) + k_x^2 V_A{}^2}, \tag{37}$$

where $\upsilon = \frac{\mu}{\rho}$, $\upsilon' = \frac{\mu'}{\rho}$ and $V_A^2 = \frac{\mu_e H_0^2}{4\pi\rho}$ (square of the Alfvén's velocity).

Substituting the value of ζ_z in equation (35), we get

$$\frac{\rho}{\in} n\, v = -ik_y \delta p - \frac{1}{k_1}(\mu - \mu' n)\,(D^2 - k^2)\, v - \frac{\mu_e H_0}{4\pi \in n}\left(\frac{2\,\Omega\, n\, Dw\;\, ik_x}{n^2 - n(\mu - \mu' n)(D^2 - k^2) + k_x^2 V_A^2}\right) + \frac{\mu_e H_0}{4\pi\, n} ik_y w\, D(H_0) - 2\frac{\rho}{\in}u\Omega. \tag{38}$$

Multiplying equations (34) and (36) by $-ik_x$ and $-ik_y$, respectively, and then adding and using (28), we obtain

$$\frac{\rho}{\in} n\, Dw = -k^2 \delta p - \frac{\rho}{k_1}(\upsilon - \upsilon' n)(D^2 - k^2)Dw + \left(\frac{2n\Omega}{n^2 - \frac{\in n}{k_1}(\upsilon - \upsilon' n)(D^2 - k^2) + V_A{}^2 k_x^2}\right)\left(\frac{\mu_e H_0^2}{4\pi \in n}k_x^2 k_y - \frac{2}{\in}\rho\right)Dw \tag{39}$$

Eliminating u, v and δp from equations (35)–(39) using equations (29), after little algebra, we get

$$\in \frac{n\rho}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)^2 w-\frac{n\rho}{k_1}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)w-\left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2-\frac{n\in}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)+k_x{}^2V_A^2}\right)-\frac{\mu_e k_x^2 D\left(H_0^2\right)}{4\pi\in n}\right]$$

$$Dw+gk^2(D\rho)w=0. \qquad (40)$$

Equation (40) is the general equation formulating the effect of variable magnetic field and uniform rotation on the stability of stratified Walters' (model B′) fluid saturating porous medium.

3. THE CASE OF EXPONENTIALLY VARYING STRATIFICATIONS:

In order to obtain the solution of the stability problem of a layer of Walters' (model B′) fluid, we suppose that the density ρ, viscosity μ, viscoelasticity μ' medium porosity $\in$ and medium permeability μ' vary exponentially along the vertical direction i.e.

$$\rho=\rho_0\, e^{\,\beta_1 z},\quad \mu=\mu_0\, e^{\beta_1 z},\quad \mu'=\mu_0'\, e^{\beta_1 z},\quad \in=\in_0\, e^{\beta_1 z}, k_1=k_{10}e^{\beta_1 z} \qquad (41)$$

where $\rho_0,\ \mu_0,\ \mu_0',\ H_1,\ \in_0,\ k_{10}$ and β_1 are constants and so the kinematic viscosity $\upsilon\left(=\frac{\mu}{\rho}=\frac{\mu_0}{\rho_0}\right)$, the kinematic viscoelasticity $\upsilon'\left(=\frac{\mu'}{\rho}=\frac{\mu_0'}{\rho_0}\right)$ and the Alfvén velocity $V_A=\left(\frac{\mu_e H_0^2}{4\pi\rho}\right)^{1/2}=\left(\frac{\mu_e H_1{}^2}{4\pi\rho}\right)^{1/2}$ are constant everywhere.

Using the stratifications of the form (41), equation (40) transforms to

$$\left(D^2-k^2\right)^3 w-\frac{2}{\frac{\in_0 n}{k_{10}}(\upsilon_0-\upsilon_0' n)}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)^2 w+\frac{1}{n^2\frac{\in_0}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[n^4+k_x^2V_A^2\left(2n^2+k_x^2V_A^2\right)-V_A^2 k_x^2\beta_1 n(\upsilon_0-\upsilon_0' n)\right.$$

$$\left.-g k^2\beta_1 n(\upsilon_0-\upsilon_0' n)\right]\left(D^2-k^2\right)w-\frac{1}{\frac{\in_0 n}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[4\Omega^2 n^2+V_A^2k_x^2\beta_1\left(n^2+k_x^2V_A^2\right)-g k^2\beta_1\left(n^2+k_x^2V_A^2\right)w\right]=0.$$

(42)

Considering the case of two free boundaries, we must have

$w=D^2w=0$ at $z=0$ and $z=d$. (43)

The appropriate solution of equation (42) satisfying the above boundary condition is

$$w=A_0\sin\frac{m\pi z}{d}, \qquad (44)$$

where m is an integer and A_0 is a constant.

Substituting the value of w from equation (44) in equation (42) we obtain dispersion relation

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\,\upsilon_0^2+\left(2k_x^2V_A^2-\frac{g k^2\beta_1}{L_3}\right)(1-\upsilon_0' L_3)+\frac{4\in_0\Omega^2k_x^2}{L_3k_{10}{}^2}-\right.$$

$$\left.\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\right]+n\left[v_0L_3\left(2k_x^2V_A^2-\frac{\in_0{}^2 g\beta_1k^2}{k_{10}{}^2L_3}\right)-\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\right]+k_x^2V_A^2\left[k_x^2V_A^2-\frac{\beta_1}{L_3}\left(g k^2+V_A^2k_x^2\right)\right]=0, \qquad (45)$$

where $L_3=\left[k^2+\frac{m_1^2\pi^2}{d^2}\right]$.

Equation (45) is biquadratic in n and is the dispersion relation governing the effects of uniform rotation, variable horizontal magnetic field, viscosity, viscoelasticity medium

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permeability and medium porosity on the stability of stratified Walters' (model B′) fluid.

4. RESULTS AND DISCUSSIONS:

(a) Case of stable stratifications (i.e. $\beta_1 < 0$) and $\left(k_{10} > 4\epsilon_0\, v_o'\right)$, Equation (45) does not admit any positive real root or complex root with positive real part using Routh–Hurwitz criterion; therefore, the system is always stable for disturbances of all wave-number.

(b) Case of unstable stratifications (i.e. $\beta_1 > 0$) and $\left(k_{10} < 4\epsilon_0\, v_o'\right)$, If $\beta_1 > 0$, $\frac{k_x^2 V_A^2}{k^2}\left(1-\frac{\beta_1}{L_3}\right) < \frac{\beta_1}{L_3} g$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part using Routh–Hurwitz criterion; so the system is unstable for all wave-numbers satisfying the inequality

$$k^2 < \frac{\beta_1 d^2 g \sec^2\theta - V_A^2\left(m_1^2\pi^2 - \beta_1 d^2\right)}{V_A^2 d^2}, \tag{46}$$

where θ is the angle between k_x and k i.e. $\left(k_x = k\cos\theta\right)$.

If $\beta_1 > 0$, (unstable stratifications) $1 > \frac{\beta_1}{L_3}$ and $V_A^2 > \frac{\beta_1\, g\, k^2}{L_3\, k_x^2\left(1-\frac{\beta_1}{L_3}\right)}$, equation (45) does not admit of any positive real root or complex root with positive real part, therefore, the system is stable. The system is clearly unstable in the absence of magnetic field, rotation and for non–viscoelastic fluid.

$$n^4\left[\left(1-\frac{2\epsilon_0}{k_{10}}\upsilon_0' L_3\right)^2\right] + n^3\left[2\upsilon_0 L_3\left(1-\frac{2\epsilon_0}{k_{10}}\upsilon_0' L_3\right)\right] + n^2\left[L_3^2\,\upsilon_0^2 - \frac{g\,k^2\beta_1}{L_3}\left(1-\upsilon_0' L_3\right) + \frac{1}{L_3}V_A^2\, k_x^2\,\beta_1\left(1-\upsilon_0' L_3\right)\right] - n\left[v_0 L_3\,\frac{\epsilon_0^{\ 2}\, g\beta_1 k^2}{L_3 k_{10}^{\ 2}}\right] = 0. \tag{47}$$

For $\beta_1 > 0$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part therefore the system is clearly unstable. The magnetic field, therefore, stabilizes potentially unstable stratifications for small wave-length perturbations

$$k^2 > \frac{\beta_1\, d^2 g \sec^2\theta - V_A^2\left(m_1^2\pi^2 - \beta_1 d^2\right)}{V_A^2 d^2}. \tag{48}$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wave-length perturbations satisfying equation (48) remain unstable and are not stabilized by magnetic field.

The behaviour of growth rates with respect to kinematic viscosity υ_0, kinematic viscoelasticity υ_0' and square of the Alfvén velocity V_A^2 satisfying equation (45) has been examined numerically using Newton–Raphson method through the software Mathcad. Figure (1) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible value of $\beta_1 = 2$, $\epsilon_0 = 0.5, k_{10} = 6, m_1 = 1$, $d = 6$ cm, $\Omega = 1$revolution/minute, $\upsilon_0' = 1$, $g = 980\text{cm/s}^2$, $k_x = k\cos 45^0$, $V_A^2 = 55$ for three values of $\upsilon_0' = 2$, 3 and 4 respectively. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [8] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscoelasticity υ_0', which indicates the destabilizing effect of viscoelasticity whereas the growth rate decreases for certain wave numbers implying thereby the stabilizing effect of kinematic viscoelasticity on the system in the presence of medium permeability and medium porosity for low wave numbers range.

Figure (2) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0' = 1,\quad g = 980\text{cm/s}^2,\ k_x = k\cos 45^0,\ V_A^2 = 55, \in_0 = 0.5, k_{10} = 6,$ for three values of $\upsilon_0 = 2,$ 4 and 6 respectively. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscosity υ_0 which indicates the destabilizing influence of kinematic viscosity, whereas the growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of kinematic viscosity on the system in the presence of medium permeability and medium porosity.

Figure (3) shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0 = 4,\quad \upsilon_0' = 2,\quad g = 980\,\text{cm/s}^2,\ k_x = k\cos 45^0, \in_0 = 0.5, k_{10} = 6$ for two values of $V_A^2 = 15$ and 55 respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in the square of the Alfvén velocity V_A^2 for certain wave number which indicates the destabilizing influence of the square of the Alfvén velocity, whereas growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of the square of the Alfvén velocity on the system in the presence of medium permeability and medium porosity.

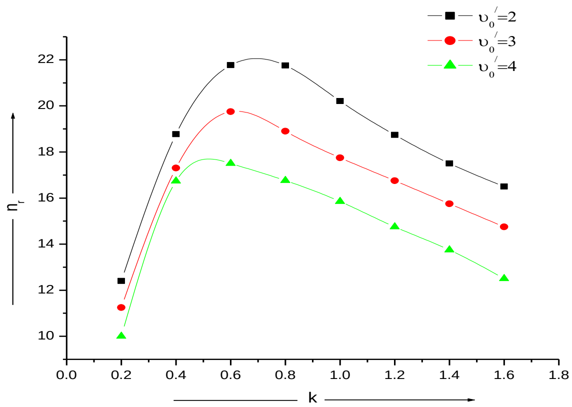

Figure 1: The variation of n_r with wave-number k for three values of $\upsilon_0' = 2,3,4$.

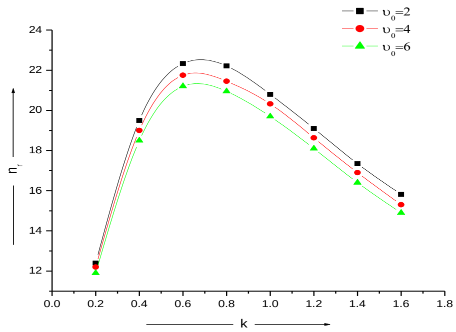

Figure 2: The variation of n_r with wave-number k for three values of $\upsilon_0 = 2, 4, 6$.

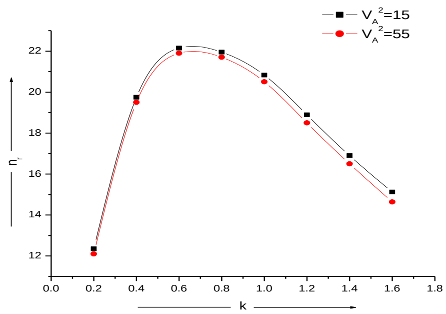

Figure 3: The variation of n_r with wave-number k for two values of $V_A^2 = 15, 55$.

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ON TRIPLY DIFFUSIVE CONVECTION IN A DENSELY PACKED POROUS MEDIUM: DARCY MODEL

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ABSTRACT:
It is mathematically established that thetriply diffusive convection in a densely packed porous medium with one of the components as heat, with diffusivity κ, cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively(with no loss of generality $\kappa > \kappa_1 > \kappa_2$), the porous parameterA and a constant P_1satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$. The result thus obtained is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection, Rayleigh number, Concentration Rayleigh number, Porous medium, Darcy Model.

INTRODUCTION:
Thermohaline convection has received much attention during some decades. It has outstanding applications in various fields of modern science like chemical engineering, nuclear engineering, astrophysics, oceanography, geophysics etc. The conditions under which thermohaline convection occurs has been extensively studied by Brandt and Fernando[1], Turner [2], Lapwood [3], Nield and Bejan [4], Radko [5], Wooding [6], Vafai [7], Abdullah and Alkazmi [8], Straughan [9] etc.

Later it is observed thatthere are many situations in nature wherein density of fluid systems depends upon more than two independently stratifying agencies having different diffusivities.Examples of such triply diffusive fluid systems include the Earth's molten core, solidification of molten alloys, ocean water, geothermally heated lakes etc. Various researchers have contributed towards the theoretical as well as analyticalstudy of the onset of convection in triply diffusive fluid systems. For a broad view of the subject one may referred to Griffiths [10], Pearlstein et al. [11], Terrones [12], Lopez et al. [13], Rionero [14], Prakash et al. [15,16,17], Ghalambaz et al. [18] etc.

Banerjee et al. [19] derived a characterization theorem for thermohaline convection problem which proves the non-existence of oscillatory motions of growing amplitude in an initially bottom heavy configuration in a certain parameter regime and paved the way for further theoretical and experimental investigations in this field of enquiry. Later, Prakash et al. [20,21] further extended these results of Banerjee et al. [19] to more general triply diffusive configurations in the domain of astrophysics, wherein liquid concern has the property of magnetic field and rotation prevalent. As a further step, in the present paper, we have derived such characterization theorem for the triply diffusive convection in a densely packed porous medium.

MATHEMATICAL FORMULATION:

Consider an infinite horizontal densely packed porous layer filled with a viscous and Boussinesq fluid, statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11}(< S_{10}), S_{21}(< S_{20})$ in the force field of gravity (seeFig.1). It is assumed that the porous medium is of constant porosity. The Darcy model has been utilized to study the triply diffusive convection in a densely packed porous medium.

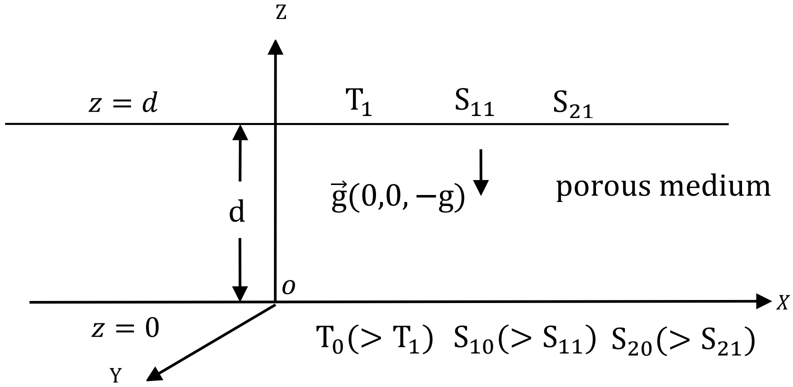

Fig.1. Physical configuration

The governing hydrodynamic equations and boundary conditions for the present problem in their non-dimensional form, are given by (Wankat and Schowalter [22], Vafai [7] and Prakash et al. [23]):

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R\, a^2\theta + R_1 a^2 \phi_1 + R_2 a^2 \phi_2, \tag{1}$$

$$(D^2 - a^2 - Ap)\theta = -w\,, \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}\,, \tag{3}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}\,, \tag{4}$$

The Eqs. (1) -(4) are to be solved by using the following boundary conditions:

$w = 0 = \theta = \phi_1 = \phi_2 = Dw$ at $z = 0$ and at $z = 1$,
(when both the boundaries are rigid) (5)

or $w = 0 = \theta = \phi_1 = \phi_2 = D^2 w$ at $z = 0$ and at $z = 1$,
(when both the boundaries are free) (6)

where z is the real independent variable such that $0 \le z \le 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number, $\tau_1, \tau_2 > 0$ are the Lewis numbers for two concentration components respectively, $R > 0$ is the Rayleigh number, $R_1 > 0$ and $R_2 > 0$ are the two concentration Rayleigh numbers, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the two concentrations, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z), \theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi_1(z) = \phi_{1r}(z) + i\phi_{1i}(z)$ and $\phi_2(z) = \phi_{2r}(z) + i\phi_{2i}(z)$ are complex valued functions of the real variable z. The governing equations also involve two more positive constants namely$P_1 = \frac{k_1}{\epsilon d^2}$ and $A = 1 + \frac{\rho_s c_s}{\rho_0 c_0}\frac{(1-\epsilon)}{\epsilon}$, where$k_1$ is the permeability,ϵ is the porosity of the porous medium, d is the depth of the fluid layer;$\rho_s$is the solid density, c_s is the heat capacity of the solid. The suffix $'0'$ denotes the values of various parameters involved in the governing equations at some properly chosen reference temperature T_0.

It may further be noted that the Eqs.(1) -(6) describe an eigen value problem for p and govern triply diffusive convectionin a densely packed porous medium with constant porosity for any combination of rigid and free boundaries.

MATHEMATICAL ANALYSIS:

Now we prove the following theorem:

Theorem 1: If $R > 0, R_1 > 0, R_2 > 0, p_r \geq 0, p_i \neq 0,$ and$R_s = R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$then a necessary condition for the existence of non-trivial solution $(w, \theta, \phi_1, \phi_2, p)$ of Eqs.(1)-(4) together with boundary conditions (5) or (6) is that$R_s = R_1 + R_2 < R$.

Proof: Multiplying Eq. (1) by w* (the superscript * henceforth denotes complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\,dz = -R\,a^2\int_0^1 w^*\,\theta\,dz + R_1 a^2\int_0^1 w^*\,\phi_1\,dz + R_2 a^2\int_0^1 w^*\,\phi_2\,dz. \quad (7)$$

Using Eqs. (2)-(4) and the fact, that w(0) = 0 = w(1), we can write

$$R\,a^2\int_0^1 w^*\,\theta\,dz = -Ra^2\int_0^1 \theta\,(D^2 - a^2 - Ap^*)\theta^* dz, \quad (8)$$

$$R_1 a^2\int_0^1 w^*\,\phi_1 dz = -R_1 a^2\tau_1\int_0^1 \phi_1\left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz, \quad (9)$$

$$R_2 a^2\int_0^1 w^*\,\phi_2 dz = -R_2 a^2\tau_2\int_0^1 \phi_2\left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz. \quad (10)$$

Combining Eqs. (7) -(10), we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\,dz = Ra^2\int_0^1 \theta\,(D^2 - a^2 - Ap^*)\theta^* dz - R_1 a^2\tau_1\int_0^1 \phi_1\left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz - R_2 a^2\tau_2\int_0^1 \phi_2\left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz \quad (11)$$

Integrating various terms of Eq. (11), by parts, for an appropriate number of times and making use of either of the boundary conditions (5)-(6), it follows that

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 (|Dw|^2 + a^2|w|^2)\,dz = Ra^2\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + Ap^*|\theta|^2)\,dz - R_1 a^2\tau_1\int_0^1\left(|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{p^*}{\tau_1}|\phi_1|^2\right)dz - R_2 a^2\tau_2\int_0^1\left(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{p^*}{\tau_2}|\phi_2|^2\right)dz \quad (12)$$

Equating the real and imaginary parts of both sides of Eq. (12) and cancelling p_i $(\neq 0)$throughout from the imaginary part, we get

$\left(\frac{p_r}{\sigma}+\frac{1}{P_1}\right)\int_0^1(|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1(|D\theta|^2+\ a^2|\theta|^2+Ap_r|\theta|^2)\,dz -$
$R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2+\ a^2|\phi_1|^2+\frac{p_r}{\tau_1}|\phi_1|^2\right)dz - R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2+\ a^2|\phi_2|^2+\right.$
$\left.\frac{p_r}{\tau_2}|\phi_2|^2\right)dz$ v (13)

and

$\int_0^1(|Dw|^2+\ a^2|w|^2)dz = -Ra^2A\sigma\int_0^1|\theta|^2dz + R_1a^2\sigma\int_0^1|\phi_1|^2\,dz + R_2a^2\sigma\int_0^1|\phi_2|^2\,dz$
(14)

We write Eq. (13) in an alternative form as

$$p_r\int_0^1(|Dw|^2+\ a^2|w|^2)\,dz+\frac{\sigma}{P_1}\int_0^1(|Dw|^2+\ a^2|w|^2)\,dz$$
$$= Ra^2\sigma\int_0^1(|D\theta|^2+\ a^2|\theta|^2)dz - R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+\ a^2|\phi_1|^2)\,dz$$
$$- R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+\ a^2|\phi_2|^2)dz$$
$$+p_r\left[Ra^2A\sigma\int_0^1|\theta|^2dz - R_1a^2\sigma\int_0^1|\phi_1|^2dz - R_2a^2\sigma\int_0^1|\phi_2|^2\,dz\right]$$
(15)

and derive the desired result from the resulting inequality obtained by replacing each term of this equation by its appropriate estimate.

We first note that since w, θ, ϕ_1 and ϕ_2 satisfy $w(0)=0=w(1)$, $\theta(0)=0=\theta(1)$, $\phi_1(0)=0=\phi_1(1)$ and $\phi_2(0)=0=\phi_2(1)$, we have by Rayleigh-Ritz inequality (Schultz[24])

$\int_0^1|Dw|^2dz \geq \pi^2\int_0^1|w|^2dz,$
(16)
$\int_0^1|D\theta|^2dz \geq \pi^2\int_0^1|\theta|^2dz,$
(17)
$\int_0^1|D\phi_1|^2dz \geq \pi^2\int_0^1|\phi_1|^2dz,$
(18)
$\int_0^1|D\phi_2|^2dz \geq \pi^2\int_0^1|\phi_2|^2dz,$ (19) Further, since $p_r \geq 0$, we note that
$p_r\int_0^1(|Dw|^2+a^2|w|^2)dz \geq 0.$ (20)

Now multiplying Eq. (2) by θ^* and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely $\theta(0)=0=\theta(1)$, we have from the real part of the final equation

$\int_0^1(|D\theta|^2+\ a^2|\theta|^2+Ap_r|\theta|^2)dz = \text{Real part of}\int_0^1\theta^*w\,dz,$
$\leq\left|\int_0^1\theta^*w\,dz\right|,$
$\leq\int_0^1|\theta^*w|dz,$
$\leq\left(\int_0^1|\theta|^2dz\right)^{1/2}\left(\int_0^1|w|^2dz\right)^{1/2}$(using Schwartz inequality). (21)

Combining the above inequality with inequality(17) and the fact that $p_r \geq 0$, we have

$$(\pi^2 + a^2)\int_0^1 |\theta|^2 dz \le \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2},$$

which gives that

$$\left(\int_0^1 |\theta|^2 dz\right)^{1/2} \le \frac{1}{(\pi^2 + a^2)}\left(\int_0^1 |w|^2 dz\right)^{1/2},$$

and thus inequality (21) gives

$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz \le \frac{1}{(\pi^2+a^2)}\int_0^1 |w|^2 dz$.(22) Further, combining the above inequality with inequality(17), we get

$\int_0^1 |\theta|^2 dz \le \frac{1}{(\pi^2+a^2)^2}\int_0^1 |w|^2 dz$.(23)

Now using inequalities (18) and (19) it follows that

$$R_1 a^2 \sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\,dz + R_2 a^2 \sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2)dz$$

$$\ge (\pi^2 + a^2)\left(R_1 a^2\sigma\tau_1 \int_0^1 |\phi_1|^2 dz + R_2 a^2 \sigma\tau_2 \int_0^1 |\phi_2|^2 dz\right)$$

$\ge (\pi^2 + a^2)\left\{\tau_1 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz + \tau_2 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz\right\}$ (utilizing Eq. (14)) (24)

and thus

$$-R_1 a^2 \sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\,dz - R_2 a^2 \sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2)dz$$

$\le (\pi^2 + a^2)\left[R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz\right] - (\pi^2 + a^2)(\tau_1 + \tau_2)\int_0^1 (|Dw|^2 + a^2|w|^2)dz.$ (25)

Again, using Eq.(14), we obtain

$R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz \le \tau_2 \int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2 A\sigma\tau_2 \int_0^1 |\theta|^2 dz,$ (26)

and

$R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz \le \tau_1 \int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2 A\sigma\tau_1 \int_0^1 |\theta|^2 dz.$ (27)

Now using inequalities (26) and (27), we can write

$R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz \le (\tau_1 + \tau_2)\int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2 A\sigma(\tau_1 + \tau_2)\int_0^1 |\theta|^2 dz.$ (28)

Using this inequality (28) in inequality (25), we get

$$-R_1 a^2 \sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\,dz - R_2 a^2 \sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2)dz$$

$\le (\pi^2 + a^2)Ra^2 A\sigma(\tau_1 + \tau_2)\int_0^1 |\theta|^2 dz,$

$\le \frac{Ra^2 A\sigma}{(\pi^2+a^2)}(\tau_1 + \tau_2)\int_0^1 |w|^2 dz.$ (using inequality (23)) (29)

Also from Eq. (14) and the fact that$p_r \ge 0$, we obtain

$p_r\left[Ra^2 A\sigma \int_0^1 |\theta|^2 dz - R_1 a^2\sigma \int_0^1 |\phi_1|^2 dz - R_2 a^2\sigma \int_0^1 |\phi_2|^2\, dz\right] < 0$ (30)

Now, if permissible, let $R_s = R_1 + R_2 \ge R$. Then, in that case, we derive from Eq. (15) and the inequalities(16), (20),(22), (29)and (30) that

$$\left[\frac{\sigma(\pi^2+a^2)}{P_1} - \frac{R_s a^2 \sigma}{(\pi^2+a^2)}\left(1 + A(\tau_1 + \tau_2)\right)\right] \int_0^1 |w|^2 dz < 0 \, , \tag{31}$$

which clearly implies that

$$R_s > \frac{\frac{1}{P_1}\left[\frac{(\pi^2+a^2)^2}{a^2}\right]}{[1+A(\tau_1+\tau_2)]} \, ,$$

so that we necessarily have

$$R_s > \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}, \tag{32}$$

since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ (for $a^2 = \pi^2$) is $4\pi^2$.

Hence if $R_s = R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$,then we must have

$$R_s = R_1 + R_2 < R. \tag{33}$$

This establishes the theorem.

The essential content of the theorem from the physical point of view are that the triply diffusive convection in porous medium with one of the components as heat with diffusivity κ cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively (with no loss of generality $\kappa > \kappa_1 > \kappa_2$),the Porous parameter A and a constant P_1 satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$. It is further established that this result is uniformly valid for the quite general nature of the bounding surfaces.

CONCLUSION:

The linear stability theory and Darcy model is used to characterize the onset of triply diffusive convection in a densely packed porous medium. The layer is considered to be heated and soluted from below. The porous medium is taken as isotropic and homogeneous. A mathematical theorem is derived which provides a classification of the neutral or unstable triply diffusive configuration into two classes namely the bottom-heavy class and the top-heavy class and then strikes a distinction between them by means of a characterization theorem which disallow the existence of oscillatory instability in the former class. It is further established that this result is uniformly valid for the quite general nature of the bounding surfaces.

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A MATHEMATICAL THEOREM IN MAGNETOHYDRODYNAMIC TRIPLY DIFFUSIVE CONVECTION IN A DENSELY PACKED POROUS MEDIUM

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ABSTRACT:
In the Present paper, it has been mathematically established that the magnetohydrodynamic triply diffusive convection in a densely packed porous medium with one of the components as heat, with diffusivity κ, cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively, the Porous parameter A and a constant P_1 satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$. It is further proved that the above result is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection, Oscillatory motions,the Lewis numbers, Concentration Rayleigh number, Porous medium, Darcy Model.

INTRODUCTION:

The problem of thermohaline instability in porous medium has been extensively studied in the recent past by many researchers on account of its importance in geophysics, in the fields of oceanography, chemical process industry, food processing industry, solidification, in the prediction of ground water movement in aquifers, in assessing the effectiveness of fibrous insulations, in the energy extraction process from the geothermal reservoirs, in nuclear engineering and due to influence of porous material (like fibre materials used for insulating purposes or metallic foams in heat transfer devices) on human life (Abdullah et al. [1], Deepika and Narayana [4], Lapwood [8], Nield and Bejan [11], Radko [18], Straughan [23],Taunton et al.[24], Vafai [25], Wooding [26], Yadav et al. [27]).

These researchers have studied the physical configurations in whichthe density depends on two stratifying agencies having different diffusivities. However, it has been found later that there are many physical configurations, in whichthe density depends on three or more stratifying agencies having different diffusivities. For example,the Earth's core, saline waters of geothermally heated lakes, sea and the oceans which contain many salts having concentrations less than a few percent of the sodium chloride concentration. For a broad view of the subject one may referred to Ghalambaz et al.[5], Griffiths [6, 7], Lopez [9]**,** Moroz[10], Prakash et al. [15,16, 17], Rionero[19], Rudraiah and Vortmeyer [20], Ryzhkov and Shevtsova [21] andZhao et al. [28**]**. These researchers have theoretically studied the onset of convection in a triply diffusive fluid layer (in porous and non porous medium, where density depends on three independently diffusing agencies with different diffusivities).

Banerjee et al. [2] derived a mathematical theorem in thermohaline convection for the nonexistence of oscillatory motions of growing amplitude in an initially bottom heavy configuration of the Veronis type and brought a fresh outlook to the subject matter of double diffusive convection which pave the way for further theoretical and experimental investigations in this field of enquiry. Later Prakash and Kumar [14] extended their result to thermohaline convection configuration in porous medium.The work of Banerjee et al. [2] have been further extended to double and triply diffusive convection configurations in a porous and non porous medium in the presence of magnetic field by Prakash et al. [12,13]. As a further step in the present problem an analogous mathematical theorem for the certain parameter regime, is derived for magnetohydrodynamic triply diffusive convection in densely packed porous medium which states that oscillatory motions (neutral or unstable) of growing amplitude cannot manifest in an initially bottom heavy configurations of magnetohydrodynamic triply diffusive convection whenever the sum of the concentration Rayleigh numbers is less than a critical value.

MATHEMATICAL FORMULATION:

Let us consider an infinite horizontal densely packed porous layer filled with a viscous and Boussinesq fluid, statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11} (< S_{10})$, $S_{21} (< S_{20})$ in the force field of gravity and in the presence of uniform vertical magnetic field, $\vec{H}$. (seeFig.1). It is assumed that the saturating fluid and the porous layer are incompressible and that the porous medium is a constant porosity medium. The Darcy's [3] modelhas been utilized to study themagnetohydrodynamic triply diffusive convection in a densely packed porous medium.

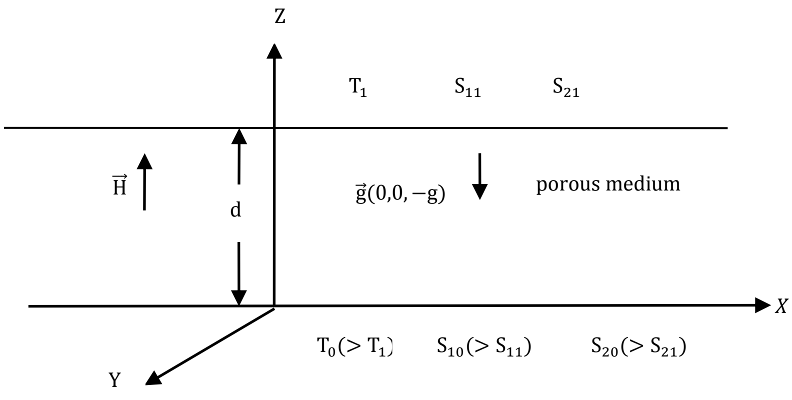

Fig.1. Physical configuration.

The equations that govern the motion of magnetohydrodynamictriply diffusive fluid layer in a densely packed porous medium (Darcy Model), in the non-dimensional form,are given by (Rudraiah and Vortmeyer [20], Prakash et al.[12,16,17]):

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R\,a^2\theta + R_1 a^2 \phi_1 + R_2 a^2 \phi_2 + Q\,D(D^2 - a^2)h_z, \tag{1}$$

$$(D^2 - a^2 - Ap)\theta = -w\,, \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}\,, \tag{3}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}\,, \tag{4}$$

$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right) h_z = -Dw,$ (5)

The Eqs. (1)-(5) are to be solved by using the following boundary conditions:

$w = 0 = \theta = \phi_1 = \phi_2 = D^2 w = h_z$ at $z = 0$ and at $z = 1$,

(when both the boundaries are free and perfectly conducting) (6)

or $w = 0 = \theta = \phi_1 = \phi_2 = Dw = h_z$ at $z = 0$ and at $z = 1$,

(when both the boundaries are rigid and perfectly conducting) (7)

where z is the real independent variable such that $0 \leq z \leq 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number,$\sigma_1 > 0$ is the magnetic Prandtl number, τ_1, $\tau_2 > 0$ are the Lewis numbers for two components respectively, $R > 0$ is the Rayleigh number, $R_1 > 0$ and $R_2 > 0$ are the two concentration Rayleigh numbers, $Q > 0$ is the Chandrasekhar number, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the two concentrations,h_z is the vertical component of the perturbation in the initially external imposed magnetic field,$p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z), \theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi_1(z) = \phi_{1r}(z) + i\phi_{1i}(z)$,$\phi_2(z) = \phi_{2r}(z) + i\phi_{2i}(z)$ and $h_z(z) = h_{z,r}(z) + ih_{z,i}(z)$ are complex valued functions of the real variable z.The governing equations also involve two more positive constants namely$P_1 = \frac{k_1}{\epsilon d^2}$ and $A = 1 + \frac{\rho_s c_s}{\rho_0 c_0}\frac{(1-\epsilon)}{\epsilon}$, where$k_1$ is the permeability,ϵ is the porosity of the porous medium, d is the depth of the fluid layer;$\rho_s$is the solid density, c_s is the heat capacity of the solid. The suffix '0' denotes the values of various parameters involved in the governing equations at some properly chosen reference temperature T_0.

It may further be noted that Eqs.(1)-(7) describe an eigen value problem for p and govern magnetohydrodynamic triply diffusive convectionin a densely packed porous mediumfor any combination of dynamically free and rigid perfectly conducting boundaries.

MATHEMATICAL ANALYSIS:

Now we prove the following theorem:

Theorem: If $R > 0, R_1 > 0, R_2 > 0, Q > 0, p_r \geq 0, p_i \neq 0$, and$R_s = R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$then a necessary condition for the existence of nontrivial solution $(w, \theta, \phi_1, \phi_2, h_z, p)$ ofEqs.(1)-(5) together with boundary conditions (6) or (7) is that$R_s = R_1 + R_2 < R$.

Proof: Multiplying Eq. (1) by w* (the superscript * henceforth denotes complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\, dz = -R\, a^2 \int_0^1 w^*\, \theta\, dz + R_1 a^2 \int_0^1 w^*\, \phi_1\, dz + R_2 a^2 \int_0^1 w^*\, \phi_2\, dz$$

$+Q\int_0^1 w^* D\,(D^2 - a^2)h_z dz.$ (8)

Using Eqs. (2)-(5) and the fact, that w(0) = 0 = w(1), we can write

$R\, a^2 \int_0^1 w^*\, \theta\, dz = -Ra^2 \int_0^1 \theta\,(D^2 - a^2 - Ap^*)\theta^* dz,$ (9)

$R_1 a^2 \int_0^1 w^*\, \phi_1 dz = -R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz,$ (10)

$R_2 a^2 \int_0^1 w^*\, \phi_2 dz = -R_2 a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz,$(11)

$$Q\int_0^1 w^* D\,(D^2-a^2)h_z dz = -Q\int_0^1 Dw^*\,(D^2-a^2)h_z dz = Q\int_0^1 \left(D^2-a^2-\frac{p^*\sigma_1}{\sigma}\right)h_z^*\,(D^2-a^2)h_z dz. \quad (12)$$

Making use of Eqs. (9) – (12), from Eq. (8)we obtain

$$\left(\frac{p}{\sigma}+\frac{1}{P_1}\right)\int_0^1 w^*\,(D^2-a^2)w\,dz = Ra^2\int_0^1 \theta\,(D^2-a^2-Ap^*)\theta^* dz$$
$$-R_1a^2\tau_1\int_0^1 \phi_1\left(D^2-a^2-\frac{p^*}{\tau_1}\right)\phi_1^* dz - R_2a^2\tau_2\int_0^1 \phi_2\left(D^2-a^2-\frac{p^*}{\tau_2}\right)\phi_2^* dz$$
$$+Q\int_0^1 \left(D^2-a^2-\frac{p^*\sigma_1}{\sigma}\right)h_z^*\,(D^2-a^2)h_z dz. \quad (13)$$

Integrating various terms of Eq. (13), by parts, for a suitable number of times and making use of either of the boundary conditions (6)-(7), it follows that

$$\left(\frac{p}{\sigma}+\frac{1}{P_1}\right)\int_0^1 (|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1 (|D\theta|^2+a^2|\theta|^2+Ap^*|\theta|^2)\,dz$$
$$-R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2+a^2|\phi_1|^2+\frac{p^*}{\tau_1}|\phi_1|^2\right)dz - R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2+a^2|\phi_2|^2+\frac{p^*}{\tau_2}|\phi_2|^2\right)dz$$
$$-Q\int_0^1 |(D^2-a^2)h_z|^2 dz - Q\frac{p^*\sigma_1}{\sigma}\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz. \quad (14)$$

Equating the real and imaginary parts of Eq. (14) form both sides and cancelling the term $p_i\ (\neq 0)$ throughout from the imaginary part, we get

$$\left(\frac{p_r}{\sigma}+\frac{1}{P_1}\right)\int_0^1 (|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1 (|D\theta|^2+a^2|\theta|^2+Ap_r|\theta|^2)\,dz$$
$$-R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2+a^2|\phi_1|^2+\frac{p_r}{\tau_1}|\phi_1|^2\right)dz$$
$$-R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2+a^2|\phi_2|^2+\frac{p_r}{\tau_2}|\phi_2|^2\right)dz$$
$$-Q\int_0^1 |(D^2-a^2)h_z|^2 dz - Q\frac{p_r\sigma_1}{\sigma}\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz, \quad (15)$$

and

$$\int_0^1 (|Dw|^2+a^2|w|^2)dz = -Ra^2A\sigma\int_0^1 |\theta|^2 dz + R_1a^2\sigma\int_0^1 |\phi_1|^2\,dz + R_2a^2\sigma\int_0^1 |\phi_2|^2\,dz + Q\sigma_1\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz. \quad (16)$$

Alternatively Eq. (15) can be written as

$$p_r\int_0^1 (|Dw|^2+a^2|w|^2)\,dz + \frac{\sigma}{P_1}\int_0^1 (|Dw|^2+a^2|w|^2)\,dz =$$
$$Ra^2\sigma\int_0^1 (|D\theta|^2+a^2|\theta|^2)dz - R_1a^2\sigma\tau_1\int_0^1 (|D\phi_1|^2+a^2|\phi_1|^2)\,dz$$
$$-R_2a^2\sigma\tau_2\int_0^1 (|D\phi_2|^2+a^2|\phi_2|^2)dz - Q\sigma\int_0^1 |(D^2-a^2)h_z|^2 dz + p_r\Big[Ra^2A\sigma\int_0^1 |\theta|^2 dz -$$

$R_1 a^2\sigma \int_0^1 |\phi_1|^2 dz - R_2 a^2\sigma \int_0^1 |\phi_2|^2\, dz - Q\sigma_1 \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz\Big],$ (17)
andderive the desired result from the resulting inequality obtained by replacing each terms of this equation by its appropriate estimate.
As w, θ, ϕ_1, ϕ_2 and $h_z$satisfy$w(0) = 0 = w(1)$, $\theta(0) = 0 = \theta(1)$,$\phi_1(0) = 0 = \phi_1(1)$,$\phi_2(0) = 0 = \phi_2(1)$and $h_z(0) = 0 = h_z(1)$,therefore, we haveby Rayleigh-Ritz inequality (Schultz[22])

$\int_0^1 |Dw|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz,$ (18)

$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz,$ (19)

$\int_0^1 |D\phi_1|^2 dz \geq \pi^2 \int_0^1 |\phi_1|^2 dz,$ (20)

$\int_0^1 |D\phi_2|^2 dz \geq \pi^2 \int_0^1 |\phi_2|^2 dz,$(21)

$\int_0^1 |Dh_z|^2 dz \geq \pi^2 \int_0^1 |h_z|^2 dz.$ (22)

Since $p_r \geq 0$, thus we can write

$p_r \int_0^1 (|Dw|^2 + a^2|w|^2)dz \geq 0.$ (23)

Now multiplying Eq. (2) by θ^* and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely$\theta(0) = 0 = \theta(1)$, we have from the real part of the final equation

$\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + Ap_r|\theta|^2)dz = \text{Real part of} \int_0^1 \theta^* w dz,$

$\leq \left|\int_0^1 \theta^* w dz\right|,$

$\leq \int_0^1 |\theta^* w| dz,$

$\leq \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2}$(using Schwartz inequality) . (24)

Making use of inequality (19) and the fact that $p_r \geq 0$, the above inequality (24) reduces to

$(\pi^2 + a^2) \int_0^1 |\theta|^2 dz \leq \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2},$

which gives that

$\left(\int_0^1 |\theta|^2 dz\right)^{1/2} \leq \frac{1}{(\pi^2 + a^2)} \left(\int_0^1 |w|^2 dz\right)^{1/2},$

and thus inequality (24) gives

$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz \leq \frac{1}{(\pi^2 + a^2)} \int_0^1 |w|^2 dz.$(25) Further, combining the above inequality with inequality(19), we get

$\int_0^1 |\theta|^2 dz \leq \frac{1}{(\pi^2 + a^2)^2} \int_0^1 |w|^2 dz\, .$ (26)

Now using inequalities (20) and (21) it follows that

$$R_1 a^2\sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\, dz + R_2 a^2\sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2)dz$$
$$\geq (\pi^2 + a^2)\left(R_1 a^2\sigma\tau_1 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_2 \int_0^1 |\phi_2|^2 dz\right)$$

$\geq (\pi^2 + a^2)\Big\{\tau_1 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz - Q\sigma_1\tau_1 \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz + \tau_2 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz - Q\sigma_1\tau_2 \int_0^1 (|Dh_z|^2 +$

$a^2|h_z|^2)dz\Big\}$, (utilizing Eq. (16)) (27)

and thus

$$-R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+a^2|\phi_1|^2)\,dz-R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+a^2|\phi_2|^2)dz$$

$\leq(\pi^2+a^2)\left[R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz+R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\right]+(\pi^2+a^2)Q\sigma_1(\tau_1+\tau_2)\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz-(\pi^2+a^2)(\tau_1+\tau_2)\int_0^1(|Dw|^2+a^2|w|^2)dz.$ (28)Further, using Eq.(16), we obtain

$R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz\leq\tau_2\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma\tau_2\int_0^1|\theta|^2dz-Q\sigma_1\tau_2\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz$, (29)

and

$R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\leq\tau_1\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma\tau_1\int_0^1|\theta|^2dz-Q\sigma_1\tau_1\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz.$ (30)

Combining inequalities (29) and (30), we have

$R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz+R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\leq(\tau_1+\tau_2)\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma(\tau_1+\tau_2)\int_0^1|\theta|^2dz-Q\sigma_1(\tau_1+\tau_2)\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz$, (31)

and thus utilizing the above inequality (31) in inequality (28), we obtain

$$-R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+a^2|\phi_1|^2)\,dz-R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+a^2|\phi_2|^2)dz$$

$\leq(\pi^2+a^2)Ra^2A\sigma(\tau_1+\tau_2)\int_0^1|\theta|^2dz,$

$\leq\frac{Ra^2A\sigma}{(\pi^2+a^2)}(\tau_1+\tau_2)\int_0^1|w|^2dz.$ (using inequality (26)) (32)

Further, using the fact that $p_r\geq 0$, we can write from Eq. (16) as

$p_r\left[Ra^2A\sigma\int_0^1|\theta|^2dz-R_1a^2\sigma\int_0^1|\phi_1|^2dz-R_2a^2\sigma\int_0^1|\phi_2|^2\,dz-Q\sigma_1\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz\right]<0.$ (33)

Now, if permissible, let $R_s=R_1+R_2\geq R$. Then, in that case, we derive from Eq. (17) and the inequalities(18), (23),(25), (32)and (33) that

$\left[\frac{\sigma(\pi^2+a^2)}{P_1}-\frac{R_sa^2\sigma}{(\pi^2+a^2)}(1+A(\tau_1+\tau_2))\right]\int_0^1|w|^2dz+Q\sigma\int_0^1|(D^2-a^2)h_z|^2dz<0$, (34)

which clearly implies that

$R_s>\frac{(\frac{(\pi^2+a^2)^2}{a^2})}{P_1(1+A(\tau_1+\tau_2)}$,

so that we necessarily have

$R_s>\frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$, (35)

since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ (for $a^2=\pi^2$) is $4\pi^2$.

Hence if $R_s=R_1+R_2\leq\frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$, then we must have

$R_s=R_1+R_2<R.$ (36)

This establishes the theorem.

The essential content of the theorem from the physical point of view are that magneto hydrodynamic triply diffusive convection in densely packed porous medium with one of the components as heat with diffusivity κ cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivitiesκ_1 and κ_2 respectively (with no loss of generality $\kappa > \kappa_1 > \kappa_2$),the Porous parameterA and a constant P_1satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$. It is further established that this result is uniformly valid for any combination of free and/or rigid perfectly boundaries.

CONCLUSION:

The onset of magnetohydrodynamic triply diffusive convectionin a horizontal layer of fluid heated and soluted from below is investigated usinglinear stability analysis. The Darcy model has been utilized to derive amathematical theorem which disapprove the existence of oscillatory motions of growing amplitude in an initially bottom heavymagnetohydrodynamic triply diffusive configurations in a densely packed porous medium.The work done in the present paper will certainly pave the way for further theoretical and experimental investigations in this field of enquiry.

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STABILITY ANALYSIS OF THERMOHALINE CONVECTION PROBLEM IN A ROTATING VISCOELASTIC FLUID LAYER

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ABSTRACT:
The eigenvalue problem governing the Veronis type thermohaline convection in a viscoelastic fluid layer in the presence of uniform rotation is mathematically investigated for both linear and nonlinear stability. The expressions for Rayleigh numbers are derived for various orders of stability problems using linear and nonlinear stability analysis, for the cases of infinite and finite amplitude of disturbances, respectively. The effects of various parameters namely; rotation, stress relaxation time, strain retardation time and concentration gradient on steady and oscillatory thermal Rayleigh numbers have been discussed numerically for infinite amplitude of distubances and the results are depicted graphically. It is observed that the stress relaxation time has destabilizing effect, however the strain retardation time, concentration gradient and rotation have stabilizing effect on the onset of oscillatory thermohaline convection in a viscoelastic fluid. Also, it is observed from expression for Rayleigh number for steady second order nonlinear stability problem for finite disturbances that the viscoelastic parameters have no effects on the onset of instability. However, the concentration gradient and rotation reinforce each other and have destabilizing effect on the onset of instability.

KEYWORDS: Thermohaline convection; Oscillatory convection; Viscoelastic fluid; Rotation; Finite amplitude; Critical Rayleigh number.

1. INTRODUCTION:

Thermohaline convection or, more generally, double diffusive convection is an important phenomenon that involves the motions driven by two different density gradients diffusing at different rates, namely; thermal and concentration gradients. Thermohaline convection has its importance in the fields of oceanography, modelling of solar ponds, geophysics, astrophysics, chemical engineering, atmospheric pollution, etc. and various applications of the problem have aroused the interest of many research workers in this subject.

Stern [1] was the first to study the thermohaline (double diffusive) convection in the context of ocean by considering stable temperature and unstable concentration gradients, now known as Stern's type configuration and derived some results concerning the instability of this configuration. He found that both steady as well as oscillatory motions of growing amplitudes occur in this configuration and steady motion is preferred one. Veronis [2] also studied the thermohaline configuration heated from below and salted from above creating destabilizing temperature gradient and stabilizing concentration gradient and derived the expressions for Rayleigh numbers for both stationary as well as oscillatory convection, when disturbances are infinitesimal small. This configuration is known as Veronis' type configuration. The thermohaline convection problems of Stern and Veronis types have been revisited by Gupta et al[3] in the context of initially bottom/top heavy configurations and derived certain conditions for the stability and the bounds for the growth rate of oscillatory convection for both bottom/top heavy configurations, which are valid for all combinations of dynamically free rigid boundaries.

Veronis [2] in his analysis also discussed the two dimensional convective rolls in thermohaline convection by considering finite amplitude disturbances and predicted that the system becomes unstable to finite amplitude steady disturbances before it becomes unstable to disturbances of infinitesimal amplitude. Gupta and Gupta [4] extended the analysis of Veronis to include the effect of uniform rotation on thermohaline convective rolls and studied the effect of Taylor's number on the onset of steady as well as oscillatory convection with both finite and infinite disturbances.

For a broad view of the thermohaline convection, one may be referred to Turner and Stommel[5], Bains and Gill [6], Turner [7], Huppert and Turner [8], Brandt and Fernando [9], Radko[10].

Fluids are classified as Newtonian or non-Newtonian on the basis of the linear or non-linear relation between shearing stress to the rate of strain, respectively. A non-Newtonian fluid may not have well defined viscosity, therefore it is inadequate to describe the behaviour of the non-Newtonian fluids and is best studied through other rheological properties. Viscoelastic fluids fall in the category of non-Newtonian fluids. Idealized viscoelastic fluids are those fluids whose behaviour at sufficiently small variable shear stresses can be characterized by three constants i.e. coefficient of viscosity, relaxation time and retardation time. A non-linear theory of class of isotropic incompressible viscoelastic fluid with the stress relaxation time ($\bar{\lambda}_1$) and strain retardation time ($\bar{\lambda}_2$) are known as Oldroydian fluid (Oldroyd [11, 12]) and the viscoelastic fluid is termed as Maxwellian fluid when strain retardation time in the constitutive relation is absent ($\text{i.e. } \bar{\lambda}_2 = 0$). However, when both stress relaxation time and strain retardation time are zero ($\text{i.e. } \bar{\lambda}_1 = \bar{\lambda}_2 = 0$), the above constitutive relation reduced to a relation applicable for Newtonian viscous fluid. Industrial fluid are basically non-Newtonian; in particular viscoelastic fluids have been relevance in industrial applications. The characteristics of heat transfer in a viscoelastic fluid layer are important in chemical processing industries. Viscosity and elasticity of the fluid have certain effects on the heat transfer rate and thus can alter the instability of the system in comparison to Newtonian fluid (*cf.* Basu and Layek [13]).

The present study is motivated by the above analysis of Veronis [2] and Gupta and Gupta [4] for thermohaline convection for Newtonian fluids. Many authors including Chandrasekhar [14], Gupta and Gupta [4] and Murthy and Ram [15] have shown that the uniform rotation have stabilizing effect on the onset of convection for Newtonian fluids. Also viscosity and the elastic properties of non-Newtonian fluids together have destabilizing effect on the stability (Rosenblat [16]). So, our aim is to study the combined effect of rotation and concentration gradient on thermohaline convection on the Oldroydian viscoelastic fluid layer heated from below. Linear stability analysis for infinitesimal amplitude disturbances and weakly non-linear stability analysis for finite amplitude disturbances are carried out and the effects of various parameters on the convection have been computed numerically.

2. Mathematical Formulation of the Physical Problem

An infinite horizontal layer of viscoelastic incompressible fluid is statically confined between two horizontal boundaries in the force field of gravity, which are respectively maintained at uniform temperatures T_0 and $T_0 - \Delta T$ and uniform concentrations S_0 and $S_0 - \Delta S$ in the presence of uniform rotation with angular velocity $\vec{\Omega}$ along vertical z-axis. We consider two-dimensional rolls, so that all the physical quantities are independent of the y-coordinate. The boundaries are taken to be *isothermal* and *dynamically free* (on which tangential stresses vanish).

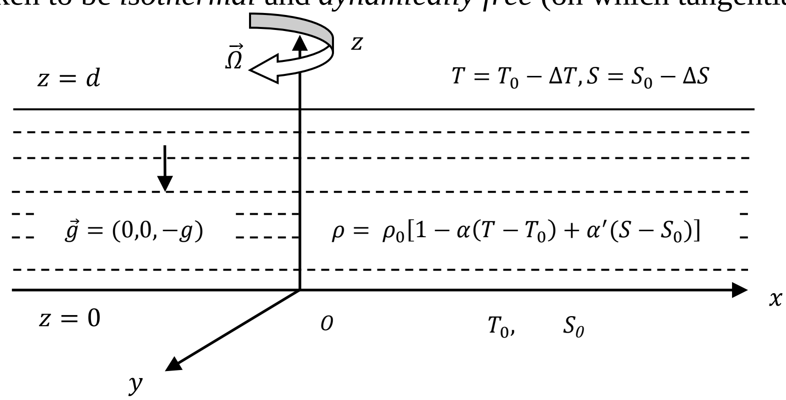

Figure 1. Schematic representation of the physical problem

The basic hydrodynamical equations governing the thermohaline convection problem in rotating viscoelastic fluid, in view of Boussinesq approximation, representing equation of continuity, equations of motion, equations of heat and mass transfer and equation of state, are respectively given by (*cf.* Veronis [2], Gupta and Gupta [4] and Sengupta and Kumar [17]);

$$\nabla.\vec{v} = 0 \tag{1}$$

$$\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}.\nabla\vec{v}\right) = -\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\nabla\left(\frac{P}{\rho_0} - \frac{1}{2}\left|\vec{\Omega} \times \vec{r}\right|^2\right) + \left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\frac{\rho}{\rho_0}\vec{g} - \left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\left(2\vec{\Omega} \times \vec{v}\right) + \left(1 + \bar{\lambda}_2 \frac{\partial}{\partial t}\right)\nu\nabla^2\vec{v} \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{v}.\nabla T = \kappa_T \nabla^2 T \tag{3}$$

$$\frac{\partial S}{\partial t} + \vec{v}.\nabla S = \kappa_s \nabla^2 S \tag{4}$$

and $\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(S - S_0)]$ (5)

In the above equations; ρ is density, $\vec{g} = (0,0,-g)$ is the external force, $\vec{v} = (u, v, w)$ are the components of velocity, T and S are respectively the temperature and the concentration at any point on the layer, P is the pressure, ν is the kinematic viscosity, c_v is the specific heat at constant volume, $\kappa_T \left(= \frac{K_0}{\rho_0 c_{v_0}}\right)$ is the thermometric diffusivity, κ_s is the coefficient of salt diffusion, $\bar{\lambda}_1$ is stress relaxation time, $\bar{\lambda}_2$ is strain retardation time, α and α' are respectively the thermal and solutal coefficients of volume expansion, ρ_0 is the reference density of the fluid at lower boundary.

Initially, when there are no motions, the basic state is assumed to be quiescent state. So, the initial state is represented by;

$$\vec{v} \equiv 0, \qquad T \equiv T(z), \qquad S \equiv S(z), \qquad \rho \equiv \rho(z), \qquad P \equiv P(z) \tag{6}$$

and hence the initial stationary state solutions are given by;

$$\vec{v} = (u, v, w) = (0,0,0), \quad T = T_0 - \beta z, \quad S = S_0 - \beta' z, \quad \rho = \rho_0[1 + \alpha\beta z - \alpha'\beta' z],$$
$$P = P_0 - g\,\rho_0 \left[z + (\alpha\beta - \alpha'\beta')\frac{z^2}{2}\right], \tag{7}$$

where, P_0 is the pressure at the lower boundary, $\beta = \frac{\Delta T}{d}$ and $\beta' = \frac{\Delta S}{d}$ are respectively the maintained uniform adverse temperature gradient and the non-adverse concentration gradient.

Imparting the *infinite amplitude* perturbations in the basic state, the perturbed state is given by;

$$\bar{\vec{v}} = 0 + \vec{v'}, \qquad \bar{S} = S_0 - \beta' z + \phi', \qquad \bar{T} = T_0 - \beta z + \theta',$$
$$\bar{P} = P_0 - g\left[z + (\alpha\beta - \alpha'\beta')\frac{z^2}{2}\right] + \delta P', \quad \rho = \rho_0[1 + \alpha(T_0 - T - \theta') - \alpha'(S_0 - S - \phi')] \tag{8}$$

where, $\vec{v'}$, ϕ', θ' and $\delta P'$ are perturbed variables.

Now, substituting the perturbed quantities given by (8) in equations (1)-(4), using the basic state solution given by equations (7) and using the stream function ψ (for two dimensional flow) defined by $u' = \partial\psi/\partial z$, $w' = -\partial\psi/\partial x$, and then non-dimensionalizing the resulting equations upon using the following non-dimensional quantities;

$$\psi = \kappa_T\hat{\psi}, \quad t = \frac{d^2}{\kappa_T}\hat{t}, \quad (x,y,z) = d(\hat{x},\hat{y},\hat{z}), \quad \theta' = \Delta T.\hat{\theta}, \quad \phi' = \Delta S.\hat{\phi},$$
$$\delta P' = \frac{\hat{P}\rho_0\nu\kappa_T}{d^2}, \quad \mathrm{v} = \frac{\kappa_T}{d}\hat{\mathrm{v}}, \quad \hat{\lambda}_1 = \bar{\lambda}_1\frac{\kappa_T}{d^2}, \quad \hat{\lambda}_2 = \bar{\lambda}_2\frac{\kappa_T}{d^2},$$

we have the following dimensionless system of equations, after eliminating $\hat{P}$ from the resulting equations of motions and dropping the caps for the convenience in writing;

$$\left[\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} - \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\psi = \left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[-R\frac{\partial\theta}{\partial x} + R_s\frac{\partial\phi}{\partial x}\right] + Y\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial z} +$$
$$\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)J(\psi, \nabla^2\psi) \tag{9}$$

$$\left[\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} - \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\mathrm{v} = -Y\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial z} + \frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)J(\psi, \mathrm{v}) \tag{10}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = J(\psi, \theta) - \frac{\partial\psi}{\partial x} \tag{11}$$

$$\left(\frac{\partial}{\partial t} - \tau\nabla^2\right)\phi = J(\psi, \phi) - \frac{\partial\psi}{\partial x} \tag{12}$$

where, $\tau = \frac{\kappa_s}{\kappa_T}$, $\sigma = \frac{\nu}{\kappa_T}$, $R = \frac{g\alpha\Delta T d^3}{\nu\kappa_T}$, $R_s = \frac{g\alpha'\Delta S d^3}{\nu\kappa_T}$ are respectively, the Lewis number, the Prandtl number, the thermal Rayleigh number and the salinity Rayleigh number. Also $Y^2 = 4\Omega^2\mathrm{d}^4/\nu^2$ is the Taylor number and v is the azimuthal component of velocity induced by rotation and $J(.,.)$ are the Jacobians which represent the nonlinear terms.

The above system of equations is to be solved under the following *stress free* and *isothermal* non-dimensional boundary conditions;

$$\psi = \frac{\partial^2\psi}{\partial z^2} = \phi = \theta = \frac{\partial \mathrm{v}}{\partial z} = 0 \text{ at } z = 0 \text{ and } 1. \tag{13}$$

3. Linearized Stability Analysis:

In order to study the effects of various parameters on the onset of convection; when the disturbances are infinite, we use the linear stability analysis theory. So, linearizing the system of equations (9)-(12) by dropping the nonlinear terms, we have the following linearized perturbation equations governing the problem;

$$\left[\frac{1}{\sigma}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\psi=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[-R\frac{\partial\theta}{\partial x}+R_s\frac{\partial\phi}{\partial x}\right]+Y\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial z} \quad (14)$$

$$\left[\frac{1}{\sigma}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\mathrm{v}=-Y\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial z} \quad (15)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta=-\frac{\partial\psi}{\partial x} \quad (16)$$

$$\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)\phi=-\frac{\partial\psi}{\partial x} \quad (17)$$

Consider the solutions of the above linearized equations (14)-(17) satisfying the boundary conditions (13) of the following form;

$$\left.\begin{array}{ll}\psi=Ae^{pt}\sin\pi ax\sin n\pi z, & \phi=Be^{pt}\cos\pi ax\sin n\pi z\,,\\ \mathrm{v}=Ce^{pt}\sin\pi ax\cos n\pi z, & \theta=De^{pt}\cos\pi ax\sin n\pi z\,.\end{array}\right\} \quad (18)$$

where, a is the wave number, $p=p_r+ip_i$ is the complex constant, n is an integer and A,B,C,D are real constants.

Substituting the above periodic solutions in equations (14)-(17), eliminating the constants A,B,C and D amongst the resulting equations after taking $p=0$ for the steady marginal state solution, we have the following equation;

$$\sigma\pi^3(n^2+a^2)^2+\frac{\sigma a^2R_s}{\tau\pi(n^2+a^2)}+\frac{n^2\sigma Y^2}{\pi(n^2+a^2)}-\frac{\sigma a^2R}{\pi(n^2+a^2)}=0 \quad (19)$$

For the lowest mode, $n=1$, we have the following expression for the Rayleigh number for steady convection;

$$R^{Steady}=\frac{\pi^4\left(1+a^2\right)^3}{a^2}+\frac{R_s}{\tau}+\frac{Y^2}{a^2}. \quad (20)$$

The critical value of above Rayleigh number for corresponding minimum value of α for steady case can be easily obtained following (Chandrasekhar [14]);

$$R_c^{Steady}=3\pi^4\left(\frac{Y^2}{2\pi^4}\right)^{\frac{2}{3}}+\frac{R_s}{\tau} \quad (21)$$

and $a_{min}=\left(\frac{1}{2}\pi^2Y^2\right)^{\frac{1}{6}}$. (22)

When $R_s=0$, the value of critical Rayleigh number R_c^{Steady} coincides with the value of critical Rayleigh number obtained by Chandrasekhar [14] for rotatory thermal convection.

Further, for the case when marginal state is oscillatory, we have $p=ip_i$, $p_i$being real. Substituting solutions (18) in equations (14)-(17) with $p=ip_i$ and eliminating constants A,B,C and D amongst the resulting equations, we obtain a single expression, the real and imaginary parts of which for $n=1$ (the lowest mode), respectively yield the following expressions for the thermal Rayleigh number for overstable case and for frequency of oscillations;

$$R^{Ov}=\left(\frac{1+a^2}{\sigma a^2}\right)\frac{[\sigma\pi^2(1+a^2)\{\pi^2(1+a^2)(1+\lambda_1\lambda_2p_i^2)-p_i^2(\lambda_2-\lambda_1)\}-p_i^2(1+\lambda_1{}^2p_i^2)]}{(1+\lambda_1{}^2p_i^2)}+R_s\frac{\left[\tau\pi^4(1+a^2)^2+p_i^2\right]}{[\tau^2\pi^4(1+a^2)^2+p_i^2]}+\left(\frac{\sigma Y^2}{a^2}\right)\frac{\left[\left\{\sigma\pi^4(1+a^2)^2-\lambda_1\pi^2(1+a^2)p_i^2(1+\sigma)+\lambda_1{}^2p_i^4\right\}+p_i^2\left\{1+\lambda_1\lambda_2\sigma\pi^4(1+a^2)^2+\lambda_1\pi^2(1+a^2)+\lambda_2\sigma\pi^2(1+a^2)\right\}\right]}{\left[\left\{\sigma\pi^2(1+a^2)-\lambda_1p_i^2\right\}^2+p_i^2\{1+\lambda_2\sigma\pi^2(1+a^2)\}^2\right]} \quad (23)$$

and $Ex^4+Fx^3+Gx^2+Hx+I=0$ (24)

where, $x=p_i^2$ is the square of frequency, and the constants occurring in (24) are given by;

$E=[\lambda_1^2(1+a^2)\{\pi^2\lambda_1^2(1+a^2)+B_1\lambda_1\lambda_2\}]$, (25)

$F=[\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}\{\lambda_1^2\pi^2(1+\alpha^2)+\lambda_1\lambda_2B_1\}(1+a^2)+\lambda_1^2\pi^2(1+a^2)^2+\lambda_1^2B_1\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+A_1^2\lambda_1^2\{\lambda_1^2\pi^2(1+a^2)+\lambda_1\lambda_2B_1\}(1+a^2)+\sigma^2Y^2\lambda_1^2\{\lambda_1\lambda_2B_1-\lambda_1^2\pi^2(1+a^2)\}+R_s(\tau-1)B_1a^2\lambda_1^4]$, (26)

$G=B_1{}^2\{\pi^2\lambda_1^2(1+a^2)+B_1\lambda_1\lambda_2\}(1+a^2)+\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}\pi^2(1+a^2)^2+B_1\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+A_1{}^2\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}\{\lambda_1^2\pi^2(1+a^2)+B_1\lambda_1\lambda_2\}(1+a^2)+A_1^2\lambda_1^2\pi^2(1+a^2)^2+A_1^2B_1\lambda_1^2\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+\sigma^2Y^2\lambda_1{}^2\{B_1+\lambda_1\pi^2(1+a^2)B_1-\pi^2(1+a^2)-\pi^2\lambda_2(1+a^2)B_1\}+\sigma^2Y^2(1+A_1^2\lambda_1^2)\{\lambda_1\lambda_2B_1-\lambda_1^2\pi^2(1+a^2)\}+R_s(\tau-1)B_1a^2\lambda_1^2\{1+(1+\lambda_2B_1)^2-2\lambda_1B_1\}$, (27)

$H=B_1{}^2\pi^2(1+a^2)^2+B_1{}^3\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+A_1^2B_1^2\{\lambda_1^2\pi^2(1+a^2)+B_1\lambda_1\lambda_2\}(1+a^2)+A_1^2\pi^2(1+a^2)^2\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}+A_1^2B_1\{(1+\lambda_2B_1)^2-2\lambda_1B_1\}\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+\sigma^2Y^2(1+A_1^2\lambda_1^2)\{B_1+\lambda_1\pi^2(1+a^2)B_1-\pi^2(1+a^2)-\pi^2\lambda_2(1+a^2)B_1\}+\sigma^2Y^2A_1{}^2\{\lambda_1\lambda_2B_1-\lambda_1^2\pi^2(1+a^2)\}+R_s(\tau-1)B_1a^2\{(1+\lambda_2B_1)^2-2\lambda_1B_1+B_1{}^2\lambda_1^2\}$, (28)

$I=[A_1^2B_1^2\pi^2(1+a^2)^2+A_1^2B_1{}^3\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2)+\sigma Y^2A_1{}^2\{B_1+\lambda_1\pi^2(1+a^2)B_1-\pi^2(1+a^2)-\pi^2\lambda_2(1+a^2)B_1\}+R_s(\tau-1)B_1{}^3a^2\,]$ (29)

$A_1=\tau\pi^2(1+a^2)$ and $B_1=\sigma\pi^2(1+a^2)$ (30)

4. Weakly Nonlinear Stability Analysis

Following Veronis [2] and Gupta and Gupta [4], for the steady convection (with $\partial/\partial t=0$), when the disturbances are of *finite amplitude*, expressing all dependent variables in powers of small perturbation parameter ε subjected to boundary conditions (13), we have;

$f=\varepsilon f_0+\varepsilon^2f_1+\varepsilon^3f_2+\cdots$, where $f\equiv(\psi,\mathrm{v},\phi,R)$.

Substituting these expansions for $\psi,\mathrm{v},\theta,\phi$ and R in governing equations (9)-(12), we obtain the following equations;

$$(-\nabla^4\psi_0)\varepsilon+(-\nabla^4\psi_1)\varepsilon^2+(-\nabla^4\psi_2)\varepsilon^3=[R_s\frac{\partial\phi_0}{\partial x}-R_0\frac{\partial\theta_0}{\partial x}+Y\frac{\partial\mathrm{v}_0}{\partial z}]\varepsilon+[R_s\frac{\partial\phi_1}{\partial x}-R_0\frac{\partial\theta_1}{\partial x}-R_1\frac{\partial\theta_0}{\partial x}+Y\frac{\partial\mathrm{v}_1}{\partial z}+\frac{1}{\sigma}J(\psi_0,\nabla^2\psi_0)]\varepsilon^2+[R_s\frac{\partial\phi_2}{\partial x}-R_0\frac{\partial\theta_2}{\partial x}-R_1\frac{\partial\theta_1}{\partial x}-R_2\frac{\partial\theta_0}{\partial x}+Y\frac{\partial\mathrm{v}_2}{\partial z}+\frac{1}{\sigma}\{J(\psi_0,\nabla^2\psi_1)+J(\psi_1,\nabla^2\psi_0)\}]\varepsilon^3 \quad (31)$$

$$(-\nabla^2\mathrm{v}_0)\varepsilon+(-\nabla^2\mathrm{v}_1)\varepsilon^2+(-\nabla^2\mathrm{v}_2)\varepsilon^3=[-Y\frac{\partial\psi_0}{\partial z}]\varepsilon+[-Y\frac{\partial\psi_1}{\partial z}+\frac{1}{\sigma}J(\psi_0,\mathrm{v}_0)]\varepsilon^2+[-Y\frac{\partial\psi_2}{\partial z}+\frac{1}{\sigma}\{J(\psi_0,\mathrm{v}_1)+J(\psi_1,\mathrm{v}_0)\}\ ]\varepsilon^3 \quad (32)$$

$$(-\nabla^2\theta_0)\varepsilon+(-\nabla^2\theta_1)\varepsilon^2+(-\nabla^2\theta_2)\varepsilon^3=[-\frac{\partial\psi_0}{\partial x}]\varepsilon+[J(\psi_0,\theta_0)-\frac{\partial\psi_1}{\partial x}]\varepsilon^2+[J(\psi_0,\theta_1)+J(\psi_1,\theta_0)-\frac{\partial\psi_2}{\partial x}]\varepsilon^3 \quad (33)$$

$$(-\tau\nabla^2\phi_0)\varepsilon + (-\tau\nabla^2\phi_1)\varepsilon^2 + (-\tau\nabla^2\phi_2)\varepsilon^3 = [-\frac{\partial\psi_0}{\partial x}]\varepsilon + [J(\psi_0,\phi_0) - \frac{\partial\psi_1}{\partial x}]\varepsilon^2 + [J(\psi_0,\phi_1) + J(\psi_1,\phi_0) - \frac{\partial\psi_2}{\partial x}]\varepsilon^3 \quad (34)$$

To obtain the first (lowest), second and third order system of equations, comparing the coefficients of ε, ε^2 and ε^3 respectively, we can have three sets of four equations each.

Now eliminating v_0, θ_0, ϕ_0 from the lowest order system obtained from equations (31)-(34), we get

$$\mathcal{L}\psi_0 = -\left[\nabla^6 + Y^2\frac{\partial^2}{\partial z^2} + \left(\frac{R_S}{\tau} - R_0\right)\frac{\partial^2}{\partial x^2}\right]\psi_0 = 0. \quad (35)$$

Similarly, eliminating v_1, θ_1, ϕ_1 from the second order and v_2, θ_2, ϕ_2 from the third order system of equations, we have the following equations;

$$\mathcal{L}\psi_1 = -R_1\frac{\partial^2\psi_0}{\partial x^2} - \frac{Y}{\sigma}\frac{\partial}{\partial z}[J(\psi_0, v_0)] + R_0\frac{\partial}{\partial x}[J(\psi_0,\theta_0)] - \frac{R_S}{\tau}\frac{\partial}{\partial x}[J(\psi_0,\phi_0)] + \frac{1}{\sigma}J(\nabla^2\psi_0, \nabla^4\psi_0) \quad (36)$$

$$\mathcal{L}\psi_2 = -R_2\frac{\partial^2\psi_0}{\partial x^2} - R_1\frac{\partial}{\partial x}\left[\frac{\partial\psi_1}{\partial x} - J(\psi_0,\theta_0)\right] + R_0\frac{\partial}{\partial x}[J(\psi_0,\theta_1) + J(\psi_1,\theta_0)] - \frac{Y}{\sigma}\frac{\partial}{\partial z}[J(\psi_0, v_1) + J(\psi_1, v_0)] - \frac{R_S}{\tau}\frac{\partial}{\partial x}[J(\psi_0,\phi_1) + J(\psi_1,\phi_0)] + \frac{1}{\sigma}[J(\nabla^2\psi_0,\nabla^4\psi_1) + J(\nabla^2\psi_1,\nabla^4\psi_0)] \quad (37)$$

Let us consider the following periodic solution of the linearized stability problem governed by the above first order system of equations satisfying the boundary conditions (13);

$$\psi_0 = -\frac{2}{\pi a}\sin\pi ax\,.\sin n\pi z, \qquad v_0 = \frac{2Y}{a\pi^2(a^2+1)}\sin\pi ax\,.\cos n\pi z,$$
$$\theta_0 = \frac{2}{\pi^2(a^2+1)}\cos\pi ax\,.\sin n\pi z, \qquad \phi_0 = \frac{2}{\tau\pi^2(a^2+1)}\cos\pi ax\,.\sin n\pi z. \quad (38)$$

Substituting solution (38), with $n = 1$ (for lowest mode), in equation (36), we get

$$\mathcal{L}\psi_1 = -R_1(2\pi a\sin\pi ax\,.\sin\pi z) = \pi^2 a^2 R_1\psi_0 \quad (39)$$

where, all the nonlinear terms represented by Jacobians are zero.

Now, equation (39) implies that $R_1 = 0$ when R_1 is to be evaluated to cancel the form of ψ_0 as it is a secular term and shall affect the periodicity of the above solution. Therefore, we must have $\mathcal{L}\psi_1 = 0$ and its solution which satisfy the boundary conditions (13) is now given by;

$$\psi_1 = 0 \quad (40)$$

Using the above value of ψ_1 in the equations obtained for second order system by comparing the coefficients of ε^2, we can deduce the following solutions satisfying the boundary conditions (13);

$$\left.\begin{aligned} v_1 &= \frac{Y}{2\pi^3a^3\sigma(a^2+1)}\sin 2\pi ax \\ \theta_1 &= -\frac{1}{2\pi^3(a^2+1)}\sin 2\pi z \\ \phi_1 &= -\frac{1}{2\tau^2\pi^3(a^2+1)}\sin 2\pi z \end{aligned}\right\} \quad (41)$$

Hence, using equation (41) in equation (37), we get

$$\mathcal{L}\psi_2 = \left[-\frac{Y^2}{\pi a^3\sigma^2(a^2+1)} + \frac{aR_c^{Steady}}{\pi(a^2+1)} - 2\pi aR_2 - \frac{aR_S}{\pi\tau^3(a^2+1)}\right]\sin\pi ax\,.\sin\pi z + \frac{Y^2}{\pi a^3\sigma^2(a^2+1)}\sin 3\pi ax\,.\sin\pi z + \left[\frac{aR_S}{\pi\tau^3(a^2+1)} - \frac{aR_0}{\pi(a^2+1)}\right]\sin 3\pi z\,.\sin\pi ax. \quad (42)$$

On comparing the value of ψ_0 given in (38), one observes that the first term on the right hand side of (42) is of the form of ψ_0 and hence must vanish. Therefore, we have

$$R_2 = \frac{R_c^{Steady}}{2\pi^2(a^2+1)} - \frac{Y^2}{2\pi^2 a^4 \sigma^2 (a^2+1)} - \frac{R_s}{2\tau^3\pi^2(a^2+1)}, \tag{43}$$

where R_c^{Steady} is the critical Rayleigh number at the onset of steady convection with respect to infinitesimal disturbances. From the above expression for Rayleigh number for second order, we observe that viscoelastic parameters have no effects on the finite amplitude steady convection, as have also been observed for the infinite amplitude by linear stability analysis for the steady case.

5. RESULTS AND DISCUSSIONS:

To study the effects of various parameters on the thermal Rayleigh number for steady and oscillatory cases, we have carried out the numerical computations of the respective expressions given by (20) and (23) for some fixed values of $\tau = 0.01$, $\sigma = 7$, $R_s = 10$, $\lambda_1 = 0.75$ and $\lambda_2 = 0.40$ (*cf.* Basu and Layek [13], Chandrasekhar [14], Kiran [18]).

The variations of thermal Rayleigh number for steady case (R^{Steady}) and overstable case (R^{Ov}) for some fixed values of other parameters given above with respect to square of wave number (a^2) is computed numerically and the results are shown in Figure 2. It is observed from the figure that oscillatory convection is the preferred mode in the thermohaline convection problem for viscoelastic fluids in the presence of rotation, when disturbances are infinite. Also, it is evident from the figure that rotation has stabilizing effect on the onset of both steady as well as oscillatory convection. Further one can easily note that in the absence of viscoelastic behaviour of the Newtonian fluid, the value of critical Rayleigh number for oscillatory case (R_c^{Ov}) is higher than for non-Newtonian fluid. Hence, we can conclude that viscoelasticity has destabilizing effect on the onset of thermohaline instability.

Further, the effect of concentration gradient represented by R_s on the Rayleigh number for same fixed values of other parameters with $Y^2 = 100$, has been studied numerically and is depicted graphically in Figure 3 and Figure 4 respectively, for R^{Steady} and R^{Ov}. It is clear from the figures that non-adverse concentration gradient postpones the onset of steady as well as oscillatory convection in viscoelastic fluid layer heated from below.

Figure 5 and Figure 6 show the effects of viscoelastic parameters λ_1 (stress relaxation time) and λ_2 (strain retardation time) on the thermal Rayleigh number (R^{Ov}) for the above fixed values of other parameters and under the condition $\lambda_1 < \lambda_2$ (*cf.* Basu and Layek [13]). It is observed from the figures that stress relaxation time has destabilizing effect whereas strain retardation time has stabilizing effect on the onset of oscillatory thermohaline convection. From Figure 5, we also observed that Oldroydian fluid model is more stable than Maxwellian fluid model ($\lambda_1 > 0, \lambda_2 = 0$).

From above analysis, we concluded that the viscoelastic parameters have the same effects on the double diffusive convection as that in single diffusive (thermal) convection. Also, we concluded that salinity gradient and rotation have usual effects on the onset of convection in viscoelastic fluid layer as in the case of thermohaline convection for Newtonian fluid.

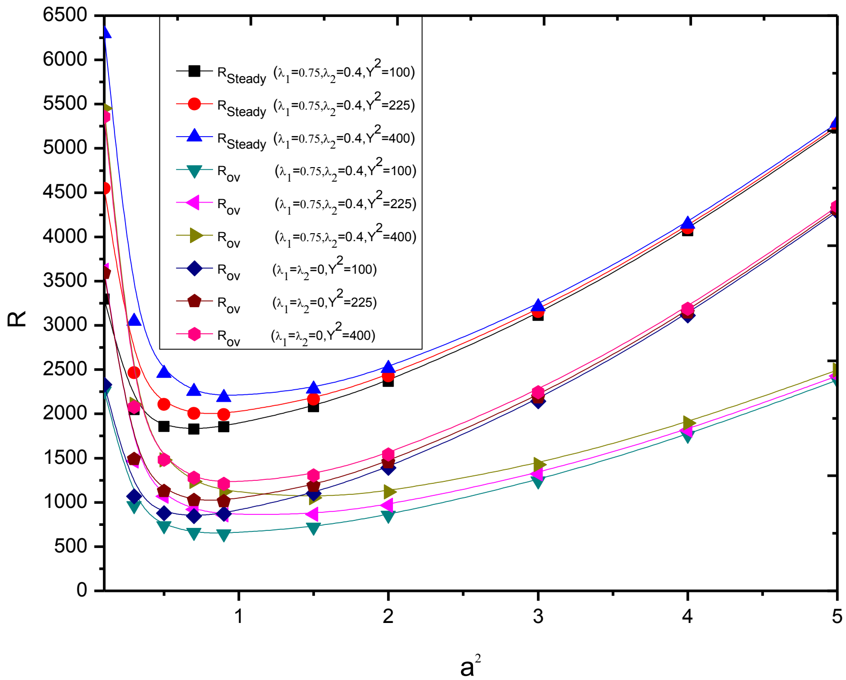

Figure 2. Variations of Rayleigh number with square of wave number for various values of Y^2 with $R_s = 10, \tau = 0.01$ and $\sigma = 7$.

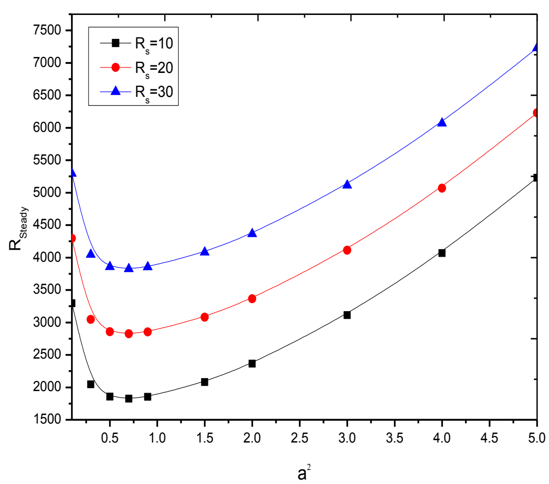

Figure 3. Variations of Rayleigh number with square of wave number for steady convection for various values of R_s with $Y^2 = 100$, $\tau = 0.01$ and $\sigma = 7$.

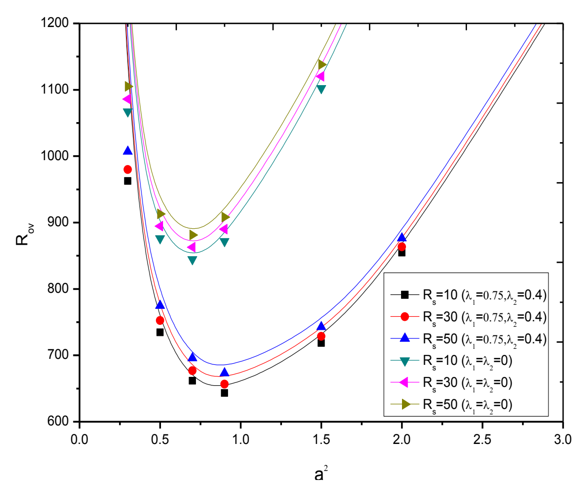

Figure 4. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of R_s with $Y^2 = 100$, $\tau = 0.01$ and $\sigma = 7$.

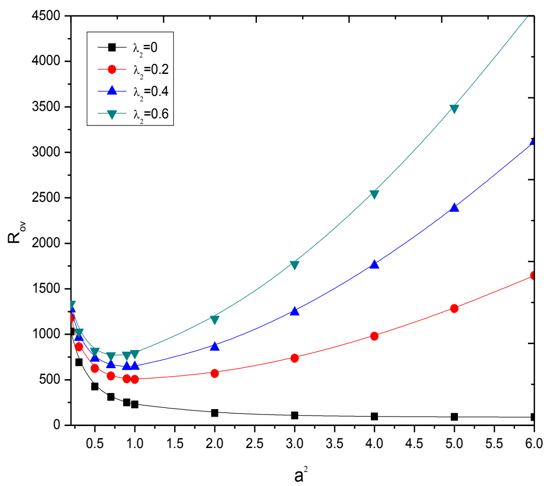

Figure 5. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of λ_2 with $Y^2 = 100$, $R_s = 10$, $\lambda_1 = 0.75$, $\tau = 0.01$ and $\sigma = 7$.

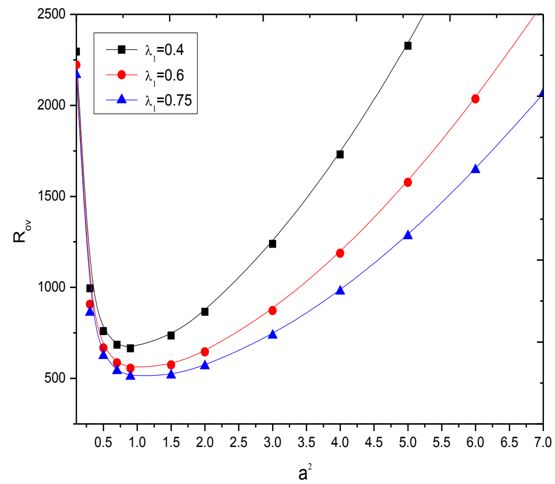

Figure 6. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of λ_1 with $Y^2 = 100$, $R_s = 10$, $\lambda_2 = 0.2$, $\tau = 0.01$ and $\sigma = 7$.

6. CONCLUSIONS:

We have analysed the effects of concentration gradient, stress relaxation time, strain retardation time and rotation in thermohaline convection problem by using linear and non-linear stability analysis against infinite and finite amplitudes, respectively. Following are the main conclusions;

Using linear stability analysis, when disturbances are of infinite amplitudes, we observed from the expressions for Rayleigh numbers for stationary and oscillatory instability in two-dimensional convective rolls that overstability is preferred mode of instability. This means that the onset of instability is through oscillations, before becomes unstable through steady infinite disturbances. Also, concentration gradient (R_S) has stabilizing effect on the onset of convection. This may be due to the enhancement of density of the fluid due to mass concentration, which decreases the heat transfer rate. The stress relaxation time (λ_1) and strain retardation time (λ_2) has no effect on stationary Rayleigh number, however, for an oscillatory mode of convection, λ_1 has destabilizing effect and λ_2 has stabilizing effect on the onset of instability. These behaviour of λ_1 and λ_2 on the onset of convection can be validated from the results obtained by Basu and Layek [13] and Kiran [18]. The Taylor number (Y), which represents the coriolis force has stabilizing effect on the onset of convection, as has also been analysed by Gupta and Gupta [4].

Using non-linear stability analysis, when disturbances are of finite amplitudes, we observed from the expression for Rayleigh number for stationary mode of instability in two-dimensional convective rolls that the viscoelastic parameters λ_1 and λ_2 have no effects on the onset of instability. This behaviour of the viscoelastic parameters is expected in the lines of linear stability analysis with infinite amplitude. Also, the concentration gradient (R_S) and rotation (Y) reinforce each other and have destabilizing effect on the onset of instability. The destabilizing behaviour of R_S contrary to the linear stability case is justified by the reason provided by Kiran[18] that the effect of solutal Rayleigh number is to increase Nusselt number, which means R_S has destabilizing effect in this case. Though the presence of a stabilizing gradient (solute concentration) prevent the onset of convection, the strong finite amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. This effect of rotation on instability has also been proved by Gupta and Gupta [4] in their analysis.

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HYDROMAGNETIC FLOW NEAR AN OSCILLATING WALL WITH CONSTANT/PERIODIC SUCTION

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ABSTRACT:
An analysis of flow of a viscous incompressible and electrically conducting fluid produced by harmonically oscillating wall of infinte extent in presence of transvese magnetic field is considered. Exact solutions for velocity, induced magnetic field, electrical current density and skin friction are obtained when the magnetic pradntl number is unity. The numerical results for velocity, induced magnetic field, electrical current density and skin friction for different parameters involved in the problem are expressed through graphs. The final results are discussed in detail at the last section of the paper.

INTRODUCTION:
The effect of a transversely applied magnetic field on the flow of an electrically conducting viscous fluid has been discussed widely owing to its physical, geophysical and engineering application. Literature survey also reveals that in an electrically conducting viscous fluid subject to an externally applied magnetic field has also received significant attention. Hartmann (1937) analyzed the effect of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid exiting through parallel stationary plates that are insulated. Rassow (1957) considered the impulsive motion of an infinite plate in a viscous incompressible magnetic fluid in the presence of an external magnetic field. The flow of a magnetic field near an infinite flat wall which oscillates in its own plate has been discussed by Ong and Nicholls (1959). Romig (1964) studied the influence of an electric and magnetic field on heat transfer to electrical conducting fluids. Soundalgekar (1965) discussed the hydromagnetic flow near an accelerated plate in the presence of magnetic field. Cramer and Pai (1973) discussed the utility of magneto fluid dynamics for engineers and applied physicists. Hydromagnetic flow near an oscillating porous flat plate under the action of body force was discussed by Kishan and Sharma (1979). An attempt has been made in this paper to study the effect of constant suction and periodic suction in hydromagnetic flow near an oscillating wall. The effect of suction consists in the removal of decelerated fluid particles from the boundary layer before they are given a chance to cause separation. A new boundary layer which is again capable of overcoming a certain adverse pressure gradient is allowed to form in the region behind the slit. With a suitable arrangement of the slits and under the favorable conditions separations can be prevented completely.

MATHEMATICAL FORMULATION OF THE PROBLEM:
We take the coordinate origin o, at an arbitrary point on an infinite wall, which is taken to be an electrical insulator. Cartesian coordinate system has been used with axis ox and oy along and perpendicular to the wall, respectively. The electrostatic system of units has been used throughout, and we assume that all physical quantities as functions of the space coordinate y and time t only. An attempt has been made in this paper to study the effect of constant suction and periodic suction in hydromagnetic flow near an oscillating wall.

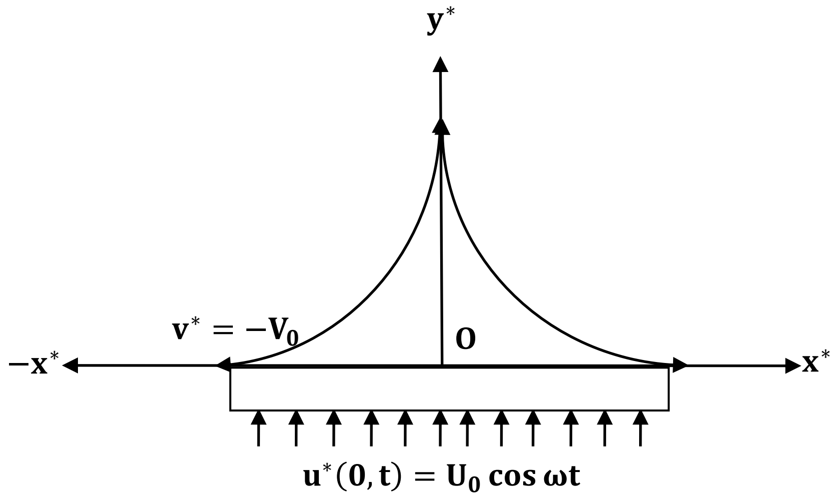

Figure 4A.1 Geometrical Interpretation of the Problem

Using Bossinesq approximation and following Pande et. al. (1975), a fluid model has been developed as follows:
Case 1: Analysis for constant suction, here we have consider the suction of the type
$v^* = -v_0$,
The basic equations governing the flows
Equation of Continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \tag{1}$$

Equations of motion

$$\frac{\partial u^*}{\partial t} - v_0 \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\mu_0}{\rho} H_y^* \frac{\partial H_x^*}{\partial y^*} \tag{2}$$

Equation of magnetic induction

$$\frac{\partial H_x^*}{\partial t} - v_0 \frac{\partial H_x^*}{\partial y^*} = \frac{1}{\sigma\mu_0} \frac{\partial^2 H_x^*}{\partial y^{*2}} + H_y^* \frac{\partial u^*}{\partial y^*} \tag{3}$$

The initial and boundary conditions for the velocity field are exactly the same as those given by Stokes (1851) for the hydrodynamic case: namely,

$$\left.\begin{array}{l} u^*(y^*, 0) = 0, \qquad \text{for } y^* \geq 0 \\ u^*(0, t) = U_0 \cos \omega t, \ u^*(\infty, t) \to 0 \end{array}\right\} \tag{4}$$

where ω denotes the frequency of oscillations of the wall and U_0 the maximum velocity of the wall.
The appropriate boundary conditions on H_x^* are

$$\left.\begin{array}{l} H_x^*(0, t) = 0, \quad H_y^*(0, t) = H_0 \\ H_x^*(\infty, t) \to 0, H_y^*(\infty, t) \to H_0 \end{array}\right\} \tag{5}$$

From Maxwell's equations the components of electrical current density are given by

$$J_x = J_y = 0 \text{ and } J_z = -\left(\frac{\partial H_y^*}{\partial y^*}\right) \tag{6}$$

And the divergence equation for the magnetic field gives

$$H_y^* = \text{constant} = H_0 \tag{7}$$

where H_0 is externally applied transverse magnetic field.

Substituting equation (7) in equation (2), (3) and (6), and introducing the following non-dimensional quantities

$$y = y^* \left(\frac{\omega}{2\nu}\right)^{1/2}, H = H_x^* \left(\frac{\mu_0}{\rho}\right)^{1/2}, \lambda = \frac{V_0}{\sqrt{2\nu\omega}}, M = H_0 \left(\frac{\mu_0}{\rho}\right)^{1/2} (2\omega\nu)^{-1/2}, P_m = \nu\rho\mu_0,$$

$$\alpha = \left(\sigma\mu_0\right)^{-1}.$$

By using above non-dimensional quantities in equations (2), (3) and (6), we get the following non-dimensional set of governing equations,

$$\frac{\partial^2 u}{\partial y^2} - \frac{2}{\omega}\frac{\partial u}{\partial t} + 2M\frac{\partial H}{\partial y} + 2\lambda\frac{\partial u}{\partial y} = 0 \tag{8}$$

$$\frac{1}{P_m}\frac{\partial^2 H}{\partial y^2} - \frac{2}{\omega}\frac{\partial H}{\partial t} + 2M\frac{\partial u}{\partial y} + 2\lambda\frac{\partial H}{\partial y} = 0 \tag{9}$$

$$J_z = -\left(\frac{\rho\omega}{2\mu_0\upsilon}\right)^{1/2}\frac{\partial H}{\partial y} \tag{10}$$

where P_m represents the magnetic prandtl number.

Following Pande et al. (1976), we seek the solution of the equation (8) and (9) as follows

$$\left.\begin{aligned} u(y,t) &= U_0\cos(\omega t)f(y) \\ H(y,t) &= U_0 os(\omega t)g(y) \end{aligned}\right\} \tag{11}$$

with periods $\frac{2\pi}{\omega}$, substituting (11) into (8) and (9), we find that f(y) and g(y) must satisfy the equation

$$\frac{\partial^2 f}{\partial y^2} - 2if + 2M\frac{\partial g}{\partial y} + 2\lambda\frac{\partial f}{\partial y} = 0 \tag{12}$$

$$\frac{1}{P_m}\frac{\partial^2 g}{\partial y^2} - 2ig + 2M\frac{\partial f}{\partial y} + 2\lambda\frac{\partial g}{\partial y} = 0 \tag{13}$$

which are to be solved under the boundary conditions

$$\left.\begin{aligned} &f(0,t) = 1, \quad g(o,t) = 0 \\ &f(\infty,t) \to 0, g(\infty,t) \to 0 \end{aligned}\right\} \tag{14}$$

If we assume the magnetic Prandtl number $P_m = 1$, then the appropriate solution of (12) and (13) satisfying the boundary conditions (14), are

$$f = \frac{1}{2}\left[e^{-(a+\lambda+M)\eta}e^{-\frac{iy}{a}} + e^{-(b+\lambda+M)\eta}e^{-\frac{iy}{b}}\right] \tag{15}$$

$$g = \frac{1}{2}\left[e^{-(a+\lambda+M)\eta}e^{-\frac{iy}{a}} - e^{-(b+\lambda+M)\eta}e^{-\frac{iy}{b}}\right] \tag{16}$$

where $2a^2 = (M+\lambda)^2 + \{(M+\lambda)^4 + 4\}^{1/2}$ and

$2b^2 = (M-\lambda)^2 + \{(M-\lambda)^4 + 4\}^{1/2}$

Thus the expression for the velocity and induced magnetic field are given by

$$\frac{u}{U_0} = \frac{1}{2}\left[e^{-(a+\lambda+M)y}\cos\left(\omega t - \frac{y}{a}\right) + e^{-(b+\lambda+M)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \tag{17}$$

$$\frac{H}{U_0} = \frac{1}{2}\left[e^{-(a+\lambda+M)y}\cos\left(\omega t - \frac{y}{a}\right) - e^{-(b+\lambda+M)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \tag{18}$$

The skin friction in presence of magnetic field

$$\tau = \mu\left(\frac{\partial u}{\partial\eta}\right)_{\eta=0} = -\frac{\mu U_0}{2}\left[(a+b+2\lambda)\cos\omega t - \left(\frac{1}{a}+\frac{1}{b}\right)\sin\omega t\right] \tag{19}$$

Ratio of skin friction at the wall in the presence and absence of magnetic field is

$$\frac{\tau}{\tau_0} = \frac{\left[(a+b+2\lambda)\cos\omega t - \left(\frac{1}{a}+\frac{1}{b}\right)\sin\omega t\right]}{2\left[(a+\lambda)\cos\omega t - \frac{1}{a}\sin\omega t\right]} \tag{20}$$

where τ and τ_0 represents, the skin friction in presence and absence of the magnetic field.

On substituting the value of H from equation (18) into (10), we get the expression for electric current density as

$$\frac{J_z}{U_0} = \frac{1}{2}\left[\left\{(a+M+\lambda)e^{-(a+\lambda+M)y} - (b+M+\lambda)e^{-(b+\lambda-M)y}\right\}\cos\left(\omega t - \frac{y}{a}\right) - \left\{\frac{1}{a}e^{-(a+\lambda+M)y} - \frac{1}{b}e^{-(b+\lambda-M)y}\right\}\sin\left(\omega t - \frac{y}{b}\right)\right] \quad (21)$$

Case 2: Analysis for periodic suction, when $v = -v_0\cos(\omega t)$,
If we assume the magnetic Prandtl number $P_m = 1$, then the appropriate solution of the equation (12) and (13) are

$$f = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{a}} + e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{b}}\right] \quad (22)$$

$$g = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{a}} - e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{b}}\right] \quad (23)$$

Where $2a^2 = \left(M + V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\right)^2 + \left\{(M + V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}})^4 + 4\right\}^{1/2}$ and

$$2b^2 = \left(M - V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\right)^2 + \left\{(M - V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\lambda)^4 + 4\right\}^{1/2}$$

Thus the expression for the velocity and induced magnetic field are given by

$$\frac{u}{U_0} = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\lambda+M\right)y}\cos\left(\omega t - \frac{y}{a}\right) + e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \quad (24)$$

$$\frac{H}{U_0} = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{a}\right) - e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \quad (25)$$

RESULTS AND DISCUSSION:

The oscillation of the wall will produce wave-like disturbances within the boundary layer. The velocity profile has the form of damped harmonic oscillations, i.e., the velocity decays exponentially as the distance from the wall increases. In the neighborhood of the wall, its amplitude decreases by the presence of the magnetic field. A fluid layer at a distance y has a phase lag given by $y/a = {y^*\left(\frac{\omega}{2\nu}\right)^{1/2}}/{a}$ with respect to the oscillations of the wall; which depends on the strength of the applied magnetic field besides the frequency of oscillation and the viscosity of the fluid. This phase lag decreases as the strength of the magnetic field increases for given ω and ν. Conversely the smaller the strength of the applied magnetic field and the viscosity of the fluid and higher the frequency, the faster the rate of decrease of velocity with the increase in y, for given U_0 . This fact is evident from figure 1 which represents the motion for several instant of time.

The induced magnetic field have phase lags given by ${y\left(\frac{\omega}{2\nu}\right)^{1/2}}/{a}$, similar to the one discussed earlier for the case of velocity profile. Also, we observe that near the wall, the velocity amplitudes increases with Hartmann number(M). Table 1 shows the periodic variation of the velocity.

It is also useful to note that, in the absence of suction parameter ($\lambda = 0$), our results become identical with the results of Pande et al. (1975) as shown in figure 6.

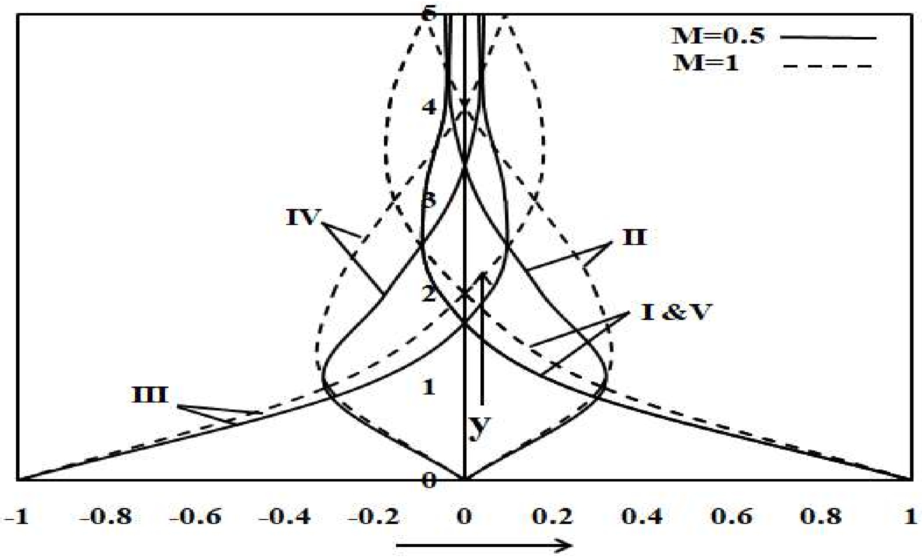

Figure 1. Velocity profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0}$.

$\mathbf{u/U_0}$

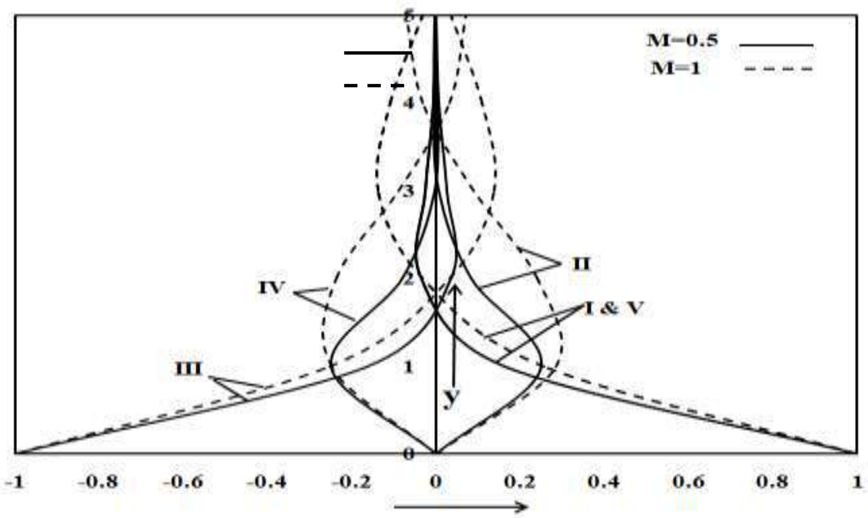

Figure 2. Velocity profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0.2}$.

$\mathbf{u/U_0}$

u/U_0

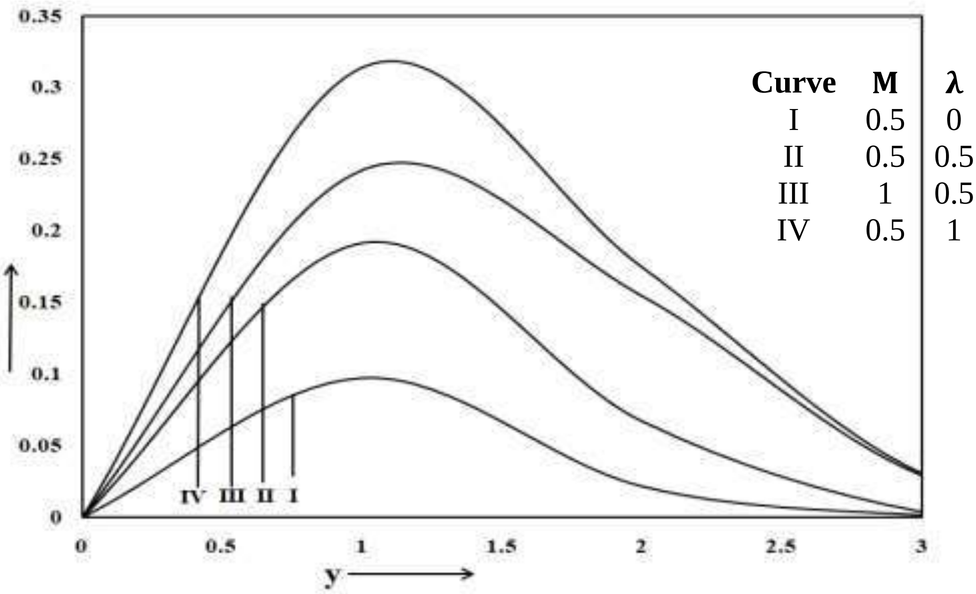

Figure 3. Velocity profiles for different values of λ and M.

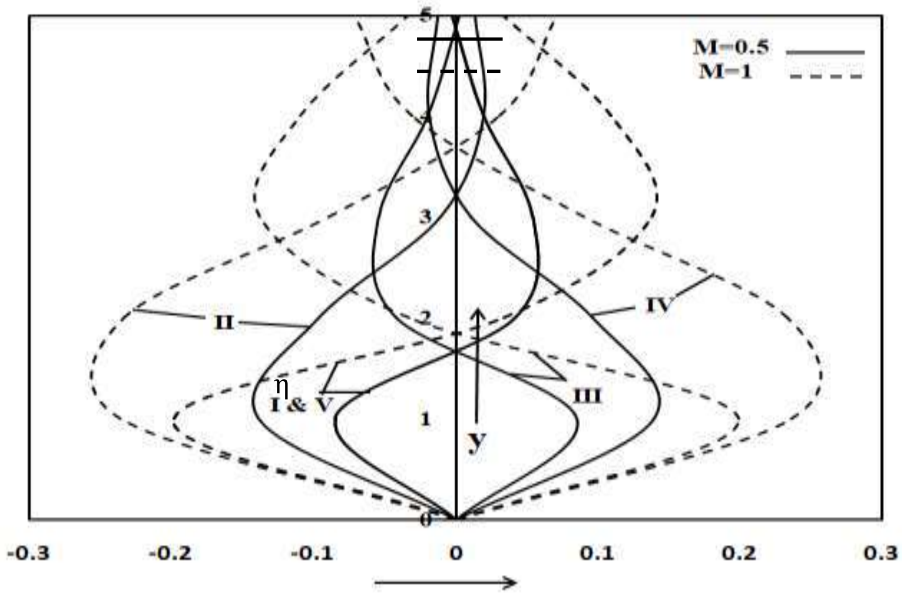

Figure 4. Induced magnetic field profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0.2}$.

H/U_0

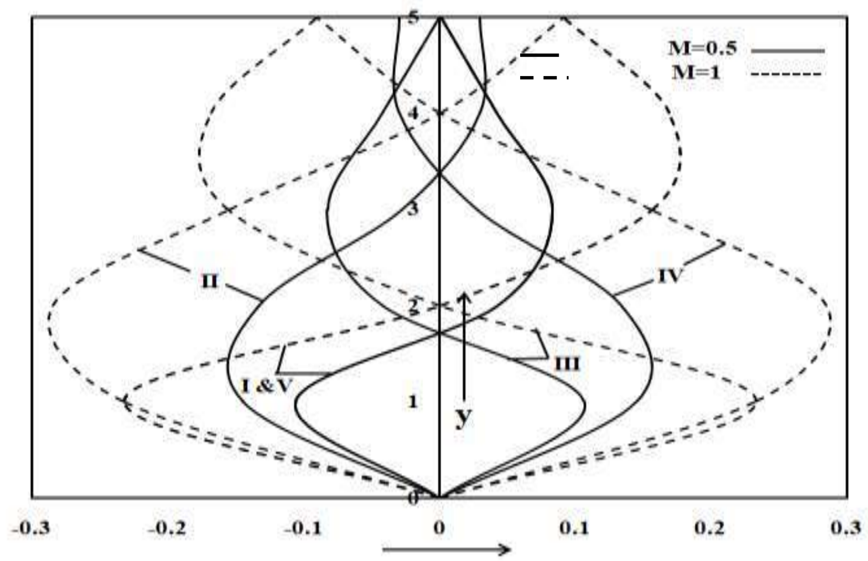

Figure 5. Induced magnetic field profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0}$.

$\mathbf{H/U_0}$

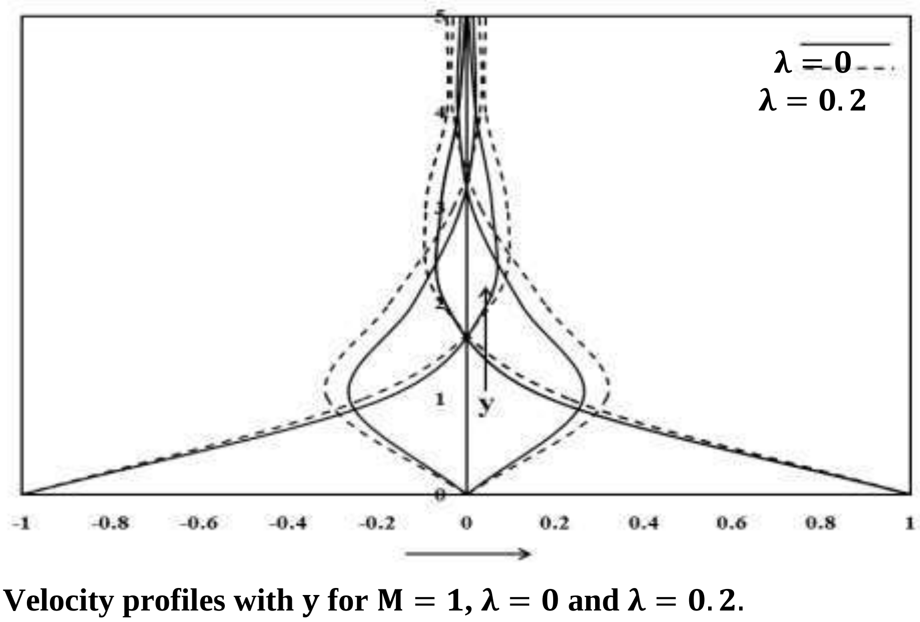

Figure 6. Velocity profiles with y for $\mathbf{M = 1}$, $\boldsymbol{\lambda = 0}$ and $\boldsymbol{\lambda = 0.2}$.

$\mathbf{u/U_0}$

Table 1. Variation of velocity for different values of t.

t/y	**0**	**1**	**2**	**3**	**4**	**5**
0	0.5	0.1274	-0.01619	-0.0689	-0.00546	-0.01673
1	0.3535	0.1984	0.08171	-0.01337	-0.05363	-0.04448
2	0	0.1530	0.07818	-0.00025	-0.04554	0.14776

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TRIPLE-DIFFUSIVE CONVECTION IN A RIVLIN-ERICKSEN FLUID LAYER IN POROUS MEDIUM

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ABSTRACT:
The onset of Triple-Diffusive convection in Rivlin-Ericksen fluid with varying gravity field is studied analytically in the presence of uniform vertical magnetic field in porous medium. For the case of stationary convection, the magnetic field, varying gravity field and the stable solute gradients have stabilizing effects whereas the medium permeability has destabilizing (or stabilizing) effect on the system under certain conditions. A linear stability analysis theory and normal mode analysis method have been carried out to study the onset convection. The kinematic viscoelasticity has no effect on the stationary convection. The solute gradients, magnetic field, varying gravity field, porosity and kinematic viscoelasticity introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained. The results are shown graphically.

KEYWORDS: Rivlin-Ericksen Fluid, Solute Gradients, Triple-Diffusive Convection, Varying Gravity Field.

1. INTRODUCTION:
The theoretical and experimental results of the onset of thermal instability in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics have been treated by Chandrasekhar[8]. Veronis [5] studied the problem of thermohaline convection in a layer of fluid heated and soluted from below.

With the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry, the investigations on such fluids are desirable. The Rivlin-Ericksen fluid [6] is one such fluid. The idealization of uniform gravity field can be hardly justified in the presence of large scale convection phenomenon occurring in atmosphere, the ocean or the mantle of the earth.

Pradhan et al [4] studied the thermal instability of the fluid layer under variable gravitational field.

A comprehensive review of the literature concerning thermosolutal convection in a fluid-saturated porous medium may be found in the book written by Nield and Bejan [3]. The thermosolutal instability in Walters' B' fluid in the presence of Hall currents in porous medium in hydromagnetics has been studied by Kango et al [9]. S. Chand [7] studied triple-diffusive convection in Walters' (Model B') fluid in porous medium in hydromagnetics.

The solution behaviour in the double-diffusive convection problem is more interesting than that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Benard problem. When temperature and two or more agencies, or two different salts, are present, the physical and mathematical situation becomes increasingly richer. Very interesting results in triply-diffusive convection have been obtained by Pearlstein et al., [1]. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker [2] derive the equation for non-Boussinesq convection in a multi-component fluid and investigate the situation analogous to that of Pearlstein et al., but allowing for a density non-linear in a temperature field. In reality the density of a fluid is never a linear function of temperature, and so the work of Straughan and Walker applies to the general situation where the equation of state is one of the density quadric in temperature. This is important, since they find that departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.

Keeping in view the recent increase in the number of non iso-thermal situations, our interest, in the present paper is to study the triple-diffusive convection in Rivlin-Ericksen Fluid with varying gravity field in porous medium.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider an infinite, horizontal, incompressible layer of thickness d of an electrically non-conducting Rivlin-Ericksen fluid heated and salted from below. The temperature T and solute concentrations C^1 and C^2 at the bottom and top surfaces $z=0,d$ are T_0 and T_1; C_0^1, C_1^1 and C_0^2, C_1^2 respectively, and a uniform temperature gradient β $(=|dT/dz|)$ and uniform solute gradients are $\beta'(=|dC^1/dz|)$ and $\beta''(=|dC^2/dz|)$ are maintained. A uniform magnetic field $\mathbf{H}=(0,0,H)$ and the gravity field $\mathbf{g}=(0,0,-g)$, where $\mathbf{g}=g_0\lambda$ $(g_0>0)$ is the value of g at z=0 and λ can be +ve or –ve according to whether gravity increases or decreases upwards from its value g_0; pervades the system. The equations relevant to the problem are:

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right]=-(\frac{1}{\rho_0})\nabla p+\mathbf{g}(1+\frac{\delta\rho}{\rho_0})-\frac{1}{k}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\mathbf{q}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\mathbf{H})\times\mathbf{H}, \tag{1}$$

$$\nabla.\mathbf{q}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T=\kappa\nabla^2 T, \tag{3}$$

$$E'\frac{\partial C^1}{\partial t}+(\mathbf{q}.\nabla)C^1=\kappa'\nabla^2 C^1, \tag{4}$$

$$E''\frac{\partial C^2}{\partial t}+(\mathbf{q}.\nabla)C^2=\kappa''\nabla^2 C^2. \tag{5}$$

The density equation of state is

$$\rho=\rho_0\left[1-\alpha(T-T_a)+\alpha'(C^1-C_a^1)+\alpha''(C^2-C_a^2)\right], \tag{6}$$

where $\rho,\rho_0,\mathbf{q},t,\mathbf{g},\nu,\nu',\kappa,\kappa',\kappa'',\alpha,\alpha',\alpha''$ are the fluid density, reference density, velocity, time, gravitational acceleration, the kinematic viscosity, the kinematic viscoelasticity, the thermal diffusivity, the solute diffusivities κ' and κ'', thermal and solvent coefficients of expansion

α' and α'' respectively. T_a is the average temperature given by $T_a = (T_0 + T_1)/2$ where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer and C_a^1, C_a^2 are the average concentrations given by $C_a^1 = (C_0^1 + C_1^1)/2$ and $C_a^2 = (C_0^2 + C_1^2)/2$, where C_0^1, C_1^1 and C_0^2, C_1^2 are the constant average concentrations of the lower and upper surfaces of the layer. Maxwell's equations yield

$$\varepsilon \frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2\mathbf{H}, \tag{7}$$

$$\nabla.\mathbf{H} = 0. \tag{8}$$

Here $E = \varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C_i}$ is a constant and E', E'' are analogous to E but corresponding to solute rather that heat. ρ_s, C_s and ρ_0, C_i stand for density and heat capacity of solid and fluid respectively. The steady state solution is

$$\mathbf{q} = (0,0,0), T = -\beta z + T_a, C^1 = -\beta' z + C_a^1, C^2 = -\beta'' z + C_a^2, \beta = (T_0 - T_1)/d,$$

$$\beta' = (C_1^1 - C_0^1)/d, \beta'' = (C_1^2 - C_0^2)/d, \rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z - \alpha''\beta'' z). \tag{9}$$

The change in density $\delta\rho$, caused by the perturbations θ in temperature and γ, γ' in concentrations, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma - \alpha''\gamma'). \tag{10}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -(\frac{1}{\rho_0})\nabla\delta p - g_0\lambda(\alpha\theta - \alpha'\gamma - \alpha''\gamma') - \frac{1}{k}(\nu + \nu'\frac{\partial}{\partial t})\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\mathbf{h})\times\mathbf{H}, \tag{11}$$

$$\nabla.\mathbf{q} = 0, \tag{12}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{13}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma, \tag{14}$$

$$E''\frac{\partial\gamma'}{\partial t} = \beta'' w + \kappa''\nabla^2\gamma', \tag{15}$$

$$\varepsilon\frac{\partial h}{\partial t} = (\mathbf{H}.\nabla) + \varepsilon\eta\nabla^2\mathbf{h}, \tag{16}$$

$$\nabla.\mathbf{h} = 0. \tag{17}$$

3. DISPERSION RELATION:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, \gamma', h_z] = [W(z), \Theta(z), K(z), \Gamma(z), \Psi(z)]\exp i(ik_x x + ik_y y + nt), \tag{18}$$

where k_x, k_y are the wave numbers along the x and y directions, respectively and $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number and n is, in general, a complex constant. Eq. (11)-(17), using (18), in non-dimensional form become

$$[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+F\sigma)](D^2-a^2)W+\frac{g_0\lambda a^2 d^2}{\nu}g_0\lambda(\alpha\Theta-\alpha'\Gamma-\alpha''\Psi)-\frac{\mu_e H}{4\pi\rho_0\nu}(D^2-a^2)DK=0, \quad (19)$$

$$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW, \quad (20)$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \quad (21)$$

$$(D^2-a^2-E'q_1\sigma)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W, \quad (22)$$

$$(D^2-a^2-E''q_2\sigma)\Psi=-\left(\frac{\beta'' d^2}{\kappa''}\right)W, \quad (23)$$

where $P_l=\frac{k_1}{d^2}$, is the non-dimensional parameter for medium permeability; $p_1=\frac{\nu}{k_1}$, is the thermal Prandtl number;

$q_1=\frac{\nu}{\kappa'}, q_2=\frac{\nu}{\kappa''}$, are the analogous Schmdit numbers accounting for two solutes, respectively and $a=kd, \sigma=\frac{nd^2}{\nu}, z=zd, \frac{d}{dz}=D.$

The appropriate boundary conditions w.r.t. which eqns. (19)-(23) must be solved (Chandrasekhar [8]).

$W=D^2W=0, \Theta=\Gamma=\Psi=0,$ at $z=0$ and $z=1$ and $K=0$ on the perfectly conducting boundaries and h_x, h_y, h_z are continues. (24)

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres.Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and $z=1$ and hence the proper solution of W characterizing the lowest mode is

$W=W_0\sin\pi z$, where W_0 is a constant. (25)

Eliminating Θ, Γ, K, Ψ between equations (19)-(23) and substituting (25), we obtained the dispersion relation

$$R_1=\frac{1+x}{x\lambda}[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-iF\sigma_1](1+x+iEp_1\sigma_1)+\frac{Q_1}{\lambda}[\frac{(1+x)(1+x+iEp_1\sigma_1)}{x(1+x+ip_2\sigma_1)}+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q_1\sigma_1)}+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE''q_2\sigma_1)}, .. \quad \textbf{(26)}$$

where $R_1=\frac{g\alpha\beta d^4}{\nu\kappa\pi^4}$, is the Rayleigh number; $S_1=\frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}, S_2=\frac{g\alpha''\beta'' d^4}{\nu\kappa''\pi^4}$ are the analogous solute Rayleigh numbers; $Q_1=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}$, is the Chandrasekhar number; $x=\frac{a^2}{\pi^2}$, is the dimensionless wave number and $P_l=\frac{P}{\pi^2}$

and $i\sigma_1=\frac{\sigma}{\pi^2}$.

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Equation (26) the required dispersion relation studying the effects of magnetic field, variable gravity field, medium permeability, kinematic viscoelasticity and stable solute gradients on the triple-diffusive convection of Rivlin-Ericksen fluid in porous medium.

4. STATIONARY CONVECTION

For the case of stationary convection, σ = 0 and Equation (26) reduces to

$$R_1 = \frac{(1+x)^2}{xP\lambda} + Q_1 \frac{1+x}{x\lambda} + S_1\lambda + S_2\lambda, \tag{27}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters Q_1, S_1, S_2 and P. For stationary convection the parameter F accounting for the kinematic viscoelasticity effect vanishes and thus the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. Equation (27) yields

$$\frac{dR_1}{dS_1} = +1, \ \frac{dR_1}{dS_2} = +1, \tag{28}$$

which imply that the stable solute gradients have a stabilizing effect on the system.

$$\frac{dR_1}{dQ_1} = \frac{1+x}{x\lambda}, \tag{29}$$

The magnetic field, therefore, has a stabilizing effect on the system when gravity is increasingly upwards ($\lambda > 0$) and destabilizes the system when gravity is decreasing upwards.

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xP\lambda}, \tag{30}$$

which is always negative, showing destabilizing effect of medium permeability and for constant varying gravity. The dispersion relation (27) is analyzed numerically.

In Fig.1, R_1 is plotted against x for $Q_1 = 30,\ P = 1,\ \lambda = 2,\ S_2 = 100\ \&\ S_1 = 100, 200, 300$. The stabilizing role of solute gradient is clear from the increase in the Rayleigh number with increasing parameter value S_1.

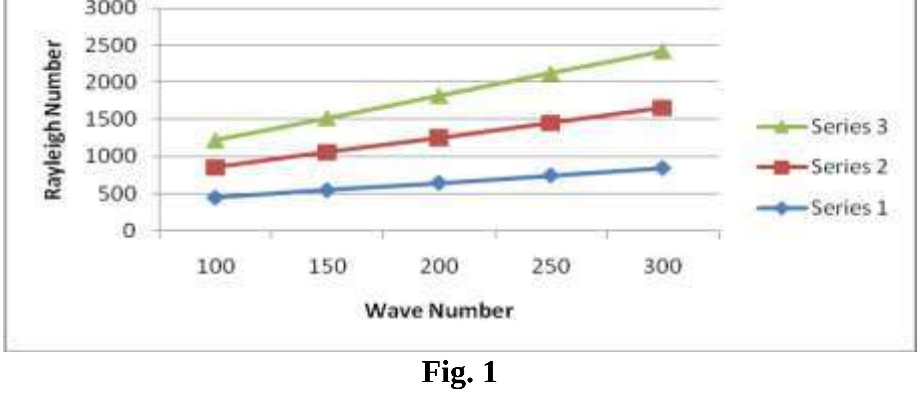

Fig. 1

In Fig.2, R_1 is plotted against x for $Q_1 = 30,\ P = 1,\ \lambda = 2,\ S_1 = 100\ \&\ S_2 = 100, 200, 300$. The stabilizing role of solute gradient is clear from the increase in the Rayleigh number with increasing parameter value S_2.

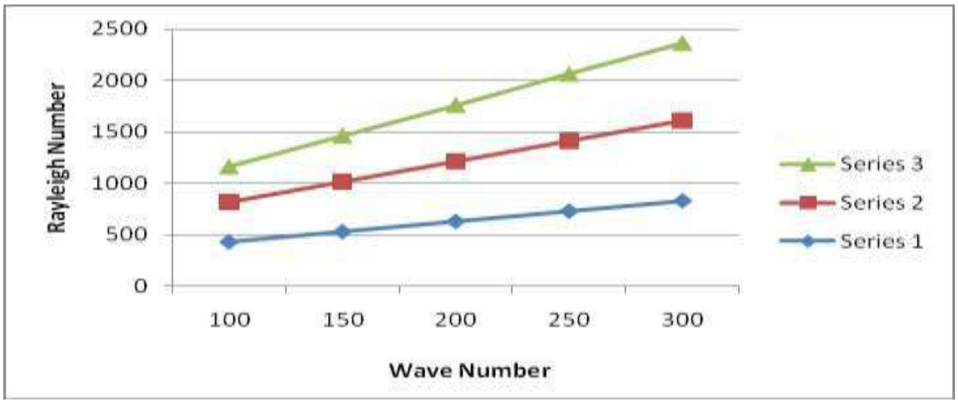

Fig. 2

In Fig.3, R_1 is plotted against x for $P=1$, $\lambda=2$, $S_1=100$, $S_2=100$ and $Q_1=30,60,90$. The stabilizing role of magnetic field is clear from the increase in the Rayleigh number with increasing parameter value Q_1.

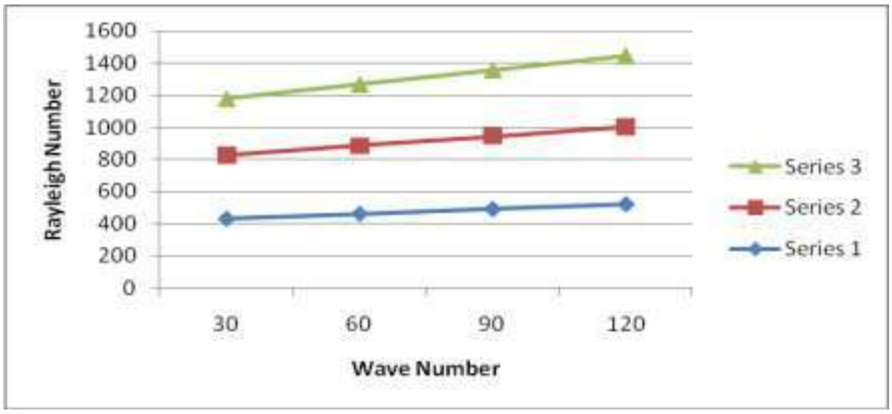

Fig. 3

In Fig.4, R_1 is plotted against x for $Q_1=30$, $\lambda=2$, $S_1=100$, $S_2=100$ and $P=2,4,6$. The destabilizing role of medium permeability is clear from the decrease in the Rayleigh number with increasing parameter value S_2.

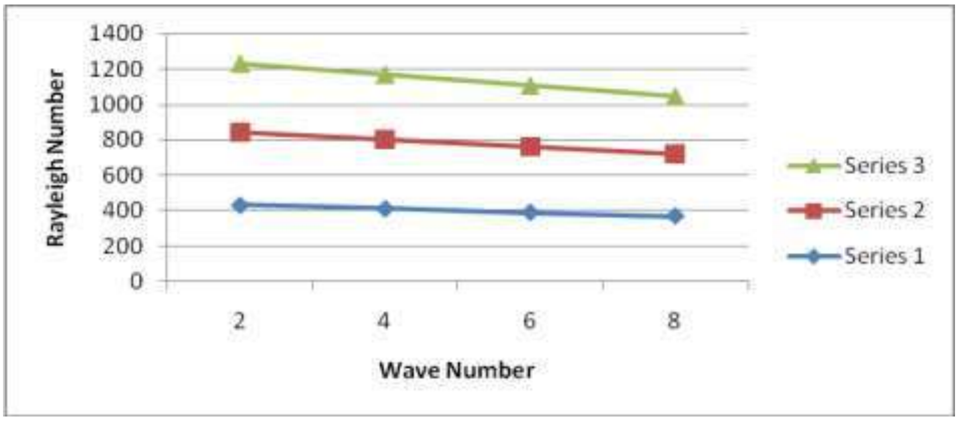

Fig. 4

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES:

Here we examine the possibility of oscillatory modes, if any, in the stability problem due to the presence of magnetic field, varying gravity field, kinematic viscoelasticity and stable solute gradients. Multiplying Eq. (19) by W*, the complex conjugate of W, integrating over the range of z and making use of Eq. (23) - (27) together with the boundary conditions (29) and putting $\sigma=\sigma_r+i\sigma_i$ & equating the real and imaginary parts, we obtain

$$[(\frac{1}{\varepsilon}-\frac{F}{P_l})I_1-(\frac{g_0\lambda\alpha\kappa a^2}{\nu\beta})Ep_1I_3+(\frac{g_0\lambda\alpha'\kappa'a^2}{\nu\beta'})E'q_1I_5+(\frac{g_0\lambda\alpha''\kappa''a^2}{\nu\beta''})E''q_2I_7+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9]\sigma_r$$
$$=-[\frac{I_1}{P_l}-\frac{g_0\lambda\alpha\kappa a^2}{\nu\beta}I_2+\frac{g_0\lambda\alpha'\kappa'a^2}{\nu\beta'}I_4+\frac{g_0\lambda\alpha''\kappa''a^2}{\nu\beta''}I_6+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8], \quad (31)$$

$$[(\frac{1}{\varepsilon}-\frac{F}{P_l})I_1+(\frac{g_0\lambda\alpha\kappa a^2}{\nu\beta})Ep_1I_3-(\frac{g_0\lambda\alpha'\kappa'a^2}{\nu\beta'})E'q_1I_5-(\frac{g_0\lambda\alpha''\kappa''a^2}{\nu\beta''})E''q_2I_7-\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9]\sigma_i=0, \quad (32)$$

The integrals $I_1,.........., I_9$ are all positive definite, where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz, I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz, I_3=\int_0^1|\Theta|^2dz, I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz, I_5=\int_0^1|\Gamma|^2dz,$$
$$I_6=\int_0^1\left(|D\Psi|^2+a^2|\Psi|^2\right)dz, I_7=\int_0^1|\Psi|^2dz, I_8=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz, I_9=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

It follows from equation (31) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (32) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, varying gravity field, magnetic field and stable solute gradients which were non-existent in their absence.

6. THE CASE OF OVERSTABILITY

Here we discuss the possibility of whether instability may occur as overstability. If we equate real and imaginary parts of (26) and eliminate R_1 between them, we obtain

$$A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \quad (33)$$

where we have put $c_1=\sigma_1^2$, $b=1+x$ and $A_3=b(E'q_1E''q_2)^2(\frac{1}{\varepsilon}-\frac{F}{P})^2[\frac{Ep_1}{P}+b(\frac{1}{\varepsilon}-\frac{F}{P})]$ (34)

$$A_0 = b^5[\frac{b}{P^2}(\frac{1}{\varepsilon}-\frac{F}{P})+\frac{Ep_1}{P^3}]+\frac{b^3}{P^2}(b-1)[S_1\lambda(Ep_1-E'q_1)+S_2\lambda(Ep_1-E''q_2)])^2+\frac{b^3}{P^2}Q_1(Ep_1-p_2) \quad (35)$$

As σ_1 is real for instability, the four values of $c_1\left(=\sigma_1^2\right)$ must be positive. The product of the roots of (33) is $-A_0 / A_3$, which is possible if $Ep_1 > E'q_1, Ep_1 > E''q_2, Ep_1 > p_2, \lambda > 0$ and $\frac{F}{P} < \frac{1}{\varepsilon}$ (36)

which implies that $E'\kappa < E\kappa', E''\kappa < E\kappa'', \kappa < E\eta, \lambda > 0$ and $\nu < \frac{k_1}{\varepsilon}$. (37)

Thus Equations (37) are, therefore, the necessary conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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EIGENVALUE APPROACH TO GERNELIZED PIEZOTHERMOELASTICITY IN TRANSVERSELY ISOTROPIC CONTINUUM WITH A CONTINOUS POINT SOURCE

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ABSTRACT:
The two-dimensional problem of piezothermoelasticity has been considered to investigate the disturbance in homogeneous, transversely isotropic (6mm class) generalized cylindrical piezothermoelastic continuum subjected to continuous mechanical load acting on thermally insulated and electrically shorted surface. Integral transform technique has been adopted, namely: Laplace transform for time variable and Hankel transform for space variables. These transforms technique have been employed to express the boundary conditions in the transformed domain. The formal solutions are employed to obtain the system of simultaneous linear algebraic equations. These systems of equations are solved by using Gauss elimination process for the unknowns. These values of unknowns are used to find the expressions of displacements, temperature change, electric potential, stresses and electric displacement in the transformed domain. The inverse transform integrals are evaluated by using numerical technique. Temperature, normal stress and shear stress so obtained in the physical domain, are computed numerically from the relevant expressions and relations code for PZT-5A material. Finally, the illustration of the results for classical and non-classical models of thermoelasticity has been presented graphically.

KEYWORDS: Generalized piezothermoelastic; Integral transforms; Relaxation time; Eigen value; Electrically shorted.

INTRODUCTION:

Composite structures can be exposed to variety of fields in the different environment. The knowledge of its response to the different loads is necessary to predict its capability to the elastodynamical stresses induced by sudden loading, which is essential for the proper and safe design. The general solution for transversely isotropic piezothermoelastic materials has been used by Hou et al. [1] to construct the three-dimensional solution of a steady point heat source acting on the apex of cone. Non-linear transient behavior of a piezothermoelastic laminate deviated from equilibrium state has been analyzed by Watanabe et al. [2]. The response of a functionally graded, radially polarized hollow cylinder under dynamic axisymmetric loadings has been studied by Babaei and Chen [3]. The one-dimensional problem of functionally graded medium excited by moving heat source has been investigated by Babaei and Chen [4].

Ailawalia and Khurana [5] studied the deformation of transversely isotropic piezoelectric medium with an overlying infinite viscous fluid due to moving load acting at the interface of both media by applying Fourier transforms. The bending behavior of a circularly curved, functionally graded piezoelectric cantilever actuator under an applied electric load and heat conduction has been studied by Zaman et al. [6]. Akbarzadeh et al. [7] investigated the dynamic response of a functionally graded piezoelectric medium (FGPM) subjected to a moving heat source. The thickness response of a piezothermoelastic panel, which is occupied by a material of hexagonal crystal class, has been studied by Montanaro [8] under quasi-static boundary conditions with one of the bounding surface subjected to prescribed temperature.

FORMULATION OF THE MODEL:

We consider a homogeneous, transversely isotropic, thermally conducting, linear generalized piezothermoelastic cylindrical medium. Initially, the medium is assumed to be undisturbed and at uniform temperature T_0. The origin of cylindrical co-ordinate system (r, θ, z) is taken at any point on the surface $(z = 0)$ and $z-$axis pointing vertically downward into the medium. It is assumed that disturbances are caused by continuous point mechanical load acting at the surface $z = 0$. Due to symmetry, all the field quantities are independent of θ co-ordinate. The governing field equations of linear piezothermoelasticity for homogeneous, transversely isotropic (6mm class) axisymmetric cylindrical body upon employing Lord and Shulman [9] and Green and Lindsay [10] models of thermoelasticity, in the absence of charge density, heat sources and body forces; are given by Thakur et al. [11]. This system of equations can be expressed in the matrix form as:

$$\left(\mathbf{A} D^2 + \mathbf{B}^{**} D + \mathbf{C}^{**}\right)\mathbf{Z} = 0 \tag{1}$$

$$\boldsymbol{\sigma}^{**} = \left(\mathbf{A}^{*} D + \mathbf{B}^{\oplus}\right)\mathbf{Z}$$

where

$$\boldsymbol{\sigma}^{**} = \begin{bmatrix}\sigma_{rr} & \sigma_{zz} & \sigma_{rz} & D_z\end{bmatrix}^T \tag{2}$$

$$\mathbf{Z} = \begin{bmatrix}u & w & \phi & T\end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} c_2 & 0 & 0 & 0 \\ 0 & c_1 & 1 & 0 \\ 0 & 1 & -\eta_3 & 0 \\ 0 & 0 & 0 & \overline{K} \end{bmatrix}$$

$$\mathbf{B}^{**} = \begin{bmatrix} 0 & c_3 \dfrac{\partial}{\partial r} & e_1 \dfrac{\partial}{\partial r} & 0 \\ c_3 \nabla_2 & 0 & 0 & -\overline{\beta}\left(1 + t_1 \delta_{2k} \dfrac{\partial}{\partial t}\right) \\ e_1 \nabla_2 & 0 & 0 & p_1\left(1 + t_1 \delta_{2k} \dfrac{\partial}{\partial t}\right) \\ 0 & -\varepsilon\overline{\beta}\left(\dfrac{\partial}{\partial t} + t_0 \delta_{1k} \dfrac{\partial^2}{\partial t^2}\right) & \varepsilon\, p_1\left(\dfrac{\partial}{\partial t} + t_0 \delta_{1k} \dfrac{\partial^2}{\partial t^2}\right) & 0 \end{bmatrix}$$

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$$\mathbf{C}^{**} = \begin{bmatrix} \left(\nabla_1^2 - \frac{1}{r^2}\right) - \frac{\partial^2}{\partial t^2} & 0 & 0 & -\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial r} \\ 0 & c_2\ \nabla_1^2 - \frac{\partial^2}{\partial t^2} & e_2\nabla_1^2 & 0 \\ 0 & e_2\nabla_1^2 & -\eta_3\bar{\varepsilon}\nabla_1^2 & 0 \\ -\varepsilon\left(\frac{\partial}{\partial t} + t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\nabla_2 & 0 & 0 & \nabla_1^2 - \left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right) \end{bmatrix}$$

$$\mathbf{A}^{*} = \begin{bmatrix} 0 & (c_3 - c_2) & (e_1 - e_2) & 0 \\ 0 & c_1 & 1 & 0 \\ c_2 & 0 & 0 & 0 \\ 0 & 1 & -\eta_3 & 0 \end{bmatrix}$$

$$\mathbf{B}^{\oplus} = \begin{bmatrix} \nabla_2 - \frac{2c_4}{r} & 0 & 0 & -\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right) \\ (c_3 - c_2)\nabla_2 & 0 & 0 & -\bar{\beta}\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right) \\ 0 & c_2\frac{\partial}{\partial r} & e_2\frac{\partial}{\partial r} & 0 \\ (e_1 - e_2)\nabla_2 & 0 & 0 & p_1\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right) \end{bmatrix}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}, \nabla_2 = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$ (3)

Initial and Regularity Conditions:

The initial and regularity conditions are given by:

$u(r,z,0) = 0 = \dot{u}(r,z,0)$, $w(r,z,0) = 0 = \dot{w}(r,z,0)$, $T(r,z,0) = 0 = \dot{T}(r,z,0)$,

$\phi(r,z,0) = 0 = \dot{\phi}(r,z,0)$, for $t = 0$, $z \geq 0$, $r \geq 0$ and (4)

$u(r,z,t) = 0$, $w(r,z,t) = 0$, $T(r,z,t) = 0$, $\phi(r,z,t) = 0$, for $t > 0$, $\sqrt{r^2 + z^2} \to \infty$. (5)

Boundary Conditions:

The following boundary conditions are to be satisfied in non-dimensional form at the surface $z = 0$ of the solid half-space.

$$\sigma_{zz}(r,0,t) = -\frac{\sigma_0^*\ \delta(r)g(t)}{2\pi r}\ , \sigma_{rz}(r,0,t) = 0,\ T_{,z}(r,0,t) = 0,\ \phi(r,0,t) = 0 \tag{6}$$

where $\sigma_0^* = \frac{\sigma_0}{\beta_1 T_0}$ denotes intensity of the mechanical load.

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FORMAL SOLUTION:

In order to solve the proposed model, we apply Laplace transform (Churchill [12]) with respect to time 't' defined by:

$$\overline{\mathbf{Z}}(r,z,\ p)=\int_0^{\infty}\mathbf{Z}(r,z,t)\,exp(-pt)\ dt\,,\ \overline{\boldsymbol{\sigma}}(r,z,\ p)=\int_0^{\infty}\boldsymbol{\sigma}(r,z,t)\,exp(-pt)\ dt \tag{7}$$

where the functions $\overline{\mathbf{Z}}(r,z,\ p)$ and $\overline{\boldsymbol{\sigma}}(r,\ z,\ p)$ represent the Laplace transforms of column vectors $\mathbf{Z}(r,z,\ t)$ and $\boldsymbol{\sigma}(r,z,\ t)$, respectively.

We define the Hankel transform with respect to 'r' as:

$$\tilde{\mathbf{Z}}(q,z,p)=\int_0^{\infty} r\,\overline{\mathbf{Z}}(r,z,p)J_n(qr)dr\,,\tilde{\boldsymbol{\sigma}}(q,\ z,\ p)=\int_0^{\infty}\ r\,\overline{\boldsymbol{\sigma}}\ (r,\ z,\ p)\ J_n(qr)dr \tag{8}$$

where $n=0$ for $\overline{w}(r,z,p)$, $T(r,z,p)$ and $n=1$ in the case of $\overline{u}(r,z,p)$. Here $J_n(qr)$ is a Bessel function of first kind and of order n. Applying Laplace and Hankel transform from equations (7) and (8) to equations (1) and (2), we obtain:

$$\left(\mathbf{A}D^2+\tilde{\mathbf{B}}^{*}D+\tilde{\mathbf{C}}^{*}\right)\tilde{\mathbf{Z}}=0\,, \tag{9}$$

$$\tilde{\boldsymbol{\sigma}}=\left(\mathbf{A}^{*}D+\tilde{\mathbf{B}}^{\oplus}\right)\tilde{\mathbf{Z}} \tag{10}$$

where $\tilde{\mathbf{B}}^{*}=\begin{bmatrix} 0 & -qc_3 & -q\,e_1 & 0 \\ c_3 q & 0 & 0-\overline{\beta}\ p\tau_1 & \\ e_1 q & 0 & 0\ p_1 p\tau_1 & \\ 0 & -\varepsilon\overline{\beta}\ p^2\tau_0^{*} & \varepsilon\,p_1\,p^2\tau_0^{*} & 0 \end{bmatrix}$

$$\tilde{\mathbf{C}}^{*}=\begin{bmatrix} -\left(p^2+q^2\right) & 0 & 0 & \tau_1\,pq \\ 0 & \left(c_2\ q^2+p^2\right) & -e_2\ q^2 & 0 \\ 0 & -e_2\ q^2 & \eta_{\,3}\overline{\varepsilon}q^2 & 0 \\ -\varepsilon\tau_0^{*}\ p^2\,q & 0 & 0 & -\left(q^2+\tau_0 p^2\right) \end{bmatrix}$$

$$\tilde{\mathbf{B}}^{\oplus}=\begin{bmatrix} q & 0 & 0 & -\overline{\beta}\,\tau_1\,p & \\ q\,(c_3-c_2) & 0 & 0 & -\overline{\beta}\,\tau_1\,p \\ 0 & -qc_2 & -qe_2 & 0 \\ (e_1-e_2)q & 0 & 0 & p\,p_1\tau_1 \end{bmatrix} \tag{11}$$

where $\tau_0=p^{-1}+t_0$, $\tau_0^{*}=p^{-1}+\delta_{1k}\,t_0$, $\tau_1=p^{-1}+t_1\delta_{2k}$.

In order to solve the matrix ordinary differential equations (9), we assume its solution as:

$$\tilde{\mathbf{Z}}(q,\ z,\ p)=\mathbf{X}^{*}(q,p)exp(mz) \tag{12}$$

Upon substituting solution (12) in equation (10), we obtain:

$$\left(\mathbf{A}m^2+\tilde{\mathbf{B}}^{*}m+\tilde{\mathbf{C}}^{*}\right)\mathbf{X}^{*}(q,p)=0 \tag{13}$$

which is a eigenvalue problem. The system of equations (13) has a non-trivial solution if and only if:

$$det\left(\mathbf{A}m^2+\tilde{\mathbf{B}}^{*}m+\tilde{\mathbf{C}}^{*}\right)=0 \tag{14}$$

On simplification this determinantal equation leads to polynomial characteristic equation given as:

$$m^8-f_1m^6+f_2m^4-f_3m^2+f_4=0 \tag{15}$$

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where

$$f_1 = \left(a_1 + \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} \right),\ f_2 = \left(a_2 + a_1 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_1 \right),$$

$$f_3 = \left(a_3 + a_2 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_2 \right),\ f_4 = \left(a_3 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_3 \right). \tag{16}$$

Quantities F, a_k and A_k $(k = 1, 2, 3)$ used in equation (16) are defined in the Appendix. The characteristic equation (15), being biquadratic in m^2 provides four pairs of characteristic roots given by

$$\sum m_1^2 = f_1,\ \sum m_1^2 m_2^2 = f_2,\ \sum m_1^2 m_2^2\, m_3^2 = f_3,\quad m_1^2 m_2^2 m_3^2 m_4^2 = f_4 \tag{17}$$

The formal solution of equation (12) can be written as:

$$\tilde{\mathbf{Z}}(q, z, p) = \sum_{k=1}^{4} B_k \mathbf{X}_k^*(q, p)\ exp(-m_k z) + B_{k+4}\ \mathbf{X}_{k+4}^*(q, p)\ exp(m_k z) \tag{18}$$

where $\mathbf{X}_k^*(q, p) = \left[\, 1 \ \ V_k^* \ \ W_k^* \ \ S_k^* \right]^T,\ k = 1, 2, 3, 4;\ m = -m_k$

$$\mathbf{X}_{k+4}^*(q, p) = \left[\, 1 \ \ -V_k^* \ -W_k^* \ \ S_k^* \right]^T,\ k = 1, 2, 3, 4;\ m = m_k \tag{19}$$

where $V_k^* = \dfrac{Q_1^*(m_k)}{Q^*(m_k)},\quad W_k^* = \dfrac{Q_2^*(m_k)}{Q^*(m_k)},\quad S_k^* = \dfrac{Q_3^*(m_k)}{Q^*(m_k)},\qquad k = 1 \text{ to } 4.$ (20)

Here, $Q^*(m_k) = -pq\tau_1 \left\{ \left(c_3 \lambda_1 + e_1 \lambda_2 - \lambda_3\right) m_k^4 + \left(\lambda_6 q^2 - \lambda_7 p^2\right) m_k^2 - q^2 \left(\bar{\varepsilon}\, \eta_3\, l_4 + e_2{}^2 q^2\right) \right\}$

$$Q_1^*(m_k) = \tau_1 p m_k \left\{ -\lambda_1\, c_2\, m_k^4 + \left(\lambda_1 p^2 + \lambda_8\, q^2\right) m_k^2 + q^2 \left(\lambda_4 l_2 + \lambda_9\, q^2\right) \right\}$$

$$Q_2^*(m_k) = \tau_1\, p\, m_k \left\{ c_2\, \lambda_2\, m_k^4 - \left(p_1 l_3 + \overline{\beta}\, l_2 + \lambda_{10}\, q^2\right) m_k^2 + l_2 \left(\lambda_5 q^2 + p_1 p^2\right) + \left(\lambda_{11} q^2 + e_1 p^2\right) q^2 \right\}$$

$$Q_3^*(m_k) = \left\{ \begin{matrix} c_2 \lambda_3 m_k^6 - \left(\lambda_{12} q^2 + \eta_3\, l_3 + l_2\right) m_k^4 \\ + \left\{ \left(\bar{\varepsilon}\, \eta_3 l_3 + \lambda_{13} e_2 q^2 + 2 e_2 p^2\right) q^2 + l_4 \left(e_1^{\,2} q^2 + \eta_3 l_2\right) \right\} m_k^2 - q^2 l_2 \left(\bar{\varepsilon}\, \eta_3 l_4 + e_2{}^2 q^2\right) \end{matrix} \right\} \tag{21}$$

The quantities λ_k $(k = 1 \text{ to } 13)$ used in equation (21) are defined in the Appendix.

The formal solution of equation (12) satisfying the requirement of boundedness is given by:

$$\tilde{\mathbf{Z}}(q, z, p) = \sum_{k=1}^{4} \mathbf{X}_k^*(q, p) B_k\ exp(-m_k z) \tag{22}$$

Upon using solution (22) in equation (10), we get:

$$\tilde{\boldsymbol{\sigma}} = \sum_{k=1}^{4} \mathbf{Y}\, \mathbf{X}_k^*(q, p) B_k\ exp(-m_k z) \tag{23}$$

where

$$\mathbf{Y} = \tilde{\mathbf{B}}^{\oplus} - m_k \mathbf{A}^* = \begin{bmatrix} q & -(c_3 - c_2) m_k & -(e_1 - e_2) m_k & -p\tau_1 \\ q(c_3 - c_2) & -c_1 m_k & -m_k & -\overline{\beta}\, p\tau_1 \\ -c_2 m_k & -qc_2 & -qe_2 & 0 \\ q(e_1 - e_2) & -m_k & \eta_3 m_k & p_1 p \tau_1 \end{bmatrix} \tag{24}$$

This gives the formal solution for piezothermoelastic (6mm class) cylindrical continua.

SOLUTION OF THE PROBLEM:

Upon applying integral transforms defined by equations (7) and (8) to the boundary conditions (6) and using the solution (22) and (23), we get a system of equations with four

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unknown arbitrary constants B_k (k = 1, 2, 3, 4) as:

$$D_{11}^{*}B_1 + D_{12}^{*}B_2 + D_{13}^{*}B_3 + D_{14}^{*}B_4 = -q\sigma_0^{*}\,\tilde{g}(p)\,, \quad D_{21}^{*}B_1 + D_{22}^{*}B_2 + D_{23}^{*}B_3 + D_{24}^{*}B_4 = 0$$

$$W_1\,B_1 + W_2\,B_2 + W_3\,B_3 + W_4\,B_4 = 0\,,\; m_1S_1\,B_1 + m_2S_2\,B_2 + m_3S_3\,B_3 + m_4S_4\,B_4 = 0 \qquad (25)$$

where $D_{1k}^{*} = q(c_3 - c_2) - c_1 m_k V_k^{*} - m_k W_k^{*} - p\bar{\beta}\tau_1 S_k^{*},\;\; D_{2k}^{*} = -c_2 m_k - qc_2 V_k^{*} - qe_2 W_k^{*}$ for $k = 1,\ 2,\ 3,\ 4$.

On solving equation (25) with the help of Gauss elimination technique, we obtain the value of B_k (k = 1, 2, 3, 4

$$B_k = (-1)^k q\,\sigma_0^{*}\,\frac{\Theta_k^{**}}{\Theta^{**}}\,\tilde{g}(p), \qquad k = 1,\ 2, 3, 4 \qquad (26)$$

where

$$\Theta_1^{**} = \begin{vmatrix} D_{22}^{*} & D_{23}^{*} & D_{24}^{*} \\ m_2S_2 & m_3S_3 & m_4S_4 \\ W_2 & W_3 & W_4 \end{vmatrix},\; \Theta_2^{**} = \begin{vmatrix} D_{21}^{*} & D_{23}^{*} & D_{24}^{*} \\ m_1S_1 & m_2\,S_3 & m_4S_4 \\ W_1 & W_3 & W_4 \end{vmatrix}$$

$$\Theta_3^{**} = \begin{vmatrix} D_{21}^{*} & D_{22}^{*} & D_{24}^{*} \\ m_1S_1 & m_2S_2 & m_4S_4 \\ W_1 & W_2 & W_4 \end{vmatrix}, \qquad \Theta_4^{**} = \begin{vmatrix} D_{21}^{*} & D_{22}^{*} & D_{23}^{*} \\ m_1S_1 & m_2S_2 & m_3S_3 \\ W_1 & W_2 & W_3 \end{vmatrix}$$

$$\Theta^{**} = D_{11}^{*}\,\Theta_1^{**} - D_{12}^{*}\,\Theta_2^{**} + D_{13}^{*}\,\Theta_3^{**} - D_{14}^{*}\,\Theta_4^{**}$$

In this case, the displacements, electric potential, temperature change, stresses and electrical displacement are obtained as:

$$\left(\tilde{u}, \tilde{w}, \tilde{\phi}, \tilde{T}\right) = \frac{q\sigma_0^{*}}{p}\sum_{k=1}^{4}(-1)^k\left(1,\, V_k^{*},\, W_k^{*},\, S_k^{*}\right)\frac{\Theta_k^{**}}{\Theta^{**}}\,exp(-m_k z)$$

which provides us the solution for this particular set of boundary conditions.

INVERSION OF THE TRANSFORMS:

According to Bradie [13], the various quadrature formulae such as Newton-Cotes, Romberg and Gaussian quadrature etc. can be used to approximate the value of an improper integral, provided the integral exists. However, some changes of variable generally must be made to achieve theoretical order of convergence, if required. Due to the existence of damping terms, the dependence of characteristic roots m_k ($k = 1, 2, 3, 4$) on p is very complicated and hence the inversion of integral transform is quite difficult task analytically. But this can be conveniently managed through numerical evaluations of the inversion integrals (Sharma and Kumar [14]; Thakur et al. [11]). In order to obtain the solution of the instant problem in the physical domain, we have to invert the integral transforms in equations (27) and (28) numerically.

NUMERICAL RESULTS AND DISCUSSION:

The piezothermoelastic material PZT-5A has been chosen for the purpose of numerical calculations, physical data for which is given by Thakur et al. [11]. For computation purpose the intensity of the mechanical load (σ_0^{*}) has been considered as unity. The non-dimensional temperature change and stresses has been computed by taking values of thermal relaxation times as $t_0 = 0.5$ and $t_1 = 0.3$ in the context of LS, CT and GL models of thermoelasticity. The computations are carried out for two instants of times $t = 0.1$ and $t = 0.25$ for thermally insulated half-space at $z = 1.0$. The computer simulated quantities have been plotted in Figs. 1 to 3.

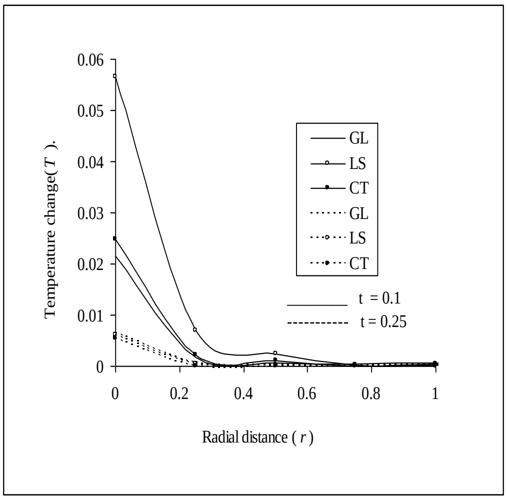

Fig. 1: Variation of non-dimensional temperature change with radial distance.

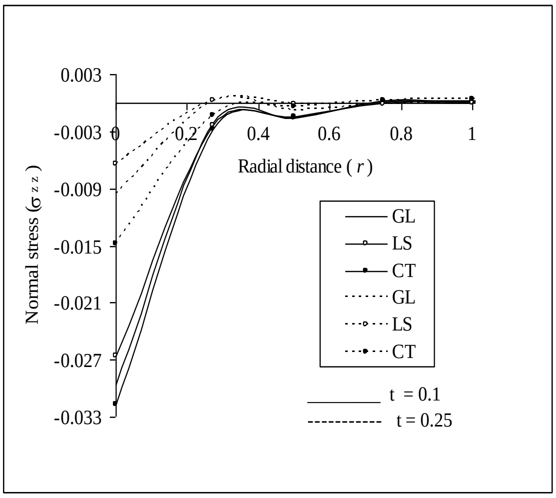

Fig. 2: Variation of non-dimensional normal stress with radial distance.

Fig. 1 presents the variation of absolute temperature change (T) in the context of GL, LS and CT models of thermoelasticity due to continuous point mechanical load. The magnitude of temperature change (T) decreases with radial distance and observes oscillating behavior to vanish at a certain value of radial distance $(r \geq 1.0)$. The oscillating behavior of the temperature change is attributed to compression and expansion of the solid.

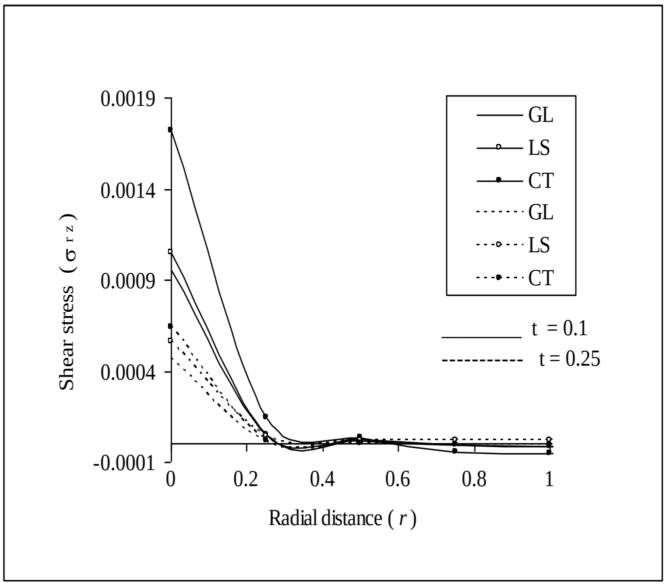

Fig. 3: Variation of non-dimensional shear stress with radial distance

From Fig. 2 it is noticed that the magnitude of normal stress (σ_{zz}) is quite large in the domain $0 \le r \le 0.2$, decreases afterward and finally dies out in an oscillating manner at certain value of radial distance $(r \ge 1.0)$ due to compression and expansion of the solid. Fig. 3 reveals that the shear stress (σ_{rz}) follows oscillatory behavior with varying amplitude due to continuous load. The shear stress has maximum magnitude near the vicinity of the load which decreases and ultimately dies out in an oscillating fashion with increasing radial distance.

CONCLUDING REMARKS:

The magnitudes of all the considered field functions have been observed to be quite large near the vicinity of point source which follow decreasing trends as we move away from the source and are noticed to vanish after certain value of radial distance, which shows the existence of wave-fronts. There exists no wave-front of temperature and stress distributions in case of coupled (CT) thermoelastic model due to diffusive nature of heat conduction equation. The shear stress development is comparatively small as compared to normal stress and temperature change. This means major portion of the energy is carried by the normal stress and thermal waves and only meager amount propagates in the form of shear stress waves as expected.

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APPENDIX

The quantities F, a_k and A_k $(k = 1, 2, 3)$ used in equation (16) are obtained as:

$$F = \frac{\left(1-\varepsilon\tau^*\varepsilon_\eta p_1^2\right)\left(c_1+\varepsilon\overline{\beta}^2\tau^*\right)+\varepsilon_\eta\left(1-p_1\varepsilon\overline{\beta}\tau^*\right)^2}{\left(c_1+\varepsilon_\eta\right)},$$

$$a_1 = \frac{l_3+\bar{\varepsilon}c_1c_2q^2+\varepsilon_\eta\left\{\left(1+c_1e_1^2+2c_2e_2-2c_3e_1\right)q^2+p^2\right\}}{l_1},$$

$$a_2 = \frac{\bar{\varepsilon}l_3q^2+l_2l_4+\varepsilon_\eta\left\{\left(c_2e_1^2+c_2e_2^2-2c_3e_1e_2\right)q^4+\left(2e_2l_2+e_1^2p^2\right)q^2\right\}}{l_1},$$

$$a_3 = \frac{q^2l_2\left\{\bar{\varepsilon}l_4+\varepsilon_\eta e_2^2q^2\right\}}{l_1},$$

$$A_1 = \frac{\left\{E_1L_3+\bar{\bar{\varepsilon}}C_1C_2q^2\right\}+\varepsilon_\eta\left\{\left(2EC_2\bar{e}_2-2C_3E\bar{e}_1+\bar{e}_1^2C_1+E^2\right)q^2+E^2\bar{p}^2\right\}}{L_1}$$

$$A_2 = \frac{\bar{\bar{\varepsilon}}q^2L_3+E_1L_2L_4+\varepsilon_\eta\left\{\left(\bar{e}_1^2C_2+\bar{e}_2^2C_2-2C_3\bar{e}_1\bar{e}_2\right)q^4+\left(2E\bar{e}_2L_2+\bar{e}_1^2\bar{p}^2\right)q^2\right\}}{L_1},$$

$$A_3 = \frac{q^2L_2\left\{\varepsilon_\eta\bar{e}_2^2q^2+\bar{\bar{\varepsilon}}L_4\right\}}{L_1}$$

Where

$$\varepsilon_\eta = \frac{1}{\eta_3}, l_1 = c_2\left(c_1+\varepsilon_\eta\right), l_2 = q^2+p^2, l_3 = Pq^2+Jp^2, l_4 = c_2q^2+p^2, L_1 = C_2\left(C_1E_1+\varepsilon_\eta E^2\right),$$

$$L_2 = q^2+\bar{p}^2, L_3 = P'q^2+J'\bar{p}^2, L_4 = C_2q^2+\bar{p}^2,\ F = \left(1-\varepsilon\tau^*\right)^3,$$

$$C_1 = \frac{c_1-\tau^*\overline{\beta}^2\varepsilon}{1-\varepsilon\tau^*}, C_2 = \frac{c_2}{1-\varepsilon\tau^*}, C_3 = \frac{c_3-\tau^*\overline{\beta}\varepsilon}{1-\varepsilon\tau^*}, \bar{e}_1 = \frac{e_1+\varepsilon p_1\tau^*}{1-\varepsilon\tau^*}, \bar{e}_2 = \frac{e_2}{1-\varepsilon\tau^*},$$

$$E = \frac{1+\varepsilon\overline{\beta}p_1\tau^*}{1-\varepsilon\tau^*}, E_1 = \frac{1+\varepsilon p_1^2\varepsilon_\eta\tau^*}{1-\varepsilon\tau^*}, \bar{\bar{\varepsilon}} = \frac{\bar{\varepsilon}}{1-\varepsilon\tau^*},\ \bar{p}^2 = \frac{p^2}{1-\varepsilon\tau^*}, P = c_1+c_2^2-c_3^2,$$

$$J = c_1+c_2, P' = \frac{\left(1+\varepsilon\tau^*\right)\left(c_1+\varepsilon\overline{\beta}^2\tau^*\right)+c_2^2-\left(c_3+\varepsilon\overline{\beta}\tau^*\right)^2}{\left(1+\varepsilon\tau^*\right)^2}, J' = \frac{\left(J+\varepsilon\overline{\beta}^2\tau^*\right)}{\left(1+\varepsilon\tau^*\right)},$$

$$\tau^* = \begin{cases} 1, & \text{for LS and CT theory} \\ \dfrac{\tau_1}{\tau_0}, & \text{for GL theory} \end{cases}$$

The quantities λ_k $(k = 1 \text{ to } 13)$ used in equation (21) are defined as:

$$\lambda_1 = \eta_3\overline{\beta}-p_1, \lambda_2 = \overline{\beta}+p_1c_1, \lambda_3 = 1+c_1\eta_3, \lambda_4 = p_1e_2-\bar{\varepsilon}\overline{\beta}\eta_3, \lambda_5 = p_1c_2+\overline{\beta}e_2,$$

$$\lambda_6 = \eta_3\left(c_1\bar{\varepsilon}+c_2\right), \lambda_7 = p_1e_1-\eta_3, \lambda_8 = \overline{\beta}\eta_3\left(1+\bar{\varepsilon}c_2\right)-p_1\left(1+c_2e_2-c_3e_1\right)-\left(e_1+\eta_3c_3\right)+e_1^2\overline{\beta},$$

$$\lambda_9 = \left(e_1e_2+\bar{\varepsilon}c_3\eta_3\right), \lambda_{10} = \left(c_1e_1+\overline{\beta}c_2e_2-\overline{\beta}c_3e_1-c_3\right), \lambda_{11} = e_1c_2-c_3e_2,$$

$$\lambda_{12} = \bar{\varepsilon}\eta_3c_1c_2+2c_2e_2-2c_3e_1+c_1e_1^2,\ \lambda_{13} = c_2e_2-2e_1c_3+2.$$

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ON THE BOUNDS FOR OSCILLATIONS IN DOUBLE DIFFUSIVE CONVECTION WITH CROSS-DIFFUSIONS EFFECTS IN POROUS MEDIUM: DARCY MODEL

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ABSTRACT: In the present paper, the stability analysis of double-diffusive convection problems (Veronis and Stern Types) with cross-diffusions effects (Soret and Dufour effects) have been carried out mathematically in porous medium using Darcy model. The eigen values equations governing these problems have been casted into mathematically tractable forms for stability analysis using some linear transformations. The stability of the oscillatory modes and consequently the bounds for the complex growth rate for arbitrary neutral or unstable oscillatory perturbations are derived which are valid for each combinations of rigid (slip free) and dynamically free (stress-free) boundaries. Various consequences of the derived results are also worked out.

KEYWORDS: Double-Diffusive Convection; oscillatory motions; porous medium, Darcy model, complex growth rate; eigenvalues problem; Soret effect; Dufour effect;

INTRODUCTION:

A broader range of dynamical behaviour is observed in the convective motions that may occur in a gravitational field containing two components (for example, temperature and solute) of different diffusivities that affect the density of the fluid and the phenomenon is known as double-diffusive convection. These phenomena of double-diffusive convection occur when the temperature and concentration gradients are of comparable magnitude and operate on different scales and lead to large scale convection. These kinds of double-diffusion processes are found in astrophysics (big Helium-stars), the earth core, metal alloy, refilling of gas reservoirs, etc. Double-diffusive convection is also of importance in various other fields of practical importance such as high quality crystal production oceanography, production of pure medication, solidification of molten alloys, limnology and engineering.

The double diffusive process was first recognized by Arons and Stommel [1] through his '*thought experiment*' with ocean flow/circulation in mind. Two fundamental configurations have been studied in the context of thermohaline instability problem, the first one by Stern[2],wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and the second one by Veronis [3], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing. Stern found that the steady motion is the preferred mode of onset of instability whereas Veronis observed that oscillatory mode of instability is the preferred mode of convective instability. Since then numerous authors have investigated the double diffusive convection problems under varying assumptions of hydrodynamics both numerically and analytically. For a broader view of the subject of

double-diffusive convection one may refer to Turner [4], Brandt and Fernando [5], Schmitt[6] andNield [7].
The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects[8,9]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, whereas the analogous effect that arises from a concentration gradient which produces a heat flux is called the Dufour effect. When considering the isothermal coupled diffusion between a solvent and two solutes, we need to include these two cross diffusion effects as each property gradient has a significant influence on the flux of the other property. The stability of Dufour-Soret driven double-diffusion convection in a horizontal layer of a fluid subjected to thermal and solutal gradients has been investigated theoretically by means of a linear stability analysis by many authors including Groot and Mazur [8] and Fitts [9]and McDougall [10].

Convective instability in porous media has been studied with great interest for more than half century. Convection given by buoyancy has found increased applications in underground coal gasification, solar energy extraction and many more areas as discussed by Nield and Bejan [11], Ingham [12] in industrial application, harmful particles can be filtered from a fluid stream by passing it through.

When we consider flow in porous medium, we have to take into consideration some additional complexities, which are principally due to the interactions between the fluids and the porous material. When a fluid permeates a porous medium, we cannot follow analytically the actual path of an individual fluid particle, because of the fluid-rock boundary conditions, which must be considered. Thus in aporous medium one generally considered. The fluid motion in termsof volume or ensemble average of the motion of individual fluid elements over regions of space. This is usually done by famous Darcy's law and as a result of this the viscous term in the equation of fluid motion will be replaced by the resistance term $-\frac{\mu}{K_1}\vec{q}$ where, μ is the dynamic viscosity of fluid,k_1 is the permeability of the medium and $\vec{q}$ is the seepage velocity of the fluid. A macroscopic equation which describes incompressible creeping flow of a Newtonian fluid of viscosity μ through a macroscopically homogenous and isotropic porous medium of permeability is well known Darcy's equation [13]

$$-\frac{\mu}{K_1}\vec{q} = \nabla p$$

where, p is interstitially averaged pressure within the porous medium and $\vec{q}$ is the filter velocity (or Darcian velocity).

The stability of flow of a fluid through porous medium was studied by Lapwood [14] and Wooding [15] and have studied double diffusive convection in porous medium by considering the Darcy flow model which is relevant to densely packed, low permeability porous medium.

The problem of double diffusive convection in porous medium has been extensibly investigated and growing volume of work in this area is well documented by Ingham and Pop[12].In this Paper we have studied double–diffusive convection problem in porous mediumby considering the Darcy flow model which is relevant to densely packed, low permeability porous medium. The aim of the present analysis is to investigate the effect of

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cross-diffusions terms (Dufour and Soret effects) on the onset of Double Diffusive Convective instability.

However, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. Hence, non-Darcy effects on double diffusive convection in porous media have received a great deal of attention in recent years.

PHYSICAL CONFIGURATION AND EIGENVALUE PROBLEM:

Consider a horizontal layer of viscous fluid saturating a porous medium between two parallel boundaries at$z = 0$ and $z = 1$,which are heated and salted from below in the force field of gravity. The temperature and concentration difference between the bounding planes are ΔT and ΔS, respectively. We assume the Oberback-Bosssinesq approximation is valid and flow in porous medium is governed by Darcy's law. The extra effect we consider here is that of the coupled fluxes of the two properties due to irreversible thermodynamic effects. The governing equations for the Double Diffusive convection problem with cross-diffusions effects in a horizontal fluid saturating a porous medium under Boussinesq approximation are given by

$$\frac{\partial u_i}{\partial t} = 0 \tag{1}$$

$$\frac{\rho_0}{\epsilon}\left[\frac{\partial u_i}{\partial t} + \frac{u_i}{\epsilon}\frac{\partial u}{\partial x_j}\right] = -\frac{\partial p}{\partial x_i} + \rho g - \frac{\mu}{\kappa}u_i \tag{2}$$

$$E'\frac{\partial T}{\partial t} + u_i\frac{\partial T}{\partial x_j} = \kappa\frac{\partial^2 T}{\partial {x_j}^2} + S_1\frac{\partial^2 S}{\partial {x_j}^2} \tag{3}$$

$$E''\frac{\partial S}{\partial t} + u_i\frac{\partial S}{\partial x_j} = \eta_s\frac{\partial^2 S}{\partial {x_j}^2} + D_1\frac{\partial^2 T}{\partial {x_j}^2} \tag{4}$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(S - S_0)] \tag{5}$$

Here, $u_i = (u, v, w)$ is the velocity,T is the temperature, $S$is the solute concentration, p is the pressure,ρ is the density,$T_{0,}S_0$ and ρ_0 are referencetemperature, concentration and density respectively, $g$is the accerlation due to gravity,μ is the fluid viscosity, ε is the porosity, κ is the effective thermal diffusivity and η_s is the solute diffusivity. The suffix '0' denotes the values of the various parameters at some suitably reference temperature T_0 and concentration C_0. Further,

$E' = \epsilon + (1-\epsilon)\frac{\rho_s c_s}{\rho_0 c_f}$is a constant related to heat and E'' is a constant analogous to concentration κ_1 is the permeability of the porous medium, where $\rho_{S\,,}C_S$ and ρ_0, C_f stand for density and heat capacity of the solid (porous matrix) material and fluid, respectively.

Following the usual steps of linear stability theory, the non dimensional linearized perturbation equations and boundary conditions governing The aim of the present analysis is to investigate the effect of cross-diffusions terms (Dufour and Soret effects) on the onset of Double Diffusive Convective instability

The following non-dimensional linearized perturbation equations

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)(D^2-a^2)w=-R_T a^2\theta+R_S a^2\varphi \tag{6}$$

$$[D^2-a^2-E^{'}p]\theta+D_T[D^2-a^2]\varphi \tag{7}$$

$$\left[D^2-a^2-\frac{E^{"}p}{\tau}\right]\varphi+S_T[D^2-a^2]\theta=-\frac{w}{\tau} \tag{8}$$

where

$a^*=kd\,,\theta^*=\frac{\hat{\theta}\kappa}{\beta_1 d^2}\,,n=\frac{p^*\kappa}{d^2}\varphi^*=\frac{\hat{\varphi}\kappa}{\beta_2 d^2}\,,w^*=\hat{w},R_T=\frac{g\alpha\beta_1 d^4}{\vartheta\kappa}\,,R_S=\frac{g\alpha^{'}\beta_2 d^4}{\nu\kappa},S_T=\frac{S_f\beta_1}{\tau\kappa\beta_2},D_T=\frac{D_f\beta_2}{\kappa\beta_1}$

together with the boundary conditions

Case1:Both boundaries are dynamically free

$$w=\theta=\varphi=0=D^2\text{w at z}=0\text{ and z}=1 \tag{9}$$

Case2: Both boundaries are rigid

$$w=\theta=\varphi=0=Dw\text{ at z}=0\text{ and z}=1 \tag{10}$$

Case3: Lower boundary rigid and upper boundary free

$$w=\theta=\varphi=0=Dw\ at\ z=0\ and\ w=\theta=\varphi=0=D^2w\ at\ z=1 \tag{11}$$

or

$$w=0=\theta=\varphi=Dw\ at\ z=0\ and\ w=0=\theta=\varphi=D^2w\ at\ z=1 \tag{12}$$

we use the linear transformations to transform the above eigenvalue problem to a mathematically tractable form.

Let us define the linear transformation given by;

$$w^{'}=\left(S_T E^{"}+BE^{'}\right)w,\theta^{'}=E\theta+F\varphi,\varphi^{'}=S_T\theta+B\varphi \tag{13}$$

where, E=$\left(\frac{S_T E^{"}+BE^{'}}{E^{'}D_T+AE^{"}}\right)A\,,F=\left(\frac{S_T E^{"}+BE^{'}}{AE^{"}+E^{'}D_T}\right)D_T,B=-\frac{AE^{'}}{\tau E^{"}}$

and A is any positive root of equation

$$A^2E^{"}+A\left(\tau E^{'}-E^{"}\right)-\tau S_T D_T E^{'}=0$$

The system of equations (6)-(8) together with boundary conditions (9)-(12) upon using the linear transformation defined by (13) assume the following forms;

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)(D^2-a^2)w=-\hat{R}_T a^2\theta+\hat{R}_S a^2\varphi \tag{14}$$

$$\left[\hat{K}_1(D^2-a^2)-E^{'}p\right]\theta=-w \tag{15}$$

$$\left[\hat{K}_2(D^2-a^2)-\frac{E^{"}p}{\tau}\right]\varphi=-\frac{w}{\tau} \tag{16}$$

and boundary conditions remains the same as described above by(9)-(12).

Here, $\hat{K}_1=1+\frac{\tau S_T D_T E^{'}}{AE^{"}}\ and\ \hat{K}_2=1+\frac{S_T D_T E^{"}}{\tau E^{'}B}$ are non-negative constant, since $0\le\frac{S_T D_T}{A}\le 1$ as $S_T\ and\ D_T>0$are Soret and Dufour numbers and A is a constant as defined earlier.

Also,$\hat{R}_T=\frac{(D_T E^{'}+AE^{"})(R_T B+R_S S_T)}{AB-S_T D_T}\,,\hat{R}_S=\frac{(S_T E^{"}+BE^{'})(R_T D_T+R_S A)}{AB-S_T D_T}$ is the modified thermal Rayleigh number and modified concentration Rayleigh number respectively.

Further, the dashes have been dropped in writing the perturbed quantities in the above equations for the sake of convenience.

Remarks A.. We note that system of equations (14)- (16) yields the following non-dimensional linearized perturbation equations governing;

(i) Dufour Driven Double-Diffusive Convection**(DDDDC)** problem with porous media, whenS_T=0.We have

$$\hat{R}_T = R_T\left(E'' + \frac{D_T E'}{A}\right), \hat{R}_S = E'\left(R_S + \frac{R_T D_T}{A}\right), \hat{K}_1 = 1\ \hat{K}_2 = 1$$

And equations (14)-(16) reduce to

$$\left(\frac{p}{\sigma\varepsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R_T\left(E'' + \frac{D_T E'}{A}\right)a^2\theta + E'\left(R_S + \frac{R_T D_T}{A}\right)a^2\varphi \qquad (17)$$

$$[(D^2 - a^2) - E'p]\theta = -w \qquad (18)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \qquad (19)$$

(ii) Soret Driven Double-Diffusive Convection (**SDDDC)** problem with porous media, when $D_T = 0$. We have

$$\hat{R}_T = E''\left(R_T + \frac{R_S S_T}{B}\right), \hat{R}_S = \frac{R_S}{B}\left(S_T E'' + BE'\right), \hat{K}_1 = 1, \hat{K}_2 = 1$$

And equations (14)- (16) reduce to

$$\left(\frac{p}{\sigma\epsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -E''\left(R_T + \frac{R_S S_T}{B}\right)a^2\theta + \frac{R_S}{B}\left(S_T E'' + BE'\right)a^2\varphi \qquad (20)$$

$$[(D^2 - a^2) - E'p]\theta = -w \qquad (21)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \qquad (22)$$

(iii) Double-Diffusive Convection (**DDC)** problem with porous media, When $S_T = 0, D_T = 0$.We have $\hat{R}_T = E'' R_T$, $\hat{R}_S = R_S E'$,

equations (14)- (16) reduces to

$$\left(\frac{p}{\sigma\epsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R_T E'' a^2\theta + R_S E' a^2\varphi \qquad (23)$$

$$[(D^2 - a^2) - E'p]\theta = -w \qquad (24)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \qquad (25)$$

MATHEMATICAL ANALYSIS:

On the Stability of Oscillatory modes:

We shall now investigate the stability of the oscillatory modes for Double-Diffusive convection problem with cross-diffusions for Darcy model in porous medium.

Theorem1. If $(p, w, \theta, \varphi), p = p_r + ip_i, p_i \neq 0$, $\hat{R}_T > 0, \hat{R}_S > 0$ is a non-trivial solution of equations (14)-(16)together with one of the boundary conditions(9)-(12) and $\hat{R}_T \leq \frac{27\pi^4 P_1 \tau \hat{K}_2 \hat{K}_1 + 16\pi^2 \sigma K_1 \in E''}{4\epsilon E'' \sigma P_1}$,then$p_r < 0$.

Proof: Multiplying both side of equation (14) by w^* and then integrating the resulting equation over the range of $z\epsilon[0,1]$, we have

$$\left(\frac{p}{\sigma\varepsilon} + \frac{1}{P_1}\right)\int_0^1 w^*(D^2 - a^2)w dz = -\hat{R}_T a^2 \int_0^1 \theta w^* dz + \hat{R}_S a^2 \int_0^1 \varphi w^* dz \qquad (26)$$

Taking the complex conjugate of both side of equations (15) and (16) and then using the resulting equations in equation (26), we get

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1 w^*(D^2-a^2)wdz=\hat{R}_T a^2\int_0^1\theta\left[\hat{K}_1(D^2-a^2)-E^{'}p^*\right]\theta^*dz-\hat{R}_S a^2\tau\int_0^1\varphi\left[\hat{K}_2(D^2-a^2)-\frac{E^{"}p^*}{\tau}\right]\varphi^*dz \quad (27)$$

Now, integrating the various terms of equation (27) by parts an appropriate number of times and making use of either of boundary conditions (9)-(12), we get

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1[|Dw|^2+a^2|w|^2]\,dz-\hat{R}_T a^2\int_0^1\left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)+E^{'}p^*|\theta|^2\right]dz+\hat{R}_S a^2\tau\int_0^1\left[\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)+E^{'}p^*|\varphi|^2\right]dz=0 \quad (28)$$

Equating the real and imaginary parts of both side of equation (28)and cancelling$p_i\neq 0$ throughout from the obtained imaginary part of the equation, wehave

$$\left(\frac{p_r}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1[|Dw|^2+a^2|w|^2]\,dz-\hat{R}_T a^2\int_0^1\left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)+E^{'}p_r|\theta|^2\right]dz+\hat{R}_S a^2\int_0^1\left[\tau\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)+E^{"}p_r|\varphi|^2\right]dz=0 \quad (29)$$

and

$$\left[\frac{1}{\varepsilon\sigma}\int_0^1(|Dw|^2+a^2|w|^2)dz+\hat{R}_T a^2E^{'}\int_0^1|\theta|^2dz-\hat{R}_S a^2E^{"}\int_0^1|\varphi|^2\,dz\right]=0 \quad (30)$$

If permissible,$p_r\geq 0$.Multiplying equation (30) by $p_r$and adding the resulting equation to equation (27), we have

$$\left(\frac{2p_r}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1[|Dw|^2+a^2|w|^2]-\hat{R}_T a^2\int_0^1\left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)\right]dz+\hat{R}_S a^2\int_0^1\left[\tau\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)\right]dz=0 \quad (31)$$

Equation also (30) implies that

$$\frac{1}{\sigma\varepsilon}\int_0^1(|Dw|^2+a^2|w|^2)\,dz<\hat{R}_S a^2E^{"}\int_0^1|\varphi|^2\,dz \quad (32)$$

Since,w,θ,φ vanish at $z=0$ and $z=1$,therefore by the use of Rayleigh Ritz inequality, we have

$$\int_0^1|Dw|^2\,dz\geq\pi^2\int_0^1|w|^2dz \quad (33)$$

$$\int_0^1|D\theta|^2\,dz\geq\pi^2\int_0^1|\theta|^2\,dz \quad (34)$$

$$\int_0^1|D\varphi|^2\,dz\geq\pi^2\int_0^1|\varphi|^2\,dz \quad (35)$$

Combining inequalities (33) and (34), we get

$$\frac{(\pi^2+a^2)}{\varepsilon\sigma}\int_0^1|w|^2dz<\hat{R}_S a^2E^{"}\int_0^1|\varphi|^2\,dz \quad (36)$$

Also upon using inequality (35), we can have

$$\int_0^1[|D\varphi|^2+a^2|\varphi|^2]\,dz\geq(\pi^2+a^2)\int_0^1|\varphi|^2\,dz \quad (37)$$

Also, upon using inequality (36) and (37) we get

$$\hat{R}_S a^2E^{"}\int_0^1[|D\varphi|^2+a^2|\varphi|^2]\,dz\geq\frac{(\pi^2+a^2)^2}{\varepsilon\sigma}\int_0^1|w|^2\,dz \quad (38)$$

Further, multiplying equation (15) by its complex conjugate and then integrating the resulting equation over the range of z, we get

$$\int_0^1\left[\hat{K}_1(D^2-a^2)-E^{'}p\right]\theta dz\int_0^1\left[\hat{K}_1(D^2-a^2)-E^{'}p^*\right]\theta^*dz=\int_0^1 ww^*dz \quad (39)$$

$\int_0^1 \left|\hat{K}_1^{\,2}(D^2-a^2)\theta\right|^2 dz + 2p_r\,\hat{K}_1 E' \int_0^1 [|D\theta|^2 + a^2|\theta|^2]\,dz + (E')^2|p|^2\int_0^1 |\theta|^2\,dz = \int_0^1 |w|^2\ dz$ (40)

Since $p_r \geq 0$, therefore inequality (40) yields

$\int_0^1 \left|\hat{K}_1^{\,2}(D^2-a^2)\theta\right|^2 dz \leq \int_0^1 |w|^2 dz$ (41)

Now, in view of inequality (34), we can have

$$\int_0^1 |(D^2-a^2)\theta|^2 = \int_0^1 [|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2]\,dz \geq (\pi^2+a^2)^2 \int_0^1 |\theta|^2\,dz \quad (42)$$

Using inequality (42) in inequality (41),we have

$\hat{K}_1^{\,2}(\pi^2+a^2)^2\int_0^1 |\theta|^2\,dz \leq \int_0^1 |w|^2\,dz$ (43)

Also, we know that

$\int_0^1 |w|^2 dz = \int_0^1 \{|w|^2\}^{\frac{1}{2}}\,dz \int_0^1 \{|w|^2\}^{\frac{1}{2}}\,dz$ (44)

which upon using inequalities(41) and (43) yields

$\int_0^1 |w|^2 dz \geq (\pi^2+a^2)\hat{K}_1^{\,2}\int_0^1 \{|(D^2-a^2)\theta|^2\}^{\frac{1}{2}}\,dz \int_0^1 [|\theta|^2]^{\frac{1}{2}}\,dz$ (45)

$$\geq \hat{K}_1^{\,2}(\pi^2+a^2)\int_0^1 |-\theta^*(D^2-a^2)\theta dz|$$

$\geq\ \hat{K}_1^{\,2}(\pi^2+a^2)\int_0^1 [|D\theta|^2 + a^2|\theta|^2]\,dz$ (46)

Since, $p_r \geq 0$ and for given $\frac{p_r}{\varepsilon}$ positive,equation (31) can be written as

$\frac{1}{P_1}\int_0^1 [|Dw|^2 + a^2|w|^2]\,dz + \hat{R}_S a^2\tau\hat{K}_2\int_0^1 [|D\varphi|^2 + a^2|\varphi|^2] < \hat{R}_T\,a^2\hat{K}_1\int_0^1 [|D\theta|^2 + a^2|\theta|^2]\mathrm{dz}$ (47)

Using inequalities (33), (38), (46) in inequality (47),

$\left[\frac{\hat{K}_1(\pi^2+a^2)^2}{a^2P_1} + \frac{(\pi^2+a^2)^3\tau\hat{K}_2\hat{K}_1}{a^2\varepsilon E''\sigma} - \hat{R}_T\right]\int_0^1 |w|^2 dz < 0$ (48)

Since, minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 is $4\pi^2$, and $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$, therefore inequality (48) yields that

$\left[\frac{K_1 4\pi^2}{P_1} + \frac{27\pi^4\tau\hat{K}_2\hat{K}_1}{4\varepsilon E''\sigma} - \hat{R}_T\right]\int_0^1 |w|^2\,dz < 0$ (49)

which can be written as

$$\left[\frac{16\pi^2K_1\varepsilon E'' + 27\pi^4P_1\tau\hat{K}_2\hat{K}_1}{4\varepsilon E''\hat{R}_T P_1}\right] < 1$$

or, we have

$\hat{R}_T \geq \frac{(16\pi^2\hat{K}_1\varepsilon E''\sigma + 27\pi^4P_1\tau\hat{K}_2\hat{K}_1)}{4\varepsilon E''P_1\sigma}$ (50)

This is a contradiction to the hypothesis of the theorem.

Hence, we must have $p_r < 0$.

This completes the proof.

Using Remark A (i-iii), we have the following corollaries;

Corollary 1. Under the hypothesis of Theorem1, for DDDDC problem with porous medium,if $R_TE'' + \frac{R_TD_TE'}{A} \leq \frac{[27\pi^4\tau P_1 + 16\pi^2\epsilon E''\sigma]}{4P_1\epsilon E''\sigma}$, *then* $p_r < 0$.

Corollary 2. Under the hypothesis of Theorem 1, for SDDDC with porous medium$R_T E'' + \frac{E'' R_S S_T}{B} \leq \frac{[27\pi^4 \tau P_1 + 16\pi^2 \epsilon E'' \sigma]}{4P_1 \epsilon E'' \sigma}, p_i \neq 0,$ then $p_r < 0$.

Corollary 3. Under the hypothesis of Theorem4,For DDC with porous medium, $\hat{R}_T = R_T E'', \hat{R}_S = R_S E', p_i \neq 0, R_T E'' \leq \frac{[27\pi^4 \tau P_1 + 16\pi^2 \varepsilon E'' \sigma]}{4P_1 \varepsilon E'' \sigma}, then\; p_r < 0.$

It is remarkable to note that when we consider the compliment of the above sufficient condition as stated in Theorem above, for the stability of the oscillatory motions i.ewhen $\hat{R}_T > \frac{27\pi^4 P_1 \tau \hat{K}_2 \hat{K}_1 + 16\pi^2 \hat{K}_1 \in E'' \sigma}{4\epsilon E'' P_1 \sigma}$, the oscillatory modes of growing amplitudemay exists. Hence, it becomes important to prescribe the bounds for the growth rate of these motions. In the following Theorem, we have derived such bounds which arrest the complex growth rate of the arbitrary neutral or unstable $p_r \geq 0$ oscillatory motions $(p_i \neq 0)$.

In the following theorem, we have derived bounds which arrest the complex growth of the arbitrary neutral or unstable $p_r \geq 0$ oscillatory motions $p_i \neq 0$.

Bound for the complex Growth Rate:

Theorem 2. If (p, w, θ, φ) , $p = p_r + ip_i$, $p_r \geq 0$, $p_i \neq 0$, $\hat{R}_T > 0, \hat{R}_S > 0$, is a non-trivial solution ofequations(14)-(16) together with one of the boundary conditions(9)-(12)and for $H > 1$, then$|p| < \frac{\hat{K}_1(\pi^2+a^2)\sqrt{H^2-1}}{E'}$, where

$H = \frac{4\hat{R}_T P_1 \varepsilon \sigma E''}{[27\pi^4 \tau \hat{K}_2 \hat{K}_1 + 16\pi^2 P_1 \varepsilon E'' \hat{K}_1]}$.

Proof : Since $p_r \geq 0$, therefore from equation (42), we can have

$$\int_0^1 \left|\hat{K}_1{}^2 (D^2 - a^2)\theta\right|^2 dz + (E')^2 |p|^2 \int_0^1 |\theta|^2 dz < \int_0^1 |w|^2\, dz \tag{51}$$

Using inequality (44) in inequality (51), we get

$$\hat{K}_1{}^2 (\pi^2 + a^2)^2 \left[1 + \frac{(E')^2 |p|^2}{\hat{K}_1{}^2 (\pi^2+a^2)^2}\right] \int_0^1 |\theta|^2\, dz < \int_0^1 |w|^2 dz \tag{52}$$

Since

$$\int_0^1 [|D\theta|^2 + a^2|\theta|^2] dz = \left|-\int_0^1 \theta^* (D^2 - a^2)\,\theta\right| dz \leq \int_0^1 |\theta|\,|(D^2 - a^2)\theta| dz$$

$\leq \int_0^1 \{|\theta|^2\}^{\frac{1}{2}} dz \int_0^1 \{|(D^2 - a^2)\theta|^2\}^{\frac{1}{2}}\, dz$ (Using Schwartz inequality)

$$\leq \frac{1}{\hat{K}_1{}^2 (\pi^2+a^2)} \left[1 + \frac{(E')^2 \sigma^2 |p|^2}{\hat{K}_1 (\pi^2+a^2)}\right]^{\frac{-1}{2}} \int_0^1 |w|^2 dz \tag{53}$$

Making use of inequalities (39), (43) and(53) in equation (47) we get

$$\left(\frac{1}{P_1}\right) \int_0^1 [|Dw|^2 + a^2|w|^2]\, dz + \hat{R}_S a^2 \tau \hat{K}_2 \int_0^1 [|D\varphi|^2 + a^2|\varphi|^2]\, dz < \hat{R}_T a^2 \hat{K}_1 \int_0^1 [|D\theta|^2 + a^2|\theta|^2]\, dz \tag{54}$$

which on simplification yields

$$\left[\left(\frac{1}{P_1}\right) \frac{\hat{K}_1 (\pi^2+a^2)^2}{a^2} + \frac{\tau \hat{K}_2 (\pi^2+a^2)^3 \hat{K}_1}{\varepsilon E'' \sigma a^2}\right] \left[1 + \frac{(E')^2 |p|^2}{\hat{K}_1 (\pi^2+a^2)}\right]^{\frac{1}{2}} < \hat{R}_T \tag{55}$$

Since, minimum values of $\frac{(\pi^2+a^2)^2}{a^2}$ and $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $4\pi^2$ and $\frac{27\pi^4}{4}$, inequality (55) yields that

$$\left[\frac{4\pi^2}{P_1}+\frac{\tau\hat{K}_2 27\pi^4\hat{K}_1}{\varepsilon E'' 4\sigma}\right]\left[1+\frac{(E')^2|p|^2}{\hat{K}_1(\pi^2+a^2)}\right]^{\frac{1}{2}} < \hat{R}_T$$

which yields

$$|p| < \frac{\hat{K}_1(\pi^2+a^2)}{E'}\sqrt{H^2-1} \tag{56}$$

This completes the proof of the theorem.
Theorem above from point of view of hydrodynamic stability theory may be stated as:

The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude $(p_r \geq 0)$ lies inside a semi-circle in the right-half of the $p_r p_i$ −plane whose centre is at the origin and whose radius is $\frac{\hat{K}_1(\pi^2+a^2)}{E'}\sqrt{H^2-1}$.
Using Remarks A. (i-iii),we have the following corollaries

Corollary 4. For **DDDDC** problem with porous medium, $p_i \neq 0$, the complex growth rate is given by, $|p| < \frac{(\pi^2+a^2)}{E'}\sqrt{H'^2-1}$, where, $H' = \frac{4R_T P_1 \epsilon E''\sigma\left(E''+\frac{D_T E'}{A}\right)}{[27\pi^4\tau P_1+16\pi^2\epsilon E''\sigma]}$

Corollary 5. For **SDDDC** with porous medium, $p_i \neq 0$, the complex growth rate is given by, $|p| < \frac{(\pi^2+a^2)}{E'}\sqrt{H''^2-1}$, where, $H'' = \frac{4P_1\epsilon E''\sigma\left(R_T E''+\frac{E'' R_S S_T}{B}\right)}{[27\pi^4 P_1(\tau)+16\pi^2\epsilon E''\sigma]}$.

Corollary 6. For **DDC** with porous medium, $p_i \neq 0$, the complex growth rate is given by, $|p| < \frac{(\pi^2+a^2)}{E'}\sqrt{H'''^2-1}$. where, $H''' = \frac{4P_1\epsilon E'' R_T E'}{[27\pi^4 P_1(\tau)+16\pi^2\epsilon E'']}$.

RESULTS AND CONCLUSIONS:

We studied the Double-Diffusive Convection problem saturating porous medium (Darcy Model) in order to study the effects of cross diffusion terms. For general nature of boundary conditions, a sufficient condition for the stability of oscillatory modes is obtained. Moreover, when compliment of this condition is supposed to be true, the bounds for growth rate for arbitrary perturbations of growing amplitude are obtained and various consequences of these results for DDDDC, SDDDC, DDC are obtained in the presence of porous medium. However, the presence of porous media prepones the onset of convection.

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WAVES PROPAGATION IN HOMOGENEOUS ISOTROPIC PLATE IN CONTEXT OF TWO TEMPERATURE GENERALIZED THERMOELASTICITY

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ABSTRACT:
This paper concerns with investigation of propagation of thermoelastic waves in homogeneous isotropic plate in context of two temperature generalized theory of thermoelasticity. The surfaces of plate are subjected to stress free, rigid insulated and isothermal boundary conditions. Secular equations in closed form and isolated mathematical conditions for symmetric and skew symmetric wave mode propagation are derived. The results for uncoupled, coupled, Lord-Shulman and Green-Lindsay theories of thermoelasticity have been obtained as particular cases from the derived secular equations. The effect of two temperatures on the propagation of thermoelastic waves is presented graphically and the results are compared theoretically as well as numerically with the one temperature theory as well as existing previous literature.

Keywords: Generalized Thermoelasticity, Two Temperature, Rayleigh-Lamb waves, Secular Equations

1. INTRODUCTION:

The classical uncoupled theory of thermoelasticity, the heat conduction equation is parabolic in nature and hence it predicts infinite speed of propagation for thermal signals and the elastic deformation is independent of temperature. Biot [1] introduced the coupled theory of thermoelasticity in 1956. Generalized Theories of Thermoelasticity predict finite speed for the propagation of thermal signals. The first generalized theory of thermoelasticity has been given by introducing one relaxation time in Fourier’s heat conduction law [2]. Another generalized theory of thermoelasticity is proposed by improving the classical energy equation and the stress-strain temperature relations [3]. Other models of generalized thermoelasticity are given in [4-6]. Model proposed in [5] is non-linear and applicable in low temperatures such as laser pulse heating. The theory of heat conduction, which depends on two distinct temperatures viz the conductive temperature and the thermodynamic temperature has been described in [7] and [8]. The difference between two temperatures is proportional to heat supply for in case of no heat supply and time independent situations, the two temperatures are equal. The wave propagation in the two temperature thermoelasticity theory [9]. Harmonic plane waves in two temperature theory have been investigated in [10]. Two temperature generalized thermoelasticity theory has been formulated by Youssef [11]. The effect of thermal relaxation time on plane wave propagation under two temperature theory has also been investigated [12].

In this article, Youssef's model of two temperature generalized thermoelasticity has been used to investigate the wave propagation.

2. FORMLATION OF PROBLEM:

Let us consider an infinite homogeneous, isotropic thermoelastic plate of thickness 2d at uniform temperature T_0. The origin of the coordinate system is taken on the middle surface of the plate such that z-axis is in thickness direction.

The surfaces z = ±d are assumed to be Stress free insulated or isothermal. We have considered the x-z plane as the plane of incident and assume that the solutions are explicitly independent of y coordinate, but they depend implicitly so that the transverse component of displacement does not vanish.

The field equations in the context of generalized thermoelasticity with two temperatures in absence of body forces and heat sources are:
Equations of motions

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z}+\mu\frac{\partial^2 u}{\partial z^2}-\gamma(1+\nu\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\mu\frac{\partial^2 w}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial z\partial x}+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\gamma(1+\nu\frac{\partial}{\partial t})\frac{\partial\theta}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2})=\rho\ddot{v} \tag{3}$$

Generalized heat conduction equation is

$$\begin{aligned}&K(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2})=\rho C_E(\frac{\partial\theta}{\partial t}+\tau_0\frac{\partial^2\theta}{\partial t^2})+\\&\gamma T_0\left(\frac{\partial}{\partial t}+n_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)\end{aligned} \tag{4}$$

where ϕ and θ are given by

$$\phi-T=a\phi_{,ii}\quad \& \quad \theta=\left|T-T_0\right| \tag{5}$$

Constitutive relations for non-vanishing stresses are

$$\begin{aligned}&\sigma_{zz}=(\lambda+2\mu)\frac{\partial w}{\partial z}+\lambda\frac{\partial u}{\partial x}-\gamma(\theta+\nu\frac{\partial\theta}{\partial t})\\&\sigma_{yz}=2\mu\frac{\partial v}{\partial z},\quad \sigma_{xz}=2\mu(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})\end{aligned} \tag{6}$$

$$\sigma_{zz}=\frac{\partial w}{\partial z}+(1-2\delta^2)\frac{\partial u}{\partial x}-(\theta+\gamma\frac{\partial\theta}{\partial t})=0$$

where n_0 is constant; λ, μ are Lame's parameters, K is the thermal conductivity; ρ and C_E are the density and specific heat at constant strain respectively, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the linear thermal expansion. Taking $\tau_0 = \nu = a = 0$, one can obtain results for coupled thermoelasticity, In case of Lord & Shulman theory $n_0 = 1, \tau_0 > 0, \nu = 0, a = 0$ and Green & Lindsay theory $n_0 = 0, \tau_0 > 0, \nu > 0, a = 0$.

Introducing non dimensional quantities

$$x' = \frac{x\omega^*}{c_1},\ z' = \frac{z\omega^*}{c_1},\ u' = \frac{u\omega^* c_1 \rho}{\gamma T_0}, w' = \frac{w\omega^* c_1 \rho}{\gamma T_0}$$

$$t' = \omega^* t,\ \nu' = \omega^* \nu,\ \tau_0' = \omega^* \tau_0,\ \gamma = (3\lambda + 2\mu)\alpha_t$$

$$a' = \frac{a\omega^{*2}}{c_1^2},,\ \delta^2 = \frac{\mu}{(\lambda + 2\mu)}, \in = \frac{T_0\gamma^2}{\rho C_E(\lambda + 2\mu)}, \theta' = \frac{\theta}{T_0},\ \phi' = \frac{\phi}{T_0} \tag{7}$$

where

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ \omega^* = \frac{C_E(\lambda + 2\mu)}{K} \tag{8}$$

Equations (1), (2) and (4) are obtained as

$$\frac{\partial^2 u}{\partial x^2} + (1 - \delta^2)\frac{\partial^2 w}{\partial x \partial z} + \delta^2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial}{\partial x}(\theta + \nu \frac{\partial \theta}{\partial t}) = \frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\frac{\partial^2 w}{\partial z^2} + (1 - \delta^2)\frac{\partial^2 u}{\partial x \partial z} + \delta^2 \frac{\partial^2 w}{\partial x^2} - \frac{\partial}{\partial z}(\theta + \nu \frac{\partial \theta}{\partial t}) = \frac{\partial^2 w}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = (\frac{\partial \theta}{\partial t} + \tau_0 \frac{\partial^2 \theta}{\partial t^2}) + \in \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \tag{11}$$

Further, the mechanical boundary conditions at z = ±d in non-dimensional form are given by

$$\sigma_{zz} = \frac{\partial w}{\partial z} + (1 - 2\delta^2)\frac{\partial u}{\partial x} - (\theta + \gamma \frac{\partial \theta}{\partial t}) = 0 \tag{12}$$

$$\sigma_{xz} = \delta^2 (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) = 0,\ \sigma_{yz} = \frac{\partial v}{\partial z} = 0\ .$$

and the thermal conditions are given as $\frac{\partial \phi}{\partial z} + h\phi = 0$ (13)

where h denotes surface heat transfer coefficient. The limiting case $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ leads isothermal boundaries.

3. SOLUTION OF THE PROBLEM:

In order to solve equations (9) to (11), introducing potential functions Φ and Ψ such that

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial z},\ w = \frac{\partial \Phi}{\partial z} - \frac{\partial \Psi}{\partial x} \tag{14}$$

and consider the solutions of the form

$$\left(\Phi,\Psi,\phi\right)=\left[f(z),g(z),h(z)\right]\exp[i\xi(x-ct)] \tag{15}$$

where $c=\omega/\xi$ is the phase velocity, ω is the frequency and ξ is the wave number. Inserting (14) in equations (9) to (11), applying the solution defined by (15) in resulting equations and then solving the resulting system of differential equations, the expressions for u, v, w and ϕ are obtained as

$$\begin{aligned}u(x,z,t)=[&i\xi(C_3\cos m_1z+C_4\sin m_1z+C_5\cos m_2z+C_6\sin m_2z)\\&-\beta C_7\sin\beta z+\beta C_8\cos\beta z]\exp[i\xi(x-ct)]\end{aligned} \tag{16}$$

$$\begin{aligned}w(x,z,t)=[&-m_1C_3\sin m_1z+m_1C_4\cos m_1z-m_2C_5\sin m_2z+\\&m_2C_6\cos m_2z-i\xi(C_7\cos\beta z+C_8\sin\beta z)]\exp[i\xi(x-ct)]\end{aligned} \tag{17}$$

$$\phi(x,z,t)=\begin{bmatrix}\dfrac{1}{\alpha_1}(C_3\cos m_1z+C_4\sin m_1z)+\\\dfrac{1}{\alpha_2}(C_5\cos m_2z+C_6\sin m_2z)\end{bmatrix}\exp[i\xi(x-ct)] \tag{18}$$

$$\begin{aligned}\theta(x,z,t)=[&\frac{\{1+a(\xi^2+m_1^2)\}}{\alpha_1}(C_3\cos m_1z+C_4\sin m_1z)+\\&\frac{\{1+a(\xi^2+m_2^2)\}}{\alpha_2}(C_5\cos m_2z+C_6\sin m_2z)]\exp[i\xi(x-ct)]\end{aligned} \tag{19}$$

where

$$\alpha_1=\frac{-i\omega t_2\{1+a(\xi^2+m_1^2)\}}{\alpha^2-m_1^2},\ \alpha_2=\frac{-i\omega t_2\{1+a(\xi^2+m_2^2)\}}{\alpha^2-m_2^2} \qquad m_1^2=[\frac{1}{2}(A+\sqrt{A^2-4B})],\ m_2^2=[\frac{1}{2}(A-\sqrt{A^2-4B})]$$

$$A=\frac{2\xi^2-\omega^2\left(1+t_0-i\in\omega t_2t_3+a\left(\omega^2t_1+2i\xi^2\omega t_2t_3-2\xi^2t_1\right)\right)}{-1+a\omega^2(t_1-i\in\omega t_2t_3)},$$

$$B=\frac{-\xi^4-\omega^4t_0+\omega^2\xi^2\left(1+t_0-i\in\omega t_2t_3+a\left(-\omega^2t_1-i\in\xi^2\omega t_2t_3+\xi^2t_1\right)\right)}{-1+a\omega^2(t_1-i\in\omega t_2t_3)}$$

$$t_0=\tau_0+i\omega^{-1},t_1=1+i\omega^{-1},t_2=\nu+i\omega^{-1},t_3=n_0\tau_0+i\omega^{-1}$$

$$\alpha^2=\xi^2\left(c^2-1\right),\ \beta^2=\xi^2\left(\frac{c^2}{\delta^2}-1\right).$$

4. SECULAR EQUATIONS:

Using the above expressions (16) to (19) and applying boundary conditions (12) and (13), a homogeneous system of linear equations in terms of unknowns C_i is obtained. The obtained system of equations will have non trivial solution iff the determinant of coefficient matrix vanishes which gives the secular equations for stress free thermally insulated and isothermal boundaries, respectively are obtained as

$$\begin{aligned}\left(\frac{\tan m_1d}{\tan m_3d}\right)^{\pm1}-\frac{m_1(\alpha^2-m_1^2)(1+a(\xi^2+m_2^2))}{m_2(\alpha^2-m_2^2)(1+a(\xi^2+m_1^2))}\left(\frac{\tan m_2d}{\tan m_3d}\right)^{\pm1}\\=\frac{4\xi^2\beta m_1(m_2^2-m_1^2)(1+a(\xi^2+\alpha^2))}{(\xi^2-\beta^2)^2(\alpha^2-m_2^2)(1+a(\xi^2+m_1^2)}\end{aligned} \tag{20}$$

$$\left(\frac{\tan m_1 d}{\tan m_3 d}\right)^{\pm 1} - \frac{m_2(\alpha^2 - m_1^2)(1 + a(\xi^2 + m_2^2))}{m_1(\alpha^2 - m_2^2)(1 + a(\xi^2 + m_1^2))}\left(\frac{\tan m_2 d}{\tan m_3 d}\right)^{\pm 1}$$
$$= \frac{(\xi^2 - \beta^2)^2 (m_2^2 - m_1^2)(1 + a(\xi^2 + \alpha^2))}{4\xi^2 \beta m_1 (\alpha^2 - m_2^2)(1 + a(\xi^2 + m_1^2)}$$

In above secular equations, the superscripts +1 and −1 correspond to skew symmetric and symmetric wave modes respectively.

5. NUMERICAL R ESULT AND DISCUSSIONS

In order to describe theoretical results presented in above equations, numerical results are presented. Here, we have chosen copper material for evaluation of numerical results and physical data for which is as given below

$$\lambda = 7.76 \times 10^{10} \mathrm{Kg\,m^{-1} s^{-2}},\ \mu = 3.86 \times 10^{10} \mathrm{Kg\,m^{-1} s^{-2}}$$
$$\varepsilon = 0.0168,\ \rho = 8954 \mathrm{Kg\,m^{-3}}, C_E = 383.1 \mathrm{J\,Kg^{-1} K^{-1}}$$
$$k = 386 \mathrm{W\,m^{-1} K^{-1}}, \alpha_t = 1.78 \times 10^{-5} \mathrm{K^{-1}}, T_0 = 293\mathrm{K}$$
$$\tau_0 = 6.131 \times 10^{-13} \mathrm{s}, \nu = 8.765 \times 10^{-13} \mathrm{s}, d = 1.0$$

The phase velocities of symmetric and skew symmetric modes of wave propagation have been computed for various values of wave number from the dispersion relation (20) for stress free insulated boundary conditions for Lord and Shulman theory of thermoelasticity.

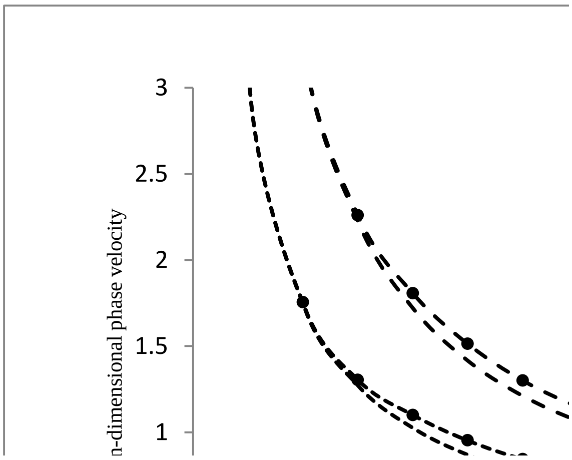

Figure 1: Variation of phase velocity with wave number

Figure 1 presents the variation of non-dimensional phase velocity with wave number for n =0, 1, 2 in case of skew symmetric mode. Curves with dark circles as markers correspond to two temperature theory in both figures. The phase velocity in all the three wave modes has higher magnitude in two temperature theory as compared to corresponding one temperature theory of thermoelasticity. Similar behavior has been observed for other theories as well. Figure 2 presents the variation of phase velocity with wave number for symmetric modes of vibrations. In symmetric modes of vibration phase velocity in case of two temperature thermoelasticity is having smaller magnitude.

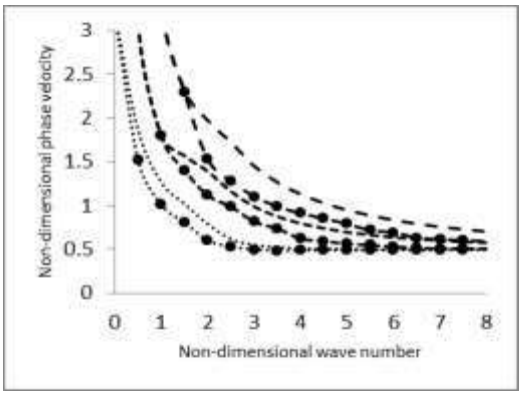

Figure 2: Variation of phase velocity with wave number

5. CONCLUSIONS:

The trends of variation of the phase velocity in two and one temperature thermoelasticity are similar however in skew symmetric modes magnitude is higher but in symmetric modes it is reversed.

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GRAVITATIONAL INSTABILITY IN VISCOELASTIC MEDIUM WITH DISSIPATIVE EFFECTS

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ABSTRACT:
In this paper, the gravitational instability of a self-gravitating viscoelastic medium has been analyzed in the presence of dissipative effects, using standard Jeans mechanism. A dispersion relation is derived for viscoelastic medium from the perturbation equations of the problem, using normal mode analysiswhen the bulk viscosity affects the first order dynamics. The instability conditions under both strongly (kinetic) and weakly (hydrodynamic) coupling limitsare obtained from the dispersion relation. It is observed for strongly coupled viscoelastic medium, the instability criterion gets modified due to the presence of dissipative effect which is represented by the Stokes-Kirchoff's factor and the critical wave number decreases, hence have stabilizing effect on the onset of instability. However, for weakly coupledfluid, the Jeans Criterion governs the onset of instability.

KEY WORDS: Gravitational instability, Jeans criteria, Dissipation energy. Normal mode analysis, viscoelastic medium, wave propagation, Coupling limits.

1. INTRODUCTION:

Generally accepted theoretical framework for the formation of the solar systems and the other planetary systems, in the domain of astrophysics and geophysics, is that of the Gravitational Instability(GI). GI is the key idea in explaining the way in which structures evolve in the universe. The star formation begins with small density fluctuations in an initially nearly uniform medium that are amplified by gravity in process called gravitational instability.During last century, the problem of self-gravitational instability has been a broad area of research in astrophysics, plasma physics and many other crucial phenomena of the interstellar medium. GI is a theory that recognizes the gravity as the only force capable of creating structures by accumulating material in space. Whenever the internal pressure of a gas is too weak to balance the self-gravitational force of a mass density perturbation, a collapse occurs; such mechanism was first studied by Jeans [1, 2] in terms of the wavelengths of a fluctuation. The dynamical mechanism responsible behind aggregation of matter and formation of various astrophysical structures, such as comets, asteroids, etc., in astro-comic environments are widely described and studied by gravitational or Jeans instability. The Jeans instability was later rigorously investigated by anumber of other authors including; Eddington [3], Mestel and Spitzer [4] and Chandrasekhar [5]. Dhiman and Dadwal [6,7] havealso studied the gravitational instability of a stratified homogeneous medium under varying assumptions of hydrodynamic and hydromagnetic, however Chandrasekhar [5] has given a detailed account of the Jeans instability under the influence of rotation or/and magnetic field and has derived the Jeans criteria for the various problems.For more details on gravitational instability, one may refer to Jeans [2].

TheJeans Instability in various kinds of plasma environment, *e.g.* gaseous plasma, fluid plasmaand dusty plasma have been investigated by many authors. In astrophysical domain, there are so many objects including interstellar medium which are composed of viscoelastic fluidsand possess both viscous and elastic properties. Elastic effects in fluid play an important role similar to thermal pressure, which lower the growth rate of gravitational instability. Janaki et al. [8] have discussed the Jeans instability of viscoelastic medium and found that the threshold for the onset of instability appears at higher wavelength in viscoelastic medium than in the case of ordinary gaseous medium. Prajapati and Chhajalni[9] have also investigated the linear self-gravitational instability of finitely conducting, magnetized viscoelastic fluid using the modified generalized hydrodynamic model.Rosenberg and Shukla [10] used the Generalized Hydrodynamic Model (GH) to describe the behavior of viscoelastic fluid and reported that the GH model describes the effects of strong correlations through the introduction of viscoelastic coefficients. Further,Kaw and Sen[11] suggested that the viscoelastic properties of the medium are characterized by the relaxation time τwhich provides a characteristic timescale to distinguish two classes of low frequency modes; one when the frequency $\sigma \ll \frac{1}{\tau}$ known as *hydrodynamic limit*(*weakly couplinglimit*) and the other frequency $\sigma \gg \frac{1}{\tau}$ known as *kinetic limit* (*strongly couplinglimit*) where $\sigma$is the wave frequency and τ is the viscoelastic relaxation time. Recently, Dhiman and Sharma [12] have investigated the effect of rotation on the growth rate of magneto gravitational instability of a viscoelastic medium under both of these limits and investigated the growth rate of the instability for both longitudinal and transverse modes of wave propagation.

Janaki [8] discussed that a central idea in the study of instability by including various factors is to find the ways of arresting the gravitational collapse.Heat in the interstellar medium play important role in the gravitational collapse which means dissipation energy effects of viscoelastic medium on GI. The viscous coefficients are not constant and we have to express their dependence on the state parameters of the fluid. Since we are interested to treat isotropic and homogeneous perturbative cosmological models. In fact, in such models there is no displacement of matter with respect to each other and this kind of viscosity represents the energy dissipation due to this effect(*cf.*Carlevaro and Montani [13]).

The present paper is motivated by the above discussions and aimed to study the effect of dissipative energy on the onset of gravitational instability of the viscoelastic fluid. For this, following [13], we shall consider the bulk viscosity expressed (in the first order)in terms of the thermo dynamical parameters of the fluid without neglecting the shear viscosity effect.We have followed the normal mode analysis to analyze linear perturbation of density in the governing equations and have obtained and a general dispersion relation. The stability criteria for both strongly and weakly coupled fluids under kinetic and hydrodynamical limitsare obtained from the dispersion relation in terms of wave numbers of the self-gravitating system,using the Hurwitz Criterion given by Guillemin[14].

2. Mathematical Formulation of the Problem:

We have considered the GH model to treat the viscoelastic properties ofan infinite homogeneous,self-gravitatingviscoelastic fluid. Under these assumptions, the generalized basic equations of continuity, motion and Poisson equation governing the physical problem (*cf*. Janaki et al. [8] and Dhiman and Sharma [12])are given by:

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\boldsymbol{u})=0 \quad (1)$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial\boldsymbol{u}}{\partial t}+(\boldsymbol{u}.\nabla)\boldsymbol{u}\right)-\rho\nabla\phi+c_s{}^2\nabla\rho\right]=\mu\nabla^2\boldsymbol{u}+\left(\zeta+\frac{\mu}{3}\right)\nabla(\nabla.\boldsymbol{u}) \quad (2)$$

$$\nabla^2\phi=-4\pi G(\rho-\rho_0) \quad (3)$$

In the above equations;$\boldsymbol{u}$and $\boldsymbol{r}$ represent respectively the velocity and position vectors, τ,ρ,ϕ,μ,G andc_s respectively denote the relaxation time, density of fluid, gravitational potential, coefficient of viscosity, the universal gravitational constant and the speed of sound in isothermal medium. Further, the term $\left(\zeta+\frac{\mu}{3}\right)$ is the coefficient of bulk viscosity and $\left(1+\tau\frac{\partial}{\partial t}\right)$denotes the Frenkel's term or viscoelastic operator which accounts for the relaxation effects(*cf*. Frenkel [15]).

Following Carlevaro and Montani [13], the fundamental hypothesis of Jeans modelfor a static and uniform solution for the zeroth-order dynamics are;

$$u_0=0,\rho_0=\text{const.}\ ,\phi_0=const. \quad (4)$$

Mathematically, we can say that when the density and pressure of the medium $\rho_0\,,p_0$ are constant, and the mean velocity v_0 is zero, it follows from equation (2) that $\nabla\phi_0=0$. On the other hand, Poisson equation (3) requires that $\nabla^2\phi_0=-4\pi G\rho_0$. This assumption contradicts the gravitational equation andthese two requirements are inconsistent unless $\rho_0\ =0$. This is known as *Jeans Swindle*. Physically, there are no pressure gradients in a homogeneous medium to balance gravitational attraction (*cf.* Larson [16]). Thus, in order to avoid the*Jeans Swindle*, we have taken Poisson equation as;

$$\nabla^2\phi=-4\pi G(\rho-\rho_0)$$

where the density $-\rho_0$is a repulsion term and may be regarded as a Newtonian analogue of Einstein's cosmological constant (*cf.*Kiessling [17]).

3.Linearized Perturbation Equations and Dispersion Relation:

To investigate the instability of the self-gravitating system governed by basic equations (1)-(3), let the initial stationary state solution (4) be slightly perturbed by giving infinitesimal small perturbations $\delta\rho(\boldsymbol{r},t),\delta\phi,\delta\boldsymbol{v}(\boldsymbol{r},t)$ in the density ρ_0, gravitational potential ϕ_0, and velocity$\boldsymbol{u}$, respectively. In treating bulk-viscosity perturbations, we use the expansions

$\zeta=\zeta_0+\delta\zeta$, where $\zeta_0=\zeta(\rho_0)=z\rho_0{}^s=constant$ and

$$\delta\zeta=\delta\rho\left(\frac{\partial\zeta}{\partial\rho}\right)+\cdots\ =zs\rho_0{}^{s-1}\delta\rho+\cdots$$

Here,$\zeta=ze^s$ defines the intensity of *dissipative effects,* with $s=constant$ and z is a parameterwhich defines the intensity of viscous effects.

Thus, the perturbations are now represented by;

$$\rho=\rho_0+\delta\rho\ ,\phi=\phi_0+\ \delta\phi\ ,\boldsymbol{u}=0+\ \delta\boldsymbol{v},\zeta=\zeta_0+\delta\zeta \quad (5)$$

Using these perturbed quantities in equations (1)-(3) and then linearizing the resulting equations, we get the following linearized perturbation equations of continuity, motion and Poisson equation respectively,for the present problem for the case of viscoelastic medium with dissipative effects;

$$\frac{\partial \delta\rho}{\partial t} + \nabla.(\rho_0 \delta\boldsymbol{v}) = 0 \tag{6}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho_0\left(\frac{\partial \delta\boldsymbol{v}}{\partial t}\right) - \rho_0\nabla\,\delta\phi + {c_s}^2\nabla\delta\rho\right] = \mu\nabla^2\delta\boldsymbol{v} + \left(\zeta_0+\frac{\mu}{3}\right)\nabla(\nabla.\delta\boldsymbol{v}) \tag{7}$$

$$\nabla^2\,\delta\phi = -4\pi G\,\delta\rho \tag{8}$$

Using equation (6) in (7), we get

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho_0\left(\frac{\partial \delta\boldsymbol{v}}{\partial t}\right) - \rho_0\nabla\,\delta\phi + {c_s}^2\nabla\delta\rho\right] = \mu\nabla^2\delta\boldsymbol{v} - \frac{\left(\zeta_0+\frac{\mu}{3}\right)}{\rho_0}\nabla\left(\frac{\partial \delta\boldsymbol{\rho}}{\partial t}\right) \tag{9}$$

Taking the divergence of both sides of the above equation and using equation (8), we get

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[-\left(\frac{\partial^2 \delta\rho}{\partial t^2}\right) + 4\pi G\rho_0\delta\rho + {c_s}^2\nabla^2\delta\rho\right] = -\frac{\mu}{\rho_0}\nabla^2\left(\frac{\partial \delta\rho}{\partial t}\right) - \frac{\left(\zeta_0+\frac{\mu}{3}\right)}{\rho_0}\nabla^2\left(\frac{\partial \delta\boldsymbol{\rho}}{\partial t}\right) \tag{10}$$

To study the properties of $\delta\rho$, we now consider a solution of the form;$\delta\rho \sim \exp\,[-i(\omega t - \boldsymbol{k}.\boldsymbol{r})]$. Here,$\omega$ is the frequency and$\boldsymbol{k}$is the wave vector of the mode under consideration.
Using the above dependence of $\delta\rho$ in the above equation, we have

$$(1-i\omega\tau)\left[\omega^2 + {\omega_j}^2 - k^2{c_s}^2\right] = -i\omega\frac{k^2}{\rho_0}\left(\zeta_0+\frac{4\mu}{3}\right) \tag{11}$$

Equation (11) can be written as

$$(1+\sigma\tau)\left[-\sigma^2 + {\omega_j}^2 - k^2{c_s}^2\right] - \sigma\frac{k^2}{\rho_0}\left(\zeta_0+\frac{4\mu}{3}\right) = 0 \tag{12}$$

where,$\sigma = -i\omega$ is frequency of harmonic disturbance.
Following Dhiman and Sharma [12], the above equation (12)under *strongly coupling limit* (*kinetic limit)* $\tau\sigma \gg 1$, yields the following dispersion relation

$$\sigma^2 + \frac{k^2}{\tau\rho_0}\left(\zeta_0+\frac{4\mu}{3}\right) + k^2{c_s}^2 - {\omega_j}^2 = 0 \tag{13}$$

where, ${\omega_j}^2 = 4\pi G\rho_0$ is the *Jean's frequency*.
Equation (13) can be written in the following form

$$\sigma^2 + k^2\left({c_s}^2 + \frac{{D_v}^2}{\tau}\right) - {\omega_j}^2 = 0 \tag{14}$$

This is the required *dispersion relation*.
Here, ${D_v}^2 = \frac{\left(\zeta_0+\frac{4\mu}{3}\right)}{\rho_0}$ is the square of *modified* Stokes-Kirchoff's factor (*cf.*García-Colín and Sandoval-Villalbazo [18]) representing the dissipative effects.

It is to note that the dispersion relation (14) is modified version of the relation obtained by Janaki et al. [8]. Now, invoking the Hurwitz criterion as discussed by Guillemin [14] for the sign of roots, which implies that if the constant term of equation (14) is negative, we have the condition for the onset of gravitationalinstability given as;

$$k^2\left({c_s}^2 + \frac{{D_v}^2}{\tau}\right) - {\omega_j}^2 < 0 \tag{15}$$

The above condition of instability is the modified form of instability criterion given byJanaki et al. [8] for the case of viscoelastic medium and now represent the effect of dissipative energy in the form of Stokes-Kirchoff's factor. Thus, the bulk and shear viscosityof the viscoelastic medium with dissipative effects modifies this criterion of Jeans instability.
The expression of critical Jeans wave number is given by;

$$k < k_j \; where \;\; k_j = \frac{\omega_j}{\sqrt{{c_s}^2 + \frac{{D_v}^2}{\tau}}}$$

It is obvious that as the viscosity with dissipative effects in the viscoelastic fluid increases the corresponding critical Jeans wave number decreases.

Further under *weakly coupling limit* (*hydrodynamic limit*)when $\tau\sigma \ll 1$, the dispersion relation (12) yields the following dispersion relation

$$\sigma^2 + k^2 \frac{{D_v}^2}{\tau} \sigma + k^2 {c_s}^2 - {\omega_j}^2 = 0 \qquad (16)$$

Again, by using Guillemin [14] criterion,we have the following condition for instability given by;

$$k^2 {c_s}^2 - {\omega_j}^2 < 0 \qquad (17)$$

which is the same Jeans Criterion as obtained by Chandrasekhar [5] for gaseous medium.

4. RESULTS AND DISCUSSION:

In the present analysis, the effect of bulk and shear viscosity on the onset of gravitational instability have been studied by considering the bulk viscosity as the first-order dynamics via a power-law function of the energy density. Here, we have derived a general dispersion relation form the linearized perturbation equations using normal mode analysis, under both strongly and weakly coupling limits for viscoelastic medium. The instability conditions are derived for both of these cases. It is observed that under the strongly coupling limit, the instability criterion obtained by Janaki et al. [8] gets modified due to the presence of dissipative effect of viscosity and is represented by the Stokes-Kirchoff's factor. Hence, for the viscoelastic medium the dissipative effect of bulk viscosity decreases the critical wave number and hence postpones the onset of instability. Further, in case of weakly coupling limit, theJeans Criterion as obtained by Chandrasekhar governs the onset of instability.

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ON THE WEIRDNESS OF SOME NUMBERS IN MATHEMATICS

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ABSTRACT:
An endless mysterious numbers are known to exist having various kind of strangeness. Kaprekar and Indian mathematician revealed an interesting property of the number 6174, which was subsequently named the Kaprekar constant. This number is truly a strange numbers. At first glance, it might not seem so obvious, but rearranging the digits of every four digit number where the digits are not all the same, also if one or more of the digits is zero to make it a four digit number, they must be embedded in the left side of minimum number to obtain the largest and smallest numbers from the digits can make. On using *Kaprekar's operation* that is subtracting the smallest number from the largest to obtain a new number, and carry on repeating the operation for each new number, reaches 6174 under Kaprekar's process, and in at most seven steps. Similarly using *Kaprekar's operation* for 3-digit numbers then most sequences (i.e., other than repeated digits such as 111) will terminate in the value 495 in at most 6 iterations. In this article Computer Algebra System's new algorithm is being used to demonstrate this operation for any number of digits for unlimited number of iterations. Various properties of the results obtained are discussed and compared with the results available in the literature. Results obtained are also exhibited graphically.

1. INTRODUCTION:

Mathematics, mainly theory of numbers, is no more complicated than any other subject. If one's have the enthusiasm, endurance, and time to work on and do the effort, then number theory is worth studying and we will find that this branch of mathematics can be amusing and entertaining. Science of numbers is for eternity at the heart of mathematics and has a natural attraction from the beginning of to the studentship. Mathematics is the conception and is part of the character of the human intelligence; it arises from the individual mind as it comes into contact with the world. Every individual demonstrate mathematical inclination and tends to guess, desires to identity, quantify, difference, similarity, and summarize, to make classify and generalize. A branch of pure mathematics.

Mathematics is not all regarding numbers–higher mathematics such as, real analysis, abstract algebra, complex analysis and topology, etc. is scarcely build upon numbers, but on definitions and concepts. However, the numbers are the most important component of mathematics since the beginning of its history.

Even after nearly seven decades Kaprekar constant continues to fascinate mathematicians, mathematics scholar, teachers and recreational mathematics. Kaprekar [1-3] revealed an exciting and amazing process, known as Kaprekar's operation. This process when functional to any positive four digit number not all the same i.e. avoiding numbers with four identical digits like 9999, converges to the same number 6174 in at most seven iterations. To understand the process, choose an four digits integer N (all digits of N are not all equal), and arranges its digits in descending and ascending order, subtract them, repeat the above procedure certain number of times (at the most 7 times). Thus the Kaprekar process for any four digit number is remarkable for two reasons. Firstly process converges to the same number 6174 and in no more than 7 iterations. Yutaka Nishiyama [4] described that at fleeting look, the number 6174 is might not seem so obvious, but it is really a mysterious number. Naranan [12] has also discussed this magical number. Patil and Shah [5] have derived the Kaprekar numbers and its analog equations. A variation on the two-digit kaprekar routine is studied by Anne Ludington Young [8]. Various authors have studied problems and intriguing questions linked with the above process. (See [6]-[11].)

In this paper Computer Algebra System's innovative algorithm is being used to demonstrate this operation for a number of any numbers of digits which can be checked for unlimited iterations. Various properties of the results obtained for a number consisting of 2-digitd, 3-digits ….. *n*-digits are discussed and compared with the results available in the literature. Results obtained are also exhibited graphically that how the results obtain in the process converges to constants (2-digitd, and 4-digits) and behaviors of all two digits number.

2. The Kaprekar Process:

Consider any number where all the digits are different, firstly reshuffle the digits to make the largest (arranging the digits in a descending order) and smallest (arranging the digits in a ascending order) numbers. Also if you consider say 5, and want to apply Kaprekar operation for three 3-digits or 4-digits number, then arrange the number as 005 or 0005 respectively. Secondly subtract the smaller number from the larger one For the resulting number, repeat the procedure, and keep repeating.

2.1 Four digit number:

Try $N = 2019$ (the current year)
9210-0129 = 9081
9810-0189 = 9621
9621-1269 = 8352
8532-2358 = 6174
7641-1467 = 6174

Interestingly it took four iterations to converge 6174 and to repeats it i.e after reaching at 6174 it creates an infinite loop on the constant 6174.
Let us try $N= 2510$
$5210 - 0125 = 5085$
$8550 - 0558 = 7992$
$9972 - 2799 = 7173$
$7731 - 1377 = 6354$
$6543 - 3456 = 3087$
$8730 - 0378 = 8532$
$8532 - 2358 = 6174$
$7641 - 1467 = 6174$

it took seven iterations (maximum) to converge 6174 and to repeats it.

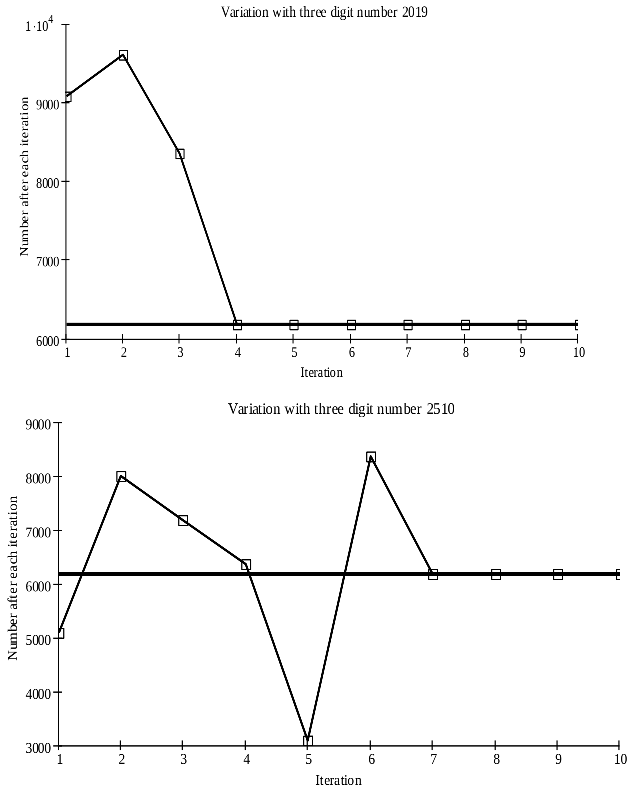

Figure 1. Variation of numbers 2019 (a) and 2510 (b) with iteration using Kaprekar's operation for 4-digits numbers.

2.2 Frequency and Number of iterations of 4-digits numbers converges to constant:

Many researchers have worked on the Kaprekar's operation by writing algorithm in various software and computer algebra systems to work out for all the 4-digits. Yutaka Nishiyama [2012] have checked all 4-digits 8991 numbers and found that every four digit number where the digits aren't all equal converges to 6174 in at most seven steps under Kaprekar's process and consequently concluded that if you do not reach 6174 after using Kaprekar's operation seven times, then categorically there is a mistake in the computation and try it again.

Number of iterations	0	1	2	3	4	5	6	7
Frequency	1	356	5192	2124	1124	1379	1508	1980

2.3 Three digit number:

On considering any three digit numbers not all the digits are same, the identical phenomenon occurs. In this case the process converges to the same number 495 and in no more than 6 iterations.

For example applying Kaprekar's operation to the three digit number 753 gives the following:

$$753 \rightarrow 396 \rightarrow 594 \rightarrow 495$$

The number 495 is the unique kernel for the operation on three digit numbers, and all three digit numbers reach 495 using the operation. Why don't you check it yourself?

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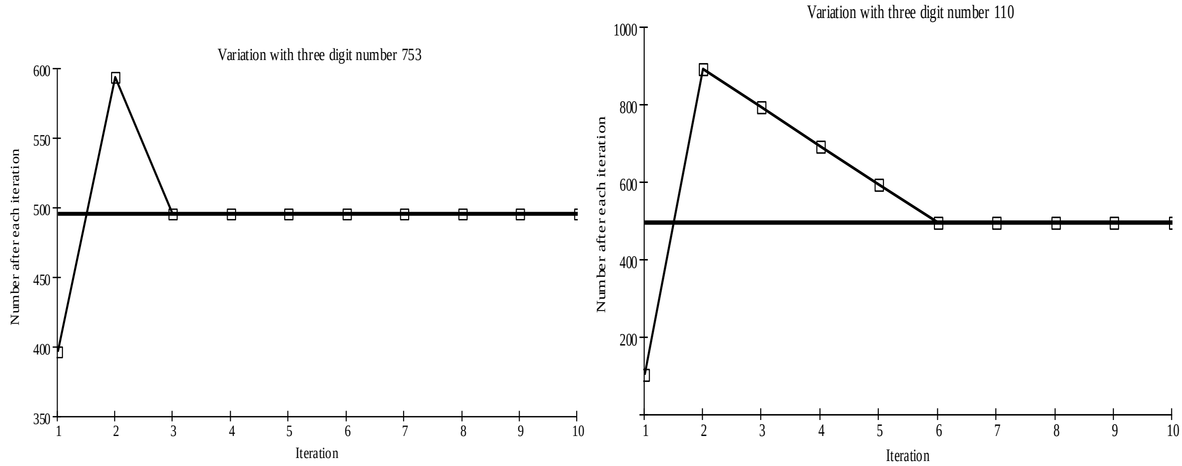

Figure 2. Variation of numbers 753 (a) and 110 (b) with iteration using Kaprekar's operation for 3-digits numbers.

2.2 Two digit number:

In case of four and three digit numbers reach a unique kernel 6174 and 495 respectively. On attempting the process for a two digit number, unlike for three and four digit numbers, there is no unique kernel for two digit numbers. Outcome is given in the following table:

2.3 Kaprekar operation on two digits number:

For all numbers 01-98 except where the digits are same (that is not 11, 22,,...,99).

Table:1 For all numbers 01-98 except where the digits are same (that is not 11, 22,,...,99) Kaprekar's operation for 2-digits numbers.

Pattern outcome	**Number (01-98) except where the digits are same**									
{9,81,63,27,45}	**01**	**12** **10**	**23** **21**	**34** **32**	**45** **43**	**56** **54**	**67** **65**	**78** **76**	**89** **87**	**98**
{**18**,63,27,45,9,81}	**02**	**13**	**20, 24**	**31,35**	**42,46**	**53.57**	**64, 68**	**75, 79**	**86**	**97**
{27,45,9,81,63}	**03**	**14**	**25**	**30,36**	**41,47**	**52,58**	**63, 69**	**74**	**85**	**96**
{**36**,7,45,9,81,63 }	**04**	**15**	**26**	**---,37**	**40,48**	**51,59**	**62**	**73**	**84**	**95**
{45,9,81,63,27}	**05**	**16**	**27**	**--, 38**	**---,49**	**50,---**	**61**	**72**	**83**	**94**
{**54**,9,81,63,27,45}	**06**	**17**	**28**	**--,39**	**Ä**	**Ä**	**60**	**71**	**82**	**93**
{63,27,45,9,81}	**07**	**18**	**29**	**Ä**	**Ä**	**Ä**	**Ä**	**70**	**81**	**92**
{**72**,27,45,9,81,63}	**08**	**19**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**80**	**91**
{81,63,27,45,9}	**09**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**90**

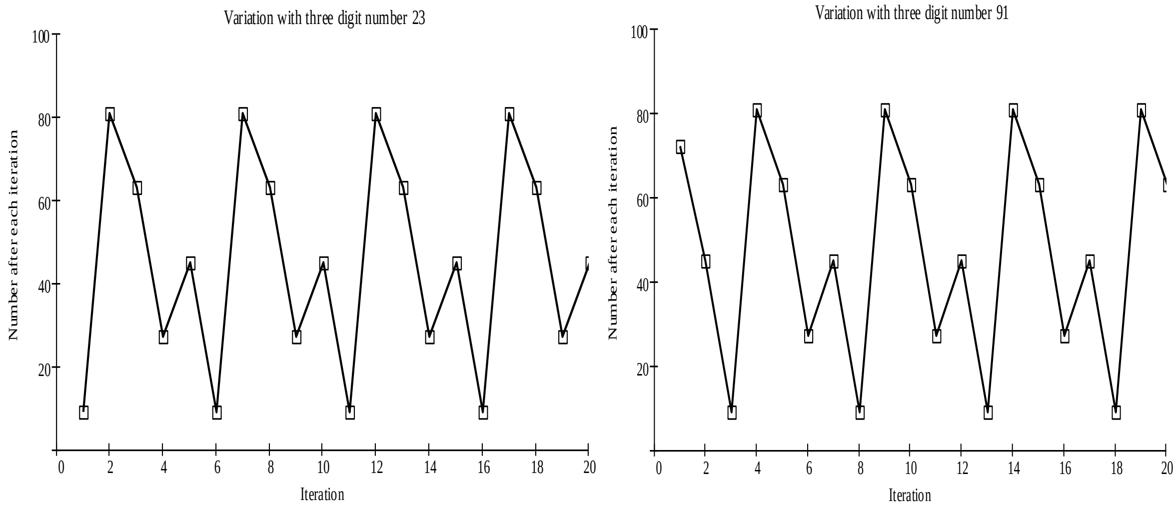

Figure 3. Variation of numbers 23 (a) and 91 (b) with iteration using Kaprekar's operation for 2-digits numbers.

2.4 Five digits, six and for any digit number...

Thus there are unique kernel for 3-digit and 4-digit numbers. Algorithm in the computer algebra system take no time to check what happens for six or more digits, and output becomes extremely monotonous, and found that there is no kernel for 2-digits, 5-digits, 7-digit numbers where as there are two kernels for 6-digits, 8-digits, 9-digit ……and 10-digit numbers having three kernels, which is in agreement with algorithm by

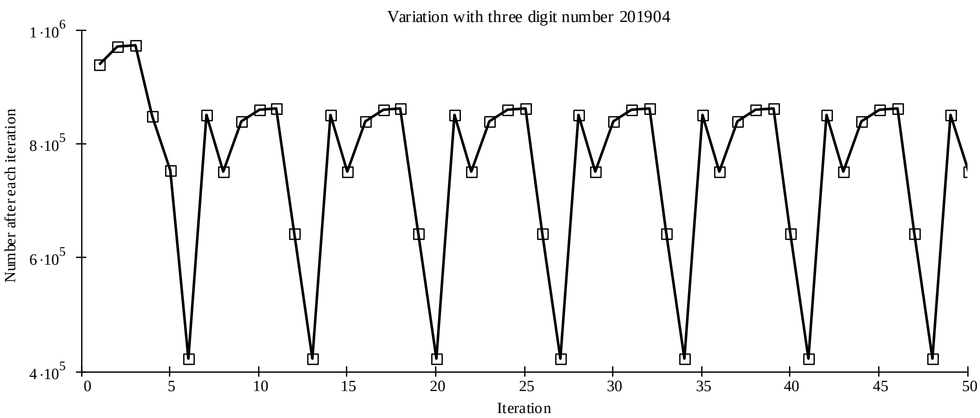

Figure 4. Variation of number 201904 with iteration using Kaprekar's operation for 6-digits numbers.

The computer algebra system (Mathcad) implementation of the method developed here is shown below:

```
ORIGIN:= 1

KLVerma(N1, n) := | P ← N1
                  | dgt ← length(P)
                  | sum ← 0
                  | Diff ← 1
                  | for j ∈ 1..n
                  |   | L ← sort(P)
                  |   | NumA ← Σ_{j=1}^{dgt} L_j · 10^(dgt−j)
                  |   | M ← reverse(sort(P))
                  |   | NumD ← Σ_{j=1}^{dgt} M_j · 10^(dgt−j)
                  |   | NNum ← NumD − NumA
                  |   | for i ∈ 1..dgt
                  |   |   | d_i ← mod(trunc(NNum / 10^(i−1)), 10)
                  |   |   | i ← i + 1
                  |   | for i ∈ 1..dgt
                  |   |   for p ∈ 1..1
                  |   |     | M_{i,p} ← d_i
                  |   |     | M
                  |   | P ← M
                  |   | Sum_j ← Σ_{k=1}^{dgt} d_k
                  |   | Diff_j ← Σ_{i=1}^{dgt} d_{dgt−i+1} · 10^(dgt−i)
                  |   | j ← j + 1
                  | ((Sum_j, Diff_j, j))
```

3. RESULTS AND CONCLUSIONS:

From the algorithm written in computer algebra system for the Kaprekar's operation and the graphical representation in figures 1-4, of the results obtained it appears that Kaprekar's operation every number converges to a unique kernel only for three and four digit numbers.

Whereas for two digits numbers under the Kaprekar's operation it is observed that Number and on reversing the digit of a number has the same representation. Numbers which are congruent to ±2, ±4, ±6, ±8 show extra two digit number in beginning before settling for a repeating sequence. First and last two digit numbers and numbers preceding and succeeding (11, 22,,...,99) also have same representation. Sequence of each number which is repeating in any pattern has sum 9. i.e repeating two digit numbers with digit sum 9 are only {9, 27, 36, 45, 63, 81} whereas 18, 36, 54 and72 are not repeating. Interestingly non-repeating two digit numbers 18, 36, 54 and72 of sequences (congruent to ±2, ±4, ±6, ±8) has property that 20-02 =18, 40-04=36, 60-06-54 and 80-08=72. Any two digit number, if the sum is not 9 then it is not in any repeating sequence. Number of two digits which has representation I to IX has terms in AP are 18,16.,14,…,2. In j^{th} Pattern each number is $\pm j$ (modulo 11), j =1,2,…9 . Every column of the table has 9 numbers. Variation of number 6-digit number 201904 upto 50 iteration using Kaprekar's exhibited in Figure 4. Various progresses in science and mathematics have been driven by 'mistakes'. Mathematicians, all over the world are still working that some great theory of mathematics might lie behind this process also.

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A SHORT STUDY OF PRIME NUMBERS

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ABSTRACT:
In this paper, a review of some general properties of prime numbers has been given. Distance between two neighboring prime numbers and simple distance between two prime numbers have been defined and some results about the distance between two prime numbers have been presented. All prime numbers have one of the digits 1, 3, 7, 9 at unit place ; a short explanation of this concept has been considered, Fermat's theorems play an important role in the study of prime numbers, thus some results have been derived using a Fermat's theorem and application of these results has been given. Idea of Mersin's numbers has also been considered. A study of sum of consecutive prime numbers has also been done.

KEYWORDS: prime, composite, neighboring, consecutive, odd, even, divisible.

INTRODUCTION:

Here in this paper, we shall study some simple properties of prime numbers. The natural number system has been the main topic of human interest for the beginning of the civilization. The natural number system can be considered as the seed of mathematics. Once begin to play with natural number system, you will across various interesting results of natural numbers. Once dive deeply in the natural number system, you will dive in this system again and again because innumerable interesting results attract you greatly.

Among the natural numbers, there exists special type of numbers called prime numbers. A prime number is that natural which is divisible by one and itself. 1 is not considered a prime number, 2 is the only even prime number and all other primes are odd. Here one thing is note worthy that the existence of even numbers is due to the even prime 2 because if there were not 2, there won't have been even natural numbers. We can easily decide that natural numbers which are divisible by 2, 3, 5, and 11, because all even numbers are divisible by 2, all those numbers, the sum of whose digits is divisible by 3 , are divisible by 3, all those natural numbers ending with 5 or 0 are divisible by 5, and all those natural numbers, the difference of the sums of alternating digits of that number, is 0, are divisible by 11. We can call them separable prime numbers. There exists innumerable conjectures about prime numbers, but there exists no short cut formula for determining that a particular number is a prime number, there exist only long and time consuming algorithms for deciding a particular natural number to be a prime number. Prime numbers are infinite in numbers and their distribution does not follow any rule. Now we study some interesting results regarding prime numbers.

SOME GENERAL PROPERTIES OF PRIME NUMBERS:

First and last even prime number is 2, and all other prime numbers are odd. So we can write every odd prime number as $2n + 1$. Further every prime greater than or equal to 5 can be expressed as $6n \pm 1$, this can be proved as:
Consider three consecutive naturals $2n, 2n + 1, 2n + 2$. If any one of them is prime then that

will be $2n+1$. Any one of three consecutive naturals is divisible by 3. If $2n+1$ is prime then it will not be divisible by 3. Thus if $2n$ is divisible by 3, then $n = 3k\ , k \in N.$ Therefore $2n+1 = 6k+1 \ \forall \ k \in N$
If $2n+2$ is divisible by 3, then $n+1 = 3k\ , k \in N \ \Rightarrow n = 3k-1\ , k \in N$.
Therefore $2n+1 = 2(3k-1)+1 = 6k-2+1 = 6k-1 \ \forall \ k \in N$.
So we find that every prime number ≥5, can be expressed in the form $6n \pm 1 \ \forall \ n \in N$.
Therefore if p≥ 5 be a prime number then $p = 6n \pm 1 \ \forall \ n \in N$.

Further each prime number ≥ 5 is sum of two consecutive natural numbers such that one of them is divisible by 3. This can be easily verified as $6k+1 = 3k+3k+1 \ \forall \ k \in N$ and $6k-1 = 3k-1+3k \ \forall \ k \in N$.
Now consider $(6n \pm 1)^2 = 36n^2 \pm 12n + 1.$

$$= 12n(3n \pm 1) + 1. \tag{1}$$

If n is even let $n = 2k, k \in N,$ then $3n \pm 1 = 6k \pm 1.$ (2)
Using the values given in (2) in (1), we have

$$(6n \pm 1)^2 = 24k(6k \pm 1) + 1 \tag{3}$$

If n is odd let $n = 2k-1, k \in N,$ then $3n \pm 1 = 3(2k-1) \pm 1$

$$= 6k-2 \ or \ 6k-4$$
$$2(3k-1) or 2(3k-2)$$

∴ if $n = 2k-1$, then $3n \pm 1 = 2(3k - 1 or 2)$ (4)
Using the values given in (4) in (1), we have

$$(6n \pm 1)^2 = 24(2k-1)(3k - 1 or 2) + 1 \tag{5}$$

From (3) and (5), we conclude that if $p \geq 5$ be a prime number then $p^2 - 1$ is always divisible by 24.

One interesting property of the natural numbers is that all those naturals which end with 0 or 2 or4 or 6 or 8 , that are always divisible by 2 and all those naturals which end with 0 or 5 , that are always divisible by 5. We can find associated natural number by placing 0 or 2 or 4 or 6 or 8 at the end of a prime numbers and dividing the resulting number again and again by 2 until the obtained number has not factor 2. Similarly we can find associated natural number by placing 0 or 5 at the end of a prime numbers and dividing the resulting number again and again by 5 until the obtained number has not factor 5.

Consider a prime number 5, place 0 at the end of 5, we get 50, divide 50 by 2, we get 25, therefore 25 is the associated natural of prime 5 under the division by 2. Similarly, divide 50 by 5, we get 10, divide 10 by 5, we get 2, therefore 2 is the associated natural of prime 5 under the division by 5.

If we get a prime number on subtracting a prime number from a even number then both the primes are called complementary primes of the used even number. For example 11 and 13 are complementary primes of 24. There may be more than one pair of complementary primes. For example, 5, 19 and 7, 17 are other complementary primes of 24.

DISTANCE BETWEEN TWO NEIGHBORING PRIME NUMBERS:
Two prime numbers are called neighboring prime numbers if there exists no other prime number between them. For example, 2 and 3, 3 and 5, 5 and 7, 7 and 11 etc are neighboring

prime numbers.
Distance between two neighboring prime numbers, is defined as number of composite numbers between them. Distance between neighboring prime numbers 2 and 3 is 0, distance between neighboring prime numbers 3 and 5 is 1, distance between neighboring prime numbers 5 and 7 is 1, distance between neighboring prime numbers 113 and 127 is 13 and so on.

It can be easily seen that if p_1 and p_2 $(p_1<p_2)$ are two neighboring prime numbers then the distance between them is $p_2 - p_1 - 1$ or $|p_1 - p_2| - 1$, let $d_p(p_1, p_2) = |p_1 - p_2| - 1$.
2 and 3 are the only neighboring prime numbers such that distance between them is 0, minimum distance between any two odd neighboring prime numbers, is 1, but it is not yet known that what is maximum distance between any two odd neighboring prime numbers.

Maximum distance between any two neighboring prime numbers up to 100, is 7, and maximum distance between any two neighboring prime numbers between 100 and 200, is 13 and it is between two neighboring prime numbers 113 and 127.

It can be proved that the maximum distance between any two neighboring odd primes, is finite. Suppose, if possible, the distance $d = d_p(p_1, p_2) = p_2 - p_1 - 1$, between any two neighboring odd primes p_1 and p_2 $(p_1<p_2)$, is infinite, this implies $p_2 - p_1 - 1 = d = \infty$
$\Rightarrow p_2 = \infty$ $\Rightarrow$ there exists no prime number after p_1
$\Rightarrow$ there are finite number of prime numbers which is a contradiction as there are infinite number of prime numbers. Therefore our supposition that the distance between any two neighboring prime numbers is infinite, is wrong and hence the distance between any two neighboring prime numbers is finite.

DISTANCE BETWEEN ANY TWO PRIME NUMBERS:
Distance between any two prime numbers is defined as the number of natural numbers occurring between them whether primes or composites. If $p \; and \; q (p < q)$ are two prime numbers, then distance between them is denoted by $d_p(p, q)$ and defined as $d_p(p, q) = q - p - 1 = |p - q| - 1$. Distance between prime numbers 7 and 29 is $d_p(7, 29) = 29 - 7 - 1 = |7 - 29| - 1 = 21$.

$d_p(3, 5) = 1, d_p(5, 7) = 1$, thus 3, 5, 7 are equidistant prime numbers and 5 is the central prime of 3 and 5. The numbers 5, 11, 17, 23, 29 are equidistant prime numbers as $d_p(5, 11) = d_p(11, 17) = d_p(17, 23) = d_p(23, 29) = 6$ here 17 is the central prime. There are infinite equidistant triples of prime numbers such as 3, 5, 7; 3, 7, 11; 3, 11, 19; 3, 13, 23; 3, 23, 43 etc. are equidistant triples of primes with 3 as first prime. We can call the ordered set of equidistant prime numbers as a chain of equidistant prime numbers and number of prime numbers in this chain, is called the length of the chain.

SIMPLE DISTANCE BETWEEN ANY TWO PRIMES:
Simple distance between any two prime number is the difference of the larger and smaller primes i.e. if $p > q$ are two primes then simple distance between them is $d(p, q) = p - q$. Equidistant prime numbers remain equidistant under any of the two definitions of distance between any two primes.

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Now a question arises that what will be the maximum length of the equidistant chain of prime numbers, it may be 5 because if we assume that first prime of the chain is $6n-1$ and d is the common difference then equidistant chain of prime numbers will be of the form $6n-1,\ 6n-1+d, 6n-1+2d, 6n-1+3d, 6n-1+4d, 6n-1+5d$, then one of these prime will be divisible by 3 or 5. Thus the maximum length of equidistant chain of primes may be 5.

DIGITS AT THE UNIT PLACE OF THE PRIME NUMBERS:

1, 3, 7, 9 are the digits which are at unit place in prime numbers, 5 has been excluded from odd digits. 13, 17, 37, 79 are prime numbers when their order is reversed; we get primes 31, 71, 73, 97. Consider all the permutation of 1, 3, 7; which are 137, 173, 317, 371, 713, 731; then 137, 173, 317 are primes but 371, 713, 731 are not primes. Consider now the permutations of 1, 3, 9 ; which are 139, 193, 319, 391, 913, 931; then 139, 193, 319 are primes but 391, 913, 931 are not primes. . Consider now the permutations of 1, 7, 9; which are 179, 197, 719, 791, 917, 971; then 179, 197, 719, 971 are primes but 791, 917 are not primes. Consider now the permutations of 3, 7, 9; which are 379, 397, 739, 793, 937, 973; then 379, 397, 739, 937 are primes but 793, 973 are not primes. Similarly we can check the all the permutations of 1, 3, 7, 9 for prime numbers.

RESULTS RELATED TO FERMAT' S THEOREM:

There are many results related to prime numbers due to Fermat 's theorems (Hardy and Wright [1]) one of them is the following theorem stated as: "if p is a prime number and n is any positive integer, then $n^p \equiv n(mod\ p)$". Some more results relating to prime numbers can be derived from this theorem.

Theorem-1: If $m,n \in N$such that $m+n=p\ (prime), m^p+n \equiv 0(mod\ p)$.

Proof: By the Fermat's theorem as stated above, we have

$$m^p \equiv m(mod\ p)$$

$\Rightarrow m^p - m = kp\ ,\ k \in N$

$\Rightarrow m^p - m + m + n = kp + m + n\ ,\ k \in N$

$\Rightarrow m^p + n = kp + p\ ,\ k \in N$

$\Rightarrow m^p + n = (k+1)p\ ,\ k \in N$

$\Rightarrow m^p + n \equiv 0(mod\ p)$.

Theorem-2: If $m,n \in N$such that $m+n=p\ (prime), n^p+m \equiv 0(mod\ p)$.

Proof: By the Fermat's theorem as stated above, we have

$$n^p \equiv n(mod\ p)$$

$\Rightarrow n^p - n = kp\ ,\ k \in N$

$\Rightarrow n^p - n + n + m = kp + n + m\ ,\ k \in N$

$\Rightarrow n^p + m = kp + p\ ,\ k \in N$

$\Rightarrow n^p + m = (k+1)p\ ,\ k \in N$

$\Rightarrow n^p + m \equiv 0(mod\ p)$.

Above stated Fermat's theorem can be stated in another way as ""if p is a prime number and n is any positive integer, then $n^{p-1} \equiv 1(mod\ p)$". From this, we can derive the following result.

Theorem-3: if $k, l,\ m, n \in N\ and\ p$ be a prime such that $m^k + n \equiv 0(mod\ p)$, then

$$m^{k+l(p-1)} + n \equiv 0(mod\ p)$$

Proof: Given $m^k + n \equiv 0(mod\ p)$
$\Rightarrow m^k + n = rp\ ,\ r \in N$
$\Rightarrow m^k = rp - n\ ,\ r \in N$ (6)
Also $m^{p-1} \equiv 1(mod\ p)$
$\Rightarrow\ m^{l(p-1)} \equiv 1(mod\ p)$
$\Rightarrow\ m^{l(p-1)} = 1 + sp, s \in N$ (7)
Therefore, by (6) and (7), we get
$m^k\, m^{l(p-1)} = (rp - n\ )(1 + sp)$
$\Rightarrow m^{k+l(p-1)} = rp + rsp^2 - n - nsp$
$\Rightarrow m^{k+l(p-1)} + n = rp + rsp^2 - nsp$
$\Rightarrow m^{k+l(p-1)} + n = p(r + rsp - ns)$
$\Rightarrow m^{k+l(p-1)} + n \equiv 0(mod\ p)$ (8)
If we take $k = l = 1\ in$ (8), we get $m^p + n \equiv 0(mod\ p)$ which is the result of theorem-1
Application of these results is given below as:
$5 + 2 = 7(a\ prime) \Rightarrow 5^7 + 2\ and\ 2^7 + 5$ are divisible by 7, this can be easily verified.
By Fermat's theorem, $10^{p-1} \equiv 1(mod\ p)$
$\Rightarrow$ a prime number p divides a number in which any one of the digit out of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; is repeated $p - 1$ times.
e.g. consider the number 111111 in which digit I is repeated $7 - 1 = 6$ times, thus 111111 is divisible by 7 and $\frac{111111}{7} = 15873$ or $15873 \times 7 = 111111$
Similarly $\frac{777777777777}{13} = 59829059829$
$\Rightarrow 59829059829 \times 13 = 777777777777$
$65359477124183 \times 17 = 1111111111111111$
It should be noted that if $\frac{p-1}{2} + 1 = \frac{p+1}{2}$ is again a prime then the prime number p divides the number in which any one of the digit out of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; is repeated $\frac{p-1}{2}$ times.
e.g. $\frac{13-1}{2} + 1 = \frac{13+1}{2} = 7$ and 7 is prime therefore 13 divides 111111, 222222,….,999999 each containing 6 digits. Since $\frac{111111}{13} = 8547$
Another example of this type is $p = 37$ such that $\frac{37-1}{2} + 1 = \frac{37+1}{2} = 19$ and 19 is prime therefore 37 divides 111111111111111111, 222222222222222222,…., 999999999999999999 each containing 18 digits. Since $\frac{111111111111111111}{37} = 3003003003003003$. In fact 37 divides all numbers of type 111, 111111, 111111111, ….
Now we express the natural numbers as a geometrical expression which has fixed repetition i.e. number of the form 237237237237, 825382538253 etc.
Now

$$1 + 10^k + 10^{2k} + \cdots + 10^{(n-1)k} = \frac{10^{nk}-1}{10^k-1}, p \neq 2, 5.$$

Therefore $1010101 = 1 + 10^2 + 10^4 + 10^6 = \frac{10^8-1}{10^2-1}$
Using Fermat's theorem, we can derive the following result:
$10^{k(p-1)} \equiv 1(mod\ p)$
$\Rightarrow 10^{k(p-1)} - 1 \equiv 0(mod\ p)$

$\Rightarrow (10^k - 1)\left(\frac{10^{k(p-1)}-1}{10^k-1}\right) \equiv 0(mod\ p)$
$\Rightarrow p$ divides $\frac{10^{k(p-1)}-1}{10^k-1}$ (9)
From (9), we find that $\frac{10^{2(7-1)}-1}{10^2-1} = 10101010101$ is divisible by 7 and $\frac{10101010101}{7} = 1443001443$. Similarly, that $\frac{10^{2(13-1)}-1}{10^2-1} = 101010101010101010101$ is divisible by 13 and $\frac{101010101010101010101}{13} = 7770007770007770007777$.
Similarly 13 divides 1001001001001001 as $\frac{1001001001001001}{13} = 77000077000077$
If p divides $\frac{10^{k(p-1)}-1}{10^k-1}$, then p divides $n\left(\frac{10^{k(p-1)}-1}{10^k-1}\right)$, $n \in N$.
Therefore 171717171717, 191919191919, 232323232323, 179179179179179179, 175317531753175317531753 etc. are divisible by 7 and 13.
Prime numbers of the form $2^{2^n} + 1, n \in N$ are called Fermat's numbers and prime numbers of the form $2^n - 1$ are called Mersin's numbers (Hardy and Wright [1]). Thus $2^{2^1} + 1 =$5, $2^{2^2} + 1 =$17, $2^{2^3} + 1 =$257 etc. are Fermat's numbers. Similarly $2^2 - 1 =$3, $2^3 - 1 =$7, $2^5 - 1 = 31$, $2^7 - 1 = 127$ etc. are Mersin's numbers.
Now consider the numbers of the form 11111, 1111111, 11111111111, ….etc. where 1 is repeated as a prime number of times i.e. 5, 7, 11, 13,… times.
Then 11111=41$\times$ 271, here 1 is repeated 5 times and $\frac{41-1}{8} = 5$, some relation exists among 11111, 41 and 5.
Similarly 1111111=239$\times$ 4649
From this it seems that 11111111111=5 digits prime number $\times$ 6 digits prime number
And 1111111111111=6 digits prime $\times$ 7 digits number prime
There is possibility that 5 digits prime factor of 11111111111 will start from 4 and end with 1, 6 digits prime factor of 11111111111 will start from 2 and end with 1, 6 digits prime factor of 1111111111111 will start from 2 and end with 9, 7 digits prime factor of 1111111111111 will start from 4 and end with 9.

Also 101010101, 1001001001001,... are divisible by 41 and 271. Similarly 1010101010101, 1001001001001001001,… are divisible by 239 and 4649. We notice that $41 - 1 = 40$, $271 - 1 = 270$ both are divisible by 5, similarly $239 - 1 = 238$, $4649 - 1 = 4648$ both are divisible by 7.

SUM OF CONSECUTIVE PRIME NUMBERS:
Let us check the sum of prime numbers from the first prime number 2 to succeeding consecutive primes as follows:

Let S_p, denotes sum of all prime numbers from 2 to p so that
$S_2 = 2, S_3 = 2 + 3 = 5, S_5 = 2 + 3 + 5 = 10, S_7 = 2 + 3 + 5 + 7 = 17,$ $S_{11} = 2 + 3 + 5 + 7 + 11 = 28, S_{13} = 2 + 3 + 5 + 7 + 11 + 13 = 41$, $S_{17} = 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58$, $S_{19} = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 58$, etc. we prepare table to write these sums conveniently as follows

Prime numbers	**Cumulative sum**	**Prime numbers**	**Cumulative sum**	**Prime numbers**	**Cumulative sum**
2	2=S_2	59	440= S_{59}	137	1988= S_{137}
3	5= S_3	61	501= S_{61}	139	2127= S_{139}
5	10= S_5	67	568= S_{67}	149	2276= S_{149}
7	17= S_7	71	639= S_{71}	151	2427= S_{151}
11	28= S_{11}	73	712= S_{73}	157	2584= S_{157}
13	41= S_{13}	79	791= S_{79}	163	2747= S_{163}
17	58= S_{17}	83	874= S_{83}	167	2914= S_{167}
19	77= S_{19}	89	963= S_{89}	173	3087= S_{173}
23	100= S_{23}	97	1060= S_{97}	179	3266= S_{179}
29	129= S_{29}	101	1161= S_{101}	181	3447= S_{181}
31	160= S_{31}	103	1264= S_{103}	191	3638= S_{191}
37	197= S_{37}	107	1371= S_{107}	197	3835= S_{197}
41	238= S_{41}	109	1480= S_{109}	199	4034= S_{199}
43	281= S_{43}	113	1593= S_{113}	211	4245= S_{211}
47	328= S_{47}	127	1720= S_{127}	223	4468= S_{223}
53	381= S_{53}	131	1851= S_{131}	227	4695= S_{227}

From this table of sum of all primes from 2 to p(prime), we find some interesting facts e.g. 10, 100 are sum of consecutive primes; the primes 5, 17, 41, 197, 281 are sum of consecutive primes, but it is interesting that there is no prime after 281, which is the sum of the consecutive primes up to S_{227} or perhaps further, beginning from 2.

CONCLUSION:

There are innumerable interesting facts, conjectures about natural numbers and prime numbers which can be explored. In this paper, there are unsolved problems like maximum distance between any two neighboring prime numbers, maximum length of the chain of a prime numbers with same common difference (distance). One can try to solve these problems. More results on prime numbers can be derived by using the property that if p divides $\frac{10^{k(p-1)}-1}{10^k-1}$, then p divides $n\left(\frac{10^{k(p-1)}-1}{10^k-1}\right)$, $k, n \in N$. Two more problems are that there may be no prime of the form 111, 1111, 11111, 111111, 1111111,… and 111, 11111, 1111111, 11111111111,.. have only two prime factors.

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CALIBERATING MICROPOLARITY ON TRANSFERENCE OF SH-WAVES IN AN ELASTIC LAYER OVERLYING SIZE-DEPENDENT SUBSTRATE

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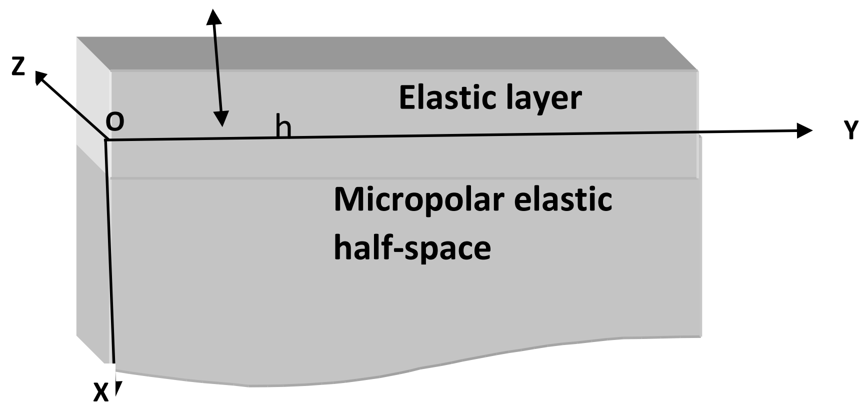
Research aim:
Considering micro structure of modern sensing devices the aim of this present study is to analyse the propagation behavior of horizontally polarized shear waves in an elastic layer overlying semi-infinite micropolar elastic substrate. Here micropolar theory is preferred instead of classical model due to its capability to expound size effects on micro scale. The closed form expression of dispersion relation for the existence of horizontally polarized shear waves is obtained analytically. Possible particular case has been derived and validated with the existing result of the classical Love wave equation. To exemplify the competence of the problem numerical computations are executed by considering aluminium-epoxy as a micropolar elastic material. The substantial effects of micropolarity and thickness of elastic layer have been examined and depicted graphically on the phase velocity profiles of the wave for the considered structure.

Brief literature survey:
Surface wave propagation in a composite structure has numerous applications in geoscience, engineering, ocean acoustics, NDE techniques, as well as in designing of surface acoustic wave (SAW) devices. Numerous researchers have investigated the propagation behavior of SH-waves in layered structure comprising of elastic material (Liu et al., 2001, Sahu et al., 2014). Detailed investigations have been done on the propagation behavior of SH-waves in different materials based on classical model. But due to inadequacy of classical model to explain the behaviour of materials viz. aluminium epoxy, cellular solids, platelet composites, polymers and many other having complex microstructures, micropolar theory is preferred as it has ability to explain size effects on microscale by auditing the additional degree of freedom. Certain discrepancies were occurred and unexplained by the classical theory. In order to remove these shortcomings Voigt (1887) introduced the concept of couple stresses in elasticity by introducing additional couple vector to explicate particle interactions in a body. Eringen and Suhubi (1964) introduced the theory of linear and nonlinear micropolar elastic continua and Eringen (1966) generalized the classical theory of elasticity by considering three rotational degrees of freedom in addition to three classical displacement degrees of freedom. Due to the practical applicability in various fields of science and technology such as acoustics, seismology, aerospace and marine structures an extensive study is available on the wave propagation phenomenon in micropolar solids.Propagation of Love-type surface waves in homogeneous micropolar elastic media were studied by Midya (2004). Tomar (2005) investigated wave propagation in a micropolar elastic plate with voids. Kaur et al. (2017) examined shear wave propagation in vertically homogeneous viscoelastic layer over a micropolar elastic half-space. Kundu et al. (2017) investigated Love wave propagation in heterogeneous micropolar media.

Problem formulation:
This paper studies the propagation characteristics of horizontally polarized shear waves in a structure comprised of elastic layer perfectly bonded over semi-infinite micropolar elastic substrate. Both materials are isotropic in nature. An elastic layer of thickness h overlying semi-infinite micropolar elastic substrate is considered. The Cartesian coordinate system is considered such that shear wave is propagating in the y-direction and x-axis is positive in vertically downward direction as shown in fig. 1. The elastic material is polarized along z-direction perpendicular to x-y plane. The displacement components will be independent of z-coordinate. Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ are the mechanical displacement components for an elastic layer and the micropolar substrate respectively.



The governing equation for the elastic layer in the absence of body forces is given by:

$$\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} = \frac{1}{\alpha_1^2}\frac{\partial^2 u_3}{\partial t^2} \tag{1}$$

The governing equations and constitutive relations for micropolar elastic half-space (Eringen, 1966) in the vector form in the absence of body forces are

$$(\lambda + \mu)\nabla(\nabla.\vec{v}) + (\mu + \kappa)\nabla^2\vec{v} + \kappa\left(\nabla \times \vec{\phi}\right) = \rho\frac{\partial^2\vec{v}}{\partial t^2}, \tag{4}$$

$$(\alpha + \beta + \gamma)\nabla\left(\nabla.\vec{\phi}\right) - \gamma\nabla \times \left(\nabla \times \vec{\phi}\right) + \kappa(\nabla \times \vec{v}) - 2\kappa\vec{\phi} = \text{ј}\rho\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{5}$$

$$\sigma_{ij} = \lambda v_{k,k}\delta_{ij} + \mu(v_{i,j} + v_{j,i}) + \kappa(v_{j,i} - \epsilon_{ijk}\phi_k), \tag{6}$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{7}$$

where i, j, k=1, 2, 3.α, β, γ, κ are micropolar material parameters. λ and μ are Lame's constants. ρ is the mass density, ј is micro inertia.$\sigma_{ij}$and m_{ij} are the stress tensor and couple stress tensor. δ_{ij} is kronecker delta. ϵ_{ijk} is alternating tensor.$\vec{\phi} = \left(\vec{\phi}_1, \vec{\phi}_2, \vec{\phi}_3\right)$ is the microrotation vector.

The admissible boundary conditions will be applied and dispersion relation will be obtained by taking the determinant of the coefficients of unknown variables equal to zero, for the propagation of SH-waves in an elastic layered structure. In the absence of micropolar constants the classical Love wave equation (Love, 1911) is obtained from the dispersion relation for the sake of validation.

Solution methodology:
The solutions are obtained analytically by using general differential equations with corresponding boundary conditions.

Significant conclusions:

- Existence of shear waves in the composite structure consisting of an elastic layer bonded perfectly to the micropolar elastic material and micropolarity has significant effect on the propagation of SH-waves. As micropolarity constant $\left(\frac{\kappa}{\mu}\right)$increases phase velocity of the wave increases.
- Thickness of elastic layer also has eminent effect on the phase velocity profile of the SH-wave. As the thickness of elastic layer is increasing, the phase velocity of the wave also increases.

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ON THE REGION OF COMPLEX GROWTH RATES IN TRIPLY DIFFUSIVE CONVECTION IN POROUS MEDIUM

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ABSTRACT:
In the present paper, the problem of triply diffusive convection in porous medium is studied by using Darcy-Brinkman model. Upper bounds for the complex growth rates of an arbitrary oscillatory disturbance of growing amplitude, neutral or unstable, are obtained which are important especially when both the boundaries are not free so that exact solutions in the closed form are not obtainable. It is further proved that the result obtained herein is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection; Porous Medium; Darcy-Brinkman model; Rayleigh number; Concentration Rayleigh number.

INTRODUCTION:

The convective fluid motion under the simultaneous action of a uniform vertical temperature gradient and a gravitationally opposite uniform vertical concentration gradient (known as double diffusive convection) has been an area of great research activity for the last many decades. Double diffusive convection is now well known. For a broad view of the subject one may refer to Stern (1960), Veronis (1965), Nield (1967), Baines and Gill (1969), Turner (1974),Huppert and Turner (1981),Banerjee et al. (1981, 1993a,b), Gupta et al. (1982, 1983, 2001), Brandt and Fernado (1996), Basu and Layek (2013), Radko (2013), Kellner and Tilgner (2014), Yang et al. (2015,2016), Chen et al. (2016,2017), Safi and Benissaad (2017) etc.

The presence of more than two components in a fluid having different diffusive properties, can lead to convective instabilities, often well before a fluid system would become statically unstable. It is now well established that (Griffiths (1979a), Moroz (1989), Pearlstein et al. (1989), Lopez et al. (1990), Terrones (1993), Prakash et al. (2014a,b,c,d,e, 2015a,b,c,d, 2016a), Patil et al. (2018a,b),etc.) the small concentration of a third component with a smaller mass diffusivity can have a significant effect upon the nature of instability; and ‘diffusive convection’ (oscillatory modes) and direct ‘salt finger’ modes (steady convection) may simultaneously exist under a wide range of conditions, even if the over-all density distribution is gravitationally stable.

The subject of systems having more than two components in porous medium has also attracted many researchers recently due to its importance in the study of geothermally heated lakes, earth core, solidification of molten alloys, underground water flow, natural phenomena such as contaminant transport, warming of stratosphere and magmas and their laboratory models and sea water etc. For the broad view of the subject one may be referred to Vafai (2005), Straughan et al. (2008),Rionero (2013), Abdullah et al. (2018),Zhao et al. (2014), Ghalambaz et al. (2016), Prakash et al. (2016b,c, 2017), Roy (2017), Hewitt (2017), Raghunatha et al. (2018)etc.

Since instability in triply diffusive configurations in porous and non-porous media may occur in the form of oscillatory motions, the problem of deriving the upper limits for the linear growth rate of an arbitrary neutral or unstable oscillatory disturbance of growing amplitude in triply diffusive convection has its own importance in fluid dynamics, especially when at least one of the boundaries is rigid so that exact solutions in the closed form are not derivable as was possible for the cases treated by Griffiths (1979a), Poulikakos (1985) and Rudraiah and Vortmeyer (1982).

In the present paper we have studied triply diffusive convection (with heat and one concentration component as destabilizing agent and the second concentration component as stabilizing agent) in a sparsely distributed porous medium by using Darcy-Brinkman model. Darcy flow model is relevant only to densely packed, low permeability porous medium. Also, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. The Brinkman (1947) extension of the Darcy’s law has overcome these difficulties by adding a viscous like term to the equations.

Mathematical Formulation and Analysis:

Consider a viscous finitely heat conducting Boussinesq fluid layer, saturating a sparsely distributed porous medium, of infinite horizontal extension statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1(< T_0)$ and uniform concentrations S_{10},S_{20} and$S_{11}(< S_{10})$,$S_{21}(> S_{20})$(see Fig.1.). It is assumed that the saturating fluid is incompressible and the porous medium is a constant porosity medium. It is further assumed that the cross-diffusion effects of the stratifying agencies can be neglected. The Darcy-Brinkman model has been used to investigate the triply diffusive convection in porous medium.

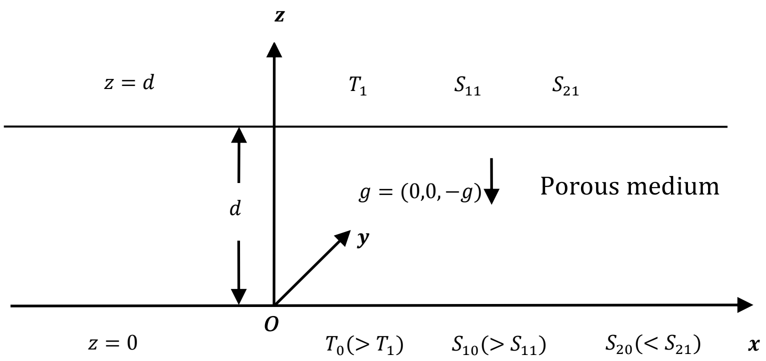

Fig.1. Physical configuration.

The basic hydrodynamics equations that govern the motion of a triply diffusive fluid layer in porous medium in their non-dimensional form with $R > 0, R_1 > 0,\ R_2 < 0$ are given(Vafai (2005), Prakash et al. (2017)):

$$\Lambda(D^2 - a^2)^2 w - (p + D_a^{-1})(D^2 - a^2)w = Ra^2\theta - R_1 a^2 \phi_1 + |R_2| a^2 \phi_2, \quad (1)$$

$$(D^2 - a^2 - E\sigma p)\theta = -w, \quad (2)$$

$$\left(D^2 - a^2 - \frac{E_1 \sigma\, p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1} \quad (3)$$

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and

$$\left(D^2 - a^2 - \frac{E_2\sigma\, p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}. \tag{4}$$

Equations (1) – (4) are to be solved by using the following boundary conditions

$w = \theta = \phi_1 = \phi_2 = Dw = 0$ at $z = 0$ and $z = 1$(both the boundaries are rigid) (5)

Or $w = \theta = \phi_1 = \phi_2 = D^2w = 0$ at $z = 0$ and $z = 1$(both the boundaries are free) (6)

or $\left.\begin{array}{l} w = \theta = \phi_1 = \phi_2 = Dw = 0 \text{ at } z = 0 \text{ (lower boundary is rigid)} \\ \text{and } w = \theta = \phi_1 = \phi_2 = D^2w = 0 \text{ at } z = 1 \text{ (upper boundary is free)} \end{array}\right\}$ (7)

or $\left.\begin{array}{l} w = \theta = \phi_1 = \phi_2 = D^2w = 0 \text{ at } z = 0 \text{ (lower boundary is free)} \\ \text{and } w = \theta = \phi_1 = \phi_2 = Dw = 0 \text{ at } z = 1 \text{ (upper boundary is rigid)} \end{array}\right\}$, (8)

where z is the real independent such that $0 \leq z \leq 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ the Prandtl number, $\tau_1 > 0$ and $\tau_2 > 0$ are the Lewis numbers for two concentration components with mass diffusivity κ_1, κ_2 respectively, $D_a > 0$ is the Darcy number, $R > 0$ is the Rayleigh number, $R_1 > 0$ and $R_2 < 0$ are the two concentration Rayleigh numbers, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the respective concentrations of the two components, $E_1 > 0$ and $E_2 > 0$ are constants.

It may further be noted that equations (1) – (8) describe an eigenvalue problem for p and govern triply diffusive convection in a porous medium for any combination of dynamically free and rigid boundaries.

MATHEMATICAL ANALYSIS:

Now we derive upper bounds for the complex growth rate in triply diffusive convection in porous medium. We prove the following theorem.

Theorem. If $R > 0,\ R_1 > 0, R_2 < 0,\ \tau_1 > \tau_2,\ E_1 > 0,\ E_2 > 0, \sigma > 0, p_r \geq 0, p_i \neq 0$, then a necessary condition for the existence of non-trivial solution $(w,\ \theta,\ \phi_1,\ \phi_2,\ p)$ of equations (1) – (4) together with either of the boundary conditions (5) – (8) is that

$$|p|^2 < \frac{R_1}{E_1\sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{E_1^2\sigma^2}.$$

Proof: Multiplying equation (1) by w^* (the superscript $*$ hence forth denotes the complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\Lambda \int_0^1 w^*(D^2 - a^2)^2 w\, dz - (p + D_a^{-1}) \int_0^1 w^*(D^2 - a^2) w\, dz = Ra^2 \int_0^1 w^*\theta\, dz - R_1 a^2 \int_0^1 w^*\phi_1\, dz + |R_2| a^2 \int_0^1 w^*\phi_2\, dz. \tag{9}$$

Making use of equations (2) – (4), and the fact, that $w(0) = 0 = w(1)$, we can write

$$Ra^2 \int_0^1 w^*\theta\, dz = -Ra^2 \int_0^1 \theta\,(D^2 - a^2 - E\,\sigma\, p^*)\theta^* dz, \tag{10}$$

$$R_1 a^2 \int_0^1 w^*\phi_1 dz = -R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{E_1\sigma\, p^*}{\tau_1}\right)\phi_1^* dz \tag{11}$$

and

$$|R_2| a^2 \int_0^1 w^*\phi_2 dz = -|R_2| a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{E_2\sigma\, p^*}{\tau_2}\right)\phi_2^* dz. \tag{12}$$

Combining equations (9) – (12), we obtain

$$\Lambda \int_0^1 w^*(D^2 - a^2)^2 w\, dz - (p + D_a^{-1}) \int_0^1 w^*(D^2 - a^2) w\, dz = -Ra^2 \int_0^1 \theta\,(D^2 - a^2 - E\,\sigma\, p^*)\theta^* dz + R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{E_1\sigma\, p^*}{\tau_1}\right)\phi_1^* dz - |R_2| a^2 \tau_2 \int_0^1 \phi_2 \Big(D^2 - a^2 -$$

$\left.\frac{E_2\sigma\, p^*}{\tau_2}\right)\phi_2^* dz.$ (13)

Integrating various terms of equation (13), by parts, for an appropriate number of times and making use of the boundary conditions (5) – (8) and the equality

$\int_0^1 \psi^* D^{2n}\psi dz = (-1)^n \int_0^1 |D^2\psi|^2 dz,$ (14)

$\psi = w (n = 1,2)$ or $\psi = \theta,\ \phi_1, \phi_2\ (n = 1),$

we may rewrite equation (13) in the form

$$\Lambda \int_0^1 (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)\, dz + (p + D_a^{-1}) \int_0^1 (|Dw|^2 + a^2|w|^2)\, dz = Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + E\sigma p^*|\theta|^2)\, dz - R_1 a^2 \tau_1 \int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{E_1\sigma\, p^*}{\tau_1}|\phi_1|^2\right) dz + |R_2| a^2 \tau_2 \int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{E_2\sigma\, p^*}{\tau_2}|\phi_2|^2\right) dz. \quad (15)$$

Equating imaginary parts of both sides of equation (15), and cancelling $p_i (\neq 0)$ throughout, we have

$$\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz = -Ra^2 E\sigma \int_0^1 |\theta|^2 dz + R_1 a^2 E_1 \sigma \int_0^1 |\phi_1|^2 dz - |R_2| a^2 E_2 \sigma \int_0^1 |\phi_2|^2 dz. \quad (16)$$

Since $w,\ \theta,\ \phi_1, \phi_2$ satisfy $w(0) = 0 = w(1),\ \theta(0) = 0 = \theta(1),\ \phi_1(0) = 0 = \phi_1(1)$ and $\phi_2(0) = 0 = \phi_2(1)$, by Rayleigh-Ritz Inequality, we have

$\int_0^1 |Dw|^2\, dz \ \geq\ \pi^2 \int_0^1 |w|^2 dz,$ (17)

$\int_0^1 |D\theta|^2\, dz \ \geq\ \pi^2 \int_0^1 |\theta|^2 dz,$ (18)

$\int_0^1 |D\phi_1|^2\, dz \ \geq\ \pi^2 \int_0^1 |\phi_1|^2 dz$ (19)

and

$\int_0^1 |D\phi_2|^2\, dz \ \geq\ \pi^2 \int_0^1 |\phi_2|^2 dz.$ (20)

Using inequality (17), we have

$\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz \ \geq (\pi^2 + a^2) \int_0^1 |w|^2 dz.$ (21)

Combining equations (16) and (21), we have

$R_1 E_1 \sigma a^2 \int_0^1 |\phi_1|^2 dz \ \geq (\pi^2 + a^2) \int_0^1 |w|^2 dz.$ (22)

Now, multiplying equation (3) by its complex conjugate and integrating the resulting equation for a suitable number of times and using the boundary conditions on ϕ_1, namely, $\phi_1(0) = 0 = \phi_1(1)$, we obtain

$$\int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2)\ dz + \frac{2E_1\sigma p_r}{\tau_1} \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz + \frac{E_1^2\sigma^2|p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz = \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz. \quad (23)$$

Now, we have (Banerjee et al (1993b))

$\int_0^1 |D^2\phi_1|^2\, dz \ \ \geq\ \pi^4 \int_0^1 |\phi_1|^2\, dz.$ (24)

Using (19) and (24), we have

$\int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2)\ dz \ \geq (\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz.$ (25)

Substituting (25) in (23), we have

$$(\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz + \frac{2E_1\sigma p_r}{\tau_1} \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz + \frac{E_1^2\sigma^2|p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz \leq \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz. \quad (26)$$

Since $p_r \geq 0$, it follows from equation (26) that

$$(\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz + \frac{E_1^2 \sigma^2 |p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz \leq \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz \quad (27)$$

which gives

$$R_1 E_1 \sigma a^2 \int_0^1 |\phi_1|^2 dz \leq \frac{R_1}{E_1 \sigma} \frac{a^2}{|p|^2} \int_0^1 |w|^2 dz - \frac{(\pi^2+a^2)^2}{|p|^2} \frac{\tau_1^2}{E_1^2 \sigma^2} . R_1 E_1 \sigma a^2 \int_0^1 |\phi_1|^2 dz. \quad (28)$$

Using (22) in (28), we have

$$R_1 E_1 \sigma a^2 \int_0^1 |\phi_1|^2 dz \leq \frac{R_1}{E_1 \sigma} \frac{a^2}{|p|^2} \int_0^1 |w|^2 dz - \frac{(\pi^2+a^2)^3}{|p|^2} \frac{\tau_1^2}{E_1^2 \sigma^2} \int_0^1 |w|^2 dz. \quad (29)$$

Utilizing (29) in (16), we obtain

$$\int_0^1 |Dw|^2 dz + Ra^2 E\sigma \int_0^1 |\theta|^2 dz + |R_2| a^2 E_2 \sigma \int_0^1 |\phi_2|^2 dz + a^2 \left\{1 - \frac{R_1}{E_1 \sigma} \frac{1}{|p|^2} + \frac{(\pi^2+a^2)^3}{a^2} \frac{\tau_1^2}{|p|^2 E_1^2 \sigma^2}\right\} \int_0^1 |w|^2 dz \leq 0. \quad (30)$$

Since minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$ for $a^2 = \frac{\pi^2}{2}$, inequality (30) yields

$$\int_0^1 |Dw|^2 dz + Ra^2 E\sigma \int_0^1 |\theta|^2 dz + |R_2| a^2 E_2 \sigma \int_0^1 |\phi_2|^2 dz + a^2 \left\{1 - \frac{R_1}{E_1 \sigma} \frac{1}{|p|^2} + \frac{27\pi^4}{4} . \frac{\tau_1^2}{|p|^2 E_1^2 \sigma^2}\right\} \int_0^1 |w|^2 dz \leq 0. \quad (31)$$

Inequality (31) clearly implies that

$$|p|^2 < \frac{R_1}{E_1 \sigma} - \frac{27\pi^4}{4} \frac{\tau_1^2}{E_1^2 \sigma^2}. \quad (32)$$

This proves the theorem.

The above theorem states, from the physical point of view that the complex growth rate (p_r, p_i) of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triply diffusive fluid layer saturating a porous medium using Darcy-Brinkman model with one of the components as heat with diffusivity κ must lie inside a semicircle in the right half of the (p_r, p_i) – plane whose centre is origin and radius equals $\sqrt{\frac{R_1}{E_1 \sigma} - \frac{27\pi^4}{4} \frac{\tau_1^2}{\sigma^2 E_1^2}}$, where $R_1 (> 0)$ and $R_2 (< 0)$ are the Rayleigh numbers for the two concentration components with diffusivities κ_1 and κ_2 , σ is the Prandtl number, E_1 and E_2 are constants. This result is uniformly valid for any combinations of rigid or free boundaries.

A general plot showing the region of complex growth rate is given below:

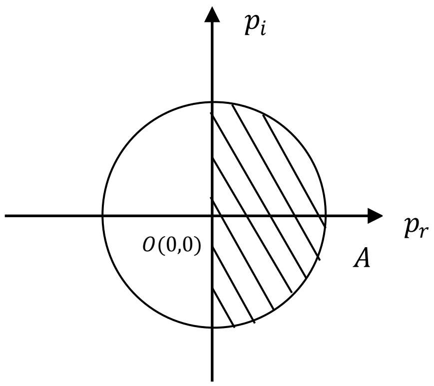

Fig.2. Shaded region shows that the region of complex growth rate

$$(OA)^2 < \frac{R_1}{E_1\sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{\sigma^2 E_1^2}.$$

CONCLUSIONS:

The linear stability theory is used to derive the upper bounds for the complex growth rate in triply diffusive convection of a fluid layer in porous medium heated from below. The Darcy-Brinkman model has been used which is more compatible for the flow through high porosity medium. These bounds are important especially when both the boundaries are not free so the exact solutions in the closed form are not obtainable. Further, since the result derived herein involve only the non-dimensional quantities and are independent of the wave number are, thus, of uniform applicability.

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THE MATHEMATICS OF CRYPTOGRAPHY AND NETWORK SECURITY

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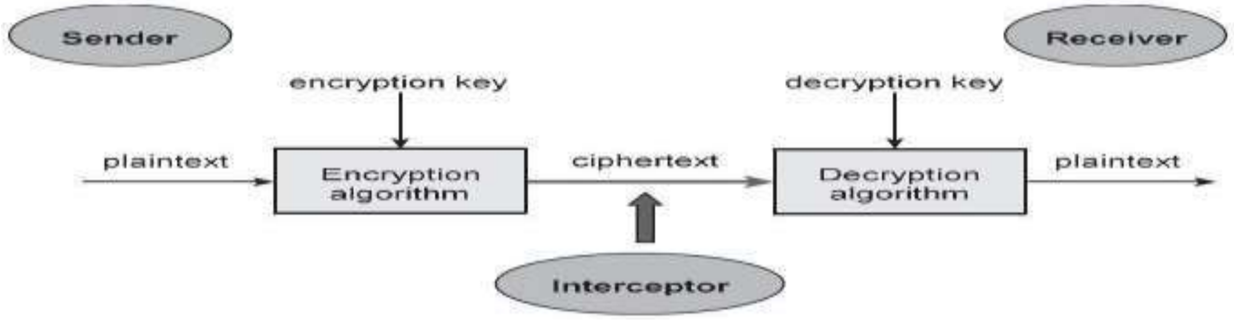
INTRODUCTION:

In the current orientation of the world, the technologies have advanced so much that most of the individuals prefer using the internet as the primary mean to relegate data from one end to another across the world. There are many possible ways to broadcast data using the internet: via e-mails, chats, etc. The data transition is made very cinch, fast and accurate using the internet. However, one of the main challenge with sending data over the internet is the "security threat" it poses i.e. the personal or privy data can be bagged or hacked in many ways. Therefore, it becomes very important to take data security into consideration, as it is one of the most necessary factors that need attention during the process of data transferring [1].

Plain or normal text, which is send over the network, is firstly get transformed into cipher text so that only the sender and the recipient can use the information.
Cipher Text, it is a data that has been encrypted. Cipher text is unreadable until it has been converted into Plain text with key.
Encryption, the process of encoding plain text messages into cipher text messages.
Decryption, transformation process of cipher text again into plain text.

Decryption is just opposite to encryption. In computer-to-computer communications, the computer at sender's end usually transforms a plain text messages into cipher text messages by performing encryption. Then this message is sent to the receiver over the network. The receiver's computer takes the encrypted message and performs the decryption process to obtain plain text.

The process of encryption and decryption is known as **Cryptography**. In general, cryptography is the art and science of achieving security by encoding message to make them non-readable. It can be used to hide the meaning of information in any form. It can also be applied to software, graphics or voice.



Cryptography Process

Cryptography is the art of secret coding. The basic service provided by cryptography is the ability to send the information between participants in a way that prevents others reading it.

The main purpose of the cryptography is used not only to provide confidentiality, but also to provide solutions for other problems like: data integrity, authentication, non-repudiation [2].

Network Security:

System and Network Technology is a key technology for a wide variety of applications. It is a critical requirement in current situation networks, there is a significant lack of security methods that can be easily implemented. There exists a "communication gap" between the developers of security technology and developers of networks. Network design is a developed process that is depends on the Open Systems Interface (OSI) model. The OSI model has several advantages when designing network security. It offers modularity, ease-of-use, flexibility, and standardization of protocols. The protocols of different layers can be easily combined to create stacks, which allow modular development. In contrast to secure network, design is not a well- developed process. There is not a methodology to manage the complexity of security requirements. When considering about network security, it should be emphasized that the complete network is secure. It does not only concern with the security in the computers at each end of the communication chain. When transferring from one node to another node data the communication channel should not be vulnerable to attack. A hacker will target the communication channel, get the data, and decrypt it and re‐insert a duplicate message. Though securing the network is just as important as securing the computers and encrypting the message. While developing a secure network, the following needs to be considered [3].

Network Security Services:

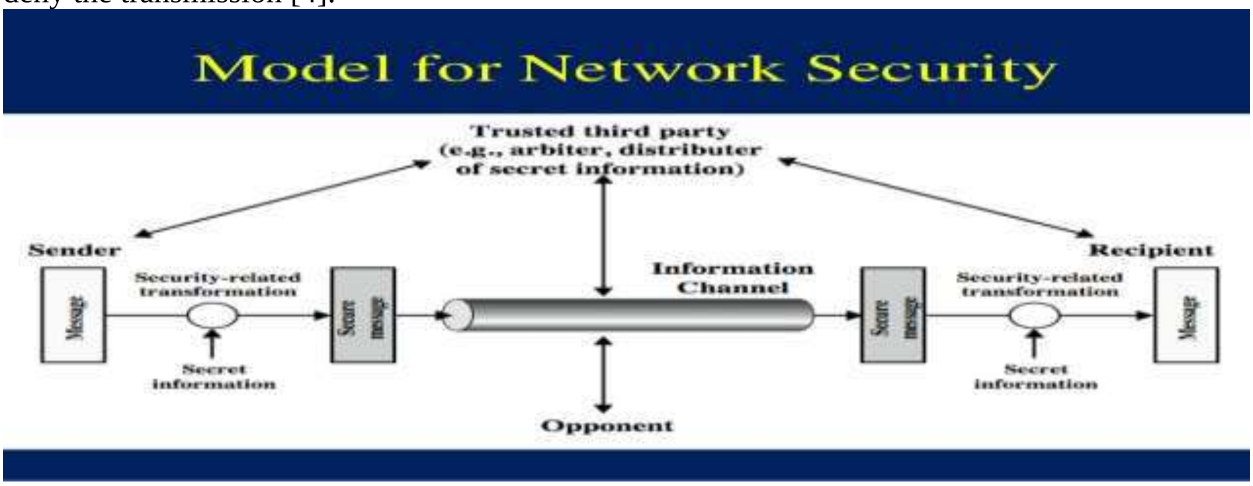
The classification of security services are as follows:

Confidentiality: Ensures that the information in a computer system and transmitted information are accessible only for reading by authorized parties. e.g. printing, displaying and other forms of disclosure.

Authentication: Ensures that the origin of a message or electronic document is correctly identified, with an assurance that the identity is not false.

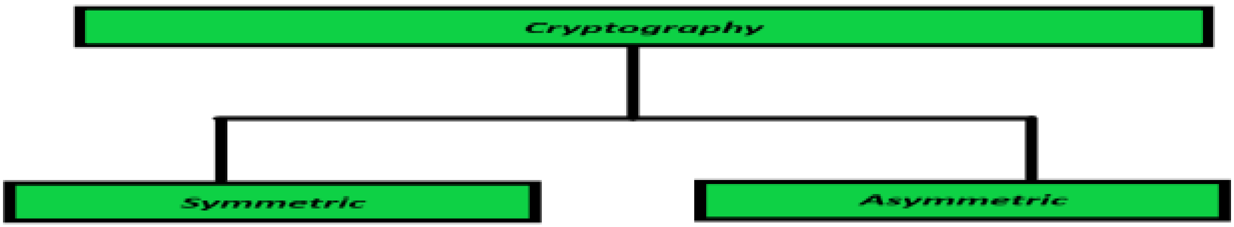
Data Integrity: Ensures that only authorized parties are able to modify computer system assets and transmitted information. Modification includes writing, changing status, deleting, creating and delaying or replaying of transmitted messages.

Non-repudiation: Requires that neither the sender nor the receiver of a message be able to deny the transmission [4].

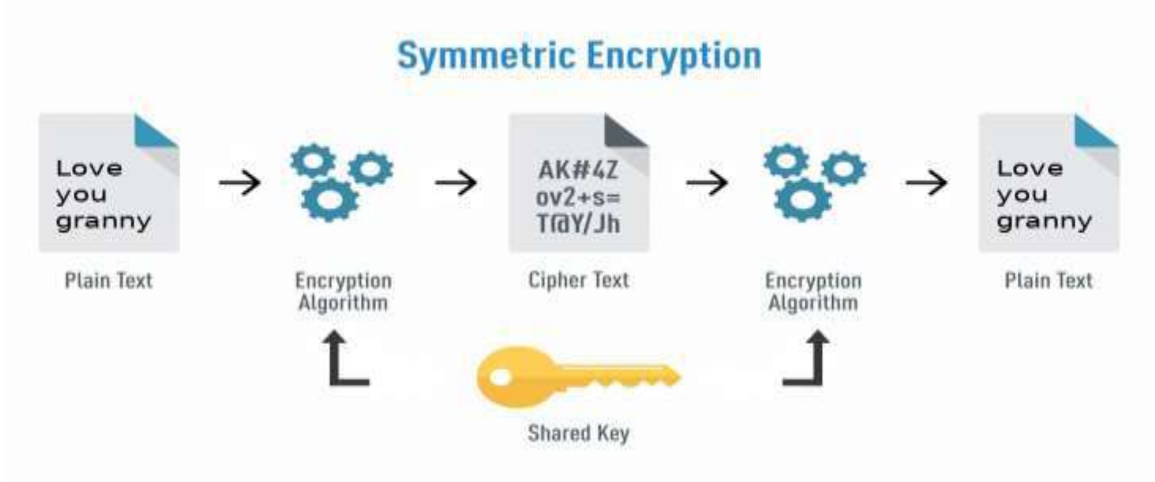


Cryptography is the methods that allow information to be sent in a secure from in such a way that the only receiver able to retrieve this information. Presently continuous researches on the new cryptographic algorithms are going on. However, it is a very difficult to find out the specific algorithm, because we have already known that they must consider many factors like security, the features of algorithm, the time complexity and space complexity [2].

Techniques/ Methods/ Types of Cryptography:
There are two main categories of cryptography depending on the type of security keys used to encrypt/decrypt the data. These two categories are asymmetric and Symmetric encryption techniques i.e. ***Symmetric and Asymmetric Encryptions***:



1) Symmetric Encryption:
It is also called as single key cryptography. It uses a single key. In this encryption process the receiver and the sender has to agree upon a single secret (shared) key. Given a message (called plaintext) and the key, encryption produces unintelligible data, which is about the same length as the plaintext was. Decryption is the reverse of encryption, and uses the same key as encryption [5].



ADVANTAGES AND DISADVANTAGES OF SYMMETRIC CRYPTOSYSTEM ADVANTAGES:

- A symmetric cryptosystem is faster.
- In Symmetric Cryptosystems, encrypted data can be transferred on the link even if there is a possibility that the data will be intercepted. Since there is no key transmitted with the data, the chances of data being decrypted are null.
- A symmetric cryptosystem uses password authentication to prove the receiver's identity.
- A system only, which possesses the secret key, can decrypt a message.

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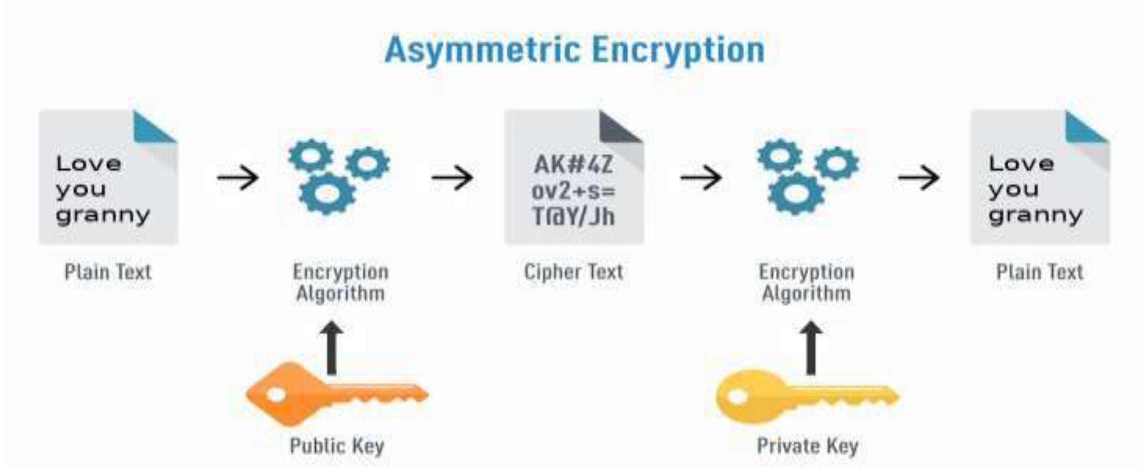
DISADVANTAGES:

- Symmetric cryptosystems have a problem of key transportation. The secret key is to be transmitted to the receiving system before the actual message is to be transmitted. Every means of electronic communication is insecure as it is impossible to guarantee that no one will be able to tap communication channels. Therefore, the only secure way of exchanging keys would be exchanging them personally.
- Cannot provide digital signatures that cannot be repudiated [6].

2) Asymmetric Encryption:

It is also called as public key cryptography. It uses two keys: public key, which is known to the public, used for encryption and private key, which is known only to the user of that key, used for decryption. The public and the private keys are related to each other by any mathematical means. In other words, data encrypted by one public key can be encrypted only by its corresponding private key. Encryption and decryption procedure [5].



ADVANTAGES AND DISADVANTAGES OF ASYMMETRIC CRYPTOSYSTEM
ADVANTAGES

- In asymmetric or public key, cryptography there is no need for exchanging keys, thus eliminating the key distribution problem.
- The primary advantage of public-key cryptography is increased security: the private keys do not ever need to be transmitted or revealed to anyone.
- Can provide digital signatures that can be repudiated.

DISADVANTAGES

A disadvantage of using public-key cryptography for encryption is speed: there are popular secret-key encryption methods which are significantly faster than any currently available public-key encryption method [6].

MAJOR TYPES OF ATTACKS

Many attacks are possible over any ongoing communication within a network.
Some major types of attacks are explained below: -
(a) Security Threats: - Security threats are attacks where the system of the user is hampered in some manner that leads to loss of confidential data. This includes activities like service denying, attacking with viruses, malwares, spywares and Trojan horses. In addition, activities

like intruding database or accessing Internet without permission.

(b) Data capturing and cryptanalysis: - This attack is performed while data is travelling in communication channels. The confidential data is captured or stolen from the channels and cryptanalysis is performed on it to extract the original data.

(c) Unauthorized Installing of Applications: - Installing unauthorized or uncertified applications within the system leads to virus intrusion and security breaching. To avoid it only certified applications must be allowed and unwanted applications such as audios, videos, games or other Internet applications must be avoided.

(d) Unauthorized Access: - Intrusion of any unauthorized person within the network resources or in data records leads to loss of confidential information. Hence proper authentication techniques for user's identity must be used and only resources must be monitored and checked from time-to-time.

(e) Virus Infection: - When network or resources are attacked with viruses, malware, Trojan horses or spywares leads to loss or manipulation of confidential data. It may sometimes destroy different resources and components of the network by effecting their source codes or hardware.

CYBER SECURITY TECHNIQUES:

To overcome or undo the attacks on networks different technologies are used these days. Some of the major techniques are given below:-

(a) **Authentication:** - All data and documents received must be authenticated if they are sent by trusted sender or not. They must also be checked for unwanted breaching or alterations within data.

(b) **Antivirus: -** Antivirus software must be installed and updated on regular time intervals. Also network and systems checks must be conducted regularly.

(c) **Firewalls:** - This software keeps track of inward and outward traffic of any system. It also inform user about unpermitted access and usage.

(d) **Access Control:** - Each user must have their particulars like username and passwords so that only intended users may log in.

(e) **Cryptography:** - It is the technique of encoding plain text into cipher text before transmitting it over channel for avoiding stealing of confidential data [7].

CONCLUSION AND FUTURE WORK:

With the explosive growth in the Internet, network and data security have become an inevitable concern for any organization whose internal private network is connected to the Internet. The security for the data has become highly important. User's data privacy is a central question over cloud. With more mathematical tools, cryptographic schemes are getting more versatile and often involve multiple keys for a single application.

In this concern, Cryptography plays a vital role in computer science area because the amount of work done is only kept secret. There are various techniques and algorithm studied and different types of research have been done. This paper further studied that symmetric key cryptography are faster than asymmetric systems. However, asymmetric key cryptography are more scalable and provide more authentication and non- repudiation easily.

In the future, work can be done on key distribution and management as well as optimal cryptography algorithm for data security over clouds.

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A TECHNIQUE TO SOLVE SIMULTANEOUS LINEAR EQUATIONS.

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ABSTRACT:
Linear equations play an important role in Science and Technology, Engineering and various social problems.Here in this paper, simple method has been developed to solve the problems occurring in the simultaneous linear equations having two or three unknowns. Some problems based on simultaneous linear equations have been solved also.

KEYWORDS :- Linear equations, simultaneous equations, co-efficients.

INTRODUCTION :-
There are so many methods to solve simultaneous linear equations having two or three unknowns like Graphical Method, Substitution Method, Elimination Method, Cross multiplication Methods, Crammer Rule, Matrix Method, Gauss Jacob's Method, Gauss Jordan Method, Gauss elimination method, Gauss Seidal Method etc. In this paper, we introduce a technique to solve simultaneous linear equations having two or three unknowns which is the modification of crammer's rule.

1. **Method to solve the simultaneous linear equations having two unknowns (Variables)**.
Consider two linear equations having two variables such that
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$

Method :
Write constants which is as follows :

a_1	b_1	b_1	c_1	c_1	a_1
b_2	a_2	c_2	b_2	a_2	c_2

(1) $\Delta = a_1b_2 - b_1a_2$
(2) $\Delta_1 = b_1c_2 - c_1b_2$
(3) $\Delta_2 = c_1a_2 - a_1c_2$

Now to find the value of $x \; and \; y$
$\therefore \; x = -\frac{\Delta_1}{\Delta}, \quad y = -\frac{\Delta_2}{\Delta}$

***Some problems based on two un-knowns:**
Example 1. Solve the following system of equations : $2x + 3y = 9\,,\; 3x - 2y = 7$.
Solution : Given system of equations are $2x + 3y = 9\,,\; 3x - 2y = 7$
Here $a_1 = 2 \;\; b_1 = 3 \;\; c_1 = 9$ and $a_2 = 3 \;\; b_2 = -2 \;\; c_2 = 7$

Method :	a_1	b_1	b_1	c_1	c_1	a_1
	b_2	a_2	c_2	b_2	a_2	c_2
i.e.	2	3	3	9	9	2
	-2	3	7	-2	3	7

$\Delta = a_1b_2 - b_1a_2 = 2\times -2 - 3 \times 3 = -4 - 9 = -13$

$\Delta_1 = b_1c_2 - c_1b_2 = 3\times 7 - 9 \times -2 = 21 + 18 = 39$
$\Delta_2 = c_1a_2 - a_1c_2 = 9\times 3 - 2 \times 7 = 27 - 14 = 13$
Now to find the value of $x\ and\ y$
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{39}{-13} = 3$
$y = -\frac{\Delta_2}{\Delta} = -\frac{13}{-13} = 1$
Hence the solution.

Example 2. Solve the following system of equations : $x - 2y = 4 , -3x + 5y = -7$.
Solution : Given system of equations are $x - 2y = 4 , -3x + 5y = -7$
Here $a_1 = 1 \quad b_1 = -2 \quad c_1 = 4$ and $a_2 = -3 \quad b_2 = 5 \quad c_2 = -7$

Method :	a_1	b_1	b_1	c_1	c_1	a_1
	b_2	a_2	c_2	b_2	a_2	c_2
i.e.	1	-2	-2	4	4	1
	5	-3	-7	5	-3	-7

$\Delta = a_1b_2 - b_1a_2 = 1\times 5 - (-2) \times (-3) = 5 - 6 = -1$
$\Delta_1 = b_1c_2 - c_1b_2 = -2\times -7 - 4 \times 5 = 14 - 20 = -6$
$\Delta_2 = c_1a_2 - a_1c_2 = 4\times -3 - 1 \times -7 = -12 + 7 = -5$
Now to find the value of $x\ and\ y$
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{-6}{-1} = -6$
$y = -\frac{\Delta_2}{\Delta} = -\frac{-5}{-1} = -5$
Hence the solution.

Example 3. Solve the following system of equations : $3x - 2y = 4, 4x - 3y = 5$.
Solution : Given system of equations are $3x - 2y = 4 ,\ 4x - 3y = 5$
Here $a_1 = 3 \quad b_1 = -2 \quad c_1 = 4$ and $a_2 = 4 \quad b_2 = -3 \quad c_2 = 5$

Method :	a_1	b_1	b_1	c_1	c_1	a_1
	b_2	a_2	c_2	b_2	a_2	c_2
i.e.	3	-2	-2	4	4	3
	-3	4	5	-3	4	5

$\Delta = a_1b_2 - b_1a_2 = 3\times -3 - (-2) \times 4 = -9 + 8 = -1$
$\Delta_1 = b_1c_2 - c_1b_2 = -2\times 5 - 4 \times -3 = -10 + 12 = 2$
$\Delta_2 = c_1a_2 - a_1c_2 = 4\times 4 - 3 \times 5 = 16 - 15 = 1$
Now to find the value of $x\ and\ y$
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{2}{-1} = 2$
$y = -\frac{\Delta_2}{\Delta} = -\frac{1}{-1} = 1$
Hence the solution.

2. **Method to solve the simultaneous linear equations having three unknowns (Variables)**.

Consider three linear equations having three variables such that

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

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Method :

(1) First of all Write constants i.e. a , b , c which is as follows :

a_1 a_1 b_1 b_1 c_1 c_1
b_2 b_3 c_2 c_3 a_2 a_3
c_3 c_2 a_3 a_2 b_3 b_2

(1) $\Delta = (a_1b_2c_3 - a_1b_3c_2) + (b_1c_2a_3 - b_1c_3a_2) + (c_1a_2b_3 - c_1a_3b_2)$.

(II) Write constants i.e. b , c , d which is as follows :

b_1 b_1 c_1 c_1 d_1 d_1
c_2 c_3 d_2 d_3 b_2 b_3
d_3 d_2 b_3 b_2 c_3 c_2

$\Delta_1 = (b_1c_2d_3 - b_1c_3d_2) + (c_1d_2b_3 - c_1d_3b_2) + (d_1b_2c_3 - d_1b_3c_2)$.

(III) Write constants i.e. c , d , a which is as follows :

c_1 c_1 d_1 d_1 a_1 a_1
d_2 d_3 a_2 a_3 c_2 c_3
a_3 a_2 c_3 c_2 d_3 d_2

$\Delta_2 = (c_1d_2a_3 - c_1d_3a_2) + (d_1a_2c_3 - d_1a_3c_2) + (a_1c_2d_3 - a_1c_3d_2)$.

(IV) Write constants i.e. d , a , b which is as follows :

d_1 d_1 a_1 a_1 b_1 b_1
a_2 a_3 b_2 b_3 d_2 d_3
b_3 b_2 d_3 d_2 a_3 a_2

$\Delta_3 = (d_1a_2b_3 - d_1a_3b_2) + (a_1b_2d_3 - a_1b_3d_2) + (b_1d_2a_3 - b_1d_3a_2)$.
Now to find the value of x , y and z
$\therefore x = \frac{\Delta_1}{\Delta}$, $y = -\frac{\Delta_2}{\Delta}$, $z = \frac{\Delta_3}{\Delta}$.

*** Some problems based on three un-knowns:**

Example 1. Solve the following system of equations : $3x + 2y - z = 4, -x - y + 3z = 6, 5x - 3y + z = 2$.
Solution :- Given system of equations are $3x + 2y - z = 4, -x - y + 3z = 6, 5x - 3y + z = 2$
Here $a_1 = 3, b_1 = 2, c_1 = -1, d_1 = 4$, $a_2 = -1, b_2 = -1, c_2 = 3, d_2 = 6$, $a_3 = 5, b_3 = -3, c_3 = 1, d_3 = 2$

Method

(I)

a_1 a_1 b_1 b_1 c_1 c_1
b_2 b_3 c_2 c_3 a_2 a_3
c_3 c_2 a_3 a_2 b_3 b_2

i.e.

3 3 2 2 -1 -1

$-1 \quad -3 \quad 3 \quad 1 \quad -1 \quad 5$
$1 \quad 3 \quad 5 \quad -1 \quad -3 \quad -1$

$\Delta = (3 \times -1 \times 1 - 3 \times -3 \times 3) + (2 \times 3 \times 5 - 2 \times 1 \times -1) + \{-1 \times -1 \times -3 - (-1) \times 5 \times -1\}$
$= (-3 + 27) + (30 + 2) + (-3 - 5)$
= 24+32-8
= 56-8 = 48
Δ=48

(II) $b_1 \quad b_1 \quad c_1 \quad c_1 \quad d_1 \quad d_1$
$c_2 \quad c_3 \quad d_2 \quad d_3 \quad b_2 \quad b_3$
$d_3 \quad d_2 \quad b_3 \quad b_2 \quad c_3 \quad c_2$
i.e.
$2 \quad 2 \quad -1 \quad -1 \quad 4 \quad 4$
$3 \quad 1 \quad 6 \quad 2 \quad -1 \quad -3$
$2 \quad 6 \quad -3 \quad -1 \quad 1 \quad 3$

$\Delta_1 = (2 \times 3 \times 2 - 2 \times 1 \times 6) + \{-1 \times 6 \times -3 - (-1) \times 2 \times -1\} + (4 \times -1 \times 1 - 4 \times -3 \times 3)$
$= (12 - 12) + (18 - 2) + (-4 + 36)$
$= 0 + 16 + 32 = 48$
$\Delta_1 = 48$

(III) $c_1 \quad c_1 \quad d_1 \quad d_1 \quad a_1 \quad a_1$
$d_2 \quad d_3 \quad a_2 \quad a_3 \quad c_2 \quad c_3$
$a_3 \quad a_2 \quad c_3 \quad c_2 \quad d_3 \quad d_2$

i.e.

$-1 \quad -1 \quad 4 \quad 4 \quad 3 \quad 3$
$6 \quad 2 \quad -1 \quad 5 \quad 3 \quad 1$
$5 \quad -1 \quad 1 \quad 3 \quad 2 \quad 6$

$\Delta_2 = \{-1 \times 6 \times 5 - (-1) \times 2 \times -1\} + (4 \times -1 \times 1 - 4 \times 5 \times 3) + (3 \times 3 \times 2 - 3 \times 1 \times 6)$
$= (-30 - 2) + (-4 - 60) + (18 - 18)$
= -32-64+0 = -96+0 = -96
$\Delta_2 = -96$

(IV) $d_1 \quad d_1 \quad a_1 \quad a_1 \quad b_1 \quad b_1$
$a_2 \quad a_3 \quad b_2 \quad b_3 \quad d_2 \quad d_3$
$b_3 \quad b_2 \quad d_3 \quad d_2 \quad a_3 \quad a_2$
i.e.
$4 \quad 4 \quad 3 \quad 3 \quad 2 \quad 2$
$-1 \quad 5 \quad -1 \quad -3 \quad 6 \quad 2$
$-3 \quad -1 \quad 2 \quad 6 \quad 5 \quad -1$

$\Delta_3 = (4 \times -1 \times -3 - 4 \times 5 \times -1) + (3 \times -1 \times 2 - 3 \times -3 \times 6) + (2 \times 6 \times 5 - 2 \times 2 \times -1)$
$= (12 + 20) + (-6 + 54) + (60 + 4)$
$= 32 + 48 + 64 = 144$
$\Delta_3 = 144$

Now to find the value of x , y and z
$\therefore x = \frac{\Delta_1}{\Delta} = \frac{48}{48} = 1$
$y = -\frac{\Delta_2}{\Delta} = -\frac{-96}{48} = 2$
$z = \frac{\Delta_3}{\Delta} = \frac{144}{48} = 3$
$\therefore x = 1, y = 2, z = 3$
Hence the solution.

Example 2. Solve the following system of equations : $6x - 9y - 20z = -4, 4x - 15y + 10z = -1, 2x - 3y - 5z = -1$.
Solution :- Given system of equations are $6x - 9y - 20z = -4, 4x - 15y + 10z = -1, 2x - 3y - 5z = -1$
Here $a_1 = 6, b_1 = -9, c_1 = -20, d_1 = -4$, $a_2 = 4, b_2 = -15, c_2 = 10, d_2 = -1$, $a_3 = 2, b_3 = -3, c_3 = -5, d_3 = -1$
Method
(I)

a_1	a_1	b_1	b_1	c_1	c_1
b_2	b_3	c_2	c_3	a_2	a_3
c_3	c_2	a_3	a_2	b_3	b_2

i.e.

6	6	-9	-9	-20	-20
-15	-3	10	-5	4	2
-5	10	2	4	-3	-15

$\Delta = (6 \times -15 \times -5 - 6 \times -3 \times 10) + (-9 \times 10 \times 2 - (-9) \times -5 \times 4) + \{-20 \times 4 \times -3 - (-20) \times 2 \times -15\}$
$= (450 + 180) + (-180 - 180) + (240 - 600)$
$= 630 - 360 - 360$
$= 630 - 720$
$\Delta = -90$

(II)

b_1	b_1	c_1	c_1	d_1	d_1
c_2	c_3	d_2	d_3	b_2	b_3
d_3	d_2	b_3	b_2	c_3	c_2

i.e.

-9	-9	-20	-20	-4	-4
10	-5	-1	-1	-15	-3
-1	-1	-3	-15	-5	10

$\Delta_1 = (-9 \times 10 \times -1 - (-9) \times -5 \times -1) + \{-20 \times -1 \times -3 - (-20) \times -1 \times -15\} + (-4 \times \quad -15 \times -5 - (-4) \times -3 \times 10)$

$= (90 + 45) + (-60 + 300) + (-300 - 120)$

$= 135 + 240 - 420 = 375 - 420$

$\Delta_1 = -45$

(III)
$$\begin{array}{cccccc} c_1 & c_1 & d_1 & d_1 & a_1 & a_1 \\ d_2 & d_3 & a_2 & a_3 & c_2 & c_3 \\ a_3 & a_2 & c_3 & c_2 & d_3 & d_2 \end{array}$$

i.e.
$$\begin{array}{cccccc} -20 & -20 & -4 & -4 & 6 & 6 \\ -1 & -1 & 4 & 2 & 10 & -5 \\ 2 & 4 & -5 & 10 & -1 & -1 \end{array}$$

$\Delta_2 = \{-20 \times -1 \times 2 - (-20) \times -1 \times 4\} + (-4 \times 4 \times -5 - (-4) \times 2 \times 10) + (6 \times 10 \times -1 - 6 \times \quad -5 \times -1)$

$= (40 - 80) + (80 + 80) + (-60 - 30)$

$= -40 + 160 - 90 = -130 + 160 = 30$

$\Delta_2 = 30$

(IV)
$$\begin{array}{cccccc} d_1 & d_1 & a_1 & a_1 & b_1 & b_1 \\ a_2 & a_3 & b_2 & b_3 & d_2 & d_3 \\ b_3 & b_2 & d_3 & d_2 & a_3 & a_2 \end{array}$$

i.e.
$$\begin{array}{cccccc} -4 & -4 & 6 & 6 & -9 & -9 \\ 4 & 2 & -15 & -3 & -1 & -1 \\ -3 & -15 & -1 & -1 & 2 & 4 \end{array}$$

$\Delta_3 = (-4 \times 4 \times -3 - (-4) \times 2 \times -15) + (6 \times -15 \times -1 - 6 \times -3 \times -1) + (-9 \times -1 \times 2 - \quad (-9) \times -1 \times 4)$

$= (48 - 120) + (90 - 18) + (18 - 36)$

$= -72 + 72 - 18 = -18$

$\Delta_3 = -18$

Now to find the value of x, y and z

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-45}{-90} = \frac{1}{2}$

$y = -\frac{\Delta_2}{\Delta} = -\frac{30}{-90} = \frac{1}{3}$

$z = \frac{\Delta_3}{\Delta} = \frac{-18}{-90} = \frac{1}{5}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Hence the solution.

Example 3. Solve the following system of equations : $3x - 4y + 5z = 25, 2x + 3y + z = 4, x - y + 2z = 9$.

Solution :- Given system of equations are $3x - 4y + 5z = 25, 2x + 3y + z = 4, x - y + 2z = 9$

Here $a_1 = 3, b_1 = -4, c_1 = 5, d_1 = 25$, $a_2 = 2, b_2 = 3, c_2 = 1, d_2 = 4$, $a_3 = 1, b_3 =$

$-1, c_3 = 2, d_3 = 9$

Method

(I)

a_1	a_1	b_1	b_1	c_1	c_1
b_2	b_3	c_2	c_3	a_2	a_3
c_3	c_2	a_3	a_2	b_3	b_2

i.e.

3	3	-4	-4	5	5
3	-1	1	2	2	1
2	1	1	2	-1	3

$\Delta = (3 \times 3 \times 2 - 3 \times -1 \times 1) + (-4 \times 1 \times 1 - (-4) \times 2 \times 2) + (5 \times 2 \times -1 - 5 \times 1 \times 3 \})$

$= (18 + 3) + (-4 + 16) + (-10 - 15)$

$= 21 + 12 - 25$

$= 33 - 25$

$\Delta = 8$

(II)

b_1	b_1	c_1	c_1	d_1	d_1
c_2	c_3	d_2	d_3	b_2	b_3
d_3	d_2	b_3	b_2	c_3	c_2

i.e.

-4	-4	5	5	25	25
1	2	4	9	3	-1
9	4	-1	3	2	1

$\Delta_1 = (-4 \times 1 \times 9 - (-4) \times 2 \times 4) + (5 \times 4 \times -1 - 5 \times 9 \times 3) + (25 \times 3 \times 2 - 25 \times -1 \times 1)$

$= (-36 + 32) + (-20 - 135) + (150 + 25)$

$= -4 - 155 + 175 = -159 + 175$

$\Delta_1 = 16$

(III)

c_1	c_1	d_1	d_1	a_1	a_1
d_2	d_3	a_2	a_3	c_2	c_3
a_3	a_2	c_3	c_2	d_3	d_2

i.e.

5	5	25	25	3	3
4	9	2	1	1	2
1	2	2	1	9	4

$\Delta_2 = (5 \times 4 \times 1 - 5 \times 9 \times 2) + (25 \times 2 \times 2 - 25 \times 1 \times 1) + (3 \times 1 \times 9 - 3 \times 2 \times 4)$

$= (20 - 90) + (100 - 25) + (27 - 24)$

$= -70 + 75 + 3 = -70 + 78 = 8$

$\Delta_2 = 8$

(IV)

d_1	d_1	a_1	a_1	b_1	b_1
a_2	a_3	b_2	b_3	d_2	d_3

$$\begin{array}{cccccc} b_3 & b_2 & d_3 & d_2 & a_3 & a_2 \end{array}$$

i.e.

$$\begin{array}{cccccc} 25 & 25 & 3 & 3 & -4 & -4 \\ 2 & 1 & 3 & -1 & 4 & 9 \\ -1 & 3 & 9 & 4 & 1 & 2 \end{array}$$

$\Delta_3 = (25 \times 2 \times -1 - 25 \times 1 \times 3) + (3 \times 3 \times 9 - 3 \times -1 \times 4) + (-4 \times 4 \times 1 - (-4) \times 9 \times 2)$

$= (-50 - 75) + (81 + 12) + (-16 + 72)$

$= -125 + 93 + 56 = -125 + 149$

$\Delta_3 = 24$

Now to find the value of $x\ , y\ and\ z$

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{16}{8} = 2$

$y = -\frac{\Delta_2}{\Delta} = -\frac{8}{8} = -1$

$z = \frac{\Delta_3}{\Delta} = \frac{24}{8} = 3$

$\therefore x = 2\ , y = -1\ , z = 3$

Hence the solution.

CONCLUSION:

This method is another technique which is helpful to find the solution of the simultaneous linear equations of two or three un-knowns or variables as compared to other methods. This is also the simplest approach to find the solution of the simultaneous linear equations of two or three un-knowns or variables.

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A CHARACTERIZATION THERMOSOLUTAL CONVECTION IN COUPLE-STRESS FLUID IN A POROUS MEDIUM IN THE PRESENCE OF A ROTATION AND MAGNETIC FIELD

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ABSTRACT:
The thermosolutal instability of a couple-stress fluid heated from below in a porous medium acted upon by uniform vertical magnetic field and rotation is investigated. Following the linearized stability theory and normal mode analysis, the paper mathematically established the conditions for characterizing the oscillatory motions which may be neutral or unstable for rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, and a constant vertical adverse temperature gradient, are necessarily non-oscillatory in the regime

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1 ,$$

where the R_s is the thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, ε is the porosity, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter. The result is important since it holds for all wave numbers and the exact solutions of the problem investigated are not obtainable in closed form when both the boundaries are perfectly conducting and rigid. A similar characterization theorem is also proved for Stern type of configuration.

KEY WORDS: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Rayleigh number; Chandrasekhar Number; Taylor number.
MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05.

1. INTRODUCTION:
A detailed account of the theoretical and experimental study of the onset of thermal instability in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1] and the Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion, except in the external force term. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [2]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can name the food processing industry, the chemical processing industry, solidification, and the centrifugal casting of metals. The development of geothermal

power resources has increased general interest in the properties of convection in a porous medium. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [3]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat Tabor and Matz [4]and some Antarctic lakes Shirtcliffe [5]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust Lister [6]. Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice - versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context Mc Donnel [7]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov [8] and Linden [9] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in their

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formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [10] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [11] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge.

The theory of couple-stress fluid has been formulated by Stokes [12]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze - film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded – bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [12], couple-stresses appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [13] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [14] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [15] have considered a couple - stress fluid with suspended particles heated from below. In another study, Sunil et al. [16] have considered a couple- stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [17] have considered the thermal instability of a layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Pellow and Southwell [18] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [19] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [20] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [21]. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [22] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in porous medium in the presence of uniform rotation and uniform magnetic field has been considered in the present paper. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, and a constant vertical adverse temperature gradient, are necessarily non-oscillatory in the regime

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1\ ,$$

where the R_s is the thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, ε is the porosity, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter. The result is important since it holds for all wave numbers and the exact solutions of the problem investigated are not obtainable in closed form when both the boundaries are perfectly conducting and rigid. A similar characterization theorem is also proved for Stern [23] type of configuration.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta^{/}\ \left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

Let p, ρ, T, C, α, $\alpha^{/}$, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid (Chandrasekhar [1]; Joseph [2]; Stokes [12]) are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T\ , \quad (3)$$

$$E^{/}\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)C=\kappa^{/}\nabla^{2}C, \tag{4}$$

$$\rho = \rho_0 [1 - \alpha (T-T_0) + \alpha^{/} (C-C_0)], \tag{5}$$

Where the suffix zero refers to values at the reference level z = 0 and in writing equation (1), use has been made of Boussinesq approximation. Here E = $\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^{/}$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o, C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The magnetic permeability μ_e, the kinematic viscosityν, couple-stress viscosity $\mu^{/}$, the thermal diffusivity κ and the solute diffusivity $\kappa^{/}$ are all assumed to be constants.

The Maxwell's equations yield

$$\in\frac{d\vec{H}}{dt}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\in\eta\nabla^{2}\vec{H}, \tag{6}$$

and $\nabla\cdot\vec{H}=0,$ (7)

where $\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\in^{-1}\vec{q}\cdot\nabla$ stands for the convective derivative.

The steady state solution is

$\vec{q}(u,v,w)=(0,0,0)$, $T = T_0-\beta z$, $C = C_0 - \beta^{/}z$,

$$\rho = \rho_0 (1+ \alpha\beta z - \alpha^{/}\beta^{/}z). \tag{8}$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, $\delta\rho$, θ, γ, $\vec{h}(h_x,h_y,h_z)$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,0)$ and velocity $\vec{q}(0,0,0)$. The change in density$\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = - \rho_0 (\alpha\theta - \alpha^{/} \gamma). \tag{9}$$

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\vec{g}(\alpha\theta-\alpha^{/}\gamma)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^{2}\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{10}$$

$$\nabla.\vec{q}=0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^{2}\theta, \tag{12}$$

$$E^{/}\frac{\partial\gamma}{\partial t}=\beta^{/}w+\kappa^{/}\nabla^{2}\gamma, \tag{13}$$

$$\in\frac{\partial\vec{h}}{\partial t}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\in\eta\nabla^{2}\vec{h}, \tag{14}$$

and $\nabla.\vec{h}=0.$ (15)

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3. NORMAL MODES ANALYSIS:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$[w,\theta,h_z,\gamma,\varsigma,\xi] = [W(z),\Theta(z),K(z),\Gamma(z),Z(z),X(z)]$ exp(ik$_x$x + ik$_y$y + nt), (16)

where k_x, k_y are the wave numbers along the x- and y- directions respectively, k= ($\sqrt{k_x^2 + k_y^2}$) is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively. $W(z), K(z), \Theta(z), Z(z)$ and $X(z)$ are the functions of z only.

Using (16), equations (10)-(15), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]W = Ra^2\Theta - R_s a^2\Gamma + T_A DZ - Q\left(D^2 - a^2\right)DK, \tag{17}$$

$$\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]Z = -DW - QDX, \tag{18}$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W, \tag{19}$$

$$\left(D^2 - a^2 - E' p_3\sigma\right)\Gamma = -W, \tag{20}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW, \tag{21}$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ, \tag{22}$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number;, $p_3 = \frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu'/(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $R_s = \frac{g\alpha'\beta' d^4}{\kappa'\nu}$ is the thermosolutal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus, \Gamma = \frac{\beta' d^2}{\kappa'}\Gamma_\oplus$, $Z = \frac{2\Omega d}{\nu\varepsilon}Z_\oplus$, $K = \frac{Hd}{\varepsilon\eta}K_\oplus$, $X = \left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, are perfectly conducting and

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maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (17)--(22), must possess a solution are

W = 0 =DW =Z=DX =K= $\Theta = \Gamma$, on both the horizontal boundaries, (23)

Equations (17)-(22), along with boundary conditions (23), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, K and Z satisfy $W(0)=0=W(1)$, $K(0)=0=K(1)$ and $Z(0)=0=Z(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz,\ \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz\ ,\ \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz,$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \text{ and } \int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz, \tag{24}$$

Further, for $W(0)=0=W(1)$, $K(0)=0=K(1)$ and $Z(0)=0=Z(1)$, Banerjee et al. [25] have shown that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz,\ \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz. \tag{25}$$

4. MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^4} \int_0^1 |DW|^2 dz \quad .$$

Proof: Multiplying equation (19) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Θ namely $\Theta(0)=0=\Theta(1)$, it follows that

$$\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2\right\}dz + E\sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of}\left\{\int_0^1 \Theta^* W dz\right\},$$

$$\leq \left|\int_0^1 \Theta^* W dz\right| \leq \int_0^1 |\Theta^* W| dz \leq \int_0^1 |\Theta^*||W| dz,$$

$$\leq \int_0^1 |\Theta||W| dz \leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{26}$$

(Utilizing Cauchy-Schwartz-inequality),

So that the fact that $\sigma_r \geq 0$, we obtain from the above that

$$a^2 \int_0^1 |\Theta|^2 dz \leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{a^2}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{27}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (26) on utilizing (27) and (24), gives

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz, \tag{28}$$

This completes the proof of lemma 1.

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz \ .$$

Proof: Multiplying equation (20) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{|D\Gamma|^2 + a^2|\Gamma|^2\right\}dz + E^{/}\sigma_r p_3 \int_0^1 |\Gamma|^2 dz = \text{Real part of}\left\{\int_0^1 \Gamma^* W dz\right\},$$

$$\leq \left|\int_0^1 \Gamma^* W dz\right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*||W| dz,$$

$$\leq \int_0^1 |\Gamma||W| dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{29}$$

(Utilizing Cauchy-Schwartz-inequality),

So that, since $\sigma_r \geq 0$, we obtain from the above that

And thus, we get

$$a^2\int_0^1 |\Gamma|^2 dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{a^2}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{30}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (29) on utilizing (30) and (24), gives

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz, \tag{31}$$

This completes the proof of lemma 2.

Lemma 3: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\}dz \leq \frac{\pi^2}{(2\pi^2-1)}\int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (21) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0) = 0 = K(1)$, it follows that

$$\int_0^1 \left\{ |DK|^2 + a^2 |K|^2 \right\} dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of} \left\{ \int_0^1 K^* DW dz \right\} \leq \left| \int_0^1 K^* DW dz \right|$$

$$\leq \int_0^1 \left| K^* DW \right| dz ,$$

$$\leq \int_0^1 |K^*| |DW| dz \leq \frac{1}{2} \int_0^1 \left(|K|^2 + |DW|^2 \right) dz , \tag{32}$$

This gives that

$$\int_0^1 |DK|^2 dz \leq \frac{1}{2} \int_0^1 \left(|K|^2 + |DW|^2 \right) dz , \tag{33}$$

Inequality (33) on utilizing (24), gives

$$\int_0^1 |K|^2 dz \leq \frac{1}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz , \tag{34}$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (32) on utilizing (34), give

$$\int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz \leq \frac{\pi^2}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz , \tag{35}$$

This completes the proof of lemma 3.

Lemma 4: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \leq \frac{P_l}{(2\pi^2 F - P_l)} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (18) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (22) and appropriate boundary conditions (23), it follows that

$$\frac{F}{P_l} \int_0^1 \left\{ |DZ|^2 + a^2 |Z|^2 \right\} dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 |Z|^2 dz + Q \int_0^1 \left\{ |DX|^2 + a^2 |X|^2 \right\} dz + Q p_2 \sigma_r \int_0^1 |X|^2 dz$$

$$= \text{Real part of } \left\{ \int_0^1 DW^* Z dz \right\} \leq \left| \int_0^1 DW^* Z dz \right| ,$$

$$\leq \int_0^1 |DW^* Z| dz \leq \int_0^1 |DW^*| |Z| dz ,$$

$$= \int_0^1 |DW| |Z| dz \leq \frac{1}{2} \int_0^1 \left(|Z|^2 + |DW|^2 \right) dz , \tag{36}$$

This gives that

$$\frac{F}{P_l}\int_0^1 |DZ|^2 dz \le \frac{1}{2}\int_0^1 \left(|Z|^2 + |DW|^2\right) dz, \tag{37}$$

inequality (36) on utilizing (37), gives

$$\int_0^1 |Z|^2 dz \le \frac{P_l}{(2\pi^2 F - P_l)}\int_0^1 |DW|^2 dz, \tag{38}$$

This completes the proof of lemma 4**.**
Now we prove the following theorems:

Theorem 1: If $R\rangle\, 0$, $R_s \rangle 0$ $F\rangle\, 0$, $Q\rangle 0$, $T_A\rangle\, 0$, $P_l\rangle 0$, $p_1\rangle 0$, $p_2\rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, K, Z, X)$ of equations (17) – (22), together with boundary conditions (23) is that

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Q p_2 \pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1 \ .$$

Proof: Multiplying equation (17) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*\left(D^2 - a^2\right) W dz$$

$$= Ra^2\int_0^1 W^*\Theta dz - R_s a^2 \int_0^1 W^*\Gamma dz + T_A\int_0^1 W^* DZ dz - Q\int W^* D(D^2 - a^2)K dz\ , \tag{39}$$

Taking complex conjugate on both sides of equation (19), we get

$$\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* = -W^*, \tag{40}$$

Therefore, using (40), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz, \tag{41}$$

Taking complex conjugate on both sides of equation (20), we get

$$\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* = -W^*, \tag{42}$$

Therefore, using (42), we get

$$\int_0^1 W^*\Gamma dz = -\int_0^1 \Gamma\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* dz, \tag{43}$$

Also taking complex conjugate on both sides of equation (18), we get

$$\frac{F}{P_l}\left(D^2 - a^2\right)Z^* - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)Z^* + QDX^* = -DW^*, \tag{44}$$

Therefore, using (44), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2 - a^2\right)Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 Z^* Z dz + Q\int_0^1 ZDX^* dz\,,$$

Integrating by parts the third term on left hand side and using equation (22), and appropriate boundary condition (23), we get

$$\int_0^1 W^* DZdz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 Z^* Zdz + Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz\,, \qquad (45)$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^* = -DW^*\,, \qquad (46)$$

Therefore, equation (46), using appropriate boundary condition (23), we get

$$\int_0^1 W^* D\left(D^2-a^2\right)Kdz = -\int_0^1 DW^*\left(D^2-a^2\right)Kdz = \int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz\,, \qquad (47)$$

Substituting (41), (43), (45) and (47), in the right hand side of equation (39), we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2 Wdz - \left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 W^*(D^2-a^2)Wdz = -Ra^2\int_0^1 \Theta\left(D^2-a^2-Ep_1\sigma^*\right)\Theta^* dz$$

$$+R_s a^2\int_0^1 \Gamma(D^2-a^2-E^/ p_3\sigma^*)\Gamma^* dz + \frac{T_A F}{P_l}\int_0^1 Z(D^2-a^2)Z^* dz - T_A\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 ZZ^* dz$$

$$+T_A Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz - Q\int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz\,, \qquad (48)$$

Integrating the terms on both sides of equation (48) for an appropriate number of times and making use of the appropriate boundary conditions (23), we get

$$\frac{F}{P_l}\int_0^1\left\{\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right\}dz + \left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2+Ep_1\sigma^*|\Theta|^2\right)dz - R_s a^2\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2+E^/ p_3\sigma^*|\Gamma|^2\right)dz$$

$$-\frac{T_A F}{P_l}\int_0^1\left\{|DZ|^2+a^2|Z|^2\right\}dz - T_A\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1|Z|^2 dz - T_A Q\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$$

$$-T_A Qp_2\sigma\int_0^1|X|^2 dz - Q\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1\left(|DK|^2+a^2|K|^2\right)dz\,, \qquad (49)$$

now equating imaginary parts on both sides of equation (49), and cancelling $\sigma_i\,(\neq 0)$ throughout from imaginary part, we get

$$\frac{1}{\varepsilon}\int_0^1\left\{|DW|^2+a^2|W|^2\right\}dz = -Ra^2Ep_1\int_0^1|\Theta|^2 dz + R_s a^2 E^/ p_3\int_0^1|\Gamma|^2 dz$$

$$+\frac{T_A}{\varepsilon}\int_0^1|Z|^2 dz + Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz - T_A Qp_2\int_0^1|X|^2 dz\,, \qquad (50)$$

Now R $\rangle$ 0, $Q\rangle 0$ $\varepsilon\rangle 0$ $p_2\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (31), (35) and (38), the equation (50) gives,

$$\frac{1}{\varepsilon}\left[1-\left\{\frac{P_l T_A}{(2\pi^2 F-P_l)}+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\varepsilon\left(\frac{R_s E^/ p_3}{\pi^4}\right)\right\}\right]\int_0^1|DW|^2 dz + I_1\langle 0\,, \qquad (51)$$

Where $I_1 = \frac{a^2}{\varepsilon}\int_0^1|W|^2 dz + Ra^2Ep_1\int_0^1|\Theta|^2 dz + T_A Qp_2\int_0^1|X|^2 dz$, is positive definite.

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and therefore , we must have

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\rangle 1 . \tag{52}$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0 \text{, then } \varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\rangle 1 . \tag{53}$$

And this completes the proof of the theorem. Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 1, can be put in the form as follow:-

Corollary 1: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and rotation in a porous medium heated from below is that, $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, where R_s is the Thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium heated from below, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, for rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 2: If $(W,\Theta,\Gamma,K,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (17) – (22), with $R\rangle 0$ and,

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1 ,$$

Then $\sigma_i=0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \le 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Veronis type, we can state the above theorem as follow:-

Corollary 3: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium heated from below is that the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1$, for rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in couple-stress fluid heated from below, i. e. when $T_A = 0 = R_s =$ Q. (Sunil et al $[16]$).

(ii). Magneto-rotatory-thermal convection in couple-stress fluid heated from below (R_s =0), if

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \le 1 ,$$

(iii) Magneto-thermosolutal convection of Veronis (1965) type in couple-stress fluid heated from below (T_A = 0), if

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right) \le 1 .$$

(iv) Rotatory-thermosolutal convection of Veronis (1965) type in couple-stress fluid heated from below (Q= 0), if

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1$$

A similar theorem can be proved for thermosolutal convection in couple-stress fluid configuration of Stern type in a porous medium as follow:

Theorem 2: If R$\langle$ 0 , $R_s \langle 0$, F$\rangle$ 0, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, Z, \sigma)$ of equations (17) – (22), together with boundary conditions (23) is that

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\rangle 1.$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (17) – (22) and proceeding exactly as in Theorem 1 and utilizing the inequality (28), we get the desired result.
Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 2, can be put in the form as follow:-

Corollary 4: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium is that, $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1$, where R is the Thermal Rayliegh number, the Taylor number T_A, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number, p_1 is the thermal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermal Rayliegh number R, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1$, for rigid boundaries at the top and bottom of the fluid
The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 5: If $(W,\Theta,\Gamma,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r\ge 0$ is a solution of equations (17) – (22), with $R\rangle\ 0$ and,

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1 \quad ,$$

Then $\sigma_i=0$.
In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r=0\Rightarrow\sigma_i=0$ if $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1.$

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Stern's type, we can state the above theorem as follow:-

Corollary 6: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium is that the Thermal Rayliegh number R, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\rangle 1$, for rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in couple-stress fluid i. e. when $T_A = 0 = R = Q$.

(ii). Magneto-rotatory thermal convection couple-stress fluid (R=0), if

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1,$$

(iii). Magneto-thermal convection of Stren (1960) type in couple-stress fluid (T_A = 0), if

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)\leq 1\ .$$

(iv). Rotatory-thermal convection of Stren (1960) type in couple-stress fluid (Q= 0), if

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1$$

5. CONCLUSIONS:

Theorem 1 mathematically established that the onset of instability in a thermosolutal couple-stress fluid configuration of Veronis (1965) type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F satisfy the inequality $\varepsilon\left(\frac{R_sE^{/}p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1$, for rigid boundaries at the top and bottom of the fluid

The essential content of the theorem 1, from the point of view of linear stability theory is that for the thermosolutal configuration of Veronis (1965) type of couple-stress fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for rigid boundaries at the top and bottom of the fluid, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\varepsilon\left(\dfrac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\dfrac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\dfrac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, and in particular PES is valid.

The similar conclusions can be drawn for the thermosolutal configuration of Stern (1960) type of couple-stress fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for rigid boundaries at the top and bottom of the fluid from Theorem 2.

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APPLICATION OF NUMERICAL ANALYSIS IN VARIOUS ENGINEERING FIELDS

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ABSTRACT:
Mathematics is an important part of the most branches of engineering fields. In this paper, several examples of application of Numerical Analysis in chemical engineering, mechanical engineering, electrical engineering and computer engineering. Numerical methods in chemical engineering deals with a broad range of problems related to quantum mechanical calculation of atoms or molecules. In mechanical engineering, numerical method deals with problems relating to linearization, finding root of the function, solving system of equation and optimization. Illumination problems in electrical engineering and in computer engineering to solve problem by using MATLAB and Mathematica.

KEYWORDS: Numerical methods, illumination, linearization, quantum mechanical.

INTRODUCTION:
Numerical analysis provide a way to solve problems quickly and easily.Whether the goal is integration or solution of complex differential equation. In engineering, application of numerical analysis in quantum mechanics of atoms and molecules, in first order irreversible series reaction, in illumination problems, in linearization problem and optimization.

Applications of Numerical analysis in chemical engineering:-

1. Quantum mechanics of atoms and molecules, Monte Carlo and Dynamics

Energy of an n-electron can be portioned into energy of one electron moving in the average field of (HF-SCF) electrons and nuclei. This is called Hartree-Fock Model. This approach allows that two electrons with different spins can found at the same spatial point. In this approach, the total Hamiltonian of the system is portioned into two pieces, zeroth order H_0 which is the Hartree-Fock Hamiltonian and a perturbation V. The exact energy is then expressed as an infinite sum of contribution of increasing complexity. The energy E_R of a molecular system is obtained as a solution of the electronic part of Schrodinger for a fixed configuration R of a nuclei

$$H(r, R)\,\psi(r) = E_R \psi_R(r)$$

The n- electron wave function $\psi_R(r)$ describes the motion of the electron in the nuclei. Due to electron-electron interaction term in Hamiltonian, this equation cannot be solved without approximations. The Hamiltonian function approximation assumes that n-particles wave function $\Psi(r)$ can be written as antisymmtrized product of n- electron function $\Psi i\ (r_i)$

$$\Psi(r_1, r_2, r_3 \ldots r_n) = \begin{vmatrix} \Psi_1(r_1) & \psi_2(r_1) & \ldots\ldots & \Psi n(r_1) \\ \psi_1(r_2) & \psi_2(r_2) & \ldots\ldots & \Psi n(r_2) \\ \vdots & \vdots & & \vdots \\ \Psi_1(r_n) & \psi_2(r_n) & \ldots\ldots & \Psi n(r_n) \end{vmatrix}$$

Equation (1) is called Slater determinant. The set of orbitals that yield the lowest energy of a molecular system in the sense of vibrational principle is given by the following set of H.F. integro differential equations

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$F(r_i)\,\Psi_i(r_i) = E_i \Psi_i(r_i)$

The orbitals $\Psi_i(r_i)$are called molecular orbitals and $F(r_i)$ is the Fock operator which comprises the differential operator of the kinetic energy and the electron-electron interaction term. The expansion of molecularorbitals $\Psi i\ (r_i)$ into a finite series of the basic function $\chi_i(r_i)$

$$\Psi_i(r_i) = \chi_i(r_i)\,C_i = (\chi_1(r_i), \chi_2(r_2), \chi_3(r_3) \ldots\ldots \chi_m(r_i)) \begin{pmatrix} C_{1i} \\ C_{2i} \\ \vdots \\ C_{mi} \end{pmatrix}$$

(2)First order irreversible series reactions:

Numerical Analysis applied in diffusion/reaction problems. Consider the first order irreversible series reactions

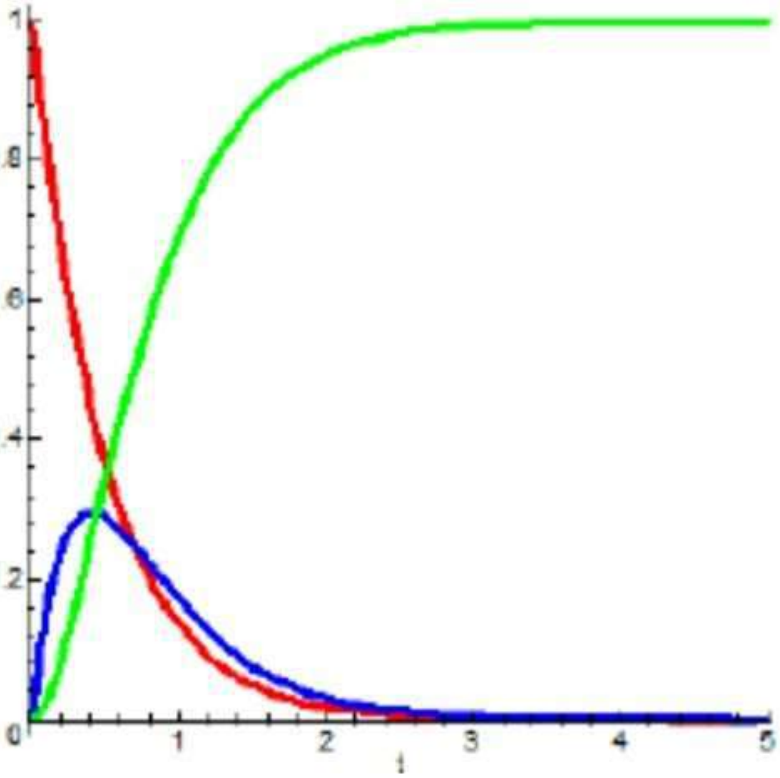
$$A \xrightarrow{K_1} B \xrightarrow{K_2} C$$

governing equation for the scheme are given as

$$\frac{dC_A}{dt} = -K_1 C_A \qquad \text{(i)}$$

$$\frac{dC_B}{dt} = K_1 C_A - K_2 C_B \qquad \text{(ii)}$$

Where $K_1$and K_2 are rate constants and the initial conditions are $C_A(0) = 1$ mol /L, $C_B(0) = 0, C_C(0) = 0$. The concentration of species C(t) at any time is given by the material balance equation, $C_C = 1 - C_A - C_B$



Using Maple, the solution of equation (i) and (ii) given by

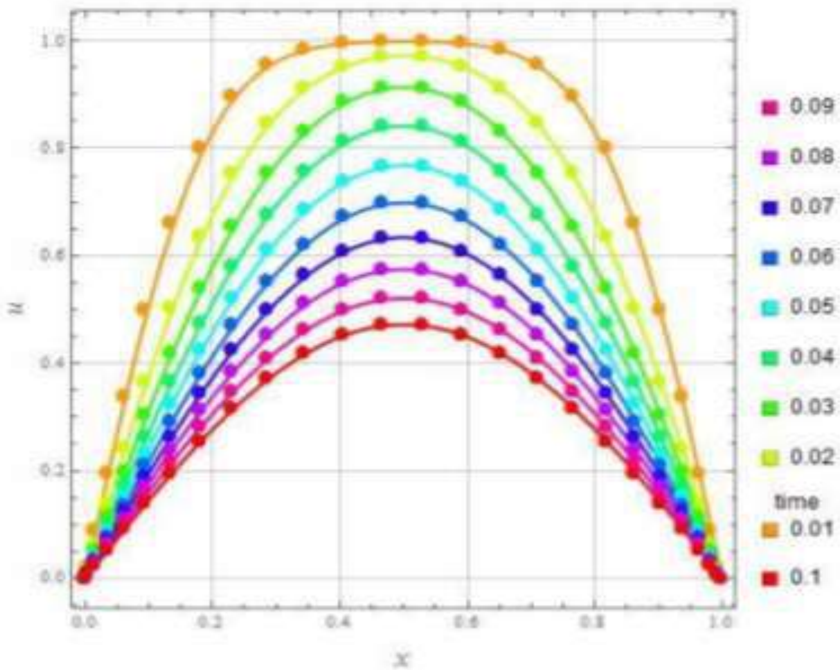
$C_A = e^{-k_1 t}$
$C_B = k\,(e^{-k_1 t} - e^{-k_2 t}) / k_2 - k_1$
As t approaches infinity, both C_A and C_B will approaches zero and C_C approaches 1

(3) Transient One Dimensional Heat Conduction:
Consider one dimensional heat equation given by

$$\frac{1}{\alpha}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{with } t \geq 0 \text{ and } 0 \leq x \leq 1$$

This equation represents heat conduction in a rod. The boundary conditions are such that the temperature u is equal to 0 at both ends of the rod.
$u(x=0, t) = 0$ and $u(x=1, t) = 0$, fort ≥ 0

One can take the thermal diffusivity α equal to 1 cm^2/s the condition is given by
$u(x, 0) = 1$ for$0 \leq x \leq 1$
The temperature can be found ND Solve (solid colored curve or Chebyshev Collocation dots) In various values of time, t ranging from 0.01 to 0.1 with a span of 0.01. One can set the number of interior pointsused by the Chebyshev Collocation dots method.



Temperature profile for t ranging from 0.01 to 0.1 with a span of 0.01

(4) Transient Two Dimensional Heat Conduction
Consider two dimensional heat conduction given by

$$\frac{1}{\alpha}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \text{with } t \geq 0 \text{ and } 0 \leq x, y \leq 1$$

which represents heat conduction in a two dimensional domain. The boundary conditions are s.t. temperature u is equal to 0 on all the edges of the domain
$u(x=0, y, t) = 0$ and $u(x=1, t) = 0$, for $t \geq 0$ and
$u(x=0, y, t) = 0$ and $u(x, y=1, t) = 0$, for $t \geq 0$

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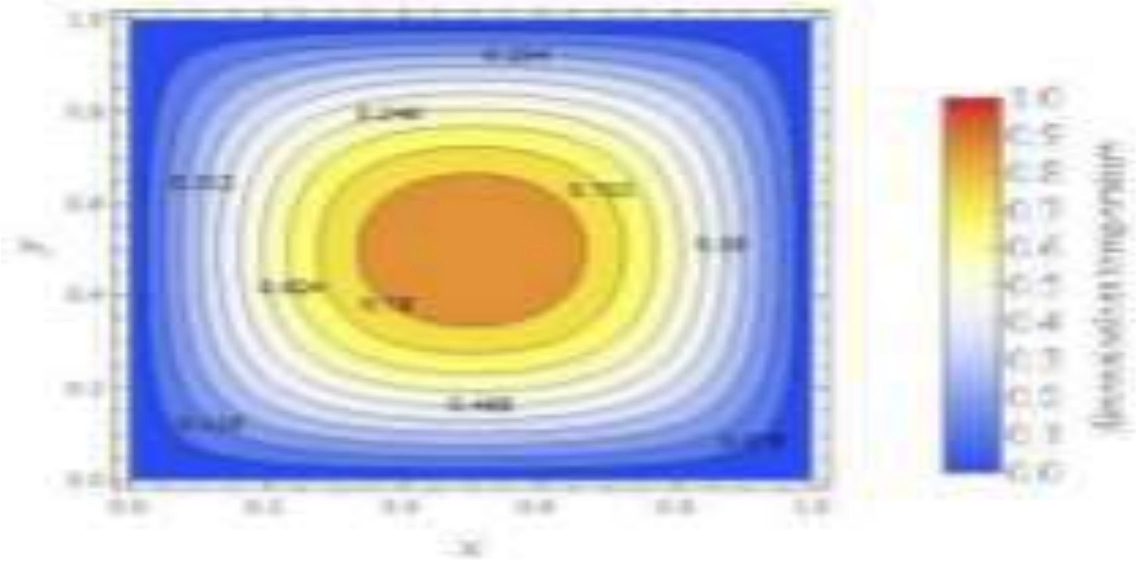
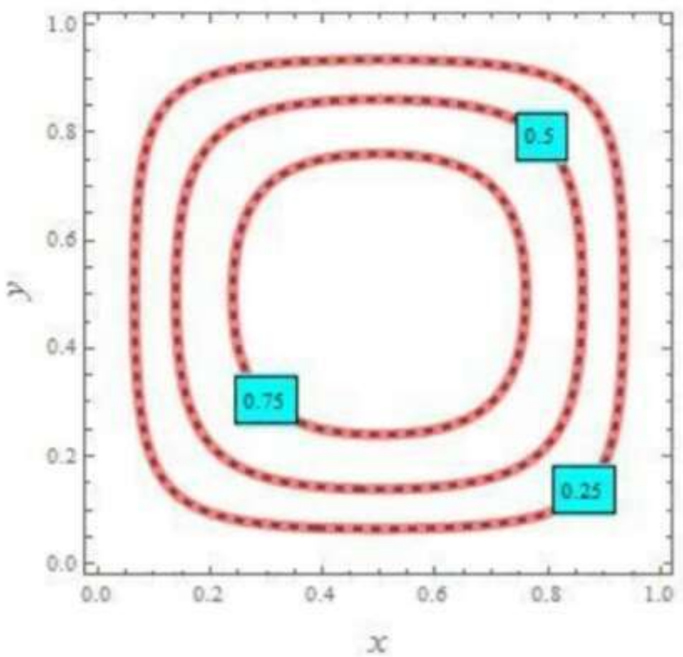
One can take the thermal diffusivity α equal to 1 cm^2/s. The condition is given by and u(x, y, 0) =0, for $0 \le x, y \le 1$

The dimensional temperature can be found by using either ND Solve (solid colored curve or Chebyshev Collocation dots)



Contour plot of the solution obtained using Chebyshev Collocation dot



Three contours of the dimensionless temperature, (i.e. 0.25, 0.5, and 0.75) at t =0.02

Application in Mechanical Engineering:

1. Linearization System: Taylor's series expansion provides a convenient way to approximate a nonlinear equation or function with a linear equation.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots\ldots\ldots\ldots\ldots$$

Linearization System in Swinging Pendulum:

Sum of forces in tangential direction:

$\sum F_t = W_t = mg\sin\theta = ma_t = m \frac{d^2\theta}{dt^2} L = m\ddot{\theta}$

$$\ddot{\theta} - g\sin\theta/L = 0$$

Linearize $\sin\theta$

$$\mathrm{Sin}\theta \approx \mathrm{Sin}(0) + \mathrm{Cos}(0)(\theta - 0) = \theta$$

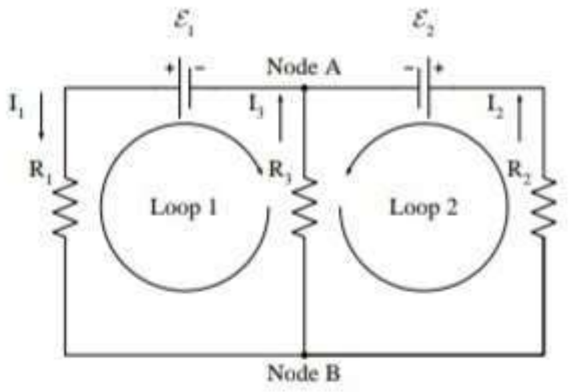
Linear System in circuit analysis:

Kirchhoff's Laws: - The sum of all voltages changes around any closed loop is zero.

$\sum_{i=1}^{ne} \Delta V_i = 0$

The sum of all currents at any node is zero

$\sum_{i=1}^{nb} \Delta I_i = 0$



Application of these two laws to an electrical circuit facilitates the formulation of a system of n-linear equation when n-unknown quantities exist. Given that $R_1=2\Omega$, $R_2=4\Omega$, $R_3=1\Omega$, $\varepsilon_1=6v$, $\varepsilon_2=9v$ and using equation from loop-1, loop-2 and node A we find

$$6 - 2I_1 - I_3 = 0$$
$$9 + 4I_2 - I_3 = 0$$
$$-I_1 + I_2 + I_3 = 0$$

$$\begin{pmatrix} -2 & 0 & -1 \\ 0 & 4 & -1 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -9 \\ 0 \end{pmatrix}$$

$$AX = B$$

2. Nonlinear Systems Example: Turbine Blade Analysis

- Turbine blades are components of gas turbine engines (used for aircraft and electricity generation)
- Subject to high temperature, high inertial forces and high drag forces
- Structural and thermal analyses must be performed simultaneously (coupled non-linear equations).

Methods apply to arbitrary non-linear equations (black-box function)

$$T(x) = f_1(L)$$
$$L = f_2(T(x))$$

3. Optimization:

To find the input variable to a function such that the function is minimized (or maximized), possibly subject to constraints.

Negative Null Form: min f(x)

subject to $g(x) \leq 0$

$h(x) = 0$

4. Engineering Design:

1. Maximize performance criteria subject to failure constraints: Minimize bicycle frame weight subject to structural failure constraints by varying frame shape and thickness
2. Minimize cost subject to performance and failure constraint. Minimize vehicle cost subject to acceleration, top speed, handling and comfort and safety constraints by varying vehicle design variables.

Application in Electrical Engineering:

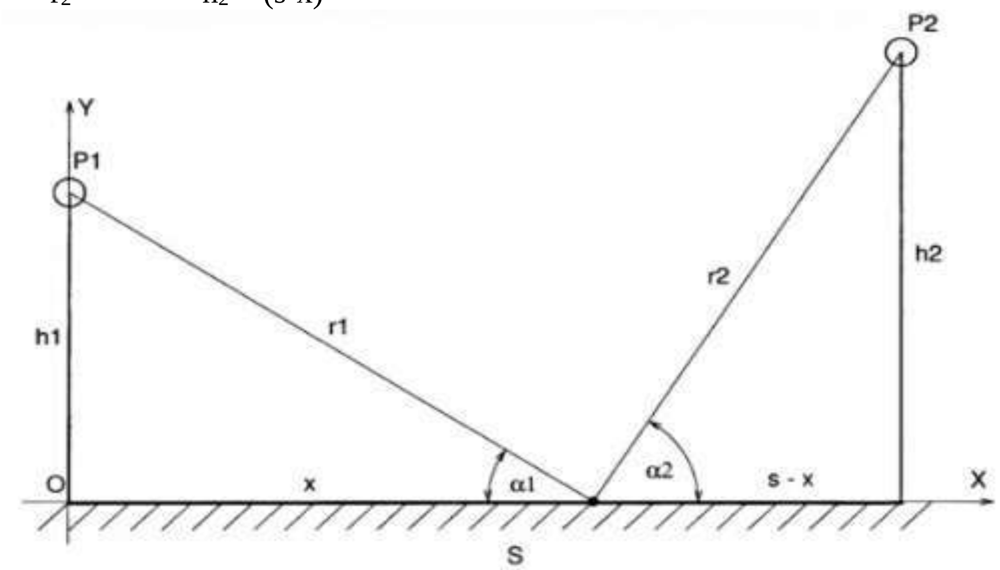
Illumination Problems: a courtyard is illuminated by two lights, where P_i is the illumination power and h_i is the height of lamp. The coordinates of the lamp are $(0,h_1)$ and (s,h_2),where s is the horizontal distance between two light sources. Let X= (x,0) be a point on the courtyard somewhere between two lights. We have to find a point X which will get minimum illumination rom two lamps.

$$r_1^2 = h_1^2 + x^2, \qquad r_2^2 = h_2^2 + (s-x)^2$$

The light intensities from the two lamps at X are given by

$$l_1(x) = \frac{P_1}{r_1^2} = \frac{P_1}{h_1^2 + x^2}$$

$$l_2(x) == \frac{P_2}{r_2^2} = \frac{P_1}{h_2^2 + (s-x)^2}$$



The illumination $ll_1(x)$ at point x from each lamp is given by

$ll_i(x) = l_i(x)$

$$ll_i(x) = \frac{P_1 h_1}{(h_1^2 + x^2)½}$$

$$ll_i(x) = \frac{P_2 h_2}{(h_1^2 + (s-x)^2)½}$$

writing $P_1 = p$, $P_2 = q$, $h_1 = a$, $h_2 = b$.
The total illumination is given
'Remove the radicals by squaring the expression

$$\frac{P^2a^2x^2}{(a^2+x^2)^5} = \frac{q^2a^2(s-x)^2}{(b^2+(s-x)^2)^5}$$

Expanding this will produce a degree-12 polynomial. For an illustration, take p=1 kW, q= 2 kW, a =4 m, b= 5m and s = 10m. Substituting the values to the above equation produces after expanded and arranging the result. $F(x) = 21x^{12} - 100x^{12}$ - ………….. 52428000 +2621440000 = 0

Numerical solution produces three roots between $0 \le x \le 10$.
The value that makes C(x) minimum can be found by testing the first derivative of F(x) with x.
If F'(x)>0, C(x) will be minimum.

Application in Computer Engineering:-
MATLAB is a highly level language and interactive environment that enables you to perform computationally intensive tasks faster with traditional programming languages such as C,C^{++}. MATLAB (for matrix laboratory) is a numerical computing environment. MATLAB supports matrix generation operation.

Matrix Generation:
Entering a vector: A vector is a special case of a matrix. The elements of vectors in MATLAB are enclosed by square brackets and are separated by spaces or columns. For example to enter a row vector v type

```
>>v  = [1 4 7 10 13]
v    =
          1   4   7    10   13
```

Columns vectors are created in a similar way, however, semicolon (;) must separates the components of a column vector,

```
> w  = [1; 4;7;10; 13]
              1
              4
              7
 10
 13
```

On the other hand, a row vector is converted to a column vector using the transpose operator. Thetranspose operation is denoted by an apostrophe or a single quote (').

```
>>w =v'
        w   =
             1
             4
             7
             10
  13
```

Thus v (1) is the first element of vector, v (2) is the second element and so forth. To access blocks of elements, we use MATLAB's colon notation (:). For example, to access the first

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three elements of v,
>>v (1:3)

ans =
1 4 7
All elements from the third through the last elements,
>> v (3, end)
ans =
7 10 13
Where end signifies the last element in the last element in the vector. If v is a vector, writing
>> v (:)
produces a column vector,whereas writing
>>v(1: end)
produces a row vector.

Entering a matrix:
A matrix is an array of numbers.To type a matrix into MATLAB you must

- Begin with a square bracket, [
- Separate elements in a row with spaces or commas (,)
- Use a semicolon (;) to separate rows
- End the matrix with another square bracket,].

For example, to enter a matrix A, such as

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Type,
>> A = [1 2 3; 4 5 6; 7 8 9]
MATLAB then displays the 3× 3 matrix as follows

```
A =
    1  2  3
    4  5  6
    7  8  9
```

Matrix indexing:
The element of row i and column j of the matrix A isdenoted by A (i, j). Thus, A (i, j) in MATLAB refersto the element A_{ij} of the matrix A. The first index is the row number and the second index is the column number .For example, A(1,3) is an element of first row and third column .Here, A(1, 3) =3 .

Correcting any entry is easy through indexing .Here we substitute A(3,3) =9 by A(3,3) =0. The result is
>>A (3, 3) =0

A =

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{matrix}$$

Single elements of a matrix are accessed as A (i, j), where $i \geq 1$ and $j \geq 1$ .Zero or negative subscripts are not supported in MATLAB.

MATHEMATICA:

It is a modern technical computing system spanning most area of technical computing including geometry, data science and visualization.Mathematica is a program for symbolic computation.

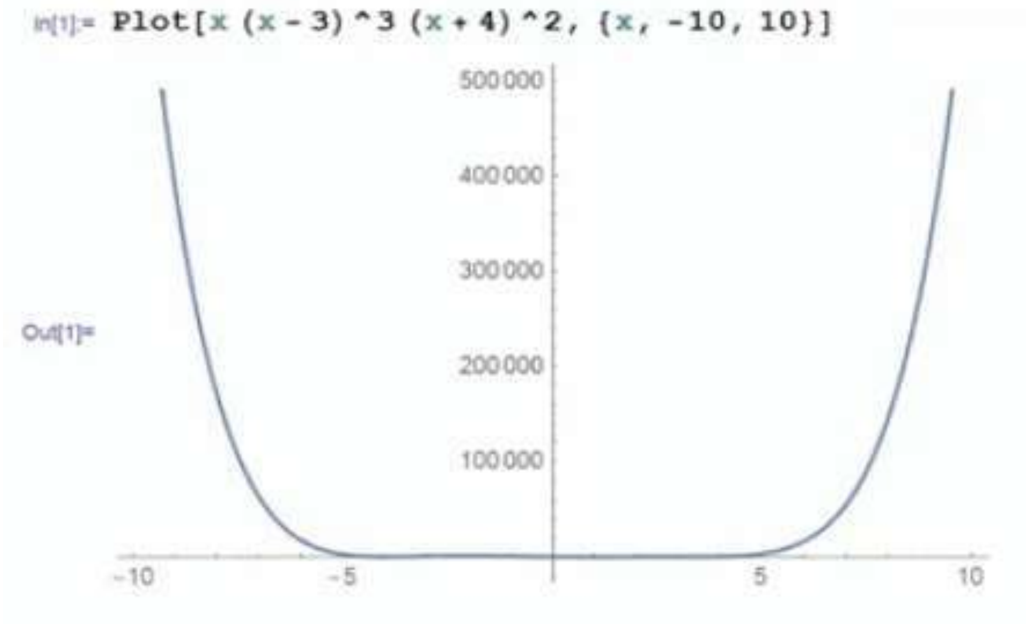
It does symbolic manipulation. Solve [$ax^2 + bx + c = 0$, x >0

{ {$x \rightarrow -b - \sqrt{b^2\text{-}4ac} /2a$} , {$x \rightarrow -b + \sqrt{b^2\text{-}4ac} /2a$ } }

It solves math problems Dsolve[{

x'' [t] +3 x'[t] +2 x [t] =0 ,x [0] = 1, x' [0] =0}, x [t], t]

Plotting graph: Plot [xˆ 2, {(x, -3, 3)],plot range –{[-3, 3), (-4, 9)] }



CONCLUSION:

In this paper, the applications of Numerical Analysis in engineering fields have been presented. The problems are from real life .It is expected that the problems presented in this paper can motivate engineering students to understand mathematics better. Mathematics should be enjoyable as it has helped engineering evolved.

Engineering + Math = Everything.

Engineering –Math = Nothing.

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ROLE OF MATHEMATICS IN MANAGEMENT

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ABSTRACT:
Research paper aims to describe the scope of mathematics in management; authors described the role of mathematics earlier and modern context in management. The role of Management science is also discussed. The authors also attempted to explore the importance of mathematics in various fields of management and effective decision making by means of using mathematics techniques, models and theories. Secondary data is used to get insight and discuss the role of mathematics in Management. Mathematics plays a major role in business management because it helps maximize profit by using techniques such as analyzing production costs, determining ideal pricing, discerning sales patterns and projecting future sales. Having strong skills in mathematics means an individual can analyze all of a company's finances and make changes to save the company money and time, and ultimately make a higher profit. Thus we concluded Mathematics is an important part of managing business

KEYWORDS: Mathematics, Management, Techniques and Decision Making

INTRODUCTION:
Management is the process of reaching organisational goals by working with and through people and other organisational resources. Mathematics is used in most aspect of daily life and Management. Many of the top jobs such as business consultants, computer consultants, airline pilot’s company directors require a solid understanding of basic mathematics for effective performance. It also play important role in business like business mathematics by commercial enterprises to record and manage business operations.

Until the mid 1950's, the role of mathematics (and statistics to a lesser extent) in Business and Management programs was minimal: some elementary business mathematics which included a smattering of the mathematics of finance using interest tables and some algebra, as well as some cookbook statistics. Beginning roughly at that time, with the infusion of funds from various sources, business schools entered the scientific age. Numerous mathematical techniques -- some new, some not so new -- were assembled to help solve various problems of management.

Mathematics typically used in management includes elementary arithmetic’s such as fractions Decimal and percentages, elementary algebra, statistics and probability. Business management can be made more effective by use of more advanced mathematics such as calculus and linear programming

OBJECTIVES OF STUDY:

- To describe the role of Mathematics in Management
- To provide a overview on use of mathematics in Business and management programme and processes

Managerial Decision models-A Decision making Process:

The role of mathematics in Business decisions has very important now days in the process of managerial decision models. To turn to the specific aspects of the quantitative decision making process, it is possible to recognize three distinct phases in every decision situation. First is carefully defined the problem, second is a conceptual model to be generated and third is the selection of the appropriate quantitative model they may lead to a solution. Lastly a specific algorithm is selected. Algorithms are the orderly delineated sequences of mathematical operations that lead to a solution. The algorithms generate the decision which is subsequently implemented managerial action program. The entire process is shown :

Defined problem ——> Conceptual model —> Quantitative Model —-> Algorithms—-> Decision —- Action programs.

Some basic question that reveals the use of math in daily life are-

You need to use math to calculate compound interest rates (to see how much your savings can grow).

You also need to use math to understand the monthly percentages, which are added to your credit cards or bank loans,

or you could end up paying Rs10,000 in 5 year's time for borrowing Rs2,000 today! This is a good reason to understand mathematics

Another meaning of Business mathematics sometimes called commercial math, consumer math is group of practical subjects used in management and everyday life in schools, these subjects are often taught to students who are not planning a university education, in the United States they are typically offered in high schools and in schools that grants associate degree.

A Business Organisation Manager should have quite knowledge of elementary arithmetic, including fraction decimals and percentages. Elementary algebra is often important as well as in the context of solving practical business problems, the practical applications typically includes checking accounts , price discounts mark-up and mark down , payroll calculation , simple and compound interest , consumer and business credit and mortgages.

Role of Applied Mathematics in Management:

Management is mainly dependent on applied mathematics a branch of mathematics, such as statistics is used to reach effective decision. The various statistical techniques such as calculation of positional averages and calculated averages are of meanwhile importance in business management. Some other techniques are discussed below

Regression and Correlation are statistical techniques used to make prediction about Demand, production, investment, consumption, prices, profit and sales in the business. it also used to estimate birth rate , death rate and tax rate etc. Through correlation we can predict about the future eg. If there is heavy rainfall, than we can expect increase in sale of umbrellas thus enable us to study the relationship between two variables.

Index Numbers are very helpful in framing suitable policies by government and business to study prices and cost of living index for future planning and progress of country.

Index numbers of import and export prices are used to measure change in the trade of a country so economic and business policies are guided by indexes. Index numbers are also important to comparing living standard of people and forecasting future economic trends in the demand of commodity, volume of production etc. These are also widely used in adjustment of wages earned over a time and suggest a possible increase in wages which the worker accept, and useful in deflating the national income in inflation situations and help the government in adjusting its policies in such situation.

Wholesale price index number gives us the change in the value of money. Cost of living index number measure real income of people so that dearness allowances may be adjusted .Index number of stock prices measure economic changes in purchasing power of money over a stock.

Probability is widely used in business decision. The theory of probability has its origin in betting and games of chance but now it is used to take many managerial decisions. Probability theory is the backbone insurance companies because life tables are based of these theories. It is also widely used in time and motion studies, marketing decisions, input- output analysis, business forecasting, trade cycles, and analysis of population.

In the current time customer attitude are changing, their expectations from the business organisations are also changing. In such demanding situations when business manager feel the pressure from all side, the solution to their problem can be extracted through business research. Business research refers to systematic collection and analysis of data with purpose of finding answers to problem faced by management. Business research can be carried out to with the objective to explore, to describe, to diagnose phenomenon. Quantitative research one of the branch of business research involves the measurement of phenomenon in quantitative terms which mainly dependent on quantitative data and application of parametric and non- parametric test to test the significance of hypothesis and to solve the business problem.

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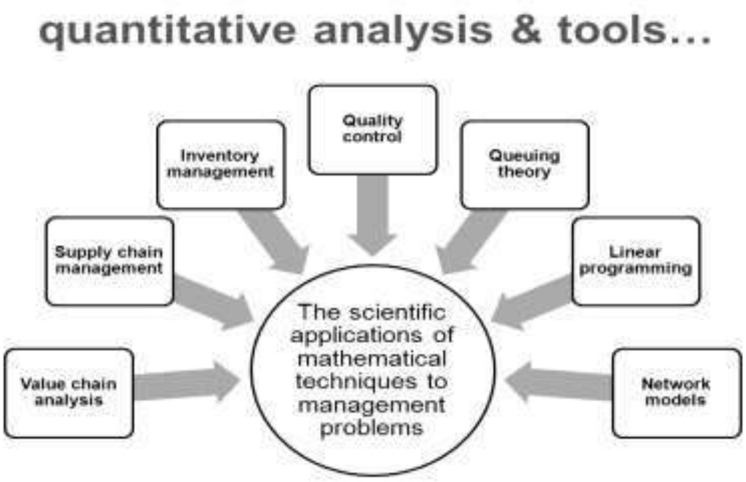
Management Science:

A particular discipline called Management Science sprang up. It incorporated various techniques -- mathematical programming, linear algebra, network methods, queuing theory, stochastic processes, statistics, recursive relations, and computer simulation to solve various management problems. Drawing upon the above techniques, management science has as its philosophy the solving of a problem. As mathematics began to be established in business schools, applications of mathematics to various management problems became prevalent. Among them were the applications of quadratic programming to financial portfolio analysis and to the planning of production, inventories, and work force as well as the application of linear programming to advertising media selection. Queuing theory was used to analyze service facilities, such as restaurants and banks.

Other applications include the transportation method of linear programming, the economic lot size formula trading off inventory and order/set-up costs, CPM-PERT networks in project management, the use of learning curves, exponentially-weighted moving averages as a means of forecasting, and gravity models for site selection.

Models that are used most in practice include linear programming and computer simulation. These models have been applied in industry for approximately twenty years. Other models are also in use, but not nearly as wide-spread a use as linear programming and simulation. Business schools differ in the extent to which they employ quantitative techniques. Some business school programs painstakingly avoid any reference to concepts of calculus and awkwardly work around instances where such concepts would be useful. For example, they might illustrate a simple maximization by enumerating all possibilities or by graphing the appropriate function. My own background (undergraduate Medical, Post graduate in management, and M.Phil in Mangement, Persuing PhD. in management) and experiences make me a staunch supporter of the mathematical end of the spectrum to solve the business Problem and Business management



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I would now like to describe our BBA program at the HPU Shimla and describe what I see to be its strengths and weaknesses. In our undergraduate program, students take a two-semester sequence in Statistics for business decision and Quantitative techniques and linear algebra offered by the BBA department. They also take a two semester sequence in statistics and computing which covers BASIC programming, probability and statistics, and the use of the computer for statistical analysis. These courses are followed by a course titled Production and Operations Management, which includes a treatment of the production activities of a firm and an introduction to the philosophy and methodology of Management Science. In this course students use packaged computer programs for solving problems; they also write computer programs in the solution of certain other problems. Subsequent undergraduate courses build on the above foundation. For example, economics makes use of the calculus (both differential and integral), certain functional areas (such as marketing and finance) use some of the models used in the above courses, and several courses use the computer skills.

USE OF MATHEMATICS IN BUSINESS MANAGEMENT:

Business and mathematics go hand in hand this is because business deals with money and money encompasses everything in itself. There is a need for everyone to manage money as some point or the other to take decisions which requires everyone to know mathematics. Business mathematics is used by commercial enterprises to record and manage business operations. Commercial organizations use mathematics in accounting,, marketing, sales forecasting, and financial analysis. It helps you know the financial formulas, fractions; measurements involved in interest calculation, hire rates, salary calculation, tax calculation etc. which help complete business tasks efficiently. Business mathematics also includes statistics and provides solution to business problems.

Business is always surrounded with challenges which need to be dealt with in a proper fashion so that they do not arise in future. These problems that occur on a daily basis can be effectively solved with the help of mathematical models. Hence mathematics not only helps to calculate but also analyze business problems and work upon them. Learning and using business mathematics enables a person to think out of the box, sharpens one’s thinking and helps in precisely formulating and structuring relationships.

In order to known a business it requires skill more than the developing a product or providing a service. If a business has to survive ad succeed it needs to look after the finances and make necessary arrangements for it to prosper as well. Understanding business mathematics is important to maintain profitable operations and accurate keeping of records. It is required right from the start for pricing products/services till the end when we need to check if the budget was met. Let’s look at situations where business mathematics is required:

PRODUCTION COSTS CALCULATION:

Before one formally starts production and establishes its business it is very important to estimate the costs that would be incurred in relation to the manufacturing such as the cost of raw materials, machinery, rent, administrative expenses etc. In addition to these basic expenses there are other associated costs such as marketing, warehousing, interest and repayment of loans etc. Once all he expenses relating to production have been included I would be easy to estimate the profit from it to sustain and remain competitive in the market. Accurately determining the cost associated with each item will make the base for the business strong.

PRICE DETERMINATION:

When you have successfully determined the costs, the next task is to price the products correctly so that it generates right amount of cash flows for future requirements of the business. Charging the correct selling price would ensure that the product remains profitable.

PROFIT MEASUREMENT:

These require determining the net profit by subtracting the operating costs from the total amount of sales/revenue during a period of time. What also needs to deducted are the tax, depreciation, discount expenses. This helps to find out if the products are being charged enough to continue the business operations and expand.

FINANCIAL ANALYSIS:

You need to project the revenue and expenses of a business if we need to analyze the financial health of a business. We need to do sensitivity analysis of how an increase or decrease in sales figure or pricing could affect the business. It helps in determining how each employee contributes to the business and how I would affect. Using business mathematics helps in making these interpretations ad take the business to a higher level.

Learn marketing concepts and apply in real world. Plan and implement promotional campaigns. Become a successful marketing manager with our practical training.

Since all corporations require managing their money and business many mathematicians find employment in these fields. Mathematicians follow a logical thinking; follow a problem-solving approach to business.

CONCLUSION:

Mathematics plays a major role in business management because it helps maximize profit by using techniques such as analyzing production costs, determining ideal pricing, discerning sales patterns and projecting future sales. Having strong skills in mathematics means an individual can analyze all of a company's finances and make changes to save the company money and time, and ultimately make a higher profit.

The study of mathematics is essentially just studying number patterns, and in business, this means knowing how to manipulate numbers and make meaning out of large data sets. All companies need some sort of mathematician to look at the company's expenses, sales and cash flow. If a company has good documentation of where their money is coming from and going to, using mathematics, an individual is able to see inefficiencies in the company's operations and make important changes. Thus we conclude that Mathematics is an important part of managing business.

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EFFECT OF ASCORBIC ACID ON THE SIZE OF CUS NANOPARTICLES

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ABSTRACT:
Among other metal sulfide nanoparticles such as CdS, ZnS, PbS, HgS and SnS, CuS has attracted the attention of researchers due to its unique properties like high solar absorbance, antibacterial activity, etc. CuSnanoparticles are less toxic, environmentally benign and can be easily synthesized using simple chemical methods. In this study we have reported sonochemical synthesis and X-ray diffraction, UV-vis and Photoluminescence spectroscopic characterization of copper sulfide nanoparticles.

INTRODUCTION:
Copper sulfides (Cu_xS) have received greatword-wide attention due to their opto-electronic properties [1]. Cu_xS materials (where x >1) are p-type semiconductors and when x = 1 it exhibits metal-like behavior. Depending upon the values of x ($1 \leq x \leq 2$), the properties of Cu_xS can be tuned towards applications in different fields, such as solar cells, Li-ion rechargeable batteries, catalysts, sensors, etc. [2]. Covellitephase of copper sulfide is one of the most studied coppersulfides due to its absorption band gap in the nearinfrared region. In this study we are reporting the sonochemical synthesis of CuS nanoparticles using ethanol as reaction medium. Synthesized samples have been characterized using X-ray diffraction (XRD), UV-vis and Photoluminescence (PL) spectroscopic methods. The results obtained reveals the hexagonal structure of CuS nanoparticles.

MATERIALS & METHODS:
Copper chloride ($CuCl_2$) and sodium sulfide (Na_2S) were used as copper and sulfur precursors respectively without further purification. Calculated quantities of $CuCl_2$(0.1M) and Na_2S (0.4M) were dissolved in ethanol in to separate beakers. Copper chloride beaker was kept under sonication and sodium sulfide solution was added drop-wise to it and solution was continuously sonicated for one hour. In another reaction ascorbic acid (0.1M) was mixed with copper chloride solution before addition of sodium sulfide. The precipitates thus obtained in both reactions were filtered and dried under vacuum oven for 16 hours. In this way we have synthesized two samples of CuS labeled as CuS1 and CuS2. CuS1 was synthesized in the presence of ascorbic acid and CuS2 was synthesized in the absence of ascorbic acid under similar conditions.Bothsamples have been characterized using XRD, UV-vis and Photoluminescence spectroscopic techniques.

RESULTS AND DISCUSSION:
X-ray diffraction Study:
Both samples have been characterized withXRD method to determine the crystal structure. XRD pattern reveals the presence of (100), (101), (102), (103), (006), (110), (108) and (116) crystal planes in these nanoparticle samplesas shown in Figure 1. These crystal planes are indexed to hexagonal structure of CuS nanoparticles. The crystallite size was calculated using Debye Scherrer formula

$$D = \frac{c\,\lambda}{\beta\ cos\theta} \qquad (1)$$

Where c = 0.94is the correction factor, β is the full width at half maximum(FWHM) of the highest peak in radians, θ is the XRD peak position and λ = 1.540593 Å.

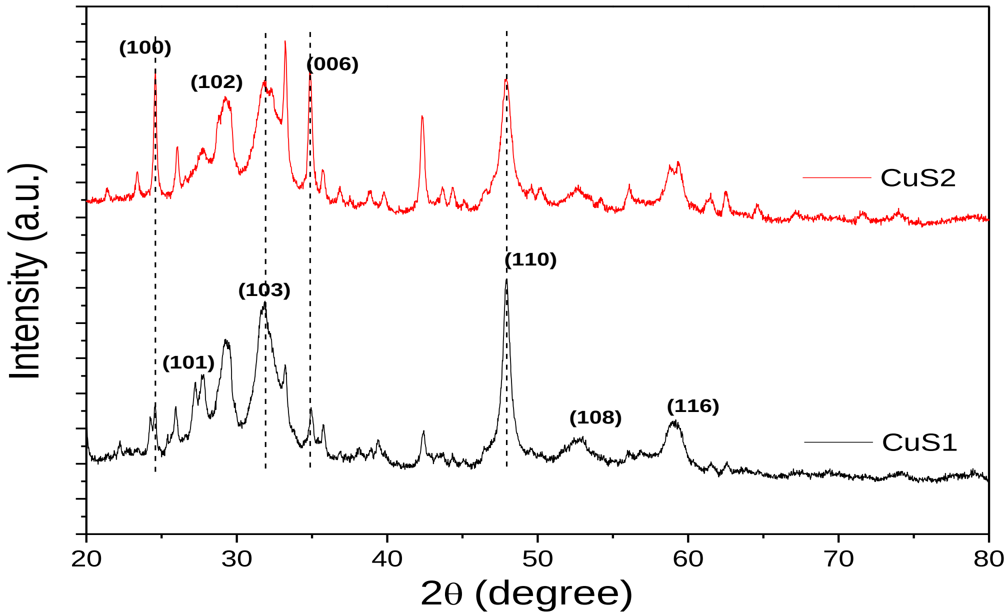

Figure 1.XRD pattern of CuS1 and CuS2 samples.

The average crystallite size for CuS1 and CuS2 are found to be ~12nm and~18nm respectively. It was observed that CuS1 has smaller crystallite size than CuS2 which indicates better nanoparticle formation in the presence of ascorbic acid.

UV-Vis and Photoluminescence spectroscopy:

The pattern of UV-vis spectrum of both samples has been shown in Figure2 (a). The bandgap of samples has been estimated using Tauc's plot and shown in inset of Figure2 (a). The bandgap value for both nanoparticle samples was found to be ~3.8eV.

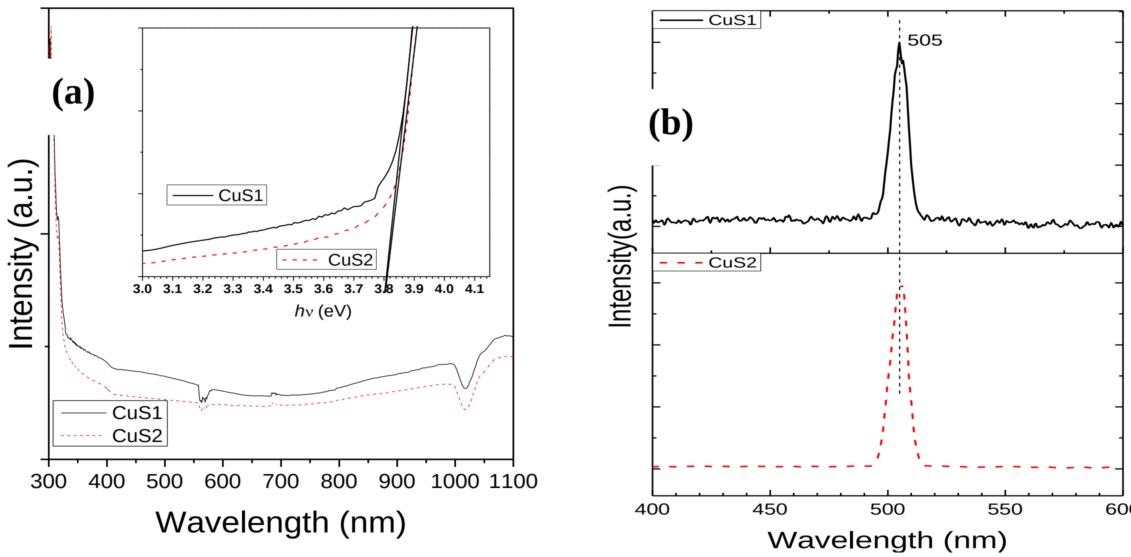

Figure 2.(a) UV-vis spectrum of CuS1 and CuS2. Inset shows bandgap estimation using Tauc's plot.(b)Photoluminescence spectrum of CuS1 and CuS2.

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PL spectrum of CuS1 and CuS2 indicates peak position at 505nm as shown in Figure 2 (b).

CONCLUSIONS:

In this work we have reported the sonochemical synthesis of CuS nanoparticles with and without ascorbic acid in ethanol as solvent. XRD characterization reveals the crystalline nature and hexagonal structure of CuS nanoparticles synthesized by two different approaches. Smaller nanoparticle size was obtained for CuS prepared in the presence of ascorbic acid. Bandgap estimation for both samples gives approximately same value 3.8eV.

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ESTABLISHING THE PRINCIPLE OF EXCHANGE OF STABILITIES OF VERONIS AND STERN TYPES PROBLEMS WITH VARIABLE GRAVITY USING POSITIVE OPERATOR

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ABSTRACT:
Weinberger used a method of a Positive Operator, a generalization of a positive matrix, to establish the Principle Of Exchange of Stabilities (PES). Wherein, the resolvent of the linearized stability operator is analyzed which is in the form of a composition of certain integral operators In the present paper, Thermohaline Problem of Veronis Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer, $\frac{\Re}{\Re_s} \geq 1$ and Lewis number $\tau < 1$. Simultaneously thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.

KEY WORDS: Veronis and Stern Types Problems, variable gravity, Principle of Exchange of Stabilities, Lewis number.

1. INTRODUCTION

In classical thermal instability problems, it has been assumed that the driving density differences are produced by the spatial variation of single diffusing property i.e. heat. Recently, it has been shown that a new phenomenon occurs when the simultaneous presence of two or more components with different diffusivities is considered. This problem has been probed when we think about ocean where both heat and salt (or some other dissolved substances) are important. This problem has been termed as *thermosolutal convection (or thermohaline convection)*. In these problems the solute is commonly, but not necessarily, a salt. Related effects have now been observed in other contexts, and the name '*double–diffusive convection*', has been used to cover this wide range of phenomena. Much of the theoretical work in this field has been developed directly from the linear stability calculations for a simple fluid heated from below. Two fundamental configurations have been studied in the context of the thermohaline instability problem, one by Stern [1960], wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing, and another by Veronis [1965], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing. The main results derived by Stern and Veronis for their respective configurations are that both allow the occurrence of a steady motion or an oscillatory motion of growing amplitude, provided the destabilizing concentration gradient or the temperature gradient is sufficiently large. Stern [1960] was the first to consider the case of linear opposing gradients of two properties between horizontal boundaries at fixed concentrations, and since then many others, including Gershuni and Zhukhovitskii [1963]. The problem of thermosolutal convection in a layer of fluid heated from below and subjected to a stable solute gradient has been studied by Veronis [1965]. The minimum requirements for the occurrence of thermosolutal convection are the following:

i) The fluid must contain two or more components having different molecular diffusivities. It is the differential diffusion that produces the density differences required to derive the motion.

ii) The components must make opposing contributions to the vertical density gradient.

Further, he found that the analogous non–dimensional parameters accounting for uniform salinity gradient are given by $S = \frac{g\alpha'\beta' d^4}{\upsilon\kappa'_T}$ and Schmidt number $q = \frac{\upsilon}{\kappa'_T}$, where α', β', κ'_T denote the coefficient of analogous solvent expansion, uniform solute gradient and solute diffusivity, respectively. The main large–scale engineering applications of double–diffusive concepts are to solar ponds, shallow artificial lakes that are density stratified. Linear calculations have also been made for a variety of boundary conditions by Nield [1967] and for an unbounded fluid by Walin [1964]. A study of the onset of convection in a layer of sugar solution, with a stabilizing concentration gradient, when the layer is heated from below, has been made by Shirtcliff [1967]. He found that the first stage of the development of convection layers similar to those described by Turner and Stommel [1964]. Nield [1967] has studied the problem of thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above. When the solute gradient is stabilizing, Sani [1965] has found that finite amplitude subcritical instability (convection at a thermal Rayleigh number less than that given by the linear theory) is possible. A direct analogue of heat/salt diffusive convection has been used to explain the properties of large stars with helium–rich core, which is heated from below and thus convecting. Spiegel [1972] has shown that variations in the helium/hydrogen ratio can produce a density gradient that limits the helium transport by double–diffusive convection, though, whether this may be in layers is still unclear. Another example of double–diffusive convection is when metals solidify, since as metals solidify, undesirable inhomogeneities on the microscopic scale can be produced by several mechanisms, among which is double–diffusive convection. It was shown by Turner [1973, 1974] that the form of the resulting motions depends on whether the deriving energy comes from the component having the higher or lower diffusivity. When one layer of fluid is placed above another (denser) layer having different diffusive properties, two basic types of convective instabilities arise, in the 'diffusive' and 'finger' configurations. In both the cases, the double–diffusive fluxes can be much larger than the vertical transport in a single–component fluid because of the coupling between diffusive and convective processes.The salinity gradient is not constant with depth and this has prompted theoretical studies (Walton [1982]) of the breakdown, which is found to occur preferentially (in agreement with observations) in a thin layer where the salinity gradient is a minimum. A recent comprehensive review of thermosolutal convection in porous media has been conducted by Nield and Bejan[1999]. For a broad and latest review of the subject one may be referred to Turner and Brandt and Fernando [1996] and Gupta *et al*.[2001, 2002]. Dhiman, *et al*.[20102] have also dealt with the problem On the Stationary Convection of Thermohaline Problems of Veronis and Stern Types.

In the present paper, Thermohaline Problem of Veronis Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number $\tau < 1$.But thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.

2. MATERIALS AND METHODS:

Mathematical Formulation of the Physical Problem.

Basic Hydrodynamical Equations Governing The Physical Configuration

The basic hydrodynamic equations that govern the above physical configurations under Boussinesq approximation for the present problem are given by (c. f. Veronis [1965]);

Equation of Continuity

$$\nabla.\vec{v} = 0 \tag{1}$$

Equations of Motion

$$\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right] = -\frac{1}{\rho_0}\nabla p + \left(1 + \frac{\delta\rho}{\rho_0} + \frac{\delta\rho'}{\rho_0}\right)\vec{X} + \upsilon\nabla^2\vec{v} \tag{2}$$

Equation of Heat Conduction

$$\frac{\partial T}{\partial t} + (\vec{v}.\nabla)T = K_T\nabla^2 T \,. \tag{3}$$

Equation of Mass Diffusion

$$\frac{\partial S}{\partial t} + (\vec{v}.\nabla)T = K_S\nabla^2 S \tag{4}$$

Equation of State $\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(S - S_0)]$ (5)

In the above equations; $\vec{v}$ is the velocity vector; p is the pressure; $\vec{X} = -g(z)\hat{\lambda}$ is the external force field (gravity); T is the temperature; S is the concentration; $\upsilon = \frac{\mu}{\rho_0}$ is the coefficient of kinematic viscosity; $K_T = \frac{K}{\rho_0 c_v}$ is the thermal diffusivity; K_S is the salt diffusivity and

$$\delta\rho = -\rho_0\alpha(T - T_0) \tag{6}$$

$$\delta\rho' = \rho_0\alpha'(S - S_0) \tag{7}$$

are the variation in density due to temperature and concentration variations.

Following the usual steps of the linearized stability theory, it is easily seen that the non dimensional linearized perturbation equations governing the physical problem described by equations (1)-(4) can be put into the following forms, upon ascribing the dependence of the perturbations of the form $exp[i(k_x x + k_y y) + \sigma t]$,

$(\boldsymbol{\sigma} = \boldsymbol{\sigma_r} + \boldsymbol{i\sigma_i})$ (c.f. Chandrasekhar [1961] and Siddheshwar and Krishna [2001]);

$$\left(D^2 - k^2\right)\left(D^2 - k^2 - \frac{\sigma}{Pr}\right)w = g(z)R_T k^2\theta - g(z)R_s k^2\phi \tag{8}$$

$$\left(D^2 - k^2 - \sigma\right)\theta = -R_T w \tag{9}$$

$$\left(D^2 - k^2 - \frac{\sigma}{\tau}\right)\phi = -\frac{R_s}{\tau} w \tag{10}$$

together with following dynamically free and thermally and electrically perfectly conducting boundary conditions

$$w = 0 = \theta = D^2 w \quad \text{at} \quad z = 0 \ and \ z = 1 \tag{11}$$

where, $\Re = R^2{}_T = \dfrac{g_0\alpha\beta d^4}{K_T\upsilon}$ is the thermal Rayleigh number, Pr= $\dfrac{\upsilon}{\kappa}$ is the Prandtl $\Re_s = R^2{}_s = \dfrac{g_0\alpha'\beta' d^4}{K_T\upsilon}$ is the salinity Rayleigh number, $\tau = \dfrac{K_S}{K_T}$ is the Lewis number.

3. THE METHOD OF POSITIVE OPERATOR:

We seek conditions under which solutions of equations (8)-(10) together with the boundary conditions (11) grow. The idea of the method of the solution is based on the notion of a 'positive operator', a generalization of a positive matrix, that is, one with all its entries positive. Such matrices have the property that they possess a single greatest positive eigenvalue, identical to the spectral radius. The natural generalization of a matrix operator is an integral operator with non-negative kernel. To apply the method, the resolvent of the linearized stability operator is analyzed. This resolvent is in the form of certain integral operators. When the Green's function Kernels for these operators are all nonnegative, the resulting operator is termed positive. The abstract theory is based on the Krein –Rutman theorem, which states that;

"If a linear, compact operator A, leaving invariant a cone $\hbar$, has a point of the spectrum different from zero, then it has a positive eigen value λ, not less in modulus than every other eigen value, and this number corresponds at least one eigen vector $\phi \in \hbar$ of the operator A, and at least one eigen vector $\varphi \in \hbar^*$ of the operator A^*". For the present problem the cone consists of the set of nonnegative functions.

To apply the method of positive operator, formulate the above equations (8) - (10) together with boundary conditions (11) in terms of certain operators as;

$$\tilde{M}\left(\tilde{M} + \frac{\sigma}{Pr}\right)w = g(z)R_T k^2\theta - g(z)R_s k^2\phi \tag{12}$$

$$\left(\tilde{M} + \sigma\right)\theta = -R_T w \tag{13}$$

$$\left(\tilde{M} + \frac{\sigma}{\tau}\right)\phi = -\frac{R_s}{\tau} w \tag{14}$$

The domains are contained in B, where

$$B = L^2(0,1) = \left\{ \phi \mid \int_0^1 |\phi|^2 \, dz < \infty \right\},$$

with scalar product $\langle \phi, \varphi \rangle = \int_0^1 \phi(z)\, \overline{\varphi(z)}\, dz$, $\phi, \varphi \in B$; and norm $\|\phi\| = \langle \phi, \phi \rangle^{1/2}$.

We know that $\boldsymbol{L}^2(\mathbf{0},\mathbf{1})$ is a Hilbert space, so, the domain of M is dom M = $\{\phi \in B / \, D\phi, m\phi \in B, \;\; \phi(0) = \phi(1) = 0\}$.

We can formulate the homogeneous problem corresponding to equations (8)-(10) by eliminating θ from (12) -(14) as;

$$w = k^2 \tilde{M}^{-1}\left(\tilde{M} + \frac{\sigma}{\Pr}\right)^{-1} g(z)\left(\Re(\tilde{M}+\sigma)^{-1} - \frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right) w \tag{15}$$

$$w = K(\sigma) w \tag{16}$$

where

$$K(\sigma) = k^2 \tilde{M}^{-1}\left(\tilde{M} + \frac{\sigma}{\Pr}\right)^{-1} g(z)\left(\Re(\tilde{M}+\sigma)^{-1} - \frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right) w \tag{17}$$

is the linearized stability operator. Further $\Re$ and $\Re_s$ respectively are the thermal and concentration Rayleigh numbers. In the present problem the linearized stability operator $K(\sigma)$ consists of three different operators, namely $\tilde{M}^{-1}, \left(\tilde{M} + \frac{\sigma}{\Pr}\right)^{-1}$ and $\left(\Re(\tilde{M}+\sigma)^{-1} - \frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right)$, however the operator $\left(\Re(\tilde{M}+\sigma)^{-1} - \frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right)$ which is the difference of two operators $\Re\left(\tilde{M}+\sigma\right)^{-1}$ and $\frac{\Re s}{\tau}(M+\frac{\sigma}{\tau})^{-1}$ needs special attention in regards to the positivity of the operator.

Defining, $T\left(\frac{\sigma}{\Pr}\right)$ exists for $\sigma \in T_{k\sqrt{\Pr}} = \{\sigma \in C \mid \operatorname{Re}(\sigma) > -k^2 \Pr, \operatorname{Im}(\sigma) = 0\}$

and $\left\| T\left(\frac{\sigma}{\Pr}\right) \right\|^{-1} > \left|\sigma + k^2 \Pr\right|$ for $\operatorname{Re}(\sigma) > -k^2 \Pr$. and define $T\left(\frac{\sigma}{\tau}\right)$ by $T\left(\frac{\sigma}{\tau}\right) f = \int_0^1 g\left(z, \xi; \frac{\sigma}{\tau}\right) f(\xi)\, d\xi$, where g $\left(z, \xi, \frac{\sigma}{\tau}\right)$ is Green's function kernel for the operator $\left(\tilde{M} + \frac{\sigma}{\tau}\right)^{-1}$

Let, $L(\sigma)$ = $\left(\Re(\tilde{M}+\sigma)^{-1}-\frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right)$. The operator $L(\sigma)$ exists for $\sigma\in L_k=\{\sigma\in C\ |\operatorname{Re}(\sigma)>\max.\{-k^2(1,\tau)\},\operatorname{Im}(\sigma)=0\}$ and $\|L(\sigma)\|^{-1}>|\sigma+k^2|$ for $\{\operatorname{Re}(\sigma)>\max\{-k^2(1,\tau)\},\operatorname{Im}(\sigma)=0\}$. $L(\sigma)$ = [$\Re T(\sigma)-\frac{\Re_s}{\tau}T(\frac{\sigma}{\tau})]$ is an integral operator with Green's function kernel

$$g'(z,\xi,\sigma)=\frac{\Re\ \cosh[r(1-|z-\xi|)]-\Re\cosh[r(-1+z+\xi)]}{2r\sinh r}-\frac{\frac{\Re_s}{\tau}\cosh[r'(1-|z-\xi|)]-\frac{\Re_s}{\tau}\cosh[r'(-1+z+\xi)]}{2r'\sinh r'}$$

for $\sigma>\max.\{-k^2(1,\tau)\}$.

$K(\sigma)$ defined in (17), which is a composition of certain integral operators, is termed as linearized stability operator. K (σ) depends analytically on σ in a certain right half of the complex plane. It is clear from the composition of K (σ) that it contain an implicit function of σ .

We shall examine the resolvent of the K (σ) defined as $[I-K(\sigma)]^{-1}$

$$[I-K(\sigma)]^{-1}=\{I-[I-K(\sigma_0)]^{-1}[K(\sigma)-K(\sigma_0)]\}^{-1}[I-K(\sigma_0)]^{-1} \tag{18}$$

If for all σ_0 greater than some a,

Remark (1) $[I-K(\sigma_0)]^{-1}$ is positive,

(2) $K(\sigma)$ has a power series about σ_0 in $(\sigma_0-\sigma)$ with positive coefficients; i.e., $\left(-\frac{d}{d\sigma}\right)^n K(\sigma_o)$ is positive for all n, then the right side of (18) has an expansion in $(\sigma_0-\sigma)$ with positive coefficients. Hence, we may apply the methods of Weinberger [1969] and Rabinowitz , to show that there exists a real eigenvalue σ_1 such that the spectrum of $K(\sigma)$ lies in the set $\{\sigma:\operatorname{Re}(\sigma)\le\sigma_1\}$. This is result is equivalent to PES, which was stated earlier as "the first unstable eigenvalue of the linearized system has imaginary part equal to zero."

4. RESULTS AND DISCUSSION:
THE PRINCIPLE OF EXCHANGE OF STABILITIES (PES)

It is clear that $K(\sigma)$is a product of certain operators. Condition (1) can be easily verified by following the analysis of Herron [2001,2002] for the present operator $K(\sigma)$, i.e. $K(\sigma)$ is a linear, compact integral operator, and has a power series about σ_0 in $(\sigma_0-\sigma)$with positive coefficients. Thus, $K(\sigma)$ is a positive operator leaving invariant a cone (set of non negative functions). Moreover, for σ real and sufficiently large, the norms of the operators $T(0)$ and $T(\Pr\sigma)$ become arbitrarily small. So, $\|K(\sigma)\|<1$. Hence, $[I-K(\sigma)]^{-1}$ has a convergent Neumann series, which implies that $[I-K(\sigma)]^{-1}$ is a positive operator. This is the content of condition (P1).

To verify condition (2), we note that. Green's function kernel g $\left(z,\xi,\frac{\sigma}{\Pr}\right)$ is the Laplace transform of the Green's function PrG $(z,\xi;\Pr t)$ for the initial-boundary value problem

$$\left(-\frac{\partial^2}{\partial z^2}+k^2+\frac{1}{\Pr}\frac{\partial}{\partial t}\right)G=\delta(z-\xi,t), \tag{19}$$

where, $\delta(z-\xi,t)$ is Dirac –delta function in two-dimension,
with boundary conditions $G(0,\xi;\Pr t)=G(1,\xi;\Pr t)=G(z,\xi;0)=0$,
then $G(z,\xi;\Pr t)\geq 0$.
and Green's function kernel $g'(z,\xi;\sigma)$ is the Laplace transform of the Green's function G' (z, $,\xi,t)$ defined by

$$G'(z,,\xi,t)=\Re G(z,\xi;t)-\Re_s G(z,\xi;\tau t), \tag{20}$$

If $\Re\geq\Re_s$ with $\tau<1$,
then $G'(z,,\xi,t)\geq 0$
With boundary conditions $G(0,\xi;t)=G(1,\xi;t)=G(z,\xi;0)=0$, (21)
Using the similar result proved in Herron [2000,2001] by direct calculation of the inverse Laplace transform, we have $K(\sigma)$ is a positive operator for all real $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and $\Re\geq\Re_s$ with $\tau<1$ together with g(z) positive in the flow domain.

Theorem. The PES holds for (12)- (14) when g(z) is nonnegative through out the fluid domain and $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and $\Re\geq\Re_s$ with $\tau<1$
Proof: As $[I-K(\sigma)]$ is a nonnegative compact integral operator for $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and for $\Re\geq\Re_s$ with $\tau<1$, which satisfied all the conditions of the Krein-Rutman theorem and hence it has a positive eigen value σ_1, which is an upper bound for the absolute values of all the eigenvalues, and the corresponding eigen function $\phi(\sigma)$ is nonnegative, which is essentially the contents of condition (2) stated in Remark 1.

We observe that
$$[I-K(\sigma)][\phi(\sigma)]=(1-\sigma_1)\phi\geq 0,$$
Thus, if $[I-K'''(\sigma)]$ is nonnegative, then $\sigma_1\leq 1$. The methods of Weinberger and Rabinowitz [1969] apply thereby showing that there exits a real eigenvalue $\sigma_1\leq 1$ such that the spectrum of $K(\sigma)$ lies in the set $\{\sigma\ |\operatorname{Re}(\sigma)\leq\sigma_1\}$.

This is equivalent to the PES.

5. CONCLUSIONS:

In this paper we have investigated the Thermohaline Convection Problem with variable gravity of Veronis Type.

It is established that if g(z) is positive; throughout the flow domain, then

PES is valid for Veronis' Thermohaline Convection if $\Re \geq \Re_s$ or $\lambda = \frac{\Re}{\Re_s} \geq 1$ and $\tau < 1$;
But thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.

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HYDROMAGNETIC UNSTEADY FLOW PAST AN INFINITE VERTICAL ACCELERATED PLATE WITH INDUCED MAGNETIC FIELD IN THE PRESENCE OF HEAT SOURCE AND VARIABLE TEMPERATURE

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ABSTRACT:
A theoretical analysis of an unsteady viscous incompressible electrically conducting fluid in an infinite vertical accelerated porous plate is presented. It is assumed that the plate is electrically non-conducting and the applied magnetic field is of uniform strength $(H_0)$and perpendicular to the plate. The plate temperature is raised with time $(t' > 0)$.The coupled non-linear partial differential equation are solved by using Laplace transform technique.The solution are expressed in terms of exponential andcomplimentary error function and the effect of flow parameters on velocity, temperature and induced magnetic field are presented through graphs.

KEY WORDS: Accelerated plate, Suction, Induced magnetic field, Heat source.

INTRODUCTION:
Free and forced convection flow of an electrically conducting fluid past a porous vertical surface under the influence of magnetic field occur in many industrial and technical applications which include plasma studies, the boundary layer control in aerodynamics , petroleum industries, MHD power generator, cooling of nuclear reactors and crystal growth. Alom[1] analyzed the Steady heat and mass transfer by mixed convection flow from induced magnetic field, constant heat and mass fluxes. Ahmed[2] presented the study of heat and mass transfer on free convective three-dimensional unsteady flow over a porous vertical plate. The analytic study of induced magnetic field with radiating fluid over a porous vertical plate was presented by Ahmed[3]. Ahmed and Chung[4] presented mixed convective three-dimensional heat and mass transfer flow with transversely periodic suction velocity. Ahmed *et al.*[5] analyzed the model of MHD mixed convective radiating fluid with viscous dissipative heat. Ahmed *et al*.[6] analyzed the MHD mixed convection and mass transfer from an infinite vertical porous plate with chemical reaction in the presence of heat source. Ahmed *et al*.[7] have presented a mathematical model of megnetohydrodynamic transient free and forced convective flow with induced magnetic field effects. Ahmed *et al*.[8] studied the effects of chemical reaction and radiation on an unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer. Beg *et al.*[9]obtained local non-similarity numerical solutions for the velocity, temperature and induced magnetic field distribution in forced convection hydromagnetic boundary layer, over an extensive range of magnetic Prandtl numbers and Hartmann numbers. Ghosh *et al.*[10] considered an exact solution for the hydromagnetic natural convection boundary layer flow past an infinite vertical flat plate under the influence of transverse magnetic field with magnetic induction effects.Hussain *et al.*[11] studied the effect of radiation on free convection from a porous vertical plate. Prakash *et al.*[12-13] investigated the Dufour effects on unsteady hydro magnetic radiative and diffusion-thermo on unsteady MHD flow through porous medium past an

impulsively started infinite vertical plate with variable temperature and mass diffusion.Raptis *et al.*[15-18] have studied MHD flow past a steadily moving infinite vertical porous plate with constant heat flux in the presence of radiation. MHD Couette flow and heat transfer in a rotating system have studied by Seth *et al.*[19] Seshaiah *et al.*[20] investigated the effects of induced magnetic field on free convective flow of radiative, dissipative fluid past a porous plate with temperature gradient heat source. Singha[21] analyzed the problem of MHD free convective flow of an electrically conducting fluid between the two heated parallel plates in the presence of an induced magnetic field. Singh and Singh[22] presented MHD effects on heat and mass transfer in flow of a viscous with induced magnetic field. MHD effects on flow of viscous fluid with induced magnetic field studied by Singh and Singh[23].

The objective of present study is to investigate the effect of induced magnetic field, heat source and variable temperature inhydromagnetic flow past a porous vertical accelerated plate.

FORMULATION OF PROBLEM:
Consider an unsteady, free convective flow of an incompressible, electrically conducting viscous fluid past an accelerated infinite vertical plate with variable temperature. We introduce a co-ordinate system with origin at the accelerated plate which is subjected to constant suction velocityV_0, the x'axis is taken along the plate in the upward vertical direction, y' axis is taken along normal to the plate directed in to the fluid region.

Let$(u', v', 0)$ be the fluid velocity and$(h'_x, h'_y, 0)$ be component of magnetic induction vector at a point(x', y', z') in the fluid. The $x'$is taken along the plate in the upward direction,y' is normal to the plate in to the fluid region. Since the plate is infinite in length inx' direction, therefore all the physical quantities except the pressure are assumed to be independent ofx'. Let$(u', v', 0)$be the fluid velocity at the$(x', y', z')$when time$(t' > 0)$. Initially$(i.e when t' \leq 0)$the plate is at rest relative to the fluid velocity $(u' = 0)$and thefluid at the plate's surface has the same temperature as at theedge of the boundary layer i.e. T'_∞ respectively. At time $(t' > 0)$the plate is accelerated with a velocity $u' = at'$ in its own planeand the temperature at the plate is raised linearly with respect to time

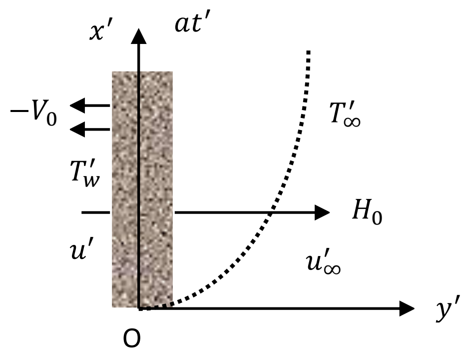

Fig.1-Geometrical configuration of the problem.

Under Boussinesq approximation the equations governing the flow are, Conservation of mass

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Gauss Law of magnetism

$$\frac{\partial h_y'}{\partial y'} = 0 \text{ which holds for } h_y' = H_0 = \text{constant} \tag{2}$$

Conservation of momentum

$$\rho \frac{\partial u'}{\partial t'} + \rho v' \frac{\partial u'}{\partial y'} = -\frac{\partial p}{\partial x'} - \rho g + \mu \frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0 \frac{\partial h'}{\partial y'} \tag{3}$$

Conservation of energy

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{S'}{\rho c_p}(T' - T_\infty') \tag{4}$$

Conservation of magnetic induction

$$\frac{\partial h_x'}{\partial t'} + v' \frac{\partial h_x'}{\partial y'} = \frac{1}{\sigma \mu_e} \frac{\partial^2 h_x'}{\partial y'^2} + H_0 \frac{\partial u'}{\partial y'} \tag{5}$$

Since there is no large velocity gradient here, the viscous term in equation (3) vanishes for small $\mu$and hence for the outer flow, beside there is no induced magnetic field along x'-direction gradient, so we have

$$-\frac{\partial p}{\partial x'} = \rho_\infty g \text{...(6)}$$

By eliminating the pressure term from equations (3) and (6), we obtain

$$\rho \frac{\partial u'}{\partial t'} + \rho v' \frac{\partial u'}{\partial y'} = (\rho_\infty - \rho) g + \mu \frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0 \frac{\partial h'}{\partial y'}. \tag{7}$$

By using Boussinesq approximation we have,

$$\rho_\infty - \rho = \rho_\infty \beta (T' - T_\infty') \tag{8}$$

On using (8) in the equation (7) and noting that is approximately equal to 1, the momentum equation reduces to

$$\rho \frac{\partial u'}{\partial t'} + \rho v' \frac{\partial u'}{\partial y'} = g\beta (T' - T_\infty') + \mu \frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0 \frac{\partial h'}{\partial y'} \tag{9}$$

The flow is governed by the following initial and boundary conditions:

$$t' \le 0 \quad u' = 0, T' \to T_\infty', = 0, h_x' = 0 \text{ for all } y \text{...(10)}$$

$$t' > 0 \begin{Bmatrix} u' = at', T' = T_\infty' + (T_w' - T_\infty')At', h_x' = 0, aty' = 0 \\ u' \to 0, T' \to T_\infty', \to 0, \; h_x' \to 0 \quad aty' \to \infty \end{Bmatrix} \tag{11}$$

whereu' and v'- denotes the velocity component in the boundary layer in direction x'-axis and y'-axis respectively; T'- the temperature inside the boundary layer;T_w'- temperature at the plate;T_∞'- the temperature of the free stream; β-thermal expansion; ρ- density of fluid, $A = \left(\frac{a^2}{\nu}\right)^{\frac{1}{3}}$;$a$- acceleration of the plate;t'- time;h_x'- induced magnetic field along x'direction;H_0-external applied magnetic field; g- acceleration due to gravity; μ_e-magnetic permeability; μ-kinematic viscosity,α- magnetic diffusivity.

From equation of continuity (1), it is clear that suction velocity normal to the plate is constant. Hence from the equation of the continuity we obtained:

$$v' = -V_0 \quad .(12)$$

Governing equations in non-dimensional form are:

$$\frac{\partial u}{\partial t} - w_0' \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + M \frac{\partial h}{\partial y} + G_r \theta. \tag{13}$$

$$P_r \frac{\partial \theta}{\partial t} - w_0 P_r' \frac{\partial u}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + P_r Q_0 \theta \text{...} \tag{14}$$

$$\frac{\partial h}{\partial t} - w_0' \frac{\partial h}{\partial y} = \frac{1}{P_{rm}} \frac{\partial^2 h}{\partial y^2} + M \frac{\partial u}{\partial y} \ldots \tag{15}$$

where the non-dimensional quantities used above are:

$$\left.\begin{array}{l} u = \frac{u'}{(\nu a)^{\frac{1}{3}}}, t = t'\left(\frac{a^2}{\nu}\right)^{\frac{1}{3}}, y = y'\left(\frac{a}{\nu^2}\right)^{\frac{1}{3}}, h = \frac{h_x'}{(\nu a)^{\frac{1}{3}}}\left(\frac{\mu_e}{\rho}\right)^{\frac{1}{2}} \\ w_0 = \frac{V_0}{(\nu a)^{\frac{1}{3}}}, M = \frac{\mu_e}{\rho}\frac{H_0}{(\nu a)^{\frac{1}{3}}}, G_r = \frac{g\beta(T' - T'_\infty)}{a}, P_r = \frac{\kappa}{\mu c_p} \\ P_{rm} = \sigma\mu_e\nu, Q_0 = \frac{s'}{\rho c_p (a^2/\nu)^{\frac{1}{3}}}, \alpha = (\sigma\mu_e)^{-1} \end{array}\right\} \ldots \tag{16}$$

The initial and boundary condition in dimensionless form are as follows:

$$t \le 0, u = 0, \theta = 0, h = 0 \quad \ldots(17)$$

$$t > 0 \begin{cases} u = t, \theta = t, h = 0 \ at y = 0 \\ u = 0, \theta = 0. h = 0 \ \ at y \to \infty \end{cases} \ldots(18)$$

Method of solution

The dimensionless governing equations (13) to (15) subjected to boundary conditions (17) and (18) are solved by using Laplace transform technique and transform to following set of equations:

$$\frac{d^2\bar{u}}{dy^2} + w_0 \frac{\partial \bar{u}}{\partial y} - S\bar{u} + M\frac{d\bar{h}}{dy} = -G_r\bar{\theta} \ldots \tag{19}$$

$$\frac{d^2\bar{\theta}}{dy^2} + w_0 P_r \frac{d\bar{\theta}}{dy} + P_r(Q_0 - S)\bar{\theta} = 0 \tag{20}$$

$$\frac{1}{P_{rm}}\frac{d^2\bar{h}}{dy^2} + w_0 \frac{\partial \bar{h}}{\partial y} - S\bar{h} + M\frac{d\bar{u}}{dy} = 0 \tag{21}$$

Corresponding boundary conditions:

$$\left.\begin{array}{l} \bar{u} = \frac{1}{S^2}, \bar{\theta} = \frac{1}{S^2}, \bar{h} = 0 \quad at y = 0 \\ \bar{u} = 0, \bar{\theta} = 0, \bar{h} = 0 \quad at y \to \infty \end{array}\right\} \tag{22}$$

Following Pande *et. al.*[14] we assume magnetic Prandtl number$P_{rm} = 1$, i.e. $\nu = \alpha$ which is a plausible assumption in most of the hydromagnetic problems, the solution of the equations (19) to (21) under the boundary condition $(22)$are given by following expressions.

$$\bar{\theta} = \frac{e^{-\left(\frac{w_0 P_r + \sqrt{w_0^2 P_r^2 - 4P_r(Q_0 - S)}}{2}\right)y}}{S^2} \ldots(23)$$

$$\bar{u} = \frac{1}{2}\left[\begin{array}{l} \frac{e^{-A_2 y}}{S^2} + \frac{G_r}{(P_r - 1)}\left\{\frac{M_1 e^{-A_2 y}}{S} + \frac{M_2 e^{-A_2 y}}{S^2} + \frac{M_3 e^{-A_2 y}}{S - \alpha_1} + \frac{M_4 e^{-A_2 y}}{S - \beta_1}\right\} \\ + \frac{e^{-A_3 y}}{S^2} + \frac{G_r}{(P_r - 1)}\left\{\frac{M_5 e^{-A_3 y}}{S} + \frac{M_6 e^{-A_3 y}}{S^2} + \frac{M_7 e^{-A_3 y}}{S - \alpha_2} + \frac{M_8 e^{-A_3 y}}{S - \beta_2}\right\} \\ - \frac{G_r}{(P_r - 1)}\left\{\begin{array}{c} (M_1 + M_5)\frac{e^{-A_1 y}}{S} + (M_2 + M_6)\frac{e^{-A_1 y}}{S^2} \\ + \frac{M_3 e^{-A_1 y}}{S - \alpha_1} + \frac{M_4 e^{-A_1 y}}{S - \beta_1} + \frac{M_7 e^{-A_1 y}}{S - \alpha_2} + \frac{M_8 e^{-A_1 y}}{S - \beta_2} \end{array}\right\} \end{array}\right] \ldots \tag{24}$$

$$\bar{h} = \frac{1}{2}\left[\begin{array}{c} \frac{-e^{-A_2 y}}{S^2} - \frac{G_r}{(P_r-1)}\left\{\frac{M_1 e^{-A_2 y}}{S} + \frac{M_2 e^{-A_2 y}}{S^2} + \frac{M_3 e^{-A_2 y}}{S-\alpha_1} + \frac{M_4 e^{-A_2 y}}{S-\beta_1}\right\} \\ + \frac{e^{-A_3 y}}{S^2} + \frac{G_r}{(P_r-1)}\left\{\frac{M_5 e^{-A_3 y}}{S} + \frac{M_6 e^{-A_3 y}}{S^2} + \frac{M_7 e^{-A_3 y}}{S-\alpha_2} + \frac{M_8 e^{-A_3 y}}{S-\beta_2}\right\} \\ + \frac{G_r}{(P_r-1)}\left\{\begin{array}{c} (M_1 + M_5)\frac{e^{-A_1 y}}{S} + (M_2 + M_6)\frac{e^{-A_1 y}}{S^2} \\ + \frac{M_3 e^{-A_1 y}}{S-\alpha_1} + \frac{M_4 e^{-A_1 y}}{S-\beta_1} - \frac{M_7 e^{-A_1 y}}{S-\alpha_2} - \frac{M_8 e^{-A_1 y}}{S-\beta_2} \end{array}\right\} \end{array}\right] \ldots \quad (25)$$

Taking inverse Laplace transforms of the equations(23)to(25),we obtained the following expression for the velocity, temperature and induced magnetic field:

$$u = \left[\begin{array}{c} -Z_4(\eta_1 e^{X_1} erfc\eta_1 - \eta_2 e^{X_2} erfc\eta_2) + Z_5(e^{X_1} erfc\eta_1 + e^{X_2} erfc\eta_2) \\ +Z_6(e^{X_3} erfc\eta_3 + e^{X_4} erfc\eta_4) + Z_7(e^{X_5} erfc\eta_5 + e^{X_6} erfc\eta_6) \\ -Z_8(\eta_7 e^{X_7} erfc\eta_7 - \eta_8 e^{X_8} erfc\eta_8) + Z_9(e^{X_7} erfc\eta_7 + e^{X_8} erfc\eta_8) \\ +Z_{10}(e^{X_9} erfc\eta_9 + e^{X_{10}} erfc\eta_{10}) + Z_{11}(e^{X_{11}} erfc\eta_{11} + e^{X_{12}} erfc\eta_{12}) + \\ Z_{12}(\eta_{13} e^{X_{13}} erfc\eta_{13} - \eta_{14} e^{X_{14}} erfc\eta_{14}) - Z_{13}(e^{X_{13}} erfc\eta_{13} + e^{X_{14}} erfc\eta_{14}) \\ -Z_6(e^{X_{15}} erfc\eta_{15} + e^{X_{16}} erfc\eta_{16}) - Z_7(e^{X_{17}} erfc\eta_{17} + e^{X_{18}} erfc\eta_{18}) \\ -Z_{10}(e^{X_{19}} erfc\eta_{19} + e^{X_{20}} erfc\eta_{20}) - Z_{11}(e^{X_{21}} erfc\eta_{21} + e^{X_{22}} erfc\eta_{22}) \end{array}\right] \ldots \quad .(26)$$

$$\theta = 2Z_3[-\eta_{13} e^{X_{13}} erfc\eta_{13} - \eta_{14} e^{X_{14}} erfc\eta_{14}] \quad (27)$$

$$h = \left[\begin{array}{c} Z_4(\eta_1 e^{X_1} erfc\eta_1 - \eta_2 e^{X_2} erfc\eta_2) - Z_5(e^{X_1} erfc\eta_1 + e^{X_2} erfc\eta_2) \\ -Z_6(e^{X_3} erfc\eta_3 + e^{X_4} erfc\eta_4) - Z_7(e^{X_5} erfc\eta_5 + e^{X_6} erfc\eta_6) \\ -Z_8(\eta_7 e^{X_7} erfc\eta_7 - \eta_8 e^{X_8} erfc\eta_8) + Z_9(e^{X_7} erfc\eta_7 + e^{X_8} erfc\eta_8) \\ +Z_{10}(e^{X_9} erfc\eta_9 + e^{X_{10}} erfc\eta_{10}) + Z_{11}(e^{X_{11}} erfc\eta_{11} + e^{X_{12}} erfc\eta_{12}) - \\ Z_{14}(\eta_{13} e^{X_{13}} erfc\eta_{13} - \eta_{14} e^{X_{14}} erfc\eta_{14}) + Z_{15}(e^{X_{13}} erfc\eta_{13} + e^{X_{14}} erfc\eta_{14}) \\ +Z_6(e^{X_{15}} erfc\eta_{15} + e^{X_{16}} erfc\eta_{16}) + Z_7(e^{X_{17}} erfc\eta_{17} + e^{X_{18}} erfc\eta_{18}) \\ -Z_{10}(e^{X_{19}} erfc\eta_{19} + e^{X_{20}} erfc\eta_{20}) - Z_{11}(e^{X_{21}} erfc\eta_{21} + e^{X_{22}} erfc\eta_{22}) \end{array}\right] \quad (28)$$

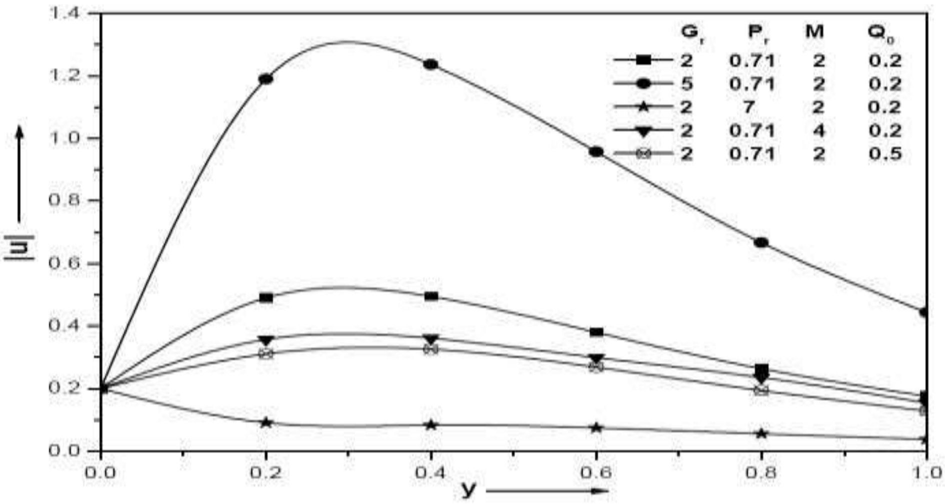

Fig.2- Variation of amplitude of velocity with $t = 0.2$..

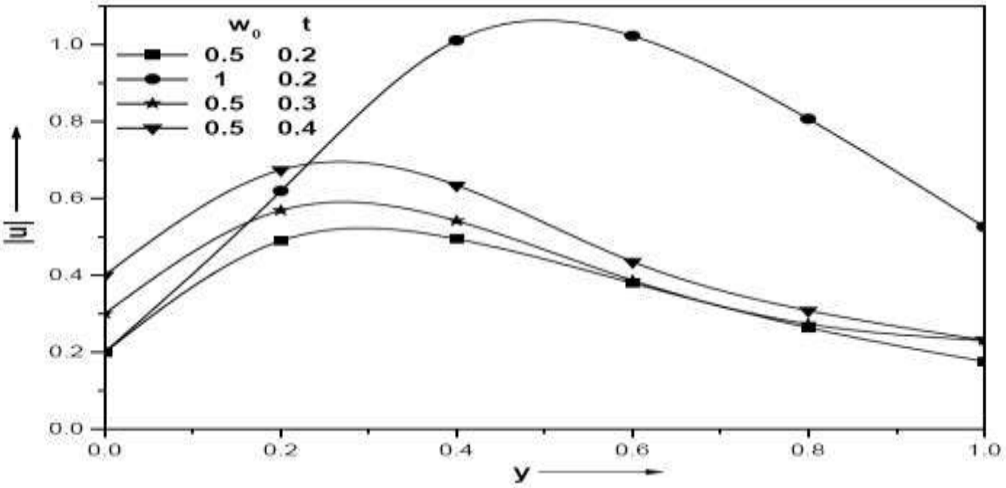

Fig.3- Variation of amplitude of velocity at $G_r = 2, P_r = 0.71, M = 2, Q_0 = 0.2$.

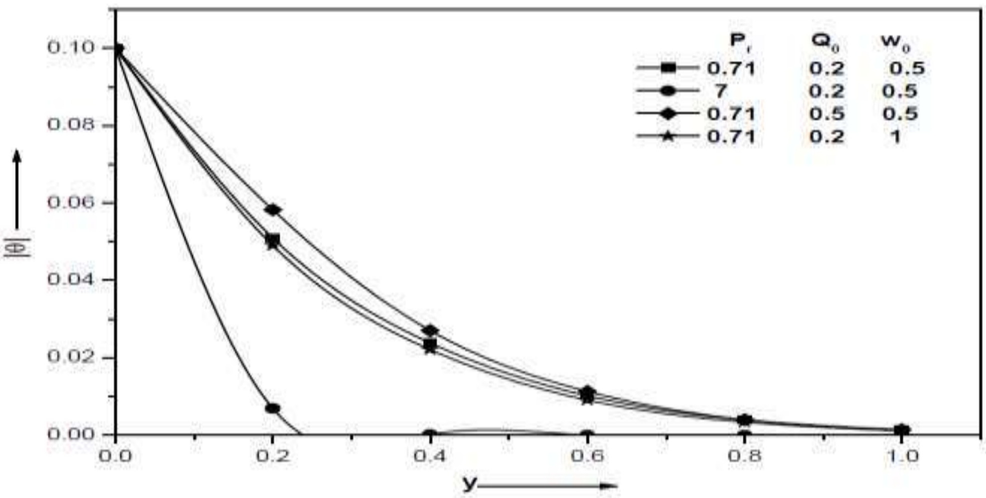

Fig.4- Variation of amplitude of temperature at $t = 0.2$.

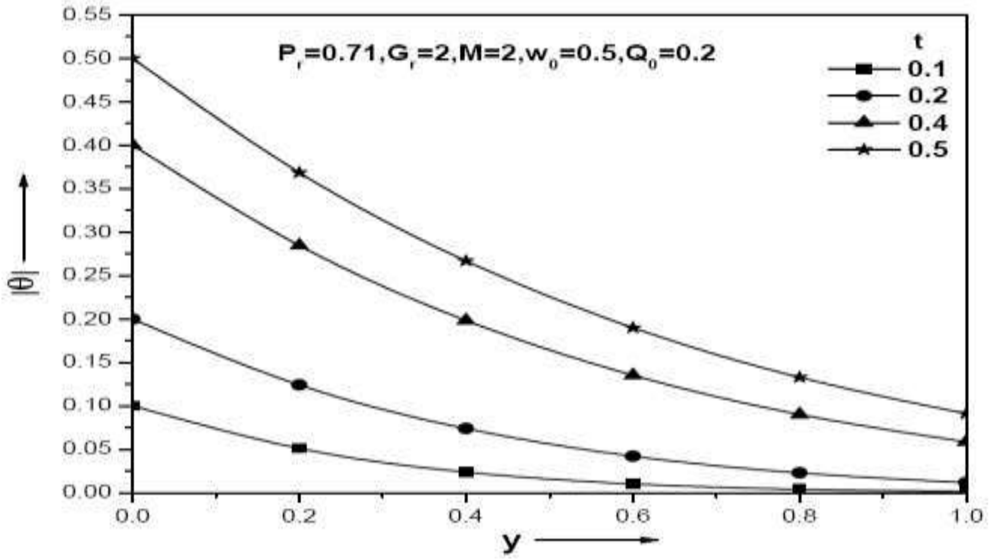

Fig.5- Variation of amplitude of temperature.

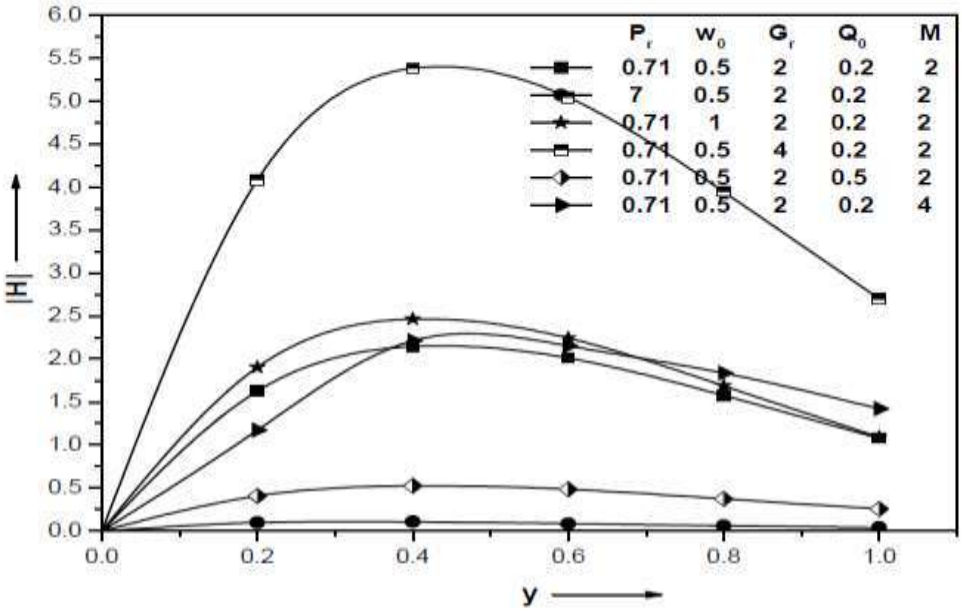

Fig.6- Variation of amplitude of induced magnetic field at $t = 0.2$.

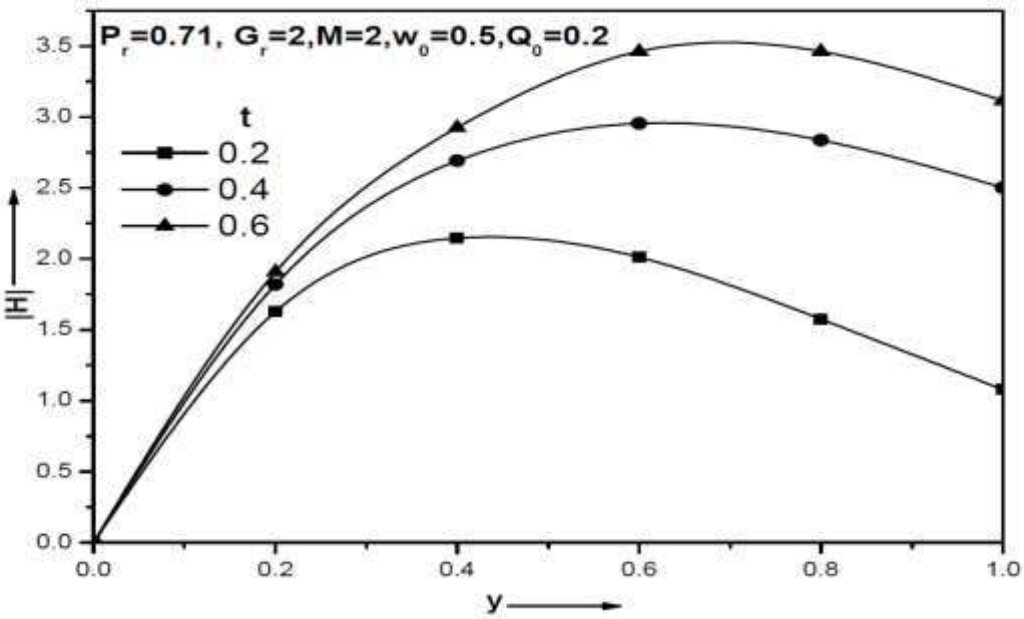

Fig.7- Variation of amplitude of induced magnetic field.

RESULTS AND DISCUSSION:

In order to have a physical view of the problem, we computed the numerical calculations for non-dimensional amplitude of velocity profile, amplitude of temperature profile and amplitude of induced magnetic field at the plate for different values of physical parameters involved and these value have been demonstrated in graphs. Our investigations are restricted to Prandtl number 0.71 which corresponds to air. The values of other parameters namely Grashoff number, Hartmann number, suction parameter, heat source parameter and timeare chosen arbitrarily. Our results are in good agreement with the result of Ahmed S[3] in the absence of mass transfer and viscous dissipation.

From fig.2 observed that with the increase of Grashoff number (G_r)amplitude of velocity increases and with increase of Prandtl number(P_r), Hartmann number and heat source parameter amplitude of velocity decreases. Fig.3. Clearly show that increase in the value of suction parameter and time cause rise in amplitude of velocity.From fig.4&5 it is noticed that fluid temperature rises due to the increasing value of heat source parameter, time and it decreases with the increase of Prandtl number $(P_r)$and suction parameter(w_0). From fig. 6 & 7 it is observed that amplitude of induced magnetic field rises with the increase in suction parameter, Grashoff number, time and it fall with the rise in Prandtl number, heat source parameter(Q).

CONCLUSIONS:

The main conclusion of this study is:

(i) An increase in Hartmann number andheat source parameter leads to decelerate the flow velocity.

(ii) It is observedthat with the increase in suction parameter the induced magnetic field and the velocity of the fluid increase.

(iii) Fluid temperature increases with the increase in heat source parameter and decrease with Prandtl number.

(iv) Fluid temperature and its velocity increases with the time.

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REMOTE SENSING AND THEIR HISTORICAL DEVELOPMENT

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ABSTRACT:
The science of remote sensing has emerged as one of the most fascinating subjects over the past three decades. Earth observation from space through various remote sensing instruments has provided a vantage means of monitoring land surface dynamics, natural resources management, and the overall state of the environment itself.
Remote sensing is defined, for our purposes, as the measurement of object properties on the earth's surface using data acquired from aircraft and satellites. Remote sensing systems, particularly those deployed on satellites, provide a repetitive and consistent view of the earth that is invaluable to monitoring the earth system and the effect of human activities on the earth.

KEYWORDS: Fascinating, Monitoring, Deployed, Vantage.

INTRODUCTION:
A huge quantity of Earth observation and geospatial data is produced daily by numerous satellites launched by several worldwide space agencies. The processing of remote sensing data requires several steps. The extraction of prominent information from remote sensing data requires a coordinated use of many applications and algorithms. Sharing computational resources among different scientists represents the sole way to approach the problem in order to achieve good performance.

In this paper I describe advantages and disadvantages of remote sensing and their problems andhistorical development. The science of remote sensing has emerged as one of the most fascinating subjects over the past three decades. Earth observation from space through various remote sensing instruments has provided a vantage means of monitoring land surface dynamics, natural resources management, and the overall state of the environment itself.

WHAT IS REMOTE SENSING:
Remote sensing is the science of obtaining information about objects or areas from a distance, typically from aircraft or satellites. Remote sensors can be either passive or active. Passive sensors respond to external stimuli. They record natural energy that is reflected or emitted from the Earth's surface.

Remote means something which is far away and sensing means getting information or getting data.It is the process of detecting and monitoring the physical characteristics of an area by measuring its reflected and emitted radiation at a distance from the targeted area. Special cameras collect remotely sensed imagesof the Earth, which help researchers "sense" things about the Earth. It is obtaining information about an area or phenomenon through a device that does not touch the area or phenomenon under study. Your eyes are a good example of remote sensing instrument. Many devices are onboard satellites that monitor the Earth from space.

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Some Examples Are

- Cameras on satellites and airplanes take images of large areas on the Earth's surface, allowing us to see much more than we can stand on the ground.
- Sonar systems on ships can be used to create images of the ocean floor without needing to travel to the bottom of the ocean.
- Cameras on satellites can be used to make images of temperature changes in the oceans

Some specific uses of remote sensing

- Large forest fires can be mapped from space, allowing rangers to see a much larger area than from the ground.
- Tracking clouds to help predict the weather or watch erupting volcanos, and help watch for dust storms.
- Tracking the growth of a city and changes in farmland or forests over several years or even decades.
- Mapping the ocean bottom - Discovery and mapping of the rugged topography of the ocean floor (e.g., huge mountain ranges, deep canyons, and the "magnetic striping" on the ocean floor)

How it work: It has seven steps to work the remote sensing.

Energy source: The first requirement for remote sensing is to have an energy source which illuminate or provide electromagnetic energy to the target of interest.

Radiation and atmosphere: As the energy travels from its source to the target, it will come in contact with or interact with the atmosphere it passes through; this interaction may take place a second as the energy travels from the target to the sensor.

Interaction with target: once the energy makes it way to the target through the atmosphere, it interacts with the target depending upon the properties of both the target and radiation.

Recording of energy by the sensor: after the energy has been scattered by, emitted from the target, we require a sensor to collect and record the electromagnetic radiation.

Transmission, reception and processing: the energy required by the sensors has to be transmitted, often in electronic form, to receive and processing section where the data are produced into an image.

Interpretation and analysis: The proceed image is interpreted in two types visually and digitally. To exact information about the target which was illuminated.
Applications: The final element of remote sensing process is achieved when we apply the information- we have been able to extract from the imagery about the target in order to better understand and reveal some new information.

SENSORS: Remote sensing instruments are of two primary types - active sensors and passive sensors:

Active sensors: They provide their own source of energy to illuminate the objects they observe. LIDAR, RADAR etc.
Passive sensors, they detect natural energy (radiation) that is emitted or reflected by the object or scene being observed. ASTER, Quickbard, Ikonos, Landsat etc.

ADVANTAGES AND DISADVANTAGES:
ADVANTAGES:
- Large area coverage.
- Some remote sensors operate in all seasons, at night and in bad weather.
- Remote sensing allows repetitive coverage which comes in handy when collecting data on dynamic themes such as water, agricultural fields and so on.
- Remote sensors "see" over a broader portion of the spectrum than the human eye.
- Remotely sensed data can easily be processed and analyzed fast using a computer and the data utilized for various purposes.

DISADVANTAGES:
- Remote sensing is a fairly expensive method of analysis especially when measuring or analyzing smaller areas.
- Remote sensing requires a special kind of training to analyze the images. It is therefore expensive in thetechnology since extra training must be accorded to the users of the technology.
- It is expensive to analyze repetitive photographs if there is need to analyze different aspects of the photography features.
- It is humans who select what sensor needs to be used to collect the data, specify the resolution of the data and calibration of the sensor, select the platform that will carry the sensor and determine when the data will be collected. Because of this, it is easier to introduce human error in this kind of analysis.

QUESTION: WHY USE SATELLITES TO STUDY THE EARTH?

ANSWER: Consistent, routine, global measurements
- Overview of information on the hemispheric, regional, national and local scales – the "big picture"
- Provide information in areas where there are no grounds – based measurements
- Advance warning of impending environmental events and disasters.
- Visual appeal: a picture is worth thousand words.

HISTORICAL DEVELOPMENT:
The history of remote sensing began with the invention of photograph. The term photograph is derived from two words 'phos' means light and 'graphy' means writing.
- In year 1038 AD AL Hazen an Arabian Mathematician explain the principle of camera obscura to observe solar eclipse.
- After 1666 Newton experimenting with a prism and found that when a white light passes through a prism then it splits into seven different colours i.e. (VIBGYOR)
- In 1800 Sir William Herschel measures the temperature of light splits with a prism into spectrum of visible colours. He had discovered thermal infrared electromagnetic radiation.

- In 1858 GFT Nadar takes the first aerial photograph from a captative balloon from an altitude of 1200 feet in Paris.
- In 1889 Arthur Batut takes the first photo by using a kite in France.
- In 1903 BP Crop uses pigeon to transmit message and take arial photograph.
- In 1957 SPUTNIK 1 launched by Russia (world first artificial satellite launched 4 October 1957).
- In 1958 EXPLORER 1 launched by USA.
- In 1970 DONG FANG HONG 1 launched by China.
- In 1975 ARYABHATTA, in 1979 BHASKAR 1, in 1980 ROHINI 1 launched by India.
- In 1982 INSAR 1A launched by India.
- In 1998 IRS (INDIAN REMOTE SENSING)
- In 2001 GSAT 1
- In 2004 EDUSAT
- In 2005 CHARTOSAT
- In 2008 CHANDRAYAAN
- In 2013 MANGALYAN launched by India (5^{th} Nov. 2013 - 24 sept. 2014)
- And recently in 2019 CHANDRAYAAN 2 launched by ISRO in India that was the biggest achievement for India
- The Chandrayaan 2 has achieved 95% of itsmission objectives, the lander's unsuccessful bud to touch-down on the lunar surface notwithstanding. And this all thanks to the chandrayaan-2 orbiter. Everything is not lost. The payloads will conduct

CONCLUSION:
Remote sensing is the gathering of information concerning the earth's surface that does not involve contact with the surface or object under study. The techniques include aerial photography, multi-spectral, and infrared imagery, and radar. With the help of remote sensing, we can able to get accurate information about the earth's surface including its components like forests, landscapes, water resources, oceans, etc. This information helps the researchers to their research activities about the earth's components concerning its sustainable management and conservation and so on.

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A CROSS SECTIONAL HOSPITAL BASED SURVEY IN NORTH INDIA ON THE KNOWLEDGE, ATTITUDE ANDPRACTICE TOWARDS CERVICAL CANCER AMONG WOMEN ATTENDING THE OUTPATIENT DEPARTMENT OF OBSTETRICS AND GYNECOLOGY

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ABSTRACT:

Cervical cancer is the second most common cancer in women worldwide after breast cancer. India contributes to one-fourth of the global burden. Cervical cancer related deaths among women in India are often due to late diagnosis of disease. Knowledge about disease and early screening is the most effective method of cervical cancer prevention. Lack of awareness, negative attitude and poor screening are the major causes to increase the incidence of disease.
AIM: to assess the knowledge, attitude and practice among the general population towards cervical cancer screening and prevention.
MATERIAL AND METHOD: Hospital based cross sectional study. A total of 200 consecutive subjects attending the Gynecology OPD in a tertiary care hospital were enrolled after informed consent.
STATISTICAL ANALYSIS: descriptive statistics were used to represent the socio-demographic characteristics and levels of knowledge, attitude and practice levels.
RESULTS: most of the subjects enrolled in the study (78%) had heard about cervical cancer and majority of them had heard from friends and family (41%) , media (12%). Only 54% women knew about symptoms , while only 33% knew the risk factors. 60% of the women knew about screening methods. More than half (52%) had a positive attitude towards screening but more than two thirds (78%) are not practicing screening methods.
CONCLUSION: Although women are having good knowledge towards cervical cancer screening and prevention, still there are many who need to be educated regarding the same. Amongst the ones who have knowledge, there is a gap to transform it into practice. There is a need for more educational programs for different strata of society for larger reforms.

INTRODUCTION:

Cervical cancer is the second most common cancer in the women around the world[1]. It is second only to breast cancer. India contributes to one fourth of the global burden. According to National Institute of Cancer Prevention and Research, one woman dies every 8 minutes in India because of Cervical Cancer[2]. Cervical cancer causes 10% of all cancer related deaths in India. The stage at diagnosis is the main factor for the outcome and 5-year survival rates. The average 5 year survival after diagnosis is 48%. In India, most of the cases are diagnosed at a much advanced stage, and hence have a poor survival rate. The main reason for delayed diagnosis is lack of awareness amongst women regarding the screening and prevention of cervical cancer. Screening for cervical cancer is essential as the women are mostly asymptomatic until the disease has far progressed. The screening methods not only make earlier diagnosis of cancer of cervix but also help in catching the premalignant lesions of the cervix. The most common symptoms are inter-menstrual bleeding (bleeding in between periods), persistent back pain, pelvic pain, foul smelling discharge, bleeding after intercourse, urinary urgency, weight loss etc. Cervical cancer is caused by Human Papilloma Virus (HPV) infection, particularly HPV 16 and 18. These HPV 16 and 18 cause 75% of the cases globally[6]. Other risk factors include multiple partners, early age of marriage, multiple child births and HIV infection.

Although there have been many advances like Pap smear test and HPV DNA analysis in the screening for cervical cancer, there have been multiple hurdles for the same. The major problems include lack of awareness and misconceptions regarding female cancers and gynecological diseases and socio-economic limitations. According to latest guidelines issued by the American College of Obstetricians and Gynecologists, women between age 21-29 years should have a Pap test done every 3 years, and those between 30-65 years should have co-testing (Pap test and HPV DNA testing) every 5 years[3].The prevention and control of disease depends upon awareness, knowledge, preventive measures and screening procedures[4]. There is lack of information regarding knowledge, attitude and practice towards cervical cancer and its screening and hence this study was conducted.

AIM : to assess the knowledge, the practice and the overall attitude of women regarding cervical cancer and screening in women attending the department of Obstetrics and Gynecology at a tertiary care hospital and to assess the relationship of these with socio-demographic characters of the population.

MATERIAL AND METHODS:
It is a hospital based cross-sectional study conducted over a period of 3 months in the outpatient department of Obstetrics and Gynecology in a tertiary care hospital between July 2019 to September 2019. 200 parous women were recruited in the study who were over 18 years of age and were willing to participate in the study. All women who participated in the study were assessed using a preformed questionnaire about cervical cancer, screening and prevention.

The questionnaire comprised of 4 parts: socio-demographic characters, knowledge, practice and attitude. The demographic details included age, residence (rural/urban), and socio-economic stage using modified Kuppuswamy's scale[5]. Knowledge regarding the disease was identified using a questionnaire which had ten questions and each question was awarded 1 mark for appropriate knowledge and nil for either misconceptions or no knowledge.

S.No.	Knowledge about	1 mark	Zero marks
1.	Awareness of Ca cervix	Yes	No
2.	Symptoms	Any 2	≤1
3.	Risk factors	Any 1	0
4.	Prevention method	Any 1	0
5.	Screening methods	Yes	No
6.	Description about screening methods	yes	No
7.	Eligibility for screening	yes	no
8.	Frequency of screening	yes	No
9.	Vaccination	yes	No
10.	Treatment available	yes	No

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Score of (A) 8 – 10 was considered good knowledge
(B) 5 – 7 = moderate knowledge
(C) <5 = Poor knowledge

Attitude was assessed using 8 statements related to cervical cancer, its screening and management. The responses were assessed using Likert scale. In Likert scale the responses ranged from strongly agree (5), agree (4), neither agree nor disagree (3), disagree (2) and strongly disagree (1).

S. no.	Statement	5	4	3	2	1
1.	Cervical cancer is highly prevalent in India					
2.	Any adult woman including you can acquire this					
3.	Cervical cancer does not spread via contact					
4.	Symptoms like intermenstrual bleeding and foul discharge pv need consultation by a doctor					
5.	Screening helps prevent it					
6.	Screening is not harmful					
7.	Screening is not expensive and is					
8.	HPV vaccination can prevent cancer					

Maximum score expected is 40 from all statements and minimum is 8. A score of ≥20 is considered a positive attitude and <20 as a negative attitude.

Practice was assessed by response towards screening in last 3 years. If patient had undergone screening in last 3 years, it was considered as regular practice, more than 3 years since last screen were considered irregular and the ones who were never screened as no practice.

STATISTICAL ANALYSIS:

Epi- info software was used to analyze collected data. Frequencies and proportions were used to represent the socio-demographic profile , knowledge, attitude and practice for carcinoma of the cervix ad its screening procedures in the study population.

RESULTS:

Among the 200 women who answered the questionnaire, most were between 35 to 45 years of age (66%). The mean age of women was 38.4 years. Majority of the women were multi-parous, i.e. with more than 1 live birth. The study population comprised of 74% women belonging to the rural area. Amongst the subjects 13% women had received no formal education, 34% were having matriculate certificate and the rest were graduate. 82% of the study population were home-makers, and 26% belonged to upper lower class, 42% into the lower middle class and 32% to the upper middle class according the modified kuppuswamy scale.

78% had heard about the cervical cancer and majority of them 41% ha heard from friends and family members, 12% from media and rest 25% from medical personnel. Regarding knowledge about signs and symptoms of the cancer of the cervix, only 54% of the women knew about 2 or more symptoms. 40% knew about foul smelling discharge from vagina and irregular spotting. Only 33% of the women had an idea about the risk factors contributing to cervical cancer. Out of these 200 women 60% of them knew about screening methods mostly Pap smear only. On assessing the attitude towards screening and prevention of the same, 52% had a positive attitude towards it, only 22% were practicing the screening procedure. The 22% women who had been previously screened , they were screened once only and had no information regarding the protocol for screening.

DISCUSSION:

The present study explored the knowledge, attitude and practice among women attending the obstetrics and gynecology department in a tertiary care hospital in northern India, with a resource limited setting. Here the responders were mostly from the lower socio-economic strata, and many of them had no formal education.

The study found out that more than two-thirds of the population had heard about cervical cancer, which was similar to the studies conducted by Chande HM et al[6]. and Abdullahi et al[7]. the results of this study are in contrast with other studies conducted in southern India by G.Narayana et al[8] and in other under-developed countries by Anorlu[9], Yifru and Asheber[10] in which more than one fourth of the population had no information regarding cervical cancer.

This study found out that more than half of the population knew about the signs and symptoms of the disease, which are consistent with the findings of Mukama et al[11]. and with G.narayana[8] et al. Still there is lack of awareness regarding the risk factors for the same.

More than two thirds of the women showed a positive attitude towards cervical cancer and more than half of the about its screening, which is again consistent with findings of G.Narayana[8] et al. Despite positive attitude among women, there is a huge difference between attitude and practice. Less than one fourth of the women had been screened in the last 3 years and only 6% had been screened more than once. None of the women knew about the screening protocol and the need for repeated screening.

Younger women i.e. between the ages 30-35 years had a positive attitude and good knowledge about signs, symptoms and screening methods. Similar findings were in the study conducted by Ogunbode and Ayinde[12].

STRENGTHSAND LIMITATIONS:

There is need to conduct such studies at greater levels with larger sample size as these data are useful to design educational programs regarding cervical cancer. The najor limitation was a smaller sample size and a shorter time frame.

CONCLUSION:

The study concludes that though women have fairly good knowledge and attitude towards cervical cancer and its screening, there is a need to bridge the gap between knowledge and practice. There also is a need to educate the masses about preventive HPV vaccination.

CONFLICTS OF INTEREST :

none.

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A REVIEW OF DEEP VEIN THROMBOSIS TREATMENT

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ABSTRACT:
Using the pretest probability score calculated from the Wells Clinical Prediction Rule, patients are stratified into three risk groups – high, moderate or low. The results from duplex ultrasound are incorporated as well. If the patient is in the high or moderate risk and the duplex ultrasound study is positive, treat for DVT. If the duplex study is negative and the patient is in low risk, DVT has been ruled out. If the patient is in high risk but the ultrasound study was negative, the patient still has a significant probability of DVT. However, if the patient is in low risk but the ultrasound study is positive, then before treating for DVT it is better to go for second confirmatory study such as a Venogram.

KEYWORDS: Deep Vein Thrombosis, Venogram.

Introduction of Deep Vein Thrombosis (DVT):
DVT usually occurs in a deep leg vein, a larger vein that runs through the muscles of the calf and the thigh.

It can cause pain and swelling in the leg and may lead to complications such as pulmonary embolism. This is a serious condition that occurs when a piece of blood clot breaks off into the bloodstream and blocks one of the blood vessels in the lungs (see below).

DVT and pulmonary embolism together are known as venous thromboembolism (VTE).

Deep vein thrombosis (DVT) is a blood clot that develops within a deep vein in the body, usually in the leg

We aim to critically review the current state of knowledge on this subject, with a view to updating clinicians on the prophylaxis and treatment of DVT.

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Symptoms of DVT:
In some cases, there may be no symptoms of DVT. If symptoms do occur they can include:
- pain, swelling and tenderness in one of your legs (usually your calf)
- a heavy ache in the affected area
- warm skin in the area of the clot
- red skin, particularly at the back of your leg below the knee

DVT usually (although not always) affects one leg. The pain may be worse when you bend your foot upward towards your knee.

Objectives of DVT treatment:
The primary **objectives** for the **treatment** of **deep venous thrombosis** (**DVT**) are to prevent pulmonary embolism (PE), reduce morbidity, and prevent or minimize the risk of developing the postthrombotic syndrome (PTS).
If left untreated, about one in 10 people with a DVT will develop a pulmonary embolism. A pulmonary embolism is a very serious condition which causes:
- breathlessness which may come on gradually or suddenly
- chest pain which may become worse when you breathe in
- sudden collapse

Both DVT and pulmonary embolism require urgent investigation and treatment.
Seek immediate medical attention if you have pain, swelling and tenderness in your leg and you develop breathlessness and chest pain.
As well as age, there are also a number of other risk factors, including:
- having a history of DVT or pulmonary embolism
- having a family history of blood clots
- being inactive for long periods such as after an operation or during a long journey
- blood vessel damage a damaged blood vessel wall can result in the formation of a blood clot
- having certain conditions or treatments that cause your blood to clot more easily than normal such as cancer (including chemotherapy and radiotherapy treatment), heart and lung disease, thrombophilia and hughes syndrome
- being pregnant your blood also clots more easily during pregnancy
- being overweight or obese

Treatment Options:
- Anticoagulation
- Thrombolytic Therapy for DVT
- Surgery for DVT
- Filters for DVT
- Compression Stockings

Anticoagulation:
Treatment for DVT usually involves taking anticoagulant medicines, which reduce the blood's ability to clot and stop existing clots getting bigger.

Heparin and warfarin are two types of anticoagulant often used to treat DVT. Heparin is usually prescribed first because it works immediately to prevent further clotting. After initial treatment, you may also need to take warfarin to prevent another blood clot forming.

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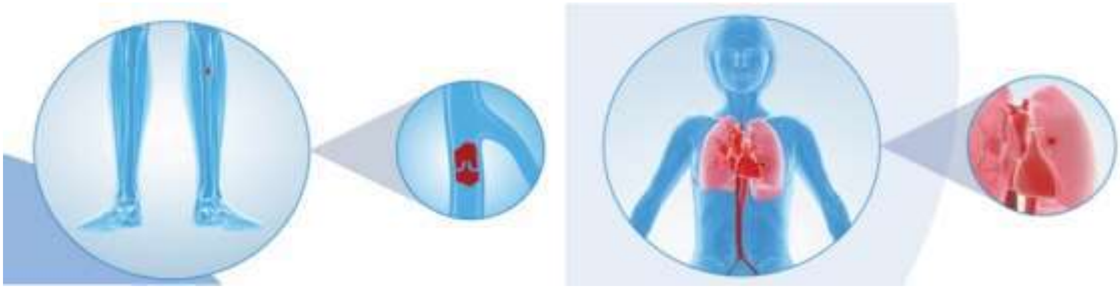
A number of anticoagulants, known as directly acting oral anticoagulants (DOACs), may also be used to treat conditions such as DVT. These medications include rivaroxaban and apixaban, and they've been shown to be as effective as heparin and warfarin with less serious side effects.

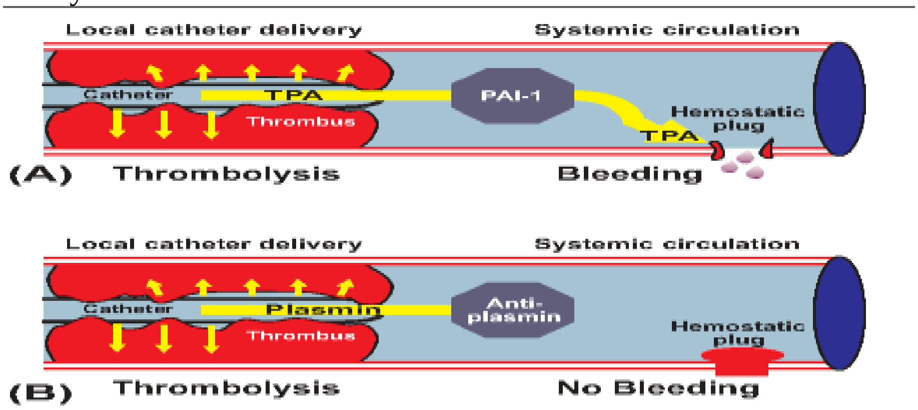


Anticoagulant Treatment for DVT

Thrombolytic Therapy for DVT:

A mechanical thrombectomy device can remove venous clots, but it is recommended only as an option when the following conditions apply: "Iliofemoral DVT, symptoms for less the seven days (criterion used in the single randomized trial), good functional status, life expectancy of greater than equal to one year, and both resources and expertise are available. Thrombolytic Therapy does not prevent

- Clot propagation
- Rethrombosis, or
- Subsequent embolisation
- Heparin therapy and oral anticoagulant therapy always must follow a course of thrombolysis



Thrombolytic Therapy for DVT:

Surgery for DVT:

Open surgical thrombectomy has a very limited role in the management of acute DVT. The limitations to the procedure are obvious, and the results are ambiguous at best. For this reason, open surgical thrombectomy for DVT is reserved as a last resort for those patients with threatened limb loss secondary to extensive DVT.

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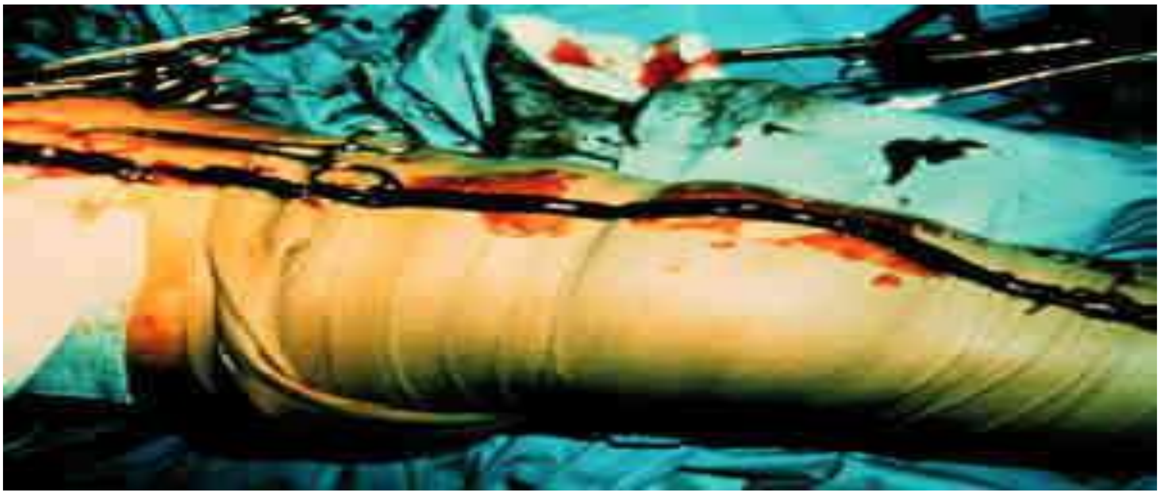
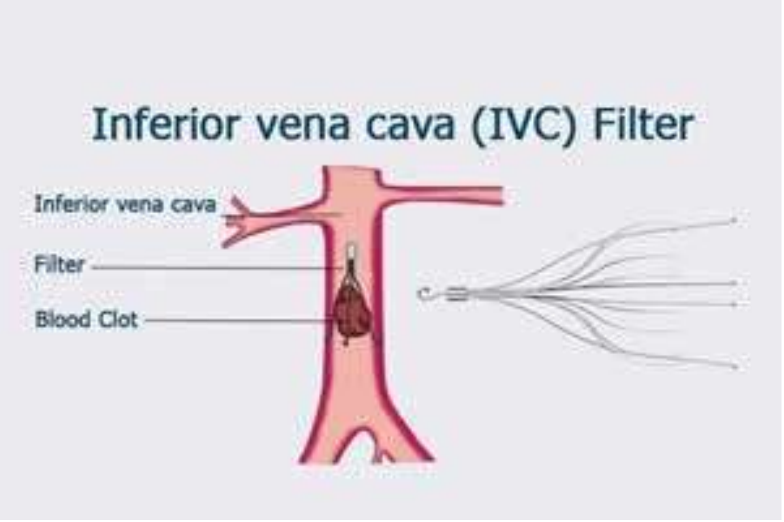


This patient underwent a thrombectomy. The thrombus has been laid over the approximate location in the leg veins where it developed

Filters for DVT:
An IVC **filter** is a small metal device that can stop blood clots in your veins from moving. It's used for conditions in which there's a chance that a blood clot could enter your lungs, such as **deep vein thrombosis** (**DVT**). It's placed in your body's main vein, called the inferior vena cava (IVC)



COMPRESSION STOCKINGS:
A compression stocking is a device that uses pressure to help prevent and ease symptoms of various thrombotic conditions such as deep vein thrombosis (DVT) and Post Thrombotic Symdrome (PTS). Compression stockings are specially made, snug-fitting, stretchy stockings that gently squeeze the leg.

Complications:

- Acute pulmonary embolism
- Postthrombotic Syndrom
- Blood clot in the kidney, called renal vein thrombosis
- Blood clot in the heart, leading to heart attack
- Blood clot in the brain, leading to stroke
- Chronic venous insufficiency

CONCLUSION:

Using the pretest probability score calculated from the Wells Clinical Prediction Rule, patients are stratified into three risk groups – high, moderate or low. The results from duplex ultrasound are incorporated as well. If the patient is in the high or moderate risk and the duplex ultrasound study is positive, treat for DVT. If the duplex study is negative and the patient is in low risk, DVT has been ruled out. If the patient is in high risk but the ultrasound study was negative, the patient still has a significant probability of DVT. However, if the patient is in low risk but the ultrasound study is positive, then before treating for DVT it is better to go for second confirmatory study such as a Venogram.

An ultrasound scan can be used to detect clots in your veins. A special type of ultrasound called a Doppler ultrasound can also be used to find out how fast the blood is flowing through a blood vessel. This helps doctors identify when blood flow is slowed or blocked, which could be caused by a blood clot. A venogram may be used if the results of a D-dimer test and ultrasound scan can't confirm a diagnosis of DVT. During a venogram, a liquid called a contrast dye is injected into a vein in your foot. The dye travels upthe leg and can be detected by X-ray , which will highlight a gap in the blood vessel where a clot is stopping the flow of blood.

TREATING DVT

Treatment for DVT usually involves taking anticoagulant medicines, which reduce the blood's ability to clot and stop existing clots getting bigger.

Heparin and warfarin are two types of anticoagulant often used to treat DVT. Heparin is usually prescribed first because it works immediately to prevent further clotting. After initial treatment, you may also need to take warfarin to prevent another blood clot forming.

A number of anticoagulants, known as directly acting oral anticoagulants (DOACs), may also be used to treat conditions such as DVT. These medications include rivaroxaban and apixaban, and they've been shown to be as effective as heparin and warfarin with less serious side effects.

You'll also be prescribed compression stockings to wear every day, which will improve your symptoms and help prevent complications.
These may include:

- not smoking
- eating a healthy, balanced diet
- taking regular exercise
- maintaining a healthy weight or losing weight if you're obese

There's no evidence to suggest that taking aspirin reduces your risk of developing DVT.
See your GP before embarking on long-distance travel if you're at risk of getting a DVT, or if you've had a DVT in the past.

When taking a long-distance journey (six hours or more) by plane, train or car, you should take steps to avoid getting DVT, such as drinking plenty of water, performing simple leg exercises and taking regular, short walking breaks.

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AN OVERVIEW OF THE APPLICATIONS OF MAGNETIC NANOPARTICLES IN THE BIOMEDICAL FIELD

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ABSTRACT:
In this paper, the biomedical applications of magnetic nanoparticles are summarized. As a result of the special physical properties, the magnetic nanoparticles offer some attractive possibilities in biomedicine. They have controllable sizes ranging from a few nanometers up to tens of nanometers, which places them at dimensions that are smaller than or comparable to those of a cell (10–100μm), a virus (20–450nm), a protein (5–50nm) or a gene (2nm wide and 10–100nm long). This means that they can 'get close' to a biological entity of interest. Firstly, a short introduction of magnetic nanoparticles is presented. Secondly, the applications of magnetic nanoparticles in biomedical field are summarized. In medicine, the application of magnetic nanoparticles in magnetic separation technique, as therapeutic agent in hyperthermia, a targeted drug delivery carrier as well as contrast agents in magnetic resonance imaging (MRI) are explained. Lastly, a perspective of the magnetic nanoparticles in biomedicine in future is also described.

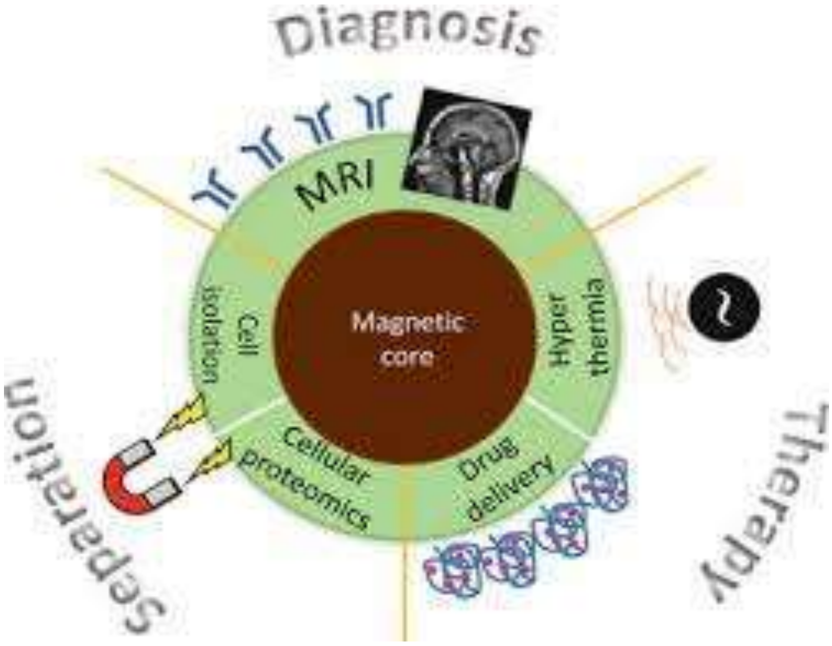
KEYWORDS: Magnetic nanoparticles, magnetic separation techniques, magnetic hyperthermia, targeted drug delivery, MRI contrast agent.

1. INTRODUCTION:

Magnetic nanoparticles consist of magnetic elements such as iron, cobalt, nickel, manganese, gadolinium, and their alloys, oxide compounds, cation complexes along with polymers etc., which shows ferromagnetism, paramagnetism or even superparamagnetism. The physical and chemical properties of magnetic nanoparticles largely rely on the chemical components, crystal structures, sizes and shapes, sometimes the source of magnetic nanoparticles synthesized. Besides the four basic effects on general nanomaterials (the quantum size effect, the surface effect, the small size effect and the macroscopic quantum tunnelling effect), the magnetic nanoparticles also possess some special magnetic properties such as superparamagnetism, high coercivity, low Curie temperature and high magnetic susceptibility.

The magnetic nanoparticles are widely used in catalysis, mineralogy (such as the selection of ores), informatics (such as data storage), environmental science (such as pollutants concentration), as well as biomedicine due to their attractive properties in physics and chemistry. With the rapid development of nanotechnology, magnetic nanoparticles are particularly attractive for biology and medicine fields. The small size is able to benefit the interaction between magnetic nanoparticles and bioentities and enhances the ability of biomolecules to cover the nanoparticle surface. The magnetic nanoparticles abide by magnetic Coulomb's law and are easily controlled by an external magnetic field. This long-distance interaction combined with the penetrativity of the magnetic field is helpful for developing applications in transportation and immobilization of magnetic nanoparticles-labelledbioentities in human tissues (such as delivering antitumor drugs or shuttling the isotope atoms to some targeted areas).The magnetic nanoparticles are able to respond the cyclic variation of the external magnetic field, obtain the energy from the excitation field and transport the heat to the targeted area, for instance different tumors. As a matter of fact, magnetic nanoparticles are able to be used as a heat enhancement agent for chemotherapy and radiotherapy, because the heated tissues can destroy effectively the malignant cancer cells.

Due to the good magnetic guidance, biocompatibility, biodegradability, and functional groups, the magnetic nanoparticles can easily be conjugated with many functional molecules or units, such as enzyme, antibody, cell, DNA and RNA. Moreover, the surface of magnetic nanoparticles is easily modifiable based on different needs. The usually used modification agents are poly (ethylene glycol) (PEG), glucosan, polyvinyl pyrrolidone (PVP), aliphatic acid, polyvinyl alcohol (PVA), peptides, gelatin, chitosan, methylsilanes and lipids, such as liposomes, etc. These modification agents improve the biocompatibility of magnetic nanoparticles, decrease aggregation, prevent protein absorption, prolong their time in blood circulation system, reduce their toxicity, and enhance their targeting. As a result, the magnetic nanoparticles are widely used in biology and medicine.



2. APPLICATIONS OF MAGNETIC NANOPARTICLES IN BIOMEDICAL FIELD

2.1. MAGNETIC BIOSEPARATIONTECHNIQUE:

Magnetic bioseparation technique is a separation technique, in which bioentities attached to the surface of nanomaterials with magnetic susceptibility are extracted from their bio-environment using an external magnetic force. At present, the usually used magnetic nanomaterials for bioseparation purpose are the superparamagnetic nanoparticles, because they are easily magnetized under an external magnetic field. After the removal of external magnetic field, the bioentities captured by the superpapramagnetic nanoparticles are immediately diffused into the solutions once again. There are two steps related to the magnetic bioseparation technique: (i) The bioentities are conjugated and labeled with the (superpara-) magnetic nanomaterials; (ii) The labeled bioentities are separated from the solutions in a magnetic separator. The magnetic bioseparation technique has been widely used in the separation and purification of bioentities of different types, for example cells, bacteria, proteins and nucleic acids.

2.2. MAGNETIC HYPERTHERMIA

Magnetic hyperthermia is a method of cancer treatment based on magnetic nanoparticles producing heat under an alternating magnetic field. Under an alternating magnetic field of appropriate amplitude and frequency, the magnetic nanoparticles which are put inside or around a tumor area will raise the tumor temperature. If the temperature is over 45˚, it could kill the tumor cells by necrosis. Or if the temperature is reached around 42˚, it could improve the efficiency of chemotherapy. The concept of magnetic hyperthermia using small particles was proposed in 1957 by Gilchrist and others, but most of the studies were inexact thermometry and poor AC magnetic field parameters in animal models. In 1993 Jordan and his colleagues found that iron oxide nanoparticles exhibit an extraordinary specific absorption rate (SAR [W/g]). This was the renaissance of a cancer treatment method using hyperthermia techniques and made a big step since then. The size and the component of nanoparticles have a great influence on their hyperthermia properties. The usually used magnetic nanoparticles for hyperthermia are composed of iron oxide nanoparticles, though some metallic nanoparticles of cobalt, iron or FeCo compared to iron oxide increase their magnetization and the maximum SAR values. Hardly were these metallic nanoparticels used because of their toxicity concerned.

2.3. TARGETED DRUG DELIVERY:

The idea of using magnetic particles as a carrier tool for drug delivery was conceived in the late 1970s by Widder, Senyei and their colleagues. The basic premise of this idea is that therapeutic agents such as targeted chemotherapy or therapeutic nucleic acid molecules are attached to, or encapsulated within a magnetic particle core. And this magnetic particle core with a layer of polymer or metal coating enables to be functionalized easily. After being functionalized, the magnetic particle-therapeutic agent conjugate is injected or administrated orally into the blood-stream system. Under the guidance of a strongly external magnetic field, the magnetic particle-therapeutic agent composites are guided to the area of the targeted sites and the drugs are delivered.

2.4. MRI CONTRAST ENHANCEMENTS:

Magnetic resonance imaging (MRI) is a non-invasive way greatly depending on the relaxation properties of proton nuclei in water and lipids showing images of the inside of an object. MRI is widely used in imaging to show pathological or other physiological changes of living tissues in the body. In comparison with CT, MRI has some advantages: (i) to employ nonionizing radio frequency (RF) signals to obtain its images and is best matched to noncalcified tis-sues in the body, (ii) to be able to detect various features in tissues via varying scanning parameters, (iii) to create cross-sectional images in any images besides oblique planes, (iv) to be superior to detect and identify tumors, (v) to be best suited for multiple times examination successively within a short period of time, (vi) to provide multiple contrastmechanisms for example: T1 weighted, T2 weighted, and T2* weighted MR images.

In MRI, T1 relaxation also called spin-lattice or longitudinal relaxation that is a time constant of nuclear spins returning to equilibrium. When nuclei are from the high-energy state to the low energy what is related to loss of energy to the surrounding nuclei. T1 relaxation is characterized by the longitudinal return of the net magnetization to its ground state of maximum length along with a direction of the main magnetic field. T1 is usually around 1s for tissue. T2 relaxation also named spin-spin or trans-verse relaxation that is a time constant

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of signal decay. T2 relaxation happens during spins in the high and low energy state exchanging energy but not releasing energy to the surrounding lattice. The magnetic moments interact with each other making a decrease in the transverse magnetization or decay after nuclei release their excess energy. T2 is usually less than 100 ms for general tissue. T2* is a time that occurs for the transverse magnetization to decay to 37% of its original magnitude. It is produced under an inhomogeneous magnetic field and happens in all magnets. It is characterized by inhomogeneous B_0 and loss of transverse magnetization at a rate greater than T2.

3. FUTURE PERSPECTIVES:

We have summarized applications of magnetic nanoparticles relied on their magnetic properties in biology and medicine of five aspects: magnetic separation, magnetic hyperthermia treatment, targeted drug delivery, MRI enhancement media and magnetofection technique.

(1)In magnetic separation field, these magnetic nanoparticles are being widely used in cells, bacteria, protein, and nucleic acids etc. based on their interactions with the above-mentioned biomoieties integrated with rapidly developingbiosensor techniques. This technique has entered many laboratories for bioanalytics, cell biology, molecular biology, biomedical analysis, and bioassay. There are some related commercial parts or devices for sale, for example magnetic separation beads, and magnetic-based micro total analysis system (μTAS), etc. In future, the magnetic separation technique should enable to integrate with the intelligent technology to carry out separation, purification, detection and analysis all in one line and the separation devices should also be minimized.

(2)As for magnetic hyperthermia treatment area, although there have been over 17 years after Jordan and their colleagues made a break-through for hyperthermia treatment of tumors based on magnetic nanoparticles, magnetic hyperthermia treatment for tumors in humans is only within a Germany hospital in Berlin, despite having been proven to be effective in different animal models from many laboratories and medical schools around the world. As a result, the magnetic hyperthermia treatment techniques will enter more and more hospitals to benefit for the patients in the near future after the face front problems such as the treatment temperature control and the magnetic nanomaterials with high SAR developed, etc., being overcome.

(3)About targeted drug delivery field based on magnetic nanoparticles, in such systems after the targeted molecules for crossing the cell membrane or traveling mechanosensitive ion channels in the membrane such as peptides, antibodies, and nucleic acids being attached on the surface of magnetic nanoparticles to form a multifunctional nanocomposite, the nanocomposite is able to shuttle the therapeutic drugs into the tumor nidi, release the drugs and make a therapy. It is a future and final tendency that the delivery drugs targeted, the release of the drugs controllable and the therapy at the tumor nidi are concentrated on a multifunctional nanocomposite or nanodevice based on magnetic nanoparticles.

(4) Although there are a lot of exciting and interesting applications of magnetic nanoparticles in biology and medicine, considerable problems remainto be resolved before they benefit more for the health of human beings, for instance (i) the precisely control of the size of magnetic nanoparticles; (ii) the solubility in aqueous solutions that is a bottleneck for their

applications both *in vitro* and *in vivo*; (iii) the biocompatibility, which is a barrier of the magnetic nanoparticles to be used in biomedicine *in vivo*; (iv) the stability, which is a key factor whether the magnetic nanoparticles enable to produce their functions; (v) the toxicity that is a gate whether the magnetic nanoparticles enable to be used in clinic later. As a result, the magnetic nanoparticles would attract much more interest and could lead to new opportunities in both biology and medicine after the above-mentioned challenges are resolved finally.

(5) To magnofection technique, it was introduced about two decades, compared with other transfection techniques based on cell biology, this technique provides a simple way to make a lot of cells at once with very high transfection rate. Up to now, magnetofection technique has been applied to all types of natural and artificial nucleic acids, nonviral transfection, viruses and been tested on many cell lines including different stem cells and primary cells. Improving the transfection rate is still the goal of magnetofection based on magnetic nanoparticles in the near future.

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FIBER OPTICS COMMUNICATION

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ABSTRACT:
Fiber optics is the major building block in the telecommunication infrastructure. Fiber optic systems are important telecommunication infrastructure for world-wide broadband networks. Wide bandwidth signal transmission with low delay is a key requirement in present day applications. Optical fibers provide enormous and unsurpassed transmission bandwidth with negligible latency, and are now the transmission medium of choice for long distance and high data rate transmission in telecommunication networks. This paper gives an overview of fiber optic communication systems including their key technologies, and also discuss their technological trend towards the next generation.

INTRODUCTION :

The major driving force behind the widespread use of fiber optics communication is the high and rapidly increasing consumer and commercial demand for more telecommunication capacity (larger than both wireless connections and copper cable). Advances in technology have enabled more data to be conveyed through a single optical fiber over long distances. The transmission capacity in optical communication networks are significantly improved using wavelength division multiplexing.

A desirable feature for future optical networks is the ability to process information entirely in the optical domain for the purpose of amplification, multiplexing, de-multiplexing, switching, filtering and correlation, since optical signal processing is more efficient than electrical signal processing. Several new classes of optical communication networks are presently emerging. For example, Code Division Multiple Access networks using optical signal processing techniques have recently being introduced.

Despite the associated benefits of utilizing optical fiber for communication (such as its high reliability over long distances, low attenuation, low interference, high security, very high information capacity, longer life span and ease of maintenance), research is still on going to further improve on the present fiber optics communication system, and also to solve some of the challenges facing it. Future optical communication systems are envisioned to be more robust than the present system.

Basic Principles of Fiber Optic Communication:

Fiber optic communication is a communication technology that uses light pulses to transfer information from one point to another through an optical fiber. The information transmitted is essentially digital information generated by telephone systems, cable television companies and computer systems. An optical fiber is a dielectric cylindrical waveguide made from low-loss materials, usually silicon dioxide. The core of the waveguide has a refractive index a little higher than that of the outer medium (cladding), so that light pulses is guided along the axis of the fiber by total internal reflection. Fiber optic communication systems consists of an optical transmitter to convert an electrical signal to an optical signal for transmission through the optical fiber, a cable containing several bundles of optical fibers, optical amplifiers to boost the power of the optical signal, and an optical receiver to reconvert the received optical signal back to the original transmitted electrical signal.

Optical fibers fall into two major categories, namely : step index optical fiber, which include single mode optical fiber and multimode optical fiber, and graded index optical fiber. Single mode step index optical fiber. Single mode step index optical fiber has a core diameter less than 10 micrometers and only allows one light path. Multimode step index optical fiber has a core diameter greater than or equal to 50 micrometers and allows several light paths, this leads to modal dispersion.

Evolution of Fiber Optics Communication:
Optical fiber was first developed in 1970 by Corning Glass Work. At the same time, GaAs semiconductor lasers were also developed for transmitting light through the fiber optic cables. The first generation fiber optic system was developed in 1975, it used GaAs semiconductor lasers, operated at a wavelength of 0.8μm, and bit rate of 45 Megabits/second with 10Km repeater spacing.

In the early 1980's, the second generation of fiber optic communication was developed, it used InGaAsP semiconductor lasers and operated at a wavelength of 1.3 μm. By 1987, these fiber optic systems were operating at bit rates of up to 1.7 Gigabits/second on single mode fiber with 50 Km repeater spacing. The third generation of fiber optic communication operating at a wavelength of 1.55 μm was developed in 1990. These systems were operating at a bit rate of up to 2.5 Gigabits/second on a single longitudinal mode fiber with 100Km repeater spacing.

The fourth generation of fiber optic systems made use of optical amplifiers as a replacement for repeaters, and utilized wavelength division multiplexing (WDM) to increase data rates. By 1996, transmission of over 11,300Km at a data rate of 5 Gigabits/second had been demonstrated using submarine cables..

Future Trends in Fiber Optics Communication:
Fiber optics communication is definitely the future of data communication. The evolution of fiber optic communication has been driven by advancement in technology and increased demand for fiber optic communication. It is expected to continue into the future, with the development of new and more advanced communication technology. Below are some of the envisioned future trends in fiber optic communication.

A. All Optical Communication Networks :
An all fiber optic communication is envisioned which will be completely in the optical domain, giving rise to an all optical communication network. In such networks, all signals will be processed in the optical domain, without any form of electrical manipulation. Presently, processing and switching of signals take place in the electrical domain, optical signals must first be converted to electrical signal before they can be processed, and routed to their destination. After the processing and routing, the signals are then re-converted to optical signals, which are transmitted over long distances to their destination. This optical to electrical conversion, and vice versa, results in added latency on the network and thus is a limitation to achieving very high data rates.

Another benefit of all optical networks is that there will not be any need to replace the electronics when data rate increases, since all signal processing and routing occurs in the optical domain. However, before this can become a reality, difficulties in optical routing, and wavelength switching has to be solved. Research is currently on going to find and effective solution to these difficulties.

B. Multi-Terabit Optical Network :

Dense wave Division Multiplexing (DWDM) paves the way for multi-terabit transmission. The world-wide need for increased bandwidth availability has led to the interest in developing multi-terabit optical networks. Presently, four terabit networks using 40Gb/s data rare combined with 100 DWDM channels exists. Researchers are looking at achieving even higher bandwidth with 100Gb/s. With the continuous reduction in the cost of fiber optic components, the availability of much greater bandwidth in the future is possible.

C. Intelligent Optical Transmission Network:

Presently, traditional optical networks are not able to adapt to the rapid growth of online data services due to the unpredictability of dynamic allocation of bandwidth, traditional optical networks rely mainly on manual configuration of network connectivity, which is time-consuming, and unable to fully adapt to the demands of the modern network. Intelligent optical network is a future trend in optical network development, and will have the following applications: traffic engineering, dynamic resource route allocation, special control protocols for network management, scalable signaling capabilities, bandwidth on demand, wavelength rental, wavelength wholesale, differentiated services for a variety of Quality of Service levels, and so on. It will take some time before the intelligent optical network can be applied to all levels of the network, It will first be applied in long-haul networks, and gradually be applied to the network edge.

D. Ultra – Long Haul Optical Transmission:

In the area of ultra-long haul optical transmission, the limitations imposed due to imperfection in the transmission medium are subject for research . Cancellation of dispersion effect has prompted researchers to study the potential benefits of soliton propagation. More understanding of the interactions between the electromagnetic light wave and the transmission medium is necessary to proceed towards an infrastructure with the most favorable conditions for a light pulse to propagate.

E. Improvements in Laser Technology:

Another future trend will be the extension of present semiconductor lasers to a wider variety of lasing wavelengths. Shorter wavelength lasers with very high output powers are of interest in some high density optical applications. Presently, laser sources which are spectrally shaped through chirp managing to compensate for chromatic dispersion are available. Chirp managing means that he laser is controlled such that it undergoes a sudden change in its wavelength when firing a pulse, such that the chromatic experienced by the pulse is reduced. There is need to develop instruments to be used to characterize such lasers.

F. Laser Neural Network Nodes:

The laser neural network is an effective option for the realization of optical network nodes. A dedicated hardware configuration working in the optical domain and the use of ultra-fast photonic sections is expected to further improve the capacity and speed of telecommunication networks become more complex in the future, the use of optical laser neural nodes can be an effective solution.

G. Polymer Optic Fibers:

Polymer optic fibers offer many benefits when compared to other data communication solutions such as copper cables, wireless communication system and glass fiber. In comparison with glass optical fibers, polymer optical fibers provide an easy and less expensive processing of optical signals, and are more flexible for plug interconnections. The use of polymer optical fibers as the transmission media for aircrafts is presently under research by different Research and Development groups due to its benefits. Also, in the future, polymer optical fiber will likely displace copper cables for the last mile connection from the telecommunication company's last distribution box and the served end consumer…

H. High – Altitude Platforms:

Presently, optical satellite links and orbit-to-ground links exists, the latter suffering from unfavorable weather conditions. Current research explores optical communication to and from high altitude platforms. High altitude platforms are airships situated above the clouds at heights of 16 to 25 Km, where the unfavorable atmospheric impact on a laser beam is less severe than directly above the ground. As shown in figure 3, optical links between high-altitude platforms, if a high-altitude platform functions as a data relay station.

I. Improvements in Optical Transmitter/Receiver Technology:

In fiber optics communication, it is important to achieve high quality transmission even for optical signals with distorted waveform and low signal to noise ratio during transmission. Research is on-going to develop optical transceivers adopting new and advanced modulation technology, with excellent chromatic dispersion and Optical Signal to Noise Ratio (OSNR) tolerance, which will be suitable for ultra-long haul communication systems. Also, better error correction codes, which are more efficient than the present BCH concatenated codes are envisioned to be available in the nearest future.

J. Improvement in Optical Amplification Technology:

Erbium Doped Amplifier (EDFA) is one of the critical technologies used in optical fiber communication system, In the future, better technologies to enhance EDFA performance will be developed. In order to increase the gain bandwidth of EDFA, better gain equalization technology for high accuracy optical amplification will be developed. Also, in order to achieve a higher output power, and a lower noise figure, high power pumping lasers that possess excellent optical amplification characteristics with outputs of more than +20dBm, and very low noise figure are envisioned to exist in the nearest future.

K. Advancement in Network Configuration of Optical Submarine Systems:

In order to improve the flexibility of network configuration in optical submarine communication systems, it is expected that the development of a technology for configuring the mesh network will be a step in the right direction. Presently, most large scale optical submarine systems adopt the ring configuration. By adopting the optical add/drop multiplexing technology that branches signals in the wavelength domain, it is possible to realize mesh network configuration that directly inter-connects the stations. Research is ongoing, and in the future such network configuration will be common.

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L. Glass Fiber Design and Component Miniaturization:

Presently, various impurities are added or remove from the glass fiber to change its light transmitting characteristic. The result is that the speed with light passes along a glass fiber can be controlled, thus allowing for the production of customized glass fibers to meet the specific traffic engineering requirement of a given route. This trend is anticipated to continue in the future, in order to produce more reliable and effective glass fibers. Also, the miniaturization of optical fiber communication components is another trend that is most likely to continue in the future.

CONCLUSION:

The fiber optics communications industry is an ever evolving one the growth experienced by the industry has been enormous this past decade. There is still much to be done to support the need for faster data rates, advanced switching techniques and more intelligent network architectures that can automatically change dynamically in response to traffic patterns and at the same time be cost efficient. The trend is expected to continue in the future as breakthroughs already attained in the laboratory will be extended to practical deployment thereby leading to a new generation in fiber optics communications. The future is bright. Just remember, the information superhighway is paved with glass.

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NANOTECH IMPROVISED LITHIUM ION BATTERY – FUTURE BATTERY FOR NEW GENERATION AUTOMOBILES: A REVIEW PAPER

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ABSTRACT:
Greater emphasis on clean and green transport technologies are because of climate and environment issues and are being well addressed by the adoption of zero emission Electric Vehicles (EVs).EVs being battery driven ensures energy security by replacing fossil fuel resources having an alarming status. Lithium Ion Batteries (LIBs)are the most preferred contenders that offer high energy density, high power density. However, issues like underperformance, fire contagiousness, high cost and slow charging speed of batteries have motivated researchers to further improve these batteries by using nanostructured materials to overcome these hurdles in its applications. Another important performance parameter is the battery SOC (State Of Charge)and its proper estimation by guiding mathematical principles enhances battery life. This review papercompares the various materials used for the fabrication of key components of LIBs and highlights the potential contribution of Nanotechnology to improve different structural and performance parameters of LIBs to make them more efficient and cost effective and finally focus on proper and accurate estimation of SOC of batteries to enhance their life cycle.Opportunities, challenges and latest researchin battery materials/ technology will alsobe discussed in this paper.

Keywords: Electric Vehicles, Lithium ion batteries, nanotechnology, SOC

1. INTRODUCTION:
Global transport emissions are still on an increasing trend despite progress in electrification of vehicles and their introduction. People still prefer to go for more efficient diesel vehicles which outweigh the impact of higher shares of electric vehicle sale globally. The initiative to move away from using fossil fuels as the energy source for transport use is necessary to ensure energy security, environmental conservation and revenue protection. The difficulties in controlling the GHG emissions and the over-dependence of fossil fuels play major roles in shaping the future of transportation. Multiple alternate energy sources are available that can power modern EVs but currently Lithium ion battery (LIB) is the preferred choice as it displays a reasonablyhigh voltage, high power density and high energy density. A major limitation of LIB is that it should be used within suitable voltage and temperature ranges as any violation in terms of operating voltage leads to safety and performance issues. Other challenges in LIB system are higher costs, durability and recharging issues of batteries that promote advancements in technology to ensure more efficiency. Moreover, according to USABC (United States Advanced Battery Consortium) present LIBs do not meet the standards of EVs. So to overcome the energy challenges, investments are being made on nanotechnology research for new and advanced batteries [1][2][3][4]. Further, LIBs require careful monitoring to avoid deterioration of battery as well as situations leading to explosions. SOC of a battery which determines the available capacity of battery is an important parameter that reflects its performance and extends battery life. So accurate SOC estimation of LIB is essential for a successful application in EVs [5]. LIB is a main component integrated in the battery management system (BMS) of an electric vehicle. To

prevent the dangerous situations during overcharging and to improve battery performance considerably BMS of EVs perform the vital operation of SOC estimation. BMS uses battery models and algorithms to determine the state of charge of battery [6].
This paper highlights keytechnological developments in LIB components to improve its performance for transport applications. This paper also discuss various mathematical techniques adopted for accurate SOC estimation of LIBs for EV applications and their recent trends.

2. LIB: A potential energy source for Electric vehicles:

Electric vehicles powered by electricity are advantageous over conventional vehicles for having lower operating costs and no tailpipe emissions.Electric vehicles are expected to increase to 22% of global share by 2030 [7]. There is a wide variety of electric energy sources that can power EVs like batteries, fuel cells, ultra capacitors and ultra-high speed flywheels etc. Among these energy sources, batteries in particular have been developed to power a diverse range of applications in transportation. In this paper we shall focus on Lithium ion batteries as the potential energy sources preferred by automation industry. The success of an Electric vehicle is governed by the efficiency of its energy storage device. John Goodenough [8] created the first lithium-ion batteries in 1980. Commercial lithium-ion batteries such as lithium cobalt oxide (LCO), lithium iron phosphate (LFP), lithium manganese oxide (LMO), lithium nickel manganese cobalt oxide (NMC), lithium nickel cobalt aluminium oxide (NCA), and lithium titanate oxide (LTO) have been widely accepted by electric vehicles in recent years [9]. LIBs have a fabulous combination of high energy and power density that makes it the technology of choice for EVs[10] [11] [12] [13] [14]. Still higher energy storage for delivering higher power for longer span, getting rid of thermal runaway problem [15], higher rate capabilities are some of the expectations for which a lot of research work is going on. New fabrication materials and techniques are being adopted to achieve the success in overcoming the barriers for widespread use of LIBs.

2.1 Battery electrodes:

The electrodes of LIB, both anode and cathode are made of materials that have the ability to be easily intercalated with lithium ions. The electrodes should also have high electrical conductivity so that the LIB can have high charging rates. Moreover the electrode material should remain consistent over a significant number of charge-discharge cycles that will ensure higher battery life and hence low costs. Also higher cell voltage is desired for achieving high energy density. Thus there has always been an effort to keep innovating and improving the existing electrode materials' chemistries to achieve higher operating voltages, high stability, long life and resistant to chemicals [16][17].Lithium cobalt oxide $LiCoO_2$ (~140mAhg^{-1}),Lithium transition metal oxides $LiMn_2O_4$(~110mAhg^{-1}), and Lithium Nickel Manganese Cobalt Oxide (NMC) $LiNiMnCoO_2$ (~154mAhg^{-1})proved to be good options for battery cathode [18][19][20][13] but Lithium iron phosphate($LiFePO_4$) (~170mAhg^{-1}) cathodes with higher specific capacity [21] are more commonly used as they are chemically stable, inherently safe and thermally stable in the operating temperature range of EVs.

Anode of LIB is mostly made up of graphite which is an abundant and low cost material. It has specific capacity of 370 mAhg^{-1}. At high charging/discharging current rate Li dendrites are formed which move through the separator and causes internal short circuit and thermal runaway(fire) in the battery. LIBs with graphite anodes are good at low temperatures only whereas the required temperature range for EV application is -30^{o}C to 60^{o}C [21]. Hunt for suitable anodic material led researchers to use Silicon electrodes, Lithium metal alloys (Li_XSn) [22], (Li_XGa) [23], Transition Metal Oxides (TMO) [24], conversion electrodes [25]. Silicon anode has a theoretical specific capacity (~4200mAhg^{-1}) but has the drawback of large volume expansion of anode during Li insertion. One of the TMO namely TiO_2 anode having specific capacity ~180mAhg^{-1}has the potential to suppress the Li dendrites and hence improves the safety and durability of LIBs. But it has low ionic diffusivity and low conductivity which are its limitations. A promising replacement for graphite is lithium metal, which has more than ten times the capacity of graphite (3860mAhg^{-1}), and would allow for the development of smaller, longer-lasting batteries. Lithium dendrites with large tortuosity often form during lithium plating, and if these dendrites penetrate the membrane separator in the middle of the battery, they can create short-circuits, raising concerns about battery safety. Due tosafety issues and low coulombic efficiency Lithium metal battery research was abandoned in favour of graphite.Li-Sulphur and Li –Air batteries are also seeking much attention due to their high energy density (~500Wh/kg) but Li-S battery suffer by capacity fade problem.Each of these materials specified above have some advantages over the other but overall efficiency, safety, high cost of battery could not be completely met with.

2.2 Battery electrolyte:

The electrolyteis an essential part of the battery which enable Li ions conduction between two electrodes. LIB electrolytes are usually based on an organic solvent with a lithium salt such as ethyl carbonate with $LiPF_6$or $LiBF_4$. But it is a flammable electrolyte so if at any given time the voltage exceeds the voltage window the battery runs the risk of catching fire. Forsafety, development of a non-flammable electrolyte or a constituent that can develop rapidly a solid electrolyte-interface (SEI) layer to prevent plating of Li on a carbon anode is required. Gel polymer electrolyte and solid polymer electrolyte are options for building safer LIBs. Currently solid state batteries with solid electrolytes are close to commercial reality but they suffer from low ionic conductivity at room temperature as compared to organic liquid electrolytes. LIBs suffer thermal runaway issue at temperatures above 130^{0}C [26]. Room Temperature Ionic Liquids (RTILs), a new development, can mitigate the flammability and volatility of organic electrolytes issue

3.Nanotech improvised LIBs:

Nanotechnology deals with materials at nano scale (10^{-9} m) and it can significantly improvethe rate capability and cyclic behaviour of electrode materials of LIB. Nanoparticles are formed through either by breaking down of larger particles or by controlled assembly processes. Nano materials are of special interest for researchers as at such a small size these materials possess distinguished properties and characteristics which have the ability for application in different fields. Nanomaterial has greater chemical reactivity due to much greater surface area to volume ratio as compared to conventional sized materials hence it greatly increases their strength. Quantum effects play an important role at nanoscale in determining the materials' properties and characteristics which result in novel magnetic, electrical and optical behaviours. Nanotechnology finds applications in many spheres like pharmaceutics, advanced materials, energy production and storage [27]. Its recent applications in transportation[28][29] include improving the efficiency of energy generation from renewable resources and energy storage in batteries by providing alternative materials and fabrication methods to produce cost effective batteries. Nanotechnology plays a vital role in battery chemistry and thus increasing the energy storage capacities, energy density, safety and durability of conventional batteries used in EVs[29] by employing Carbon Nano Tubes

(CNTs), fullerenes, quantum dots and nano composites in various components of the battery such as electrodes, catalysts[27] and electrolyte [4][30][31]. Nano composites can also be used to improve energy storage capacities of batteries. Nanotechnology companies used their own proprietary material composition to reduce the risk of the battery catching fire. Nanoscale batteries can be combined together to function as a macro battery such as within a Nano pore battery. Nano batteries which can charge 60 times faster than conventional batteries have been developed using nanotechnology. Batteries that can prevent electrode contact prior to activation have been developed which gives limitless shelf life and longer active life to them. In all nanotechnology enables more energy at less cost [1].

3.1 Nanostructured LIB Electrodes:

Faster intercalation of Li ions can be facilitated by using nanosized materials for electrodes, which offer high surface areas and short diffusion paths, and hence faster storage and delivery of energy. One prominent example is the cathode material of A123 LIBs that use nanosized lithium iron phosphate cathode [15]. Altair nano LIBs that use lithium titanate spinel electrodes do not suffer thermal runaway issue below 250^0C temperature [26].Several other nanomaterials like graphene, carbon coated silicon nanowires [32], CNTs [33], layered nanostructured vanadium oxide (V_2O_5) [34] ,manganese oxide (MnO_2), TiO_2 Nanotubes[35] are used to replace conventional graphite electrodes in batteries to increase the surface area that is accessible to battery electrolyte, low volume expansion during Lithium insertion which enhances the electricity output [36]. Graphene, a single layer of graphite, allow intercalation of Li ions on both sides forming LiC_3 thus doubles its capacity. Silicon nanowires and silicon porous structure overcomes the drawback of large volume expansion of silicon anodes and successfully help in maintaining structural integrity in addition to enhancing cyclic performance.TiO_2 based nanostructured electrodes ensure improved electrochemical performance when applied to lithium ion batteries. TiO_2 NT arrays are one group of nanostructures which basically offer high structural stability and safety during cycling, as these are crucial requirements in many. Hence nanostructured electrodes when used in batteries improve the energy storage that automatically improves battery life [37][27].

3.2 Nanostructured Electrolyte ensures battery safety:

Electrolytes in LIB conduct lithium ions between two electrodes. Instead of conventional liquid electrolytes use of solid electrolytes could ensure high-energy battery chemistries and better safety. A major challenge is to reduce interfacial resistance between the solid electrolyte and lithium based anodes and to increase lithium ion conductivity. Nano structuring of solid electrolytes has proven to improve the lithium ion conductivity. Al_2O_3, SiO_2, or ZrO_2 nanoparticles added to solid polymer gel could significantly enhance the conductivity and storage capacity of the electrolyte[39].

In case of the LIBs with lithium metal as anodes, lithium dendrite are formed across the electrolyte that leads to short circuits and overheating. Separators with nano porous structures can prevent the spreading of dendrites by acting as a mechanical barrier without hindering the ion-transport during charging and discharging cycles. Recently, a nano porous polymer-ceramic composite separator has been reported. This laminated nano porous gamma alumina sheet (pore size of 100 nm) sandwiched between macro porous polymer membranes could prevent the spreading of dendrites and hence cell failure that is caused by short circuits [15].

3.3 Nano sensors:

In each cell of the battery nano sensors are integrated which help to monitor the State Of Charge (SOC) in real time which will be helpful not only for security reason but also be useful to maximize the use of battery [38]. Accurate SOCestimation is one of the most important functions in a battery management system for battery packs used in electrical vehicles.

4. Estimation of State Of Charge of LIBs:

Electric vehicles are always equipped with a battery management system (BMS) to regulate and monitor LIB pack. The BMS has built in battery management hardware and software including algorithms which predict various states, available capacity and power of battery to ensure high efficiency and safety [40]. To estimate SOC, battery model techniques and accurate battery SOC estimation algorithms are currently of extreme importance due to their applications in electrified transportation and energy storage systems [41]. A battery model is required to establish the relationship between internal and external behaviours of battery by mathematical modelling. In any battery model the first step is to accurately identify the parameters of the model for battery state estimation. . A lithium-ion battery model can be classified as an electrochemical model, physical model, equivalent circuit model (ECMs), thermal model, coupled electro-thermal model, and so on. Among them, ECMs are the most commonly used in lithium-ion batteries SOC estimation for EV applications due to their simple model structure. Various ECMs used are the Rint model, Thevenin model, Partnership for a New Generation Vehicle (PNGV) model, nRC model, and FOM (Fractional Order Model). As hundreds of single cells are connected in parallel and series to compose a battery pack and provide energy that can meet the requirements of an application like a smart grid or EV, a whole battery pack model is required [42].

The existing SOC estimation methodologies can be categorized into five groups, which are the direct method, model based method, the adaptive filter algorithm, the learning algorithm and the hybrid algorithm.

4.1 Direct methods :

These are conventional methods which utilize battery characteristics such as voltage, current and temperature and solve some equations to estimate SOC.

(1) Coulomb Counting (CC) method also called Ampere –hour counting method is used to integrate the discharging or charging current to calculate the remaining charge in the battery[43] given by equation

$SOC(k) = SOC(0) - T/C_n \int_0^k [\eta \cdot I(t) - S_d]\, dt$

Where SOC(0) is the initial battery SOC, I(t) is the current at time t, T is the sampling period, C_n is the nominal capacity of the battery, η is the coulombic efficiency, and S_d is the self-discharging rate. This method however has short term accuracy and needs initial SOC value which pose limitations in its widespread use. The average error in estimation of SOC by this method is $<\pm$ 4%.

(2) Open Circuit Voltage (OCV) method is successful in case battery e.m.f and SOC relationship is stable which is not always the case. The relationship is expressed as

$$SOC = f\,(OCV)$$

Temperature and cycle life of battery can change this relationship which is no more reliable [44]. The average error in estimation of SOC by this method is $<\pm$ 5%.

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(3) Impedance and internal resistance method is used to describe the intrinsic electric characteristic under any current excitation, if temperature, SOC, and SOH are fixed. Electrochemical impedance spectroscopy (EIS) involve processing of large amount of data and internal resistance changes slowly so not fit for SOC estimation. In general, SOC estimation based on the impedance and internal resistance method are not suitable for use in EVs [45].

4.2 Model based methods:

Model-based methods deploy a battery model with advanced algorithms to estimate the states of a battery from its measured parameters such as voltage, current, and temperature. Electrochemical model and Equivalent circuit model (ECMs) fall under this category which use resistances and RCs to simulate the electrical characteristics for lithium-ion batteries. We can estimate SOC directly through ECM parameter identification or in association with SOC-OCV method. These methods have a high order of accuracy and are suitable for online SOC estimation[46]. However ECMs cannot simulate actual battery voltage under all current values and also have high computational load. The average error in estimation of SOC by this method is $<\pm$ 5%.

4.3 Adaptive Filter Algorithm:

Adaptive techniques combine the direct and model-based methods.The flow chart depicts that a predetermined SOC is taken as input, a battery model selected then calculate output battery voltage and then gain is calculated to update the SOC by comparing measured voltage and model voltage. The adaptive filter algorithm improves the accuracy and robustness of the battery SOC estimation and reduces the noise influence on the battery model. Various adaptive techniques include Kalman filter (KF), a linear state-space model that predicts the current state from the earlier state and updates the current state to converge it to the real value. The average error in estimation of SOC by this method is $<\pm$ 1.76%.

KF is applicable for linear systems only so to take care of non-linear systems Extended Kalman filter (EKF) is devised [47]. A linearization process using Taylor series expansion approximate the nonlinear system with a linear time varying (LTV) system then applied to KF to give EKF. But this method degrades SOC estimation accuracy so Unscented Kalman Filter (UKF) is devised to handle highly nonlinear state space model by applying a discrete-time filtering algorithm[48]. The Particle filter (PF) algorithm is used to estimate the states, which approximate the probability density function of a non-linear system by using the Monte Carlo simulation technique. The H∞ filter is a simple design model with robustness,even in the presence of parameter uncertainties and modelling errors, to restrict the effect of exogenous disturbances on output. However, hysteresis, aging, and temperature effects may deviate the accuracy of the model [49].The filter algorithms based on the equivalent circuit model with fixed model parameters are often used to estimate the battery state. However, the parameters of the equivalent circuit model are often affected by temperature, C-rate, SOC, and battery aging. Some researchers have proposed adaptive filtering (AF) methods for on-line identification of battery model parameters. Practice has proved that these methods are easier to implement in on-line applications. Besides, these methods can help to compensate for parameter values for battery variations and aging. Therefore, some hybrid estimation methods have been proposed to handle these problems.

4.4 Hybrid Methods:

The hybrid algorithm method is composed of two more algorithms. It can improve the efficiency and accuracy of the battery model and avoid the shortcomings of a single algorithm. The hybrid algorithm method not only achieves reliable and effective results, but also reduces the cost of the battery management system. However, this method has a very complex mathematical calculation, which requires a large storage memory and computing power unit. These methods are usually made up of two parts. The first part is used to identify the parameters of the model with recursive least squares (RLS) on-line. The second part is used to estimate the battery state parameters with filter algorithms [50].

4.5 Artificial Intelligence Based Learning Methods:

Recently trend is to estimate battery state using datainstead of using battery models as a lot of improvement has been done in embedded hardware performance. Over the last five years,the inclusion of some probabilistic techniques or artificial intelligence has been incorporated to improve the performance of estimation algorithms. The Artificial Intelligence (AI) based learning approach including artificial neural network (ANN) modelling as well as the support vector machine (SVM) was proposed [51], and could be very accurate depending on the training data. For EVs and HEVs, the trend is now towards the design of intelligent BMS, which involves research into intelligent-adaptive SOC estimation methods.

5. CONCLUSION:

For EVs to offer environmental benefits over conventional vehicles, technological improvements introduced by nanomaterials must be incorporated by the manufacturers in batteries of EVs. Future research will be focused on bringing down cost of nanomaterials for batteries, and making sure these stand up to the requirements of large-scale commercial applications. As nano engines, nanomotors and nanostructure are already developed, it can be widely used in future EVs to render them highly efficient vehicles. Various research efforts on nanotechnology based LIB technology has already led into the production and use of high performance LIBs (Toshiba, A123 Systems, Altair Nano, Next Alternative Inc., etc.). Altair nano LIBs provide a unique feature of recharge at minus 30^0C which is unachievable in conventional LIBs. Sothese ensure excellent cold temperature performance.

For accurate SOC estimation of real-time EV, the battery model must be as simple as possible.ECM is regarded as the most appropriate for online estimation and, based on this type of model the adaptive filter-based and artificial-intelligence-based approaches are presented to estimate the SOC with high precision. From the review, it is clear that adaptive filter-based algorithms are more suitable for EV applications, and those based on artificial intelligence are not suitable for this application due to its intensive computing. In future more attention must be given to develop less complex SOC estimation algorithm having less implementation cost for EV application.

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A STUDY OF HRD MECHANISM IN NATIONALIZE BANK (A CASE STUDY OF PNB BANK)

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ABSTRACT:
Human Resource Development plays vital role in every organization because it is now considered as a part of modern management system. HRD is helpful in development of employee's physical capacities, relationships attitudes, values, knowledge and skills. The success of the Banks depends not only on the satisfaction of their customers but also the satisfaction level of the employees working in the banks. The level of satisfaction of the bank employees can be ensured and enhanced by effective HR-practices adopted by the Banks. The study is aimed at assessing the extent of HRD Practices in Punjab National Bank. For the purpose of the study, primary data is collected from 300 employees of Punjab National Bank from Bilaspur district through a structured questionnaire. The study revealed that the Performance appraisal, Training and Development, career planning, motivation, compensation management and grievance redressal are better in Punjab National Bank.

KEYWORDS: HRD Practices, Punjab national bank, Satisfaction of the Bank Employees

I. INTRODUCTION:

In India, the origin of human resource management can be traced in the 1970s, concern for welfare shifted towards higher efficiency, a change in professional values of human resource managers was visible. During the 1980s due to new technology and other environmental changes, Human Resource Development (HRD) became a major issue. During the 1990s, the overwhelming role of human factor in industry has been realized. Growing awareness about the significance of human side of organisation has led to the development of human resource management as a distinct discipline. Focus on human values and a philosophical approach, are likely to provide this discipline the status of a profession.

Human Resource Development (HRD) is a process of developing skills, competencies, knowledge and attitudes of people in an organization. The people become human resource only when they are competent to perform organizational activities. Therefore, HRD ensures that the organization has such competent human resource to achieve its desired goals and objectives. HRD imparts the required knowledge and skill in them through effective arrangement of training and development programs. HRD is an integral part of Human Resource Management (HRM) which is more concerned with training and development, career planning and development and the organization development. The organization has to understand the dynamics of HR and attempt to cope with changing situation in order to deploy its HR effectively and efficiently. According to **Leonard Nadler,** "Human resource development is a series of organised activities, conducted within a specialised time and designed to produce behavioural changes."

Banks have to understand that the capital and technology-considered to be the most important pillars of banking are replicable, but not human capital, which needs to be viewed as a valuable resource for the achievement of competitive advantage. The long-term vision for India's banking system is to transform itself from being a domestic one to the global level may sound far-fetched at present. To take up this industry to the heights of international excellence requires combination of new technologies, better processes of credit and risk appraisal, treasury management, product diversification, internal control, external regulations and human resources at the most. The satisfaction of the employees working in the bank is predominant condition for the success of any bank. The satisfaction of the bank employees can be ensured and enhanced by effective HR-practices adopted by the Banks. An attempt to verify the HR-practices of one among the Punjab National Bank situated in Bilaspur District of Himachal Pradesh.

II Brief History of Punjab National Bank:

PNB is an Indian banking and financial services company .The Head office of PNB in New Delhi, India. It serves over 80 million customer having 6968 across 764 cities. It has 9935 ATM's across branches over the world. PNB India's first Swadeshi bank commenced its operation on April 12, 1895 from Lahore. Lala laj Path Rai played a key role in PNB's birth. When the first branch opened the Rai was its account holder with 2 lakh capital and 20000 its working capital. India's first Prime Minister Jawaharlal Nehru and Gandhi included the other customers of PNB's over the years. After the partition PNB its register its office Lahore to New Delhi. Today, PNB has nearly 7000 branches in India and this bank listed on world's biggest public companies.

III. REVIEW OF LITERATURE:

Sirca et.al.(2012) The aim of the paper is to theoretically and empirically describe the role of HR practices in the area of HR development: training and education, career development, performance management and reward management, on job satisfaction. The study has been conducted on a sample of Slovenian employees (N = 824), from medium size private and public sector organisations. Cluster analysis of participant responses on the HRM practices questionnaire shows four different groups of organisational approaches to HR development that are moderately related to job satisfaction. The results are discussed from four perspectives: individual attitudes, HRM theory, social exchange theory and the psychological climate theory.

Kesti (2012) the article suggests that the employee quality of working life can be measured by working unit collective competencies consisting leadership, team culture and processes. However, these competencies and attributes should be first validated for each organization environment, situation and strategy. This is essential because effective HRD process has to focus on the development of organization specific human drivers of performance. This seems to be vital for generating optimal workplace innovations.

Maier et. al. (2014) this paper proposes a framework for HR development for innovation, to reach the actual performance of an organization. The purpose of this framework is to offer, to all managers, a clear picture of existing HR capacity to innovate and possibilities to develop this capacity in order to improve the organization's capacity for innovation, thus increasing their chances of success in today's highly dynamic and competitive business environment.

Rakesh (2016) in his article highlighted banking sector implement the latest ideas on management regarding development of human resources. Developing the individual or human capacity is an integral element of building capacity and, in fact, capacity building initiatives are now increasingly becoming necessary. Taking the banking industry to the heights of excellence, especially in the present competitive environment, requires a combination of new technologies, better processes of credit and risk appraisal, treasury management, product diversification, internal control and external regulations and, not the least, human resources. Skilled and efficient manpower inventory is now becoming the need of the hour. So banks must start proper HRD practices to their future growth.

IV HRD PRACTICES IN INDIAN BANKS:

In the booming economy and the continuing expansion most of the banks facing challenges to do well and it outlines the fact that, contrary to public perception it is not just the new private sector banks that are doing well. There are few public sector banks are also doing well and got the place in top 10 best performing Indian banks. And it's worth mentioning that these public sector banks have performed so admirably in spite of the fact that they operate with many handicaps, such as strong unions and the inability to offer market salaries and incentives and burdened with extravagant workforce. The secret of success of any company depends on how they treat employees and keep them satisfied. For that they have to design their human resource processes like recruitment, selection, training and development, performance appraisal and other based on employee perspective in order to benefit them.

In India the banking industry is becoming more competitive than ever with private and public sector banking competing each other to perform well. The executives of the bank are now modifying their traditional human resources practice in to innovative human resource practices in order to meet the challenges from other competitive banks. Effective human resource practices relate to Bank's performance by contributing to employee and customer satisfaction, innovation, productivity and development of a favourable organizational working climate in the Banks. In order to verify HR practices adopted by the chosen public sector bank in Bilaspur District, the following functions have been chosen by the author as attributes of HRD practices. Recruitment and selection, Training and development, Performance appraisal, Career development, Motivation, Compensation management, Grievance Handling

V. OBJECTIVES OF THE STUDY:

- To explore the HRD practices in Punjab National Bank of Bilaspur District.
- To find out the satisfaction level of bank officers on HRD practices of their banks.
- To measure the relationship between the demographic characteristics of bank officers and their level of satisfaction on HRD practices.

VI. METHODOLOGY:

Research Design:

The methodology of the study is based on the primary data as well as secondary data. The study depends mainly on the primary data collected through a well-framed and structured questionnaire to elicit the well-considered opinions of the respondents.

Targeted Population & Sample:

Punjab National Bank operating in Bilaspur District has been chosen for the study. In Bilaspur District 21 Punjab National Banks are functioning. 300 officers & non officers of different age group, different gender, different designation and different length of work experience have been chosen as respondents for this study by using simple random sampling technique

Source of Data:
Data was collected from staff members of PNB through primary & secondary source of data. Questionnaire & interviews were used.

Data Collection Tool:
Required data had been enumerated from the chosen respondents by using structured schedule. The schedule contained the items related to the demographic variables of the employees and important attributes of HR-practices namely recruitment and selection, training and development, performance appraisal, career development, motivation, compensation management and grievance handling. Descriptive, t test and one way ANOVA were used as a statistical tool for the study.

VII Hypotheses of the Study:

- Gender of the Punjab National bank officers does not influence their satisfaction level on HR-practices.
- There is no significant relationship between the age of the Punjab National Bank officers and their satisfaction level on HR-practices.
- The level of satisfaction of the PNB officers based HR-practices does not differ with respect to their work experience.
- Designation of the Punjab National Bank officers does not have any influence on the satisfaction level on HR-practices.

VIII ANALYSIS AND INTERPRETATIONS:
It is found from the analysis of the data that offices of PNB functioning in Bilaspur district in Himachal Pradesh follow HR-practices. The study has been tested by using the statistical tools‘t’ test and One-Way ANOVA.

TABLE 1: **PROFILE OF THE SAMPLE UNITS**

S.no.	Name of Branches	Number of respondents		
		Male	Female	Total
1	Hatwar	12	03	15
2	Talyana	14	02	16
3	Barmana	12	03	15
4	Deoth	12	02	14
5	Mehri Kaithla	11	01	12
6	Chhakoh	12	01	13
7	Harlog	11	01	12
8	Jadukuljiar	10	01	11
9	Dabatmatari	11	02	13
10	Rani Kotla	11	02	13
11	Panjgain	12	02	14
12	Bilaspur	25	02	27
13	Ghumarwin	12	02	14
14	Jhandutta	13	02	15

15	Maloh	11	02	13
16	Chandpur	11	02	13
17	Kuthera	13	02	15
18	Majhwar	12	02	14
19	Samoh	11	03	14
20	Namhol	11	03	14
21	Dhamli	11	02	13
	Total	258	42	300

Gender wise classifications

S.no.	**Particular**	**No. of respondents**	**%age**
1	Male	258	86.0
2	Female	42	14.0
	Total	300	100

It is clear from the table that majority of respondents i.e. 86% belongs to the total sample of male category and 14 % under the category of female employees in PNB bank.

Designation

S.no	**Particular**	**No. of Respondents**	**% age**
1	Officers	142	47.33
2	Non-officers	158	52.67
	Total	300	100

It is clear from the table that majority of respondents i.e. 52.67% belongs to the total sample of non-officers category and 47.33% under the category of officers' employees in PNB bank.

Age wise classification

S.no.	**Particular**	**No. of respondents**	**Percentage**
1	20-30	110	36.7
2	31-40	134	44.7
3	41-50	45	15.0
4	51-60	11	3.7
	Total	300	100

It is clear from the table that majority of respondents i.e.44.7% belongs in the age group between 31-40, followed by 36.7% that is in the age group of 20-30 .very few respondents i.e.3.7% belongs to the age group of 51-60 years.

Work experience

S.no.	**Particular**	**No. of respondents**	**Percentage**
1	Up to 5 years	167	55.7
2	6-10 years	89	29.7
3	11-15 years	21	7.0
4	More 15 years	23	7.7
	Total	300	100

It is clear from the table that majority of employees 55.7% having up to 5 years of work experience whereas very few respondents i.e. 7% having up to 11-15 years of work experience.

Hypothesis –I

Ho: Gender & designation of PNB employees does not influence their satisfaction level on HR practices.

HR Practices	Gender				't' value	'p' value
	Male		Female			
	Mean	SD	Mean	SD	-0.57	0.57
	59.77	7.765	60.55	10.098		

The Table 2 indicates that the calculated p value (0. 57) for the variables Gender and HR practices in Punjab National Bank is found to be greater than 0.05 at 5% level of significance. Since the calculated p value for the variables Gender and HR practices is greater than 0.05, the null hypothesis H0 is accepted. Hence, it was concluded that Gender of the Punjab National Bank does not influence their satisfaction level on the HR practices followed in their banks.

HR practices	Designation				t value	p value
	officer		Non-officer			
	Mean	SD	Mean	SD		
	14.27	2.54	14.03	2.17	0.89	0.38

The Table 2 indicates that the calculated p value (0. 38) for the Designation and HR practices in Punjab National Bank is found to be greater than 0.05 at 5% level of significance. Since the calculated p value for the designation and HR practices is greater than 0.05, the null hypothesis H0 is accepted. Hence, it was concluded that designation of the Punjab National Bank does not influence their satisfaction level on the HR practices followed in their banks.

Hypothesis – II

Results of One-Way ANOVA for Demographic Characteristics and HR Practices

Age Group	Mean
Below 25	11.58
25-35	11.91
35-45	11.40
Above 45	11.27
'F' value	0.75
'p' value	0.52

Work experience	Mean
Up to 5 years	25.26
6-10 years	25.76
11-15 years	26.62
More 15 years	25.65
'F' value	1.07
'p' value	0.36

Table 3 shows that the calculated 'p' value for the variables 'Age' and HR-practices is 0.52 which is greater than the value 0.05. So, it is needless to say that H0 is accepted. Results of the One-Way ANOVA reveal that there is no significant difference between the Punjab National Bank of different age group towards their satisfaction on HR practices adopted by their Banks.

Table 3 depicts that the calculated 'p' value (0.36), for the variables "Work Experience and HR practices" is greater than 0.05 at 5% level of significance. Since, the "p" value is greater than 0.05, the H0 is not rejected for the variables work experience and HR practices. Hence, it is concluded that the work experience of the Punjab National Bank does not influence their level of satisfaction on the HR practices adopted in their banks.

IX. CONCLUSION:

The Study carried out in Punjab National Bank branches, Bilaspur District adopt HRD practices discloses various facts. However, the efficiency and the performance of HRD practices differ among banks. Though the bank officers insignificantly differ themselves in the level of satisfaction on HR practices on different functional areas of HRM of their banks, yet all respondents have same level of satisfaction over the HR practices adopted in the banks. Moreover the demographic characteristics Gender, Age, Designation and Work Experience do not influence the level of satisfaction of the Punjab National Bank on the HRD Practices.

X. SUGGESTIONS:

On the basis of the findings of the study, it can be concluded that since the employees are assumed as the inherent force to make the progress of the banks, the managements of these banks should make a concerted effort to provide greater satisfaction that boost the morale of the employees.

Bank to create more awareness among the employees that the knowledge and skill acquired through training to apply on the job.

- PNB Bank to activate HR department which, in turn, takes initiative and interest in organizing training and development programs.
- PNB Bank to organize adequate number of training programs.
- Bank needs to motivate the superiors in developing their subordinates.
- Banks are suggested to make the existing system of evaluation of superiors by subordinates more effective.
- Bank makes the use of the existing performance appraisal system for effective utilization of employees.
- PNB Bank suggested the existing performance appraisal system strong in order to identify the strengths and weaknesses relating the employees' performance.
- The Study suggested giving more importance in giving an opportunity to employees for linking their growth to the future of the organization.
- The study suggested enhancing its efforts to motivate supervisors to guide their subordinates for improved performance.
- It is proposed by the researcher to PNB to strengthen the existing feedback on positive aspects of performance appraisal.

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CONTRIBUTION OF APPLIED SCIENCES IN THE TRANSFORMATION OF BUSINESS MANAGEMENT AND COMMERCE - A 21STCENTURY BUSINESS PERSPECTIVE

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ABSTRACT:
Applied sciences have contributed significantly towards the development of discipline of business management and commerce. The Quantitative,System and Contingencyapproach has changed the business perspective in 20thcentury .The basic applied sciences behind all these was mathematics , statistics , computing etc.The fuel to the fire in 21stcentury is the technology and innovation, which is rapidly making application of existing knowledge to more practical applications resulting into application oriented devices. The key to success in any business with the rapidly evolving technology is analysis. Therefore, field of Analytics has evolved to provide useful insights and help in optimising the business activities, and that's how the Machine Learning and Artificial Intelligence has stepped in and now have a bigger role to play by using algorithms for decision making. Hence, there is a need to redesign and reengineer the business. The tomorrow's successful entrepreneurs, managers have to apply the fundamentals of business management holistically, thoughtfully keeping in view all of the applied sciences relevant or becoming relevant along with the concepts like, Artificial Intelligence , Machine Learning, Internet of Things and Block Chain etc.

KEY WORDS: Artificial Intelligence, Analytics, Block Chain, Internet of Things, Machine Learning.

INTRODUCTION:

Human being is capable of doing and creating so many things, it is well said that, thank God, we do not have wings but we have created an aeroplane which can fly faster than the speed of sound; thank God, we do not have fins, the man has created Ships and under water vehicles. We are exploring ocean as well as space. Space ships are there to travel and under water activities are numerous. We are capable of launching satellites in to the geo- stationary sphere of the earth. Satellite communication is a reality. Now we are exploring Moon and Mars. All these activities have been commercialised up to certain extent.Integrated Applied Sciences and most modern era's evolved disciplines are making things real and virtual as well for the betterment of mankind but it is bringing challenges and threats also.

STATEMENT OF THE PROBLEM:

Integrated Applied Sciences are contributing a lot for the development of existing and new disciplines. Therefore, most of the disciplines are application oriented today. Technology is also converging towards digital technologies. At present the digital technologies evolved may be listed as 5G Networks, Machine Learning and Artificial Intelligence, Augmented Analytics, Smart devices, Robotic Process Automation etc. Any scientific idea or technology which is useful formasses turns in to a business idea. Therefore, there is a focus on the contribution of applied sciences in the transformation of discipline like Business Management and Commerce. Although, the discipline of Business Management and Commerce is considered as a discipline of social science but the developments stated above has made it a discipline of science and technology oriented in this century. Still, more than eighty years are remaining of this century, the change and adaptability to change is brisk, therefore, we may perceive a big shift in 21st century business,

OBJECTIVES:

1. To identify the applied sciences those have contributed to the Business Management and Commerce.
2. To study the contribution of applied sciences in the transformation of Business Management and Commerce.
3. To study the 21st century business perspective so that quick change may be anticipated for further brisk change,

METHODOLOGY:

The study is to identify the contribution of applied sciences and it focuses on the 21st century business perspective. The methodology to be adopted here is to look at the development of management thought i.e., a theoretical perspective, through secondary sources of information from books, dallies and magazines etc.

SCOPE OF THE STUDY:

The scope of the study is very wide, since it is concerned with different individualdisciplines. In modern terms these disciplines have wider application orientation. This increases its scope manifold.

LIMITATIONS OF THE STUDY:

The study is based on the contribution then we have to look back.But the concepts are emerging quickly. Therefore, a big gap is felt in concepts, their development and emerging patterns. The primary source based study needs time and labour. This is the main limitation of the study.

REVIEW OF LITERATURE:

The development of management thought in its modern term begins with the coming of 20th century. Initially it was considered a Rule of Thumb, then policy oriented-Business/Corporate Policy, Strategy oriented-Strategic, and now known as Advanced Strategic Management.The development of thought took many turns up to the 21st century. It may be perceived that during this century these turns will certainly be more and quick. All those turns were segmented as Classical era, Neo-classical era and Most- modern era.

The classical era’s contribution may be spelled as, the concept of formal organisation, scientific management-productivity approach and management process –functional approach. The contributors were mainly from the field of engineering i.e., applied sciences. We may conclude that the genesis of modern management is through the thinkers who were from applied science – may be marked as initial contribution.

During the Neo-classical era the school of thought developed through Human Relations approach, Behavioural and social sciences. The contributors were from the fields of Logic, Ethics, Philosophy, and Social Psychology. This amounted to the contribution of social sciences.

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The most modern era begins with the application of Operations Research and Computerised Data Processing techniques. These are mainly the fields of Applied Mathematics. The technical and quantitative sciences resulted in to the development of Modern System Theory (Multidisciplinary and holistic), Contingency theories of organisations. Here, Environmental forces, Technology, Formal Structure, Behavioural forces and Decision Making processes were integrated. All from different fields of knowledge, their applied part became relevant for the development of modern management thought.

There is another most modern way to look at the phenomenon as we term it as Industry0.0, Industry 1.0, Industry 2.0, Industry 3.0 and Industry4.0. These terms also signify change and development of diverse phenomenon relevant to the human life through industrial development. Industry 1.0 identified as Industrial Revolution i.e., mechanisation. Industry 2.0 means Development of Telecommunication, Infrastructure development (Rail-Road, Electricity etc.), transformed society with the fast dissemination of goods, services and ideas. Industry 3.0 begins with the period after World War II followed by the period of technological progress, Information Communication Technologies, Personal Computers, Internet, Pager, Cellular phones etc. Industry 4.0 may have 100% Digitisation, thisperiod iscoming withsmart devices, even smart cities, smart homes,smart factories/ Dark factories so on and so forth. However, as per the Fred W. Riggs’s model of Prismatic Society- an intermediate society, India may have to deal with all Industry 1.0,2.0,3.0,simultaneously to achieve Industry 4.0. Because some part of India has not tasted the Industrial Revolution, still deprived of infrastructure, do not know about the benefits of computerisation/digitisation etc.

Whatsoever, is possible till date is due to the application of the pure and social sciences, the integration of applied science,the development of new disciplines from such integrated applies sciences. Behind all these is the basic science and spirit of mathematics.

DISCRIPTION AND EVALUATION OF THE PROBLEM:

The review of literature and the management continuum helped us to identify the disciplines which contributed to bring Business Management and Commerce to its present form and shape. We may begin with the Indian Philosophy which through its scriptures provides us unique insight for the solution of complex problems of family, society, organisations etc. Among other individual or ethnic group we may mention Sumerians, Egyptians, Chinese, Greeks, Socrates, Plato, Alexander the Great, Kautilya, there are hundreds of such names which may be mentioned here.The Ancient, Medieval, Modern and Most-modern period of History may also contribute in many ways. The Business Environment components are Social, Economic, Political, Administrative, Legal, Ethical, Scientific and Technological aspects. These have their own authority and are beyond the control of the business organisation and most volatile in nature. Although, ever changing environmental components but have impact on the business and in turn are being affected. Now there is sufficient evidence to say that right from Anthropology to present day integrated applied science subjects all have contributed a lot towards Business Management and Commerce.

The contribution of each and every subject, Philosophy, Individual and ethnic group cannot be taken or mentioned here because it is not possible to create voluminous research paper; therefore, we will take the present phenomenon relevant to the problem under study.

Now any phase of Business Organisation and Management, Decision-making, Strategic Planning and Controlcan be expressed in quantitative terms for a more exacting analysis. The number of techniques that have evolved with the quantatives and analytics related to or based on computing are going to revolutionise the field of business management and commerce. The phenomenon evolved recently may be mentioned as follows.

1. 5G:

5G is the fifth generation of cellular network technology. It is next to the 4G LTE fourth generation long term evolution. 5G network work on the 600MHz spectrum and 5G tower can transmit signals over one thousand square miles. With the advent and adoption of 5G technology, there will be elevation of the cellular network not only to interconnect people but also to interconnect and control machines. It will be 200 times faster than the 4G LTE network. It will enable new operating models instead of redesign of production line and offer manufacturers to build smart factories. The connectivity and data interoperability in IIoT projects will be possible.

2. Internet of Things (IoT) and Industrial Internet of things (IIoT):

It is a network of things which includes devices, appliances (domestic or industrial), vehicles of various mode of transport, robots and other things capable of computing applications equipped with sensors, electronic gadgets, software, having connectivity with one and all so that there is exchange of data.

It is certain that this technology may influence the areas like Safety, security, convenience, energy efficiency, operational efficiency and agricultural productivity etc. This has made the concept of smart home and smart work place possible. Building Management system for commercial buildings may be drawn and installed. The IoT will influence the present business by facilitating Data Sharing, Remote Work, to prepare skilled workers, Inventory tracking and Management, Customer Relationship Management etc. But the issue of connectivity depends upon the wired, wireless and hybrid network. This will influence the implementation strategies. However, the wired is more reliable in comparison to the wireless or hybrid. Hence, Signal strength shall matter in wireless connectivity.

3. Business Analytics:

The IoT shall generate huge data. Moreover, with the continuous increase of internet and web users and universal mobile computing for the masses led to additional data generation. Business is the interested to get every type of data for their business purposes. Therefore, there arises a field of Analytics. It is a field where there is a process involved in collating, mining, sorting, processing, drawing and studying the data by using statistical tools and techniques and mathematical models for making it purposeful. When the concept is used in business it is called Business Analytics. Here, the aim is to identify which datasets are of use for particular business and then may be used to solve the decision making problems and in turn to increase productivity, revenue and efficiency. The field of Business Analytics is classified as Descriptive Analytics, Diagnostic Analytics, Predictive Analytics and Prescriptive Analytics. Over a period of time different tools have been developed, these may be assessable from the open source platforms named as Metabase, Matomo, OmniSci, SpagoBI, Zeppelin by Apache and Birt. With these tools the businesses can manage their processes and may draw solutions in a relevant and comprehensive way. Analytics is committed to simplify the human life by doing future sales forecasting, demand planning.

However, the challenges for business Analytics may arise due to Poor coordination, Lack of commitment of decision makers, Executive Distrust and Slow Information Maturity.

4. Artificial Intelligence and Machine Learning:

Artificial Intelligence is a technological reality, it is a process through which machines/devices like computers in terms of hardware and in combination with the software simulates human activities including learning and self- correction, problem solving etc.
Machine Learning is a type of Artificial Intelligence, rather a process of applying Artificial Intelligence without explicit programming, so that the machine/device is able to learn automatically.

Now, the Artificial Intelligence is paving the way for more impactful digital business models. It will transform the Enterprise Resource Planning (ERP) by identifying the problems even before they occur, Automate routine tasks with system intelligence and Improve user experience through interaction. The Conversational Gateways has made the omnichannel businesses possible. Adoption of Artificial Intelligence and its implementation require technical literacy of business leaders only then best AI solutions may be sought. Company culture is the detrimental factor in the adoption of AI. The sectors those are attracted towards implementation of AI are retail, banking, healthcare, IT, infrastructure. However, the list may expand further. There is an evolution of a concept of Deep Learning now. It is more specific version of Machine Learning which relies on neural networks to engage in nonlinear reasoning. The neural networks are a form of AI where flow of information resembles the human brain. At this point of time we are not in a position to see the exact impacts of AI-ML technology. But we may foresee the new start-ups, displacement of certain jobs and creation of entirely new one, AI-ML businesses and business applications.

5. Robotic Process Automation (RPA):

It is the technology that allows anyone to configure computer software or a Robot to emulate and integrate the actions of a human interacting within digital systems to execute a business process. It is specialised software and Artificial Intelligence to automate repeatable tasks and business processes using programs called “bots.”

6. The Cloud:

The cloud computing means storing and accessing data and programs over the internet instead of own computer’s hard drive. Fog computing refers to extending cloud computing to the edge of an enterprise’s network. It facilitates the operation of compute, storage and networking services between end devicesand cloud computing data centers. It may connect all the industries and businesses of the world.

7. 3D Printing:

It is also called additive manufacturing, is a process by which physical objects are created by depositing material in layers based on digital model. These processes require software, hardware and materials to work together.If 3D printing get industrialised, any product one may think of could be taken out of 3D printer.

8. Block-chain Technology:

It is a technology to decentralise the web. It is a peer to peer network. It is based on Distributed Ledger Technology which is the outcome of interdisciplinary application of the fields of Accounting (Triple Entry Accounting System) and Cryptography. It is capable of making the processes more democratic, secure, transparent and efficient. Initially this technology was applied to evolve Digital Currency named Bitcoin. However, the technocrats have found and exploring other potential uses of this technology. The business applications of Block Chain are supply chain management/auditing, smart contracts, The sharing economy, crowd funding, governance, prediction markets, protection of intellectual property, Anti-money laundering, Know your customer, land title registration and stock trading etc. The challenges posed by this technology may be enumerated as, an environmental threat due to high carbon emission because of the computational resources used. The technology needs high power, processing and storage. Quantum computing is another threat to the block chain technology because the processing speed of the computers will increase to the extent, that may enable hackers to find loopholes in block chain based networks.

9. Cyber security:

All we know that in computing security is a paramount issue, however, when everything will be digitized and nothing is offline, and then imagine what may happen. Presently, cyber threats are increasing worldwide and India is no exception. This year there were so many major security breaches. Therefore, everyone has to address the issue. This month India is going to establish Defence Cyber Agency (DCA) to counter cyber threats.

A 21st Century Business Perspective:

The emerging technologies cited above evolve a fact that 21st century business activity which will transform the ethos of business.The complete digitisation will bring unprecedented benefits in efficiency and productivity. It will make the businesses more reliable, predictable and scalable. The digital transformation shall amount to the transformation of Business Management and Commerce. The picture may not be that much rosy as we may perceive today. The key to the smart systems is meaningful data/ information. The more data is available, we may have better solutions, if information is available and we are not in a position to process it, it can work counterproductive. Moreover, India is not comparatively ready to adopt such technologies quickly, because these are yet to be fully implemented the world over. Therefore, we have to address the challenges and threats first. A holistic effort is needed at every step.

CONCLUSION:

It is a proven fact that Applied Sciences and Integrated Applied Sciences have transformed so many ideas, concepts, disciplines, and Business Management and Commerce not an exception. The quantitative tools and techniques were more instrumental. There is a big change in the technological environment of the business. The technological environment had further influenced the other components of business environment. Therefore, the influence of the technology will compel the businesses to adopt and cope with the big and brisk change. Skilful and technical savvy workforce will be needed. Robots may be engaged along with the human beings. The coexistence of such machines will require more adjustments with machines because they lack emotions. The organisational culture will take a new shape. More digital technology applications are inevitable. The challenges are to be dealt with the strengths. A 21st century business activity will contain new kind of activities because the technologies and concepts discussed above are getting integrated. For example RPA plus AI plus Analytics, the integration of these three will automate every process. Moreover, if any of the components discussed under Sr. No. 1-9 is missing, the whole process shall stall. All these technologies are capable to enable Industry4.0 enterprise – A Smart Business Entity.

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INFORMATION AND COMMUNICATION TECHNOLOGY AND MATHEMATICS EDUCATION: PRACTICAL CHALLENGES AND FUTURISTIC PERSPECTIVES

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ABSTRACT
Education has been remained the most outstanding development criterion and priority area for the development of any society throughout the world.However, Education becomes dynamic and practical by use of technology in its various domains. Today, almost all of the universities and colleges are promoting the development of practical knowledge and skills within the individuals worldwide.In this regard, the role of digital technologies has been remained the subject of interest for mathematics educators for at least the past three decades. Mathematics consists of basic structure for many fields like physics, business terminologies, and architecture by means of use of calculus, algebra, statistics, and trigonometry. Thus, the responsibility of education system is to acknowledge students with the need and practical use of mathematical knowledge in their life.Technology generates the new avenues for its users especially in the various areas of mathematics education such as control theory, signal processing and cryptography etc as theuse of Information and Communication Technology (ICT) in classrooms develops the visual impact and practical environment within the classroom. In this direction, the present paper is focussing on understanding the use of ICTin mathematics classroom and also exploring the practical challenges that are faced by educators as well as students while teaching and learning mathematics. The paper is also discussing the futuristic perspectives in regard to use of ICT in Mathematics classroom.

KEYWORDS: Mathematics Education,Information and Communication Technology, Knowledge, Teaching and Learning.

INTRODUCTION

The advancement of a nation depends upon the quality and quantity of education received by its people. Education plays a vital role in shaping the future of a nation. Only education can differentiate between animals and men.

Gorge and Polya (2002) had discovered two aims of school education. First is to converting out employable adults who contributes to social and economic development of the country and second is to develop the inner resources of growing child. Lockey has aptly remarked, “Plants are developed by cultivation and men by education”.

The education system needs some tools that make learning in classroom effective for students .This purpose can be resolved by use of some visual tools such as images, videos and visual presentation. The teaching becomes effective as well as student’s interest makes class room learning more studious.

ICT stands for information and communication technology. It means diverse set of technological tools and resources used to communicate and to create, store and manage information. Since 1970's the digital technologies have been available in school mathematics by the introduction of simple forth calculators. ICT has a great impact on education as well as other aspects. It has both positive and negative impact on students and teachers. Nowadays the role of mathematics is not restricted to purely academic domain but it is widen into domain of technology and industry. Robler, Casting and King, (1988) have analysed that by the introduction of ICT many researchers and funding agencies were led to invest their resources to investigate the possibility of replacing teachers in key instructional role by computers.

Olive and Makar (2010) argued that the connection of mathematical knowledge and mathematical practices can be strengthen by the use of technologies. There are many tools and visual aids that can create interest of students for mathematics rather than regular calculation studies in class room. Mathematics has trigonometry and geometry portions that can be represents more effectively by projection technique for 3D Plane as well as 2D plane.

The Department of Higher Education proposes to closely integrate the critical components into a framework, incorporating the learning and best practices from the similar initiatives at the global level and practical needs of the evolving knowledge society.

The objective of this paper is to study the present status of usage of ICT by teachers during teaching learning process and to investigate the barriers in mathematics classroom. The main aim of this paper is to determine the key area of policies and awareness among teachers towards use of ICT.

MATHEMATICS EDUCATION:

Mathematics is very important part of our life and plays a major role in achieving the aims of education. Mathematics is science of measurement, quantity, magnitude, space and numbers. It has comprehensive applications in the subjects like social sciences, medicine, business and commerce, life, health, physical sciences and technical sciences. As we compare Mathematics with other subjects it is the most important subject because there are lots of techniques and branches for finding the formulae on the problem of various fields in mathematics.According to Napoleon"The progress and the improvement of mathematics are linked to the prosperity of the state."

Mathematics helps children to think creatively, develop his imaginations and think logically. The main goal of mathematics education in schools is the mathematisation of the child's thinking.As we compare Mathematics with other subjects it is the most important subject because there are lots of techniques and branches for finding the formulae on the problem of various fields in mathematics.

Mathematics can be used as a component in many interdisciplinary combinations. The interdisciplinary programmes in mathematics will make the subject more attractive and meaningful. We can propose a number of such interdisciplinary programmes involving mathematics for example Physical science-Mathematics, Commerce-Mathematics, Economics-mathematics, Education-Mathematics etc. We make learning more fruitful and appealing by such interdisciplinary programmes involving mathematics.

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Mathematics is used directly or indirectly to fulfil the needs of man such as in business, carpentry, shop keeping, banking, accountancy, weighing, measuring, counting etc. By the study of mathematics an individual can be able to apply his knowledge to a new situation.
In India mathematics is included as compulsory subject right from kindergarten level upto secondary level in school curriculum. Due to modernization the curriculum of mathematics changes time to time in India. So the government has to change the teaching method from chalk and talk method to student centred approach method. This helps the students to think and understand the concepts logically in different way. Ping and Hua(2015) investigated that maximum number of students are not able to solve the basic operations well due to dearth of understanding the concepts. Some factors which affectthe achievement of mathematics are social factors, personal factor, economic factors, psychological factor, etc.

ICT and Mathematics Education:
The use of ICT in the mathematics classroom has long been a topic for consideration by mathematics educators. D.D.Agyei and J.Voogt (2010) analysed some examples of use of ICT in mathematics are portables, graphic calculators and computerized graphing, specialised software, programmable toys or floor robots, spreadsheets and databases etc.Computers, wireless classroom microphones, interactive white boards, digital video on demand, online study tools, LCD projectors etc. can be used as teaching aids in classrooms. Graphic calculators, Geogebra, Dynamic geometry tools, Microsoft mathematics, Auto shapes, Mat lab are the software used for teaching and learning of mathematics. ICT-supported education can develop the acquisition of the knowledge and 21st century skills such as critical thinking, creativity and problem solving. Recent research indicates that the purposeful use of computers in classroom instruction can indeed enhance student outcomes (Archer, 1998; Milheim, 1995). Studies involving computer assisted algebra instruction (Brunner & Sheehan, 1997; O'Callaghan, 1998) and instruction with graphing calculators (Adams, 1997) have similar positive results. The National Council of Teachers of Mathematics (NCTM) concluded that technology is an important tool for teaching and learning process and it is stated by NCTM in Curriculum and Evaluation Standards for School Mathematics (1989) and Principles Standards for School Management.

In those countries where majority of children cannot afford more than one notebook, the country should provide appropriate low cost technology for teaching and learning process.

In 1972 the scheme of Educational Technology was introduced during the IVth plan.Later, in 1984-85 the Computer Literacy and Studies in Schools (CLASS) Project was introduced as a pilot project.

In VIIIth plan (1993-98),this project was considered as a Centrally Sponsored Scheme and its scope is enlarged to provide financial grants to educational institutions.

Muhammad Shahid Farooq (2006) studied the effects of teacher's professional education on students achievements in mathematics. The purpose of the study was to compare the effectiveness of teaching of professionally trained and untrained teachers. Data were collected from four public and private boys and girls high schools record. Four hundred secondary school graduates (Two hundred boys and two hundred girls) taught by trained and untrained teachers of mathematics were selected conveniently. The results of the study supported the fact that the students taught by trained teachers showed better results in

Mathematics and gender has no significant effect on achievement in mathematics.

Practical Challenges in usage of ICT in Mathematics Education:
Many studies have shown a lot of difficulties that teachers experience in the synthesis of ICT in their classrooms. Jones (2004) found a number of barriers for the integration of ICT into lessons such as low confidence among the teachers during integration, dearth of time for integration, technical problems in using software, lack of access to resources etc.

Chong, et al. (2005) has identified six major barriers in a survey to study the barriers preventing the integration and adoption of ICT in teaching mathematics. These barriers are lack of time in school schedule for project involving ICT, insufficient teacher training opportunities for ICT project, lack of knowledge about ways to integrate ICT to enhance the curriculum, difficulty in integrating and using different ICT tools in a Popoola et.al (2018) finds that importance existed in the use of ICT facilities on teaching and learning of mathematics. This showed that ICT plays a major role in teaching and learning of mathematics as a teaching tool. Also, there was sound effect of ICT facilities availability and usage on student's performance in mathematics. This also showed that ICT facilities helps students to have unlimited access to study materials and ability to relate with their correspondent in other institutions across the globe even from the comfort of their rooms and this undoubtedly helps them to improve in their academics. It was equally revealed that there exists significant influence of ICT facilities on teaching of mathematics. This equally showed that lecturers also relied to certain extent on ICT facilities on how to improve on their skills and teaching methods. Lastly, the study revealed that significant effect of ICT facilities in improving the students' learning and knowledge in mathematics existed. This implied that ICT facilities help greatly in expanding the knowledge and understanding of students in mathematics.

Some of the barriers in usage of ICT in mathematics education are lack of confidence among teachers during synthesis, lack of personal access during lesson preparation, lack of ways to integrate ICT to enhance the curriculum and insufficient teacher training opportunities for ICT project.

ICT and Mathematics Education: Futuristic Perspectives:
Ting SengEng (2005) investigated that ICT has small effects on learning outcomes and sometimes negative effects have been found. Qualitative research on how to improve the effective use of ICT through innovative methods, possibly incorporating a variety of ICT tools should be investigated as pilot studies. Such studies should be on going such that feedback can be obtained and methods modified to refine the teaching and learning process.

Lecturers should encourage the students to use internet and to visit cybercafé to solve their mathematics problems. To enhance the skills and lecture method of lecturers, they should be trained and retrained for the benefits of ICT. College management should make sufficient supply of computers into all the departments in the institution and assure that all students have unconditionalapproach to them during lectures. Management should also make wireless internet connection available in the institution and ensure free accessibility for both students and lecturers. Students must be properly orientated on the numerous benefits and opportunity that ICT offers in enhancing their academic performance in mathematics and science in general. (Popoola et. al.2017).

As considering future of mathematics with ICT the major resources is internet for lectures as well as for students .there are lot of websites containing content for different theorems and calculations. As updated form of theorems are also be available on internet. This will enhance the quality of lectures .Students may attend online lecture if they miss them in any reason during study time.

CONCLUSION:

The 21st century is based on science and technology The education system also needs advancement. The use of ICT facilitates modern education system with accuracy and updated knowledge. As concern with mathematics study, lot of new updates are there in mathematics with respect of time. The Indian teacher should be ready to face upcoming challenges for imparting new age education .Teacher has major role between students and new upcoming in mathematics. This is responsibility of them to convey these updated education to their students. They should use ICT for smooth flow of lectures.Efforts must be made by the educationist to change the process of teaching-learning in order to prepare the students to adjust themselves to the society; this could definitely create a new learning environment and information rich society.

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MAGNETO-ROTATORY THERMAL CONVECTION IN RIVLIN-ERICKSEN VISCOELASTIC FLUID HEATED FROM BELOW

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ABSTRACT:
A layer of Rivlin-Ericksen viscoelastic fluid heated from below is considered in the presence of uniform vertical magnetic field and rotation. Following the linearized stability theory and normal mode analysis, the paper mathematically established the condition for characterizing the oscillatory motions which may be neutral or unstable, for rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, opposite to gravity and a constant vertical adverse temperature gradient, are necessarily non-oscillatory, in the regime

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\le 1 ,$$

where T_A is the Taylor number, Q isthe Chandrasekhar number, p_2 is the magnetic Prandtl number and F is the viscoelasticity parameter. The result is important since it hold for all wave numbers and for horizontal rigid boundaries of infinite extension at the top and bottom of the fluid, and the exact solutions of the problem investigated in closed form, is not obtainable.

KEY WORDS: Thermal convection; Rivlin-Ericksen Fluid; Rotation; Magnetic Field PES; Rayleigh number; Chandrasekhar number: Taylor number.

1. INTRODUCTION:

Stability of a dynamical system is closest to real life, in the sense that realization of a dynamical system depends upon its stability. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors (e.g., Bénard [1], Rayleigh [2], Jeffreys [3]) under different conditions. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [4]. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Bhatia and Steiner [5] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner [6]. They have found that the magnetic field stabilizes a viscoelastic (Maxwell) fluid just as the Newtonian fluid. Sharma [7] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted upon by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. In another study Sharma [8] has studied the stability of a layer of an electrically conducting Oldroyd fluid [9] in the presence of magnetic field and has found that the magnetic field has a stabilizing influence.

There are many viscoelastic fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [9] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen [10] has proposed a theoretical model for such one class of elastic-viscous fluids. Sharma and kumar [11] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar et al. [12] considered effect of rotation and magnetic field on Rivlin-Ericksen viscoelastic fluid and found that rotation has stabilizing effect, where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics.

Pellow and Southwell [13] proved the validity of 'principle of exchange of stability' (PES) for the classical Rayleigh-Bénard convection problem. Banerjee et al [14] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [15] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [16]. However no such result existed for non-Newtonian fluid configurations, in general and for Rivlin-Ericksen viscoelastic fluid configurations in particular. Banyal [17] have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of Rivlin-Ericksen viscoelastic fluids, this paper is an attempts to study Rivlin-Ericksen viscoelastic fluid heated from below in the presence of uniform vertical magnetic field and rotation. It has been established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid in the present configuration, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2 and the viscoelasticity parameter F, satisfy the inequality $\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Q p_2}{\pi^2} \le 1$. These results hold for all wave numbers with rigid boundaries of infinite horizontal extension at the top and bottom of the fluid.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

Considered an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained.

The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ and a uniform vertical magnetic field $\vec{H}(0,0,H)$.

The equation of motion, continuity, heat conduction, and Maxwells equations governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field and rotation are (Rivlin and Ericksen [10]; Chandrasekhar [4] and Kumar et al [12]) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_o}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}$$
$$+\frac{\mu_e}{4\pi\rho_o}(\nabla\times\vec{H})\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right), \qquad (1)$$

$$\nabla.\vec{q}=0, \qquad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \qquad (3)$$

$$\nabla.\vec{H}=0, \qquad (4)$$

$$\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{q}+\eta\nabla^2\vec{H}, \qquad (5)$$

Where ρ, p, T, ν, ν' and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature, kinematic viscosity, kinematic viscoelasticity and velocity of the fluid, respectively and $\vec{r}(x,y,z)$.

The equation of state for the fluid is

$$\rho=\rho_0[1-\alpha(T-T_0)], \qquad (6)$$

Where the suffix zero refer to the values at the reference level z = 0. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and α is the coefficient of thermal expansion. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity κ, and electrical resistivity η, are all assumed to be constant.

The initial state is one in which the velocity, density, pressure, and temperature at any point in the fluid are, respectively, given by

$$\vec{q}=(0,0,0),\ \rho=\rho(z),\ p=p(z),\ T=T(z), \qquad (7)$$

Assume small perturbations around the basic solution and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$ and $\vec{h}=\left(h_x,h_y,h_z\right)$ denote respectively the perturbations in density ρ, pressure p, temperature T, velocity $\vec{q}(0,0,0)$ and the magnetic field $\vec{H}=(0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0\left[1-\alpha\left(T+\theta-T_0\right)\right]=\rho-\alpha\rho_0\theta, \text{ i.e. } \delta\rho=-\alpha\rho_0\theta . \tag{8}$$

Then the linearized perturbation equations are

$$\frac{\partial \vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\vec{g}\,\alpha\theta+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right), \tag{9}$$

$$\nabla.\vec{q}=0, \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \tag{11}$$

$$\nabla.\vec{h}=0, \tag{12}$$

$$\frac{\partial\vec{h}}{\partial t}=\left(\vec{H}.\nabla\right)\vec{q}+\eta\nabla^2\vec{h}. \tag{13}$$

Within the framework of Boussinesq approximation, equations (9) – (13), becomes

$$\frac{\partial}{\partial t}\nabla^2 w=\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^4 w+\frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right)+g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-2\Omega\frac{\partial\varsigma}{\partial z}, \tag{14}$$

$$\frac{\partial\varsigma}{\partial t}=\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\varsigma+2\Omega\frac{\partial w}{\partial z}-\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}, \tag{15}$$

$$\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta \tag{16}$$

$$\frac{\partial h_z}{\partial t}=H\frac{\partial w}{\partial z}+\eta\nabla^2 h_z \tag{17}$$

$$\frac{\partial\xi}{\partial t}=H\frac{\partial\varsigma}{\partial z}+\eta\nabla^2\xi \tag{18}$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and; $\varsigma=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ denote the z-component of vorticity and current density respectively.

3. Normal Mode Analysis

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$\left[w,\theta,h_z,\varsigma,\xi\right]=\left[W(z),\Theta(z),K(z),Z(z),X(z)\right]\exp\left(ik_x x+ik_y y+nt\right), \tag{19}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^{\,2} + k_y^{\,2}\right)^{\frac{1}{2}}$, is the resultant wave number, and n is the growth rate which is, in general, a complex constant.

Using (19), equations (14) – (18), in non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[(1+F\sigma)\left(D^2 - a^2\right) - \sigma\right]W = Ra^2\Theta + T_A DZ - Q\left(D^2 - a^2\right)DK\ , \tag{20}$$

$$\left[(1+F\sigma)\left(D^2 - a^2\right) - \sigma\right]Z = -DW - QDX\ , \tag{21}$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W\ , \tag{22}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW\ , \tag{23}$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ\ , \tag{24}$$

Where we have introduced new coordinates (x', y', z') = (x/d , y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $F = \frac{\nu'}{d^2}$, is the Rilvin-Ericksen kinematic viscoelasticity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2}$, is the Taylor number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $Z = \frac{2\Omega d}{\nu}Z_\oplus$, $K = \frac{Hd}{\eta}K_\oplus$, $X = \left(\frac{Hd}{\eta}\right)\left(\frac{2\Omega d}{\nu}\right)X_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the case where both the boundaries are rigid and perfectly conducting and are maintained at constant temperature, then the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (20)--(24), must possess a solution are

W = DW = 0, $\Theta = 0$, Z=0, K=0 and DX = 0 at z = 0 and z = 1. (25)

Equations (20)--(24), along with boundary conditions (25), poses an eigenvalue problem for σ and we wish to Characterize σ_i when $\sigma_r \geq 0$.

We first note that since W and Z satisfy $W(0) = 0 = W(1)$, K(0)=K(1) and $Z(0) = 0 = Z(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality[24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz\ ;\ \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz \text{ and } \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz\ , \tag{26}$$

Further, for $W(0)=0=W(1)$, K(0)=0 =K(1)and $Z(0)=0=Z(1)$, Banerjee et al. $[25]$ have show that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2\, dz \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2\, dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2\, dz, \qquad (27)$$

4. MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\} dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz$$

Proof: Multiplying equation (23) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0)=0=K(1)$, it follows that

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\} dz + \sigma_r p_2 \int_0^1 |K|^2\, dz = \text{Real part of} \left\{\int_0^1 K^* DW dz\right\} \leq \left|\int_0^1 K^* DW dz\right|$$

$$\leq \int_0^1 \left|K^* DW\right| dz,$$

$$\leq \int_0^1 \left|K^*\right|\left|DW\right| dz \leq \int_0^1 |K||DW| dz \leq \left\{\int_0^1 |K|^2\, dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2\, dz\right\}^{\frac{1}{2}}, \qquad (28)$$

(Utilizing Cauchy-Schwartz-inequality),

which gives that

$$\int_0^1 |DK|^2\, dz \leq \left\{\int_0^1 |K|^2\, dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2\, dz\right\}^{\frac{1}{2}}, \qquad (29)$$

inequality (28) on utilizing inequalities (26) and (29), gives

$$\left\{\int_0^1 |K|^2\, dz\right\}^{\frac{1}{2}} \leq \frac{1}{\pi^2}\left\{\int_0^1 |DW|^2\, dz\right\}^{\frac{1}{2}}, \qquad (30)$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (28) on utilizing (30), give

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right) dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz, \qquad (31)$$

This completes the proof of lemma**.**

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2\, dz \leq \frac{1}{\pi^4}\int_0^1 |DW|^2\, dz \text{ and } \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right) dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz$$

Proof: Multiplying equation (21) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (24) and appropriate boundary conditions (25), it follows that

$$(1 + F\sigma_r)\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\} dz + \sigma_r \int_0^1 |Z|^2\, dz + Q\int_0^1 \left\{|DX|^2 + a^2|X|^2\right\} dz + Qp_2\sigma_r \int_0^1 |X|^2\, dz$$

=Real part of $\left\{\int_0^1 DW^* Zdz\right\} \le \left|\int_0^1 DW^* Zdz\right|$,

$\le \int_0^1 \left|DW^* Z\right| dz \le \int_0^1 \left|DW^*\right|\left|Z\right| dz$,

$= \int_0^1 |DW||Z| dz \le \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}$, (32)

(Utilizing Cauchy-Schwartz-inequality),
which gives that

$\int_0^1 |DZ|^2 dz \le \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}$, (33)

inequality (32) on utilizing inequalities (26) and (33), gives

$\left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \le \frac{1}{\pi^2} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}$, (34)

Since $\sigma_r \ge 0$ and $p_2 \rangle 0$, hence inequality (32) on utilizing (34), give

$\int_0^1 |Z|^2 dz \le \frac{1}{\pi^4} \int_0^1 |DW|^2 dz$ and $\int_0^1 \left(|DZ|^2 + a^2 |Z|^2\right) dz \le \frac{1}{\pi^2} \int_0^1 |DW|^2 dz$, (35)

This completes the proof of lemma**.**
We prove the following theorems:

Theorem 1: If $\mathrm{R} \rangle\, 0$, $\mathrm{F} \rangle\, 0$, $Q \rangle 0$, $T_A \rangle\, 0$, $p_1 \rangle 0$, $p_2 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K, Z, X) of equations (20) – (24), together with boundary conditions (25) is that

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \rangle 1 \ .$$

Proof: Multiplying equation (20) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$(1+F\sigma)\int_0^1 W^* \left(D^2 - a^2\right)^2 Wdz - \sigma \int_0^1 W^* \left(D^2 - a^2\right) Wdz$

$= Ra^2 \int_0^1 W^* \Theta dz + T_A \int_0^1 W^* DZdz - Q \int_0^1 W^* D\left(D^2 - a^2\right) Kdz$, (36)

Taking complex conjugate on both sides of equation (22), we get

$\left(D^2 - a^2 - p_1 \sigma^*\right)\Theta^* = -W^*$, (37)

Therefore, using (37), we get

$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta \left(D^2 - a^2 - p_1 \sigma^*\right)\Theta^* dz$, (38)

Also taking complex conjugate on both sides of equation (21), we get

$(1+F\sigma^*)\left(D^2 - a^2\right)Z^* - \sigma^* Z^* = -DW^* - QDX^*$, (39) Therefore, using (39), we get

$$\int_0^1 W^* DZdz = -\int_0^1 DW^* Zdz = (1+F\sigma^*)\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \sigma^*\int_0^1 Z^* Zdz + Q\int_0^1 ZDX^* dz, \quad (40)$$

Integrating by parts the third term on left hand side and using equation (24), and appropriate boundary condition (25), we get

$$\int_0^1 W^* DZdz = (1+F\sigma^*)\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \sigma^*\int_0^1 Z^* Zdz + Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz, \quad (41)$$

Also taking complex conjugate on both sides of equation (23), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^* = -DW^*, \quad (42)$$

Therefore, equation (42), using appropriate boundary condition (25), we get

$$\int_0^1 W^* D\left(D^2-a^2\right)Kdz = -\int_0^1 DW^*\left(D^2-a^2\right)Kdz = \int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz, \quad (43)$$

Substituting (38), (41) and (43), in the right hand side of equation (36), we get

$$(1+F\sigma)\int_0^1 W^*\left(D^2-a^2\right)^2 Wdz - \sigma\int_0^1 W^*\left(D^2-a^2\right)Wdz = -Ra^2\int_0^1 \Theta\left(D^2-a^2-p_1\sigma^*\right)\Theta^* dz$$

$$+T_A(1+F\sigma^*)\int_0^1 Z(D^2-a^2)Z^* dz - T_A\sigma^*\int_0^1 Z^* Zdz + T_A Q\int_0^1 X\left(D^2-a^2-p_2\sigma^*\right)X^* dz$$

$$-Q\int_0^1 K^*\left(D^2-a^2\right)^2 Kdz - Qp_2\sigma^*\int_0^1 K^*\left(D^2-a^2\right)Kdz, \quad (44)$$

Integrating the terms on both sides of equation (44) for an appropriate number of times and making use of the appropriate boundary conditions (25), we get

$$(1+F\sigma)\int_0^1 \left\{\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \sigma\int_0^1\left(|DW|^2 + a^2|W|^2\right)dz = Ra^2\int_0^1\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz$$

$$+Ra^2 p_1\sigma^*\int_0^1 |\Theta|^2 dz - T_A(1+F\sigma^*)\int_0^1\left\{|DZ|^2 + a^2|Z|^2\right\}dz - T_A\sigma^*\int_0^1 |Z|^2 dz - T_A Q\int_0^1\left(|DX|^2 + a^2|X|^2\right)dz$$

$$-T_A Qp_2\sigma\int_0^1 |X|^2 dz - Q\int_0^1\left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1\left(|DK|^2 + a^2|K|^2\right)dz, \quad (45)$$

now equating imaginary parts on both sides of equation (45), and cancelling $\sigma_i(\neq 0)$ throughout from imaginary part, we get

$$F\int_0^1\left\{\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \int_0^1\left\{|DW|^2 + a^2|W|^2\right\}dz$$

$$= -Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A F\int_0^1\left\{|DZ|^2 + a^2|Z|^2\right\}dz + T_A\int_0^1 |Z|^2 dz - T_A Qp_2\int_0^1 |X|^2 dz + Qp_2\int_0^1\left\{|DK|^2 + a^2|K|^2\right\}dz,$$

(46)

Now R $\rangle$ 0, $p_2\rangle 0$, $p_1\rangle 0$, $Q\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (26), (27), (31) and (35), the equation (46) gives,

$$\left[(1+\pi^2 F) - \left(\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right)\right]\int_0^1 |DW|^2 dz + I_1\langle 0, \quad (47)$$

Where

$$I_1 = F\int_0^1 \left\{2a^2|DW|^2 + a^4|W|^2\right\}dz + a^2\int_0^1 |W|^2 dz + Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A Q p_2 \int_0^1 |X|^2 dz, \tag{48}$$

Is positive definite, and therefore, we must have

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\}\rangle 1. \tag{49}$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0\text{, then } \frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\}\rangle 1 . \tag{50}$$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 2: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation is that, $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$, where T_A is the Taylor number, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number and F is the viscoelasticity parameter, when both the boundaries are rigid.

or

The onset of instability in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F, satisfy the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$, when both the bounding surfaces are rigid.

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Theorem 3: If $(W, \Theta, K, Z, X, \sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (15) – (19), with R $\rangle$ 0 and,

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1,$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ is that $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state the above theorem as follow:-

Theorem 4: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation is that the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F must satisfy

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the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\rangle 1$, when both the bounding surfaces are rigid

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in Rivlin-Ericksen fluid heated from below i. e. when Q = 0 = T_A. (Sunil et al, 2002).

(ii). Magneto-thermal convection in Rivlin-Ericksen fluid heated from below (T_A=0), if $\frac{1}{(1+\pi^2 F)}\left(\frac{Qp_2}{\pi^2}\right)\leq 1$,and when F=0 then we have $\left(\frac{Qp_2}{\pi^2}\right)\leq 1$ (Gupta et al, 1986).

(iii). Rotatory-thermal convection in Rivlin-Ericksen fluid heated from below (Q = 0), if $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}\right\}\leq 1$.

(iv) When F = 0, then we retrieve the result of Newtonian fluid by (Gupta et al, 1986) in the presence of uniform vertical magnetic field and rotation i. e.

$$\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\leq 1$$

5. CONCLUSIONS:

This theorem mathematically established that the onset of instability in a Rivlin-Ericksen fluid in the presence of uniform vertical magnetic field and rotation cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F satisfy the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\leq 1$, when both the bounding surfaces are rigid.

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having rigid boundaries at the top and bottom of the fluid, in the presence of uniform vertical magnetic field and rotation, parallel to the force field of gravity, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\leq 1$, and in particular PES is valid.

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LIMITATIONS TO COMPLEX GROWTH RATE IN MAGNETO-ROTATORY THERMOSOLUTAL CONVECTION IN COUPLE-STRESS FLUID IN POROUS MEDIUM

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ABSTRACT:
The thermosolutal instability of a couple-stress fluidVeronis (1965) type configuration acted upon by uniform vertical rotation and magnetic field, heated from below in a porous medium is investigated. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of couple-stress fluid convection with a uniform vertical rotation and magnetic field in porous medium, for the case of rigid boundaries shows that the complex growth rate σ of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle

$$\sigma_r^{\,2} + \sigma_i^{\,2} \langle \frac{R_s^{\,2}\varepsilon^2}{\pi^4}\left[1-\left\{\frac{T_A P_l^{\,2}}{(1+\pi^2F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-2},$$

in the right half of a complex σ-plane, where R_s is the thermosolutal Rayleigh number, Q is the Chandrasekhar number, T_A is the Taylor number, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in a magneto-rotatory couple-stress fluidof Veronis (1965) type configuration in porous medium heated from below.A similar result is also proved for Stern (1960) type of configuration.

KEY WORDS: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Rayleigh number; Chandrasekhar Number; Taylor number**.**
MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05**.**

1. INTRODUCTION:

A detailed account of the theoretical and experimental study of the onset of thermal instability in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1] and the Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion, except in the external force term. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [2]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can name the food processing industry, the chemical processing industry, solidification, and the centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium.The problem of thermohaline convection in a layer of fluid heated from below and

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subjected to a stable salinity gradient has been considered by Veronis [3]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat Tabor and Matz [4] and some Antarctic lakes Shirtcliffe [5]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust Lister [6]. Generally it is accepted that comets consists of a dusty ‘snowball’ of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice - versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context Mc Donnel [7]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth’s core where the Earth’s mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov [8] and Linden [9] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth’s rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role

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in the extraction of energy in the geothermal regions. Brakke [10] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [11] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge.

The theory of couple-stress fluid has been formulated by Stokes [12]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze - film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded – bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [12], couple-stresses appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [13] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [14] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [15] have considered a couple - stress fluid with suspended particles heated from below. In another study, Sunil et al. [16] have considered a couple- stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [17] have considered the thermal instability of a layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Pellow and Southwell [18] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [19] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [20] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [21]. However no such result existed for non-Newtonian fluid configurations in general andin particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [22] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in porous medium in the presence of uniform rotation and uniform magnetic field has been considered.The present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible couple-stress fluidconfiguration of Veronis [3] type, in porous medium heated from below in the presence of uniform vertical rotation and magnetic field, opposite to force field of gravity, when the bounding surfaces are of infinite horizontal extension, at the top and bottom of the fluid are rigid.A similar characterization theorem is also proved for Stern [23] type of configuration.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta^{/}\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

Let p, ρ, T, C, α, $\alpha^{/}$, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid (Chandrasekhar [1];Joseph [2] ;Stokes [12]) are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T, \quad (3)$$

$$E^{/}\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)C=\kappa^{/}\nabla^2 C, \quad (4)$$

$$\rho = \rho_0\,[1 - \alpha\,(T-T_0) + \alpha^{/}\,(C-C_0)], \quad (5)$$

Where the suffix zero refers to values at the reference level z = 0 and in writing equation (1), use has been made of Boussinesq approximation.Here E = $\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^{/}$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o , C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The magnetic permeability μ_e, the kinematic viscosityν, couple-stress viscosity $\mu^{/}$, the thermal diffusivity κ and the solute diffusivity $\kappa^{/}$ are all assumed to be constants.

The Maxwell's equations yield

$$\in \frac{d\vec{H}}{dt} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in \eta\nabla^2 \vec{H}\,, \tag{6}$$

and $\nabla\cdot\vec{H} = 0$, (7)

where $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \in^{-1} \vec{q}\cdot\nabla$ stands for the convective derivative.

The steady state solution is

$\vec{q}(u,v,w) = (0,0,0)$, $T = T_0 - \beta z$, $C = C_0 - \beta^{/} z$,

$\rho = \rho_0 (1 + \alpha\beta z - \alpha^{/}\beta^{/} z)$. (8)

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, $\delta\rho$, θ, γ, $\vec{h}(h_x, h_y, h_z)$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,0)$ and velocity $\vec{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$\delta\rho = -\rho_0(\alpha\theta - \alpha^{/}\gamma)$. (9)

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}(\alpha\theta - \alpha^{/}\gamma) - \frac{1}{k_1}\left(\nu - \frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H} + \frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{10}$$

$$\nabla.\vec{q} = 0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta\,, \tag{12}$$

$$E^{/}\frac{\partial\gamma}{\partial t} = \beta^{/} w + \kappa^{/}\nabla^2\gamma\,, \tag{13}$$

$$\in\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in\eta\nabla^2\vec{h}\,, \tag{14}$$

and $\nabla.\vec{h} = 0$. (15)

3. NORMAL MODES ANALYSIS:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$[w, \theta, h_z, \gamma, \varsigma, \xi] = [W(z), \Theta(z), K(z), \Gamma(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt)$, (16)

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k = \left(\sqrt{k_x^2 + k_y^2}\right)$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively. $W(z), K(z), \Theta(z), Z(z)$ and $X(z)$ are the functions of z only.

Using (16), equations (10)-(15), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]W = Ra^2\Theta - R_s a^2\Gamma + T_A DZ - Q\left(D^2 - a^2\right)DK\,, \qquad (17)$$

$$\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]Z = -DW - QDX\,, \qquad (18)$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W\,, \qquad (19)$$

$$\left(D^2 - a^2 - E^{/} p_3\sigma\right)\Gamma = -W\,, \qquad (20)$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW\,, \qquad (21)$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ\,, \qquad (22)$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number;, $p_3 = \frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu' /(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $R_s = \frac{g\alpha'\beta' d^4}{\kappa'\nu}$ is the thermosolutal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $\Gamma = \frac{\beta' d^2}{\kappa^{/}}\Gamma_\oplus$, $Z = \frac{2\Omega d}{\nu\varepsilon}Z_\oplus$, $K = \frac{Hd}{\varepsilon\eta}K_\oplus$, $X = \left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, are perfectly conducting and maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (17)--(22), must possess a solution are

W = 0 =DW =Z=DX =K= $\Theta = \Gamma$, on both the horizontal boundaries, (23)

Equations (17)-(22), along with boundary conditions (23), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, K and Z satisfy $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz\,,\quad \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz\,,\quad \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz\,,$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \text{ and } \int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz\,, \tag{24}$$

Further, for $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, Banerjee et al. [25] have shown that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz\,,\quad \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz\,. \tag{25}$$

4. MATHEMATICAL ANALYSIS:

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^2 E p_1 |\sigma|} \int_0^1 |DW|^2 dz \quad .$$

Proof: Multiplying equation (19) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Θ namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\int_0^1 \left\{ |D\Theta|^2 + a^2 |\Theta|^2 \right\} dz + E\sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of} \left\{ \int_0^1 \Theta^* W dz \right\},$$

$$\leq \left| \int_0^1 \Theta^* W dz \right| \leq \int_0^1 |\Theta^* W| dz \leq \int_0^1 |\Theta^*||W| dz\,,$$

$$\leq \int_0^1 |\Theta||W| dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{26}$$

(Utilizing Cauchy-Schwartz-inequality),

So that the fact that $\sigma_r \geq 0$, we obtain from the above that

$$a^2 \int_0^1 |\Theta|^2 dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{27}$$

Multiplying equation (19) and its complex conjugate, and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Θ namely $\Theta(0) = 0 = \Theta(1)$, we get

$$\int_0^1 \left|\left(D^2 - a^2\right)\Theta\right|^2 dz + 2Ep_1\sigma_r \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz + E^2 p_1^2 |\sigma|^2 \int_0^1 |\Theta|^2 dz = \int_0^1 |W|^2 dz\,, \tag{28}$$

Since $\sigma_r \geq 0$ therefore the equations (28) give,

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{E^2 p_1^2 |\sigma|^2} \int_0^1 |W|^2 dz \tag{29}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (27) on utilizing (29), gives

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^2 E p_1 |\sigma|} \int_0^1 |DW|^2 dz\,, \tag{30}$$

This completes the proof of lemma 1.

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^2 E^{/} p_3 |\sigma|} \int_0^1 |DW|^2 dz\ .$$

Proof: Multiplying equation (20) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{ |D\Gamma|^2 + a^2|\Gamma|^2 \right\} dz + E^{/}\sigma_r p_3 \int_0^1 |\Gamma|^2 dz = \text{Real part of} \left\{ \int_0^1 \Gamma^* W dz \right\},$$

$$\leq \left| \int_0^1 \Gamma^* W dz \right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*||W| dz\,,$$

$$\leq \int_0^1 |\Gamma||W| dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{31}$$

(Utilizing Cauchy-Schwartz-inequality),

So that, since $\sigma_r \geq 0$, we obtain from the above that

And thus, we get

$$a^2 \int_0^1 |\Gamma|^2 dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{32}$$

Multiplying equation (20) and its complex conjugate, and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, we get

$$\int_0^1 \left| \left(D^2 - a^2\right)\Gamma \right|^2 dz + 2E^{'} p_3 \sigma_r \int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2 \right) dz + E^{'2} p_3^{\,2} |\sigma|^2 \int_0^1 |\Gamma|^2 dz = \int_0^1 |W|^2 dz\ , \tag{33}$$

Since $\sigma_r \geq 0$ therefore the equations (33) give,

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{E^{'2} p_3^{\,2} |\sigma|^2} \int_0^1 |W|^2 dz \tag{34}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (29) on utilizing (30) and (24), gives

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^2 E^{/} p_3 |\sigma|} \int_0^1 |DW|^2 dz\,, \tag{35}$$

This completes the proof of lemma 2.

Lemma 3: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{ |DK|^2 + a^2|K|^2 \right\} dz \leq \frac{1}{\pi^2} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (21) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0)=0=K(1)$, it follows that

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of} \left\{\int_0^1 K^* DW dz\right\} \le \left|\int_0^1 K^* DW dz\right| \le \int_0^1 |K^* DW| dz,$$

$$\le \int_0^1 |K^*||DW| dz \le \int_0^1 |K||DW| dz \le \left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{36}$$

(Utilizing Cauchy-Schwartz-inequality),
Inequality (36) on utilizing (24), gives

$$\left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \le \frac{1}{\pi^2}\left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{37}$$

Since $\sigma_r \ge 0$ and $p_2 \rangle 0$, hence inequality (36) on utilizing (37), give

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz \le \frac{1}{\pi^2}\int_0^1 |DW|^2 dz, \tag{38}$$

This completes the proof of lemma 3**.**

Lemma 4: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \le \frac{P_l^2}{(1+\pi^2 F)^2}\int_0^1 |DW|^2 dz \tag{39}$$

Proof: Multiplying equation (18) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (22) and appropriate boundary conditions (23), it follows that

$$\frac{F}{P_l}\int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 |Z|^2 dz + Q\int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz + Qp_2\sigma_r \int_0^1 |X|^2 dz$$

$$= \text{Real part of} \left\{\int_0^1 DW^* Z dz\right\} \le \left|\int_0^1 DW^* Z dz\right|,$$

$$\le \int_0^1 |DW^* Z| dz \le \int_0^1 |DW^*||Z| dz,$$

$$= \int_0^1 |DW||Z| dz \le \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}}, \tag{40}$$

(Utilizing Cauchy-Schwartz-inequality),
Utilizing the inequality (24), (40) gives that

$$\frac{(1+\pi^2 F)}{P_l}\int_0^1 |Z|^2 dz \le \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \tag{41}$$

Inequality (40) on utilizing (41), gives

$$\int_0^1 |Z|^2 dz \le \frac{P_l^2}{(1+\pi^2 F)^2}\int_0^1 |DW|^2 dz, \tag{42}$$

This completes the proof of lemma 4**.**

Now we prove the following theorems:

Theorem 1: If $R \rangle 0$, $R_s \rangle 0$ $F \rangle 0$, $Q \rangle 0$, $T_A \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_2 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, K, Z, X)$ of equations (17) – (22), together with boundary conditions (23) is that

$$|\sigma| \langle \frac{R_s \varepsilon}{\pi^2}\left[1 - \left\{\frac{T_A P_l^2}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Q p_2}{\pi^2}\right)\right\}\right]^{-1} .$$

Proof: Multiplying equation (17) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*\left(D^2 - a^2\right) W dz$$

$$= Ra^2 \int_0^1 W^* \Theta dz - R_s a^2 \int_0^1 W^* \Gamma dz + T_A \int_0^1 W^* DZ dz - Q\int W^* D(D^2 - a^2) K dz , \tag{43}$$

Taking complex conjugate on both sides of equation (19), we get

$$\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* = -W^*, \tag{44}$$

Therefore, using (44), we get

$$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz, \tag{45}$$

Taking complex conjugate on both sides of equation (20), we get

$$\left(D^2 - a^2 - E' p_3 \sigma^*\right)\Gamma^* = -W^*, \tag{46}$$

Therefore, using (46), we get

$$\int_0^1 W^* \Gamma dz = -\int_0^1 \Gamma\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* dz , \tag{47}$$

Also taking complex conjugate on both sides of equation (18), we get

$$\frac{F}{P_l}\left(D^2 - a^2\right)Z^* - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)Z^* + QDX^* = -DW^*, \tag{48}$$

Therefore, using (48), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2 - a^2\right)Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 Z^* Z dz + Q\int_0^1 ZDX^* dz ,$$

Integrating by parts the third term on left hand side and using equation (22), and appropriate boundary condition (23), we get

$$\int_0^1 W^* DZ dz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2 - a^2\right)Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 Z^* Z dz + Q\int_0^1 X\left(D^2 - a^2 - p_2\sigma\right)X^* dz, \tag{49}$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[D^2 - a^2 - p_2\sigma^*\right]K^* = -DW^*, \tag{50}$$

Therefore, equation (50), using appropriate boundary condition (23), we get

$$\int_0^1 W^* D\left(D^2 - a^2\right)K dz = -\int_0^1 DW^*\left(D^2 - a^2\right)K dz = \int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right)K^* dz \ , \tag{51}$$

Substituting (45), (73), (49) and (41), in the right hand side of equation (43), we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*(D^2 - a^2)W dz = -Ra^2\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz$$

$$+ R_s a^2\int_0^1 \Gamma(D^2 - a^2 - E' p_3\sigma^*)\Gamma^* dz + \frac{T_A F}{P_l}\int_0^1 Z(D^2 - a^2)Z^* dz - T_A\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 ZZ^* dz$$

$$+T_A Q\int_0^1 X\left(D^2 - a^2 - p_2\sigma\right)X^* dz - Q\int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right)K^* dz, \tag{52}$$

Integrating the terms on both sides of equation (52) for an appropriate number of times and making use of the appropriate boundary conditions (23), we get

$$\frac{F}{P_l}\int_0^1 \left\{\left|D^2 W\right|^2 + 2a^2\left|DW\right|^2 + a^4\left|W\right|^2\right\}dz + \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(\left|DW\right|^2 + a^2\left|W\right|^2\right)dz$$

$$= Ra^2\int_0^1 \left(\left|D\Theta\right|^2 + a^2\left|\Theta\right|^2 + Ep_1\sigma^*\left|\Theta\right|^2\right)dz - R_s a^2\int_0^1 \left(\left|D\Gamma\right|^2 + a^2\left|\Gamma\right|^2 + E' p_3\sigma^*\left|\Gamma\right|^2\right)dz$$

$$-\frac{T_A F}{P_l}\int_0^1 \left\{\left|DZ\right|^2 + a^2\left|Z\right|^2\right\}dz - T_A\left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left|Z\right|^2 dz - T_A Q\int_0^1 \left(\left|DX\right|^2 + a^2\left|X\right|^2\right)dz$$

$$-T_A Q p_2\sigma\int_0^1 \left|X\right|^2 dz - Q\int_0^1 \left(\left|D^2 K\right|^2 + 2a^2\left|DK\right|^2 + a^4\left|K\right|^2\right)dz - Qp_2\sigma^*\int_0^1 \left(\left|DK\right|^2 + a^2\left|K\right|^2\right)dz, \tag{53}$$

now equating imaginary parts on both sides of equation (53), and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\frac{1}{\varepsilon}\int_0^1 \left\{\left|DW\right|^2 + a^2\left|W\right|^2\right\}dz = -Ra^2 Ep_1\int_0^1 \left|\Theta\right|^2 dz + R_s a^2 E' p_3\int_0^1 \left|\Gamma\right|^2 dz$$

$$+\frac{T_A}{\varepsilon}\int_0^1 \left|Z\right|^2 dz + Qp_2\int_0^1 \left(\left|DK\right|^2 + a^2\left|K\right|^2\right)dz - T_A Q p_2\int_0^1 \left|X\right|^2 dz \ , \tag{54}$$

Now R $\rangle$ $0, Q\rangle 0$ $\varepsilon\rangle 0$ $p_2\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (35), (39) and (42), the equation (54) gives,

$$\frac{1}{\varepsilon}\left[1 - \left\{\frac{P_l^{\,2} T_A}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Qp_2}{(\pi^2}\right) + \varepsilon\left(\frac{R_s}{\pi^2\left|\sigma\right|}\right)\right\}\right]\int_0^1 \left|DW\right|^2 dz + I_1 \langle 0, \tag{55}$$

Where $I_1 = \dfrac{a^2}{\varepsilon}\int_0^1 \left|W\right|^2 dz + Ra^2 Ep_1\int_0^1 \left|\Theta\right|^2 dz + T_A Q p_2\int_0^1 \left|X\right|^2 dz$, is positive definite.

and therefore , we must have

$$\left|\sigma\right| \langle \frac{R_s\varepsilon}{\pi^2}\left[1 - \left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1} \ , \tag{56}$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then } |\sigma| \langle \frac{R_s \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}, \qquad (57).$$

And this completes the proof of the theorem.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state and prove a theorem 1 as follow:-

Theorem 2: If R $\langle$ 0 , $R_s \langle 0$ F $\rangle$ 0, Q $\rangle$ 0, $T_A \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, K, Z, X)$ of equations (17) – (22), together with boundary conditions (23) is that

$$|\sigma| \langle \frac{|R| \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}, \qquad (58)$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (14) –(17) and proceeding exactly as in Theorem 1and utilizing the inequality (30), we get the desired result.

CONCLUSIONS

The inequality (57) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as

$$\sigma_r^{\,2}+\sigma_i^{\,2} \langle \frac{R_s^{\,2}\varepsilon^2}{\pi^4}\left[1-\left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-2},$$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces rigid, in the presence of uniform vertical rotation and magnetic field parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, must lie inside a semi-circle in the right half of the $\sigma_r \sigma_i$ - plane whose centre is at the origin and radius is $\frac{R_s \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}$,where R_s is the thermosolutal Rayleigh number, Q is the Chandrasekhar number, T_A is the Taylor number, P_l is the dimensionless medium permeability of the porous medium, ε is the porosity and F is the couple-stress parameter. The inequality (58) established a similar result for Stern (1960) type of the configuration.

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HALL EFFECT ON MAGNETO-THERMAL STABILITY OF VISCOELASTIC FERROMAGNETIC FLUID SATURATING POROUS MEDIUM

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Abstract:
The effect of Hall Currents on the thermal stability of an electrically conducting ferromagnetic rheological fluid heated from below saturating a porous medium has been investigated. The rheology of the ferromagnetic fluid is described by the Walters'(model B`). The exact solutions of the eigen-value problem for stress- free boundary surfaces are obtained analytically Galerkin technique to find the thermal Rayleigh number for onset of both oscillatory and non- oscillatory modes. It is observed that thermal Rayleigh number decrease with increasing medium permeability and Hall Current, whereas magnetic field and magnetization showing opposite result for both oscillatory and non- oscillatory modes. The principle of exchange of stabilities is not valid. The oscillatory modes are introduced due to viscoelastic parameter F. in oscillatory mode viscoelastic parameter, magnetic field, magnetization as well as Hall current gives stabilizing effect for certain condition, whereas medium permeability showing destabilizing effect. These results have also been shown graphically.

Keywords: Porous medium, Hall current, Galerkin Weighted Residual method, Ferromagnetic Fluids, Viscoelasticity, Magnetic field and Magnetization.

INTRODUCTION

Magnetic fluids which are formed by dispersion of stable non - conducting colloidal suspension of magnetic nanoparticles in a conducting carrier liquid such as heptane's, water or kerosene etc., are known as ferromagnetic fluids. These fluids have some special features like polarization force and body couple which are not found in ordinary fluids. The behavior of these fluids are homogeneously continuum and exhibit various fascinating phenomena. These fluids are not found in nature but are artificially synthesized and find diverse application of ferromagnetic fluids in many science field such as instrumentation, lubrication, printing, vacuum technology, vibration damping, metal recovery, acoustics and medicine, commercial usage of these ferromagnetic fluid includes vacuum feedthroughs of semiconductor (Moskowitz, 1975) ,pressure seals used in compressors and blowers (Brooks, 1955), sealing of rotating shaft in computer disk drives(Bailey, 1983), cooling loudspeakers (Hathaway, 1979) attracted many researchers towards this field. Significant contribution is given by various researchers toward ferromagnetic fluid and their application by taking theoretical and experimental aspects.

Convection stability of ferromagnetic fluid for fluid layer heated from below in the presence of uniform vertical magnetic field has been discussed in detail by Finlayson (1970). He found that there is induction of convection due to temperature and temperature gradient across the layer which leads to magnetization of ferrofluids.Lalas and Carmi (1971) analyzed thermoconductive stability of ferrofluids and proves that linear and energies theories give identical results for stationary convection in ferrofluids. Shliomis (1974) has studied the mechanisms of relaxation of the magnetization of a suspension.

Earlier many authors considered Bernard's convection in non-porous (Siddheshwar, 1993, 1995; Sunil, Sharma and Shandil, 2006; Aggarwal and Prakash, 2009). In recent years researchers concentrate on the convective flow and stability of fluid layer in porous medium which was earlier considered by(Lapwood, 1948) and (Wooding, 1960).

Growing importance of non- Newtonian fluid in geophysical fluid dynamics chemical technology and petroleum industry aroused interest of many researchers toward studies of convective fluid motion in porous medium because of its important application in prediction of ground water movement, in atmospheric physics and recovery of crude oil from porous of rocks. There is vast variety of non- Newtonian fluids. Principle types of non- Newtonian fluid include: couple stress fluids, viscoelastic fluids (Rivlin- Ericksian fluids, Walters' (Model B′) fluid), plastic solids, power –law fluids, time dependent etc.Walters(Walters, 1960) studied the motion of oscillation flow of the viscoelastic liquid in a concentric- sphere elastoviscometer. He concludes that concentric-sphere elastoviscometer and coaxial- cylinder elastoviscometer both discriminant exclusive viscoelastic liquids in same way. Walters(1962) again studied the behaviour of non– Newtonian elasto-viscous fluids at small rates of shear which are characterized by a general linear equation of state. With growing importance of non- Newtonian viscoelastic fluids in science and technology has also been studied by many authors (Sharma and Kumar, 1997; Kumar and Sharma, 2000; Kumar*et al.*, 2004; Ali *et al.*, 2012; Pandey*et al.*, 2016).

If electric current flow through a conductor in magnetic field which reduce conductivity parallel to the electric field and hence, the current is reduced in the direction normal to both electric and magnetic field.This phenomenon in the literature is known as Hall effect.The Hall current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. The effect of Hall current on thermal instability has been studies by Aggarwal in the presence of dusty couple fluids (Aggarwal and Verma, 2016). In this research paper he has conclude that effect of Hall current on thermal convection with dusty couple particle show destabilizing effect and also found that oscillatory modes are produced in the present of Hall current. P. Narayana studied the effect of rotating micro polar fluid and radiation in Hall current (Narayana, 2013). He observed that with increase in the value of Hall current parameter micro – rotation profile decreases. There are many several authors studied the effect of Hall effect on different thermal instability(Gupta, 1967; Raghavachar and Gothandaraman, 1989; Sharma and Thakur, 2000; Sunil *et al.*, 2005; Aggarwal and Verma, 2017).

Motivated by the various application of rheology, medium porosity, Hall current and medium permeability, an attempt has been made to study on the criterion for stability of a layer of ferromagnetic viscoelastic fluid heated from below saturating a porous medium.The purpose of work is to analyze the influence of viscoelasticity in the magneto thermal convective thresholds in ferrofluids in the presence of presence of Hall current and rheology of the ferrofluids is described by the fluid Walters' (model B') saturating a porous medium in ferrofluids. In the present problem, we have studies the effect of Hall current on thermal stability of ferromagnetic fluid heated from below in porous medium in the presence of horizontal magnetic field.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Figure 1: Geometrical Configuration

Consider magnetic nanoparticles suspended in conducting various carrier liquids. This prevents on the porous matrix, which is electrically non-conducting and incompressible. We consider that horizontal magnetic field H_{mf} $(H,0,0)$ and uniform vertically downward gravity force ***g*** *(0,0,-g)* of a viscoelastic ferromagnetic porous medium is confined between the planes z = 0 and z = d. The temperatures at the lower and upper boundaries are assumed to be T_l and $T_u$respectively ($T_l > T_u$). Constant temperature difference $\Delta T\ (=\ T_l - T_u)$ is maintain between the boundaries.

The governing basic equations for the flow of an incompressible viscoelastic ferromagnetic fluids saturating a porous medium are

Equation of state

$$\rho_{mf} = \rho_l\left[1-\alpha_{mf}\left(T-T_l\right)\right], \tag{1}$$

Equation of continuity

$$\nabla.\vec{u}_{mf} = 0, \tag{2}$$

Equation of conservation of momentum are given by

$$\frac{\rho_l}{\varepsilon}\frac{D}{Dt^*}\overrightarrow{u_{mf}} = -\nabla p_{mf} + \vec{g}\rho_{mf} + \overrightarrow{M_{mf}}.\nabla\overrightarrow{H_{mf}} - \frac{\rho_l}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{u_{mf}} + \frac{\mu_{me}}{4\pi}\left(\nabla\times\overrightarrow{H_{mf}}\right)\times\overrightarrow{H_{mf}}, \tag{3}$$

where $\frac{D}{Dt^*} = \left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon}\left(\overrightarrow{q_{mf}}.\nabla\right)\right]$ is convective derivative.

Equation of energy

$$E\frac{\partial T}{\partial t^*} + \left(\overrightarrow{u_{mf}}.\nabla\right)T = k_{mf}\nabla^2 T, \tag{4}$$

Where in above equations, p_{mf} is the pressure, ρ_{mf} is density of ferromagnetic fluid, T is the temperature, α_{mf} is the thermal coefficient of expansion, $\vec{g} = (0,0,-g)$ is the gravitational acceleration acting vertically downward; t^* is the time, ρ_l is the ferromagnetic density at a

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reference temperature $T_l\left(z=0\right)$; $\vec{u}_{mf}=\left(u,v,w\right)$ is the filter velocity at coordinate x,y and z respectively; $\mu_{me}=4\pi\times10^{-7}\,m/A$ is the vacuum magnetic permeability; k_{mf} is thermal diffusivity.

To study traditional ferromagnetic concepts, the relationship between the induced field $\overrightarrow{B_{mf}}$, the external magnetic field, $\overrightarrow{H_{mf}}$ and the intensity of magnetization $\overrightarrow{M_{mf}}$, are given by following equations

$$\overrightarrow{M_{mf}}\times\overrightarrow{H_{mf}}=0 \tag{5}$$

$$\nabla.\overrightarrow{B_{mf}}=0,\nabla\times\overrightarrow{H_{mf}}=0 \tag{6}$$

$$\overrightarrow{B_{mf}}=\mu_0\left(\overrightarrow{H_{mf}}+\overrightarrow{M_{mf}}\right) \tag{7}$$

We superimpose that the magnetization is correspond to the magnetic field, while its magnitude $\left|\vec{M}_{\mathrm{mf}}\right|=M_{\mathrm{mf}}$ dependence on the magnetic field and temperature so that

$$\overrightarrow{M_{mf}}=\frac{H_{\mathrm{mf}}}{H}M_{\mathrm{mf}}\left(H,T\right) \tag{8}$$

In the presence of Hall currents Maxwell's equations reduces to

$$\frac{\partial\overrightarrow{H_{mf}}}{\partial t^*}=\frac{1}{\varepsilon}\left[\nabla\times\left(\overrightarrow{u_{mf}}\times\overrightarrow{H_{mf}}\right)\right]+\eta_{\mathrm{mf}}\nabla^2\overrightarrow{H_{mf}}-\frac{1}{4\pi N_{\mathrm{mf}}e}\nabla\times\left[\left(\nabla\times\overrightarrow{H_{mf}}\right)\times\overrightarrow{H_{mf}}\right] \tag{9}$$

$$\nabla.\overrightarrow{H_{mf}}=0 \tag{10}$$

All the quantities appeared in the above equations are already defined in nomenclature. In process $\overrightarrow{H_{mf}}$ and T are only two thermodynamics variables of $\overrightarrow{M_{mf}}$ magnetization. We assume the first approximation for temperature dependent magnetization $\overrightarrow{M_{mf}}=M_{\mathrm{mf}}\left(T\right)$ are given in the only form of temperature as

$$\frac{\overrightarrow{M_{mf}}}{M_l}=\left[1-\gamma_{mf}\left(T-T_l\right)\right] \tag{11}$$

where M_l represents the magnetization at lower temperature which known as reference temperature with $T=T_l$ at $z=0$, and

$$\gamma_{mf}=\frac{1}{M_l}\left(\frac{\partial\overrightarrow{M_{mf}}}{\partial T}\right)_H.$$

Assume basic state for the given system is

$$\overrightarrow{u_{mf}}\left(u,v,w\right)=u_b\left(0,0,0\right)=0,\ p_{mf}=p_b\left(z\right),T=T_b\left(z\right)=T_0-\beta\, z,$$
$$\rho_{mf}=\rho_0\left(1+\alpha_{mf}\beta\, z\right)=\rho_b\left(z\right),\overrightarrow{M_{mf}}=M_b\left(z\right). \tag{12}$$

Let us consider infinitesimal perturbations around the initial state in the following forms

$$\overrightarrow{u_{mf}}(u,v,w)=0+\overrightarrow{u_{mf}},T=T_b+\theta,\ p_{mf}=p_b\left(z\right)+\delta p,\rho_{mf}=\rho_b+\delta\rho,$$
$$\overrightarrow{H_{mf}}=H_b\left(H,0,0\right)+\vec{h}\left(h_x,h_y,h_z\right),\overrightarrow{M_{mf}}=M_b\left(z\right)+\delta M \tag{13}$$

Where $\delta\rho,\delta p,\delta M,\theta,h(h_x,h_y,h_z)$ and $u_{mf}(u,v,w)$ denote the perturbations in density, pressure, magnetization, temperature, magnetic field $H_{mf}(H,0,0)$ and filter velocity u_{mf} (zero initially), respectively which are superimposed into the basic state. The small change in $\delta\rho$ density and δM magnetization, caused mainly by the perturbations in concentration γ_{mf} and temperature θ, is given by

$$\frac{\delta\rho}{\rho_0}=-\alpha_{mf}\theta,\frac{\delta M}{M_0}=-\gamma_{mf}\theta \tag{14}$$

Using boussinesq approximation, linearized perturbations equations (1) – (4), (9) and (10) for ferromagnetic fluids are

$$\frac{\rho_0}{\varepsilon}\frac{\partial\overrightarrow{u_{mf}}}{\partial t^*}=-\nabla\delta p-\vec{g}\rho_0\alpha_{mf}\theta-\gamma_{mf}M_0.\nabla\overrightarrow{H_{mf}}\theta-\frac{\rho_0}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{u_{mf}}+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\overrightarrow{H_{mf}}, \tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{16}$$

$$E\left(\frac{\partial}{\partial t^*}-k_{mf}\nabla^2\right)\theta=\beta w, \tag{17}$$

$$\varepsilon\left(\frac{\partial}{\partial t^*}-\eta_{mf}\nabla^2\right)\vec{h}=\left(\overrightarrow{H_{mf}}.\nabla\right)\overrightarrow{u_{mf}}-\frac{\varepsilon}{4\pi N_{mf}e}\nabla\times\left[\left(\nabla\times\vec{h}\right)\times\overrightarrow{H_{mf}}\right], \tag{18}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0. \tag{19}$$

As we know that vector components of $\overrightarrow{u_{mf}}$, $\vec{h}$ and $\overrightarrow{H_{mf}}$ are

$$\overrightarrow{u_{mf}}=(u_x,v_y,w_z),\vec{h}=\left(h_x,h_y,h_z\right)$$

and

$\overrightarrow{H_{mf}}=H\left(H,0,0\right)$. The unknowns $u_x,v_y,w_z,h_x,h_y,h_z,\delta p$ can be reduced to w_z,h_z,φ,ψ by operating on eq. (15) and eq. (18) with $\hat{e}_z.curlcurl$ and using the identity curl operator i.e.,

$curlcurl \equiv graddiv - \nabla^2$ with the aid off eq. (16) and eq. (19),we get suitable result in the followings forms

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1}{k_1}\nu - \frac{1}{k_1}\frac{\partial}{\partial t^*}\nu'\right]\nabla^2 w = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(g\alpha_{mf} - \frac{\gamma_{mf} M_0 \nabla.\overrightarrow{H_{mf}}}{\rho_0}\right)\theta + \frac{\mu_e \overrightarrow{H_{mf}}}{4\pi\rho_0}\left(\nabla^2 \frac{\partial h_z}{\partial x}\right), \quad (20)$$

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)\right]\varphi = \frac{\mu_e H_{mf}}{4\pi\rho_0}\left(\frac{\partial \psi}{\partial x}\right), \quad (21)$$

$$\left(\frac{\partial}{\partial t^*} - \eta_{mf}\nabla^2\right)\psi = \frac{H_{mf}}{\varepsilon}\left(\frac{\partial \varphi}{\partial x}\right) + \frac{H_{mf}}{4\pi N_{mf} e}\frac{\partial}{\partial x}\left(\nabla^2 h_z\right), \quad (22)$$

$$\left(\frac{\partial}{\partial t^*} - \eta_{mf}\nabla^2\right)h_z = \frac{H_{mf}}{\varepsilon}\left(\frac{\partial w}{\partial x}\right) - \frac{H_{mf}}{4\pi N_{mf} e}\frac{\partial}{\partial x}\left(\nabla^2 \psi\right), \quad (23)$$

where $\varphi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity, $\psi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is laplacian operator on the horizontal plane.

NORMAL MODE ANALYSIS

Analyzing the arbitrary disturbance into a complete set of normal modes and examine the stability of modes individually. Thus, we assume that perturbation quantities dependence on x, y and t are of the form

$$\left[w, \theta, \psi, \varphi, h_z\right] = \left[W^*(z), \Theta^*(z), X^*(z), Z^*(z), K^*(z)\right]\exp\left(iq_x x + iq_y y + \sigma^* t\right), \quad (24)$$

Where q_x and q_y are horizontal wavenumbers in the x and y -directions, respectively, $k\left[= \left(q_x^2 + q_y^2\right)^{\frac{1}{2}}\right]$ is the resultant wave number of the disturbance and σ^* is the growth rate of disturbance (in general, a complex constant). For functions with this dependence on x, y and t, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -q^2\right)$ and $\left(\nabla^2 = \frac{\partial^2}{\partial z^2} - q^2\right)$.

Using expression (24), linearized the perturbation equations (17), (20)- (23) in non –dimensional form become

$$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} - \sigma F\right)\left(D^2 - a^2\right)W^* = -\frac{\alpha_{mf} a^2 d^2}{\nu}\left(g - \frac{\gamma_{mf} M_0 \nabla H_{mf}}{\rho_0 \alpha_{mf}}\right)\Theta^* + \frac{ik_x \mu_e H d^2}{4\pi\rho_0\nu}\left(D^2 - a^2\right)K^*, \quad (25)$$

$$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} - \sigma F\right)Z^* = \frac{ik_x \mu_e H d^2}{4\pi\rho_0\nu}X*, \quad (26)$$

$$\left(D^2-a^2-p_2\sigma\right)K^* = -\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}W^* + \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}X^*, \tag{27}$$

$$\left(D^2-a^2-p_2\sigma\right)X^* = -\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}Z^* - \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}\left(D^2-a^2\right)K^*, \tag{28}$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta^* = -\frac{\beta d^2}{\kappa}W^*. \tag{29}$$

where $a=qd, q_x=q\cos\theta, \sigma=\frac{\sigma^* d^2}{\nu}, D=\frac{d}{dz}$ and the non- dimensional parameters are $p_1=\frac{\nu}{\kappa}$ is the thermal Prandtl number, $p_2=\frac{\nu}{\eta}$ is the magnetic Prandtl number, $P_1=\frac{k_1}{d^2}$ is the dimensionless medium permeability and $F=\frac{\nu'}{k_1}$ is non – dimension viscoelastic parameter.

Eliminating Θ^* from equation (25) and (29) by using algebraic operation $\left(D^2-a^2-Ep_1\sigma\right)$ with equation (25), one gets

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)-R_f a^2\right]W - \frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)K = 0, \tag{30}$$

also we can rewrite equation (27) as

$$\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}W^* - \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}X^* + \left(D^2-a^2-p_2\sigma\right)K^* = 0, \tag{31}$$

Eliminating Z^* by operating equation (28) by $\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)$ and using equation (26), which yields

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2-p_2\sigma\right)-\frac{k_x^2 d^2 Q}{\varepsilon}\right]X^* + \frac{ik_x H}{4\pi N_{mf}\eta_{mf}e}\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)K^* = 0 \tag{32}$$

Method of solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the W, X and K of equations (31)-(33) with the corresponding boundary condition. The appropriate boundary conditions for both bounding surfaces free- free; which transform expression (25) in non- dimensional form

$$W = D^2W = 0, X = DX = 0, \Theta = 0, D^2Z = 0, K = 0 \text{ at } z=0 \text{ and } z=1 \tag{34}$$

Accordingly, the base functions W, X and K are taking in the following way:

$$W=\sum_{n=1}^{N}A_nW_n,\ X=\sum_{n=1}^{N}B_nX_n \text{ and } K=\sum_{n=1}^{N}C_nK_n \tag{35}$$

where $W_n=\left(z^n-2z^{n+2}+z^{n+3}\right), X_n=\left(z^{n+1}-2z^{n+2}+z^{n+3}\right), K_n=\left(z^n-z^{n+1}\right)$.

The trial solutions satisfying the dimensionless boundary conditions A_n, B_n and C_n are unknown coefficients, and $n=1,2,3,........,N.$

Calculating the residual or integrals in the limits from zero to unit, we obtain a set of linear homogeneous equations in the $3N$ unknowns A_n, B_n and C_n which admits a non – trivial solution only if its determinant is equal to zero. Once the determinant is equated to zero, thus it found the characteristic equation of the system in terms of the thermal Rayleigh number R_f , other parameters. For a first approximation, we take N =1; this produces the result

$$\begin{bmatrix} a_{11} & 0 & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{36}$$

where the elements of matrix are

$$a_{11}=\left(\frac{24}{5}+\frac{17}{35}\left(2a^2+Ep_1\sigma\right)+\frac{31}{630}a^2\left(a^2+Ep_1\sigma\right)\right)\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)-\frac{31}{630}R_f a^2,$$

$$a_{13}=-\frac{\iota k_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(4+\frac{2}{5}\left(2a^2+Ep_1\sigma\right)+\frac{17}{420}a^2\left(a^2+Ep_1\sigma\right)\right),$$

$$a_{21}=\frac{11}{1260}\left(\frac{\iota k_x Hd^2}{\varepsilon\eta_{mf}}\right), a_{22}=-\frac{1}{630}\frac{\iota k_x Hd^2}{4\pi N_{mf}e\eta_{mf}},$$

$$a_{23}=-\frac{1}{15}-\frac{1}{140}(a^2+p_2\sigma),$$

$$a_{32}=\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(\frac{-1}{15}-\frac{1}{140}\left(a^2+p_2\sigma\right)\right)-\frac{1}{140}\left(\frac{k_x{}^2Qd^2}{\varepsilon}\right),$$

$$a_{33}=\frac{\iota k_x Hd^2}{4\pi N_{mf}e\eta_{mf}}\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(-\frac{1}{3}-\frac{1}{30}a^2\right)$$

$$R_f=\frac{-\left(\begin{array}{c}56\left(\varepsilon+P_1\sigma-FP_1\varepsilon\sigma\right)M_3M_2+6\left(28+3a^2+3p_2\sigma\right)\left(-\varepsilon+P_1\left(-1+F\,\varepsilon\right)\sigma\right)M_2M_1\\ -33a^2P_1Q\left(17a^4+168\left(10+Ep_1\sigma\right)+a^2\left(336+17Ep_1\sigma\right)\right)M_1\cos^2\theta\end{array}\right)}{\left(62a^2P_1\varepsilon\left(-28M_3+3\left(28+3a^2+3p_2\sigma\right)M_1\right)\right)} \tag{37}$$

where $M_1=\left(28+3a^2+3p_2\sigma\right)\left(-\varepsilon+P_1\left(-1+F\,\varepsilon\right)\sigma\right)-3a^2P_1Q\cos^2\theta$,

$M_2=\left(3024+31a^4+306Ep_1\sigma+a^2\left(612+31Ep_1\sigma\right)\right)$,

$$M_3 = a^2\left(10+a^2\right)M\left(\varepsilon + P_1\sigma - FP_1\varepsilon\sigma\right)\cos^2\theta,$$

Where $R_f\left(=\left[g-\dfrac{\gamma_f M_0 \nabla .H_{mf}}{\rho_0\alpha_f}\right]\dfrac{\alpha_f \beta d^4}{\nu\kappa}\right)$ is the Rayleigh number for ferromagsnetic fluids, $Q\left(=\dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \nu \eta_{mf}}\right)$ is the Chandrasekhar number, $M\left(=\dfrac{H}{4\pi N_{mf} e\eta_{mf}}\right)^2$ is the Hall parameter and $F=\dfrac{\nu'}{k_1}$ is the viscoelastic number.

Mathematical analysis

Non-oscillatory convection

When instability sets in stationary convection, the marginal state is characterized by putting $\sigma = 0$, in the dispersion relation (37), then the Rayleigh number for the case of stationary convection is given by

$$R_f^{stat} = \frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta + 6\left(28+3a^2\right)M_4M_5\varepsilon + 33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)} \tag{38}$$

Where $M_4 = \left(28+3a^2\right)\varepsilon + 3a^2P_1Q\cos^2\theta$

$M_5 = \left(3024 + 612a^2 + 31a^4\right)$

$M_6 = \left(1680 + 336a^2 + 17a^4\right)$

which is identical with the result of R_f^{stat} derived by sharma et al.(2017) for stationary convection. For stationary convection R_f^{stat} is independent of viscoelastic parameter since it vanishes with disappearing of σ which show same result as other non- viscoelastic ferromagnetic fluids.

In order to investigate the effects of magnetic field, medium permeability and Hall currents, we examine the behavior of $\dfrac{dR_f}{dQ}, \dfrac{dR_f}{dM}$ and $\dfrac{dR_f}{dP_1}$ analytically.

$$\frac{dR_f}{dQ} = \frac{33M_6\cos^2\theta\begin{pmatrix}\left(28+3a^2\right)\left(27a^4P_1^2Q^2\left(3+4\cos[2\theta]+\cos[4\theta]\right)+28\left(10+a^2\right)M\varepsilon^2\left(4a^2+\cos[2\theta]\right)\right)+\\ \left(28+3a^2\right)^2\left(18P_1Q\varepsilon\left(4a^2+\cos[2\theta]\right)+24\left(28+3a^2\right)\varepsilon^2\right)+\\ 168\left(10+a^2\right)a^2MP_1Q\varepsilon\left(3a^2+\cos[2\theta]+a^2\cos[4\theta]\right)\end{pmatrix}}{496\varepsilon\left(3\left(28+3a^2\right)^2\varepsilon + a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2} \tag{39}$$

$$\frac{dR_f}{dM}=\frac{-231a^2M_6\left(10+a^2\right)Q\cos^4\theta\left(56\varepsilon+3a^2\left(P_1Q+2\varepsilon\right)+3a^2P_1Q\cos[2\theta]\right)}{31\left(3\left(28+3a^2\right)^2\varepsilon+a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2}, \tag{40}$$

$$\frac{dR_f}{dP_1}=\frac{-\begin{pmatrix}9\left(28+3a^2\right)^4M_5\varepsilon^2+6a^2\left(28+3a^2\right)^2M_5\varepsilon\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta+\\ a^4M_5\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)^2\cos^4\theta-1386a^6\left(10+a^2\right)M_6MP_1^2Q^2\cos^6\theta\end{pmatrix}}{31a^2P_1^2\left(3\left(28+3a^2\right)^2\varepsilon+a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2} \tag{41}$$

It is clear from equations (39) that for stationary convection the magnetic field has a stabilizing effect for all wave numbers. Equation (40) and equation (41) shows that Hall effect and medium Permeability have destabilizing effect on the system.

In order to investigating the analytical effect of magnetization replaced R_f by R_f/π^4in equation (38), which yields

$$R_f^{stat}=\frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta+6\left(28+3a^2\right)M_4M_5\varepsilon+33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)}\frac{\pi^4}{1-\frac{\gamma_{mf}M_0\nabla H}{\rho_0\alpha_{mf}\ g}} \tag{42}$$

To see the effect of magnetization, we examine the behavior of $\frac{dR_f}{dM_0}$ analytically. Equation (42) yields:

$$\frac{dR_f^{stat}}{dM_0}=\frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta+6\left(28+3a^2\right)M_4M_5\varepsilon+33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)}\frac{\pi^4}{\left(1-\frac{\gamma_{mf}M_0\nabla H}{\rho_0\alpha_{mf}\ g}\right)^2}\frac{\gamma_{mf}\nabla H}{\rho_0\alpha_{mf}\ g} \tag{43}$$

It is clear from equation (43) that the magnetization has stabilizing effect on the system.

Oscillatory convection

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of viscoelastic number parameter F. Equating the imaginary parts of equation (37) (by putting $\sigma=\iota\,\sigma_i$,), we obtain

$$\begin{aligned}&\sigma_i[(A_{14}c_1-A_{11}c_2)+\sigma_i^2(A_{15}c_1-A_{14}c_3-A_{12}c_2+A_{11}c_4)+\\&\sigma_i^4(A_{16}c_1-A_{15}c_3-A_{13}c_2+A_{12}c_4)+\sigma_i^6(A_{13}c_4-A_{16}c_3)]=0\end{aligned} \tag{44}$$

It is evident from equation (44) that σ_i may be either zero or non- zero, meaning that the modes may be either non- oscillatory or oscillatory and the principle of exchange of stabilities is not satisfied for the problem.

The case of overstability

Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (44) will admit of solutions with σ_i real. For oscillatory motions, the real part of σ *is* zero. Hence, putting $\sigma = \iota\sigma_i$, *in equation* (37) whereσ_i is real and is the dimensional frequency; one obtains

$$R_f = \Delta_{\mathrm{Re}al} + i\sigma_i\Delta_{\mathrm{Im}g} \quad , \tag{45}$$

$$\Delta_{\mathrm{Re}al} = \frac{-\begin{pmatrix} A_{11}c_1 + \sigma_i^2\left(A_{12}c_1 - A_{11}c_3 + A_{14}c_2\right) + \sigma_i^4\left(A_{13}c_1 - A_{12}c_3 + A_{15}c_2 - A_{14}c_4\right) + \\ \sigma_i^6\left(-A_{13}c_3 + A_{16}c_2 - A_{15}c_4\right) - \sigma_i^8\left(A_{16}c_4\right) \end{pmatrix}}{\left(c_1 - \sigma_i^2 c_3\right)^2 + \sigma_i^2\left(c_2 - \sigma_i^2 c_4\right)^2}$$

where (46)

$$\Delta_{Img} = [(A_{14}c_1 - A_{11}c_2) + \sigma_i^2(A_{15}c_1 - A_{14}c_3 - A_{12}c_2 + A_{11}c_4) +$$
$$\sigma_i^4(A_{16}c_1 - A_{15}c_3 - A_{13}c_2 + A_{12}c_4) + \sigma_i^6(A_{13}c_4 - A_{16}c_3)] = 0 \tag{47}$$

Equation (44) implies for $\sigma_i \neq 0$ for oscillatory modes, therefore equation (45) implies that $\Delta_{\mathrm{Im}g} = 0$ which on simplification yields a dispersion relation as

$$[(A_{14}c_1 - A_{11}c_2) + \sigma_i^2(A_{15}c_1 - A_{14}c_3 - A_{12}c_2 + A_{11}c_4) +$$
$$\sigma_i^4(A_{16}c_1 - A_{15}c_3 - A_{13}c_2 + A_{12}c_4) + \sigma_i^6(A_{13}c_4 - A_{16}c_3)] = 0 \tag{48}$$

Also equation (45) with $R_f^{OSC} = \Delta_{\mathrm{Re}al}$ on simplification gives the thermal Rayleigh number for oscillatory modes as

$$R_f^{OSC} = -\begin{pmatrix} A_{11}c_1 + \sigma_i^2\left(A_{12}c_1 - A_{11}c_3 + A_{14}c_2\right) + \\ \sigma_i^4\left(A_{13}c_1 - A_{12}c_3 + A_{15}c_2 - A_{14}c_4\right) + \\ \sigma_i^6\left(-A_{13}c_3 + A_{16}c_2 - A_{15}c_4\right) - \sigma_i^8\left(A_{16}c_4\right) \end{pmatrix}\left(\left(c_1 - \sigma_i^2 c_3\right)^2 + \sigma_i^2\left(c_2 - \sigma_i^2 c_4\right)^2\right)^{-1} \tag{49}$$

where $A_{11} = A_1 + A_3 + A_6, A_{12} = -A_2 + A_4 + A_7, A_{13} = A_5, A_{14} = B_1 + B_3 + B_6$

$$A_{15} = -B_2 + B_4 + B_7, A_{16} = -B_5, A_1 = 56a^2\left(10 + a^2\right)\varepsilon^2 M\left(3024 + 612a^2 + 31a^4\right)\cos^2\theta$$

$$A_2 = 56a^2\left(10 + a^2\right)M\left(P_1 - P_1\varepsilon F\right)\left[\left(P_1 - P_1\varepsilon F\right)\left(3024 + 612a^2 + 31a^4\right) + 2\varepsilon E p_1\left(306 + 31a^2\right)\right]\cos^2\theta$$

$$A_3 = \left(\varepsilon^2\left(28 + 31a^2\right)^2 + 3a^2\varepsilon P_1 Q\left(28 + 31a^2\right)\cos^2\theta\right)\left(3024 + 612a^2 + 31a^4\right),$$

$$A_4=\left(28\varepsilon+31a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)\left(\begin{array}{l}3P_1p_2\left(-1+F\varepsilon\right)\left(3024+612a^2+31a^4\right)+\\ \left(-3p_2\varepsilon+P_1\left(-1+F\varepsilon\right)\left(28+3a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)\end{array}\right),$$

$$A_5=3P_1p_2\left(-1+F\varepsilon\right)\left(-3p_2\varepsilon+P_1\left(-1+F\varepsilon\right)\left(28+31a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)$$

$$+3P_1p_2\left(-1+F\varepsilon\right)\left(\begin{array}{l}3P_1p_2\left(-1+F\varepsilon\right)\left(3024+612a^2+31a^4\right)+\\ \left(-3p_2\varepsilon+P_1\left(-1+F\varepsilon\right)\left(28+3a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)\end{array}\right),$$

$$A_6=33a^2P_1Q\cos^2\theta\left(17a^4+336a^2+1680\right)\left(\left(28\varepsilon+3a^2\varepsilon\right)+3a^2P_1Q\cos^2\theta\right),$$

$$A_7=-33a^2P_1Q\cos^2\theta\left(17a^2Ep_1+168Ep_1\right)\left(\left(28\varepsilon P_1+3a^2\varepsilon P_1\right)\left(-1+F\varepsilon\right)+3p_2\varepsilon\right)$$

$$\left(99a^2p_2P_1^2Q\cos^2\theta\left(17a^4+336a^2+1680\right)\left(-1+F\varepsilon\right)\right),$$

$$B_1=56a^2\left(10+a^2\right)M\left[2\varepsilon\left(P_1-P_1\varepsilon F\right)\left(3024+612a^2+31a^4\right)+\varepsilon^2Ep_1\left(306+31a^2\right)\right]\cos^2\theta,$$

$$B_2=56a^2\left(10+a^2\right)M\left(P_1-P_1\varepsilon F\right)^2Ep_1\left(306+31a^2\right)\cos^2\theta,$$

$$B_3=-\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)\left(\begin{array}{l}\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(3024+31a^4+612a^2\right)+\\ \left(-28\varepsilon-3a^2\varepsilon\right)\left(306Ep_1+31a^2Ep_1\right)\end{array}\right)+$$

$$\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(-28\varepsilon-3a^2\varepsilon\right)\left(3024+31a^4+612a^2\right),$$

$$B_4=3p_2\left(-P_1+P_1\varepsilon F\right)\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)\left(306Ep_1+31a^2Ep_1\right)-$$

$$\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(\begin{array}{l}3p_2\left(-P_1+P_1\varepsilon F\right)\left(3024+31a^4+612a^2\right)+\\ \left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(306Ep_1+31a^2Ep_1\right)\end{array}\right)$$

$$-3p_2\left(-P_1+P_1\varepsilon F\right)\left(\begin{array}{l}\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(3024+31a^4+612a^2\right)+\\ \left(-28\varepsilon-3a^2\varepsilon\right)\left(306Ep_1+31a^2Ep_1\right)\end{array}\right),$$

$$B_5=9p_2^2\left(-P_1+P_1\varepsilon F\right)^2\left(306Ep_1+31a^2Ep_1\right),$$

$$B_6=33a^2P_1Q\cos^2\theta\left(\begin{array}{l}\left(168Ep_1+17a^2Ep_1\right)\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)+\\ \left(17a^4+336a^2+1680\right)\left(\left(28+3a^2\right)\left(P_1-F\varepsilon P_1\right)+3p_2\varepsilon\right)\end{array}\right),$$

$$B_7=99a^2p_2P_1Q\cos^2\theta\left(168Ep_1+17a^2Ep_1\right)\left(-P_1+F\varepsilon P_1\right),$$

$$c_1=62a^2P_1\varepsilon\left(-3\varepsilon(28+3a^2)^2-28a^2(10+a^2)M\varepsilon cos^2\theta-9a^2P_1Q(28+3a^2)cos^2\theta\right),$$

$$c_2=62a^2P_1\varepsilon\left(\begin{array}{l}-18p_2\varepsilon(28+3a^2)+3(28+3a^2)^2\left(-1+F\varepsilon\right)-\\ 28a^2(10+a^2)MP_1cos^2\theta-27a^2p_2P_1Qcos^2\theta\end{array}\right),$$

$c_3 = 62a^2 P_1 \varepsilon \left(18 p_2 (28+3a^2)(-1+F\varepsilon) - 27 p_2{}^2 \varepsilon\right)$, $c_4 = 27 p_2{}^2 a^2 P_1 \varepsilon (-62+62F\varepsilon)$.

Now, the oscillatory neutral solutions of equation (49) are obtained by firstly determine the roots of cubic equation (48) in term of $\sigma_i{}^2$. For overstability σ_i is real, at most one positive root is required of equation (48) for which the critical thermal Rayleigh number for oscillatory modes is obtained for various values of non- dimensional wave number which is seen in table no. 2 and related graphs on overstability convection.

Table 1. Rayleigh number with respect to wave number for stationary convection with M =100, Q = 10, P_1 =1.2, p_1 =7, p_2 =1, $\varepsilon = 0.5, \theta = 45^o, \alpha_{mf} = 10, \gamma_{mf} = 0.5, \rho_o = 10, \nabla H = 10$.

a	P_1 = 1.0	P_1 = 1.2	P_1 = 1.4	M = 80	M = 120	Q = 20	Q =30	M_0 =10	M_0 =20	M_0 = 30
1	149.613	132.092	120.093	137.656	127.914	187.814	257.868	13559.7	14329.1	15192.3
2	76.8884	72.2329	69.6816	78.173	67.8964	137.835	224.062	7414.42	7835.7	8307.73
3	75.4791	73.6116	73.3578	81.2685	68.0508	161.031	276.888	7555.5	7985.27	8466.31
4	89.4442	89.0194	90.2061	99.2831	81.5768	207.791	365.562	9137.49	9656.67	10238.4
5	111.794	112.55	115.102	126.227	102.637	271.883	483.697	11552.8	12209.2	12944.7
6	140.795	142.736	146.772	160.612	129.785	351.751	629.695	14651.3	15483.8	16416.5

Table 2. Rayleigh number with respect to wave number for overstability convection with M =100, Q =10, P_1 =1.2, p_1 =7, p_2 =1, $\varepsilon = 0.5$, F = 1.005, $\theta = 45^o, \alpha_{mf} = 10, \gamma_{mf} = 0.5, \rho_o = 10, \nabla H = 10$.

a	P_1 = 1.0	P_1 = 1.2	P_1 = 1.4	M = 80	M = 120	Q = 20	Q =30	F = 0.5	M_0 =10	M_0 =20	M_0 = 30
0.5	26011.8	20214.4	17789.8	31049	17447.9	22459.3	24787.9	22024.7	2074930	2192830	2324930
1.0	6510.65	6397.78	6308.96	5904.16	6958.65	7189.23	7986.19	7714.82	656707	694020	735829
1.5	5929.21	5901	5877.08	5166.05	6658.11	6651.06	7403.32	7254.79	605808	640229	678797
2.0	6586.15	6595.49	6600.13	5652.66	7550.06	7405.42	8217.5	8214.98	677001	715467	758568
2.5	7874.8	7924.13	7961.86	6714.67	9142.57	8841.33	9759.98	9995.13	813381	859596	911379
3.0	9740.79	9847.73	9933.42	8283.55	11419.9	10906.8	11966.6	12582	1010830	1068260	1132620
3.5	12242.6	12435.4	12592.6	10404.4	14474.2	13667.1	14898.8	16094.5	1276450	1348970	1430240
4.0	15483.7	15801.6	16062.6	13165.5	18445.6	17235	18667.9	20711	1621970	1714130	1817390

NUMERICAL RESULTS AND DISCUSSION

The equations (48) and (49) have been examined numerically using the software Mathematica version- 5.2. we have plotted the variation of Rayleigh number with respect to wavenumber using equation (38) for stationary case and equation (48) and equation(49)for overstability case, for the fixed permissible vales of the dimensionless parameters $M = 100, Q = 10, P_1 = 1.2, p_1 = 7, p_2 = 1, \varepsilon = 0.5, F = 1.005, \theta = 45^o, \alpha_{mf} = 10, \gamma_{mf} = 0.5, \rho_o = 10, \nabla H = 10$ as shown in table 1 and table 2.

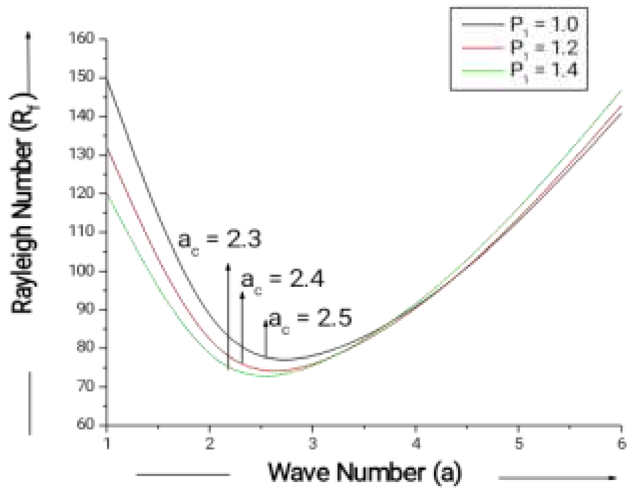

Figure 2: The variation of thermal Rayleigh Number R_f (Stationary) versus wave number **a** for three different values of the medium permeability parameter P_1 = 1.0, 1.2, 1.4.

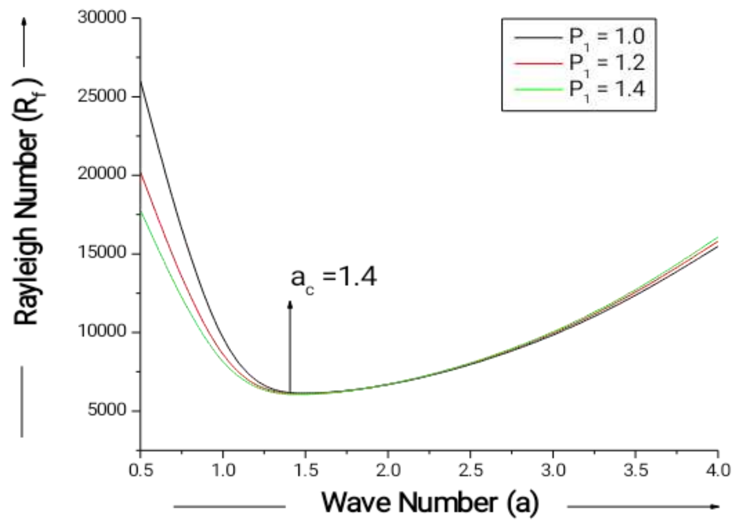

Figure 3: The variation of thermal Rayleigh Number R_f (overstability) versus wave number **a** for three different values of the medium permeability parameter P_1 = 1.0, 1.2, 1.4.

Figures 2 and 3 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of medium permeability parameter $P_1 = 1.0, 1.2, 1.4$. the Rayleigh number deceases with the increase in the parameter P_1 showing thereby the destabilizing effect of medium permeability parameter on the system but for $a \geq 4$ for stationary convection Rayleigh number increase with

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increase in the parameter P_1 and for the case of overstability wavenumber $a \geq 2$ Rayleigh number increase with increase in medium permeability thereby the stabilizing effect on the system.

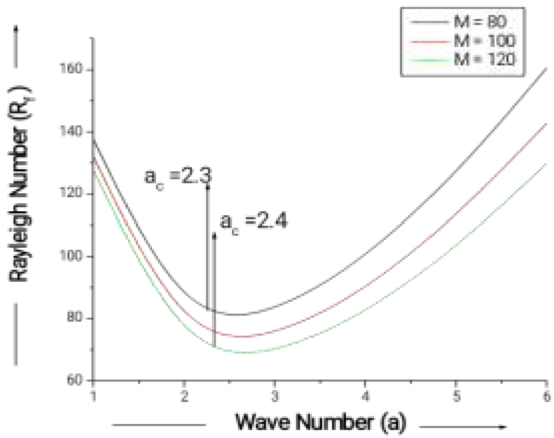

Figure 4: The variation of thermal Rayleigh Number R_f (Stationary) versus wave number **a** for three different values of the Hall current parameter M = 80, 100, 120.

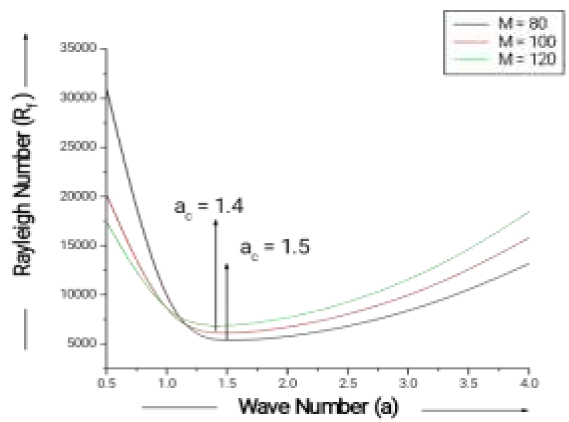

Figure 5: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the Hall current parameter **M** = 80, 100, 120.

Figures 4 and 5 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for various values of Hall Current parameter $M = 80, 100, 120$. For the case of stationary convection as well as overstability Rayleigh number deceases with the increase in the parameter M showing thereby the destabilizing effect of Hall Current parameter on the system whereas for wave number $a \geq 1.2$ in the case of overstability Rayleigh number increase with increase in the parameter M showing thereby stabilizing effect on the system.

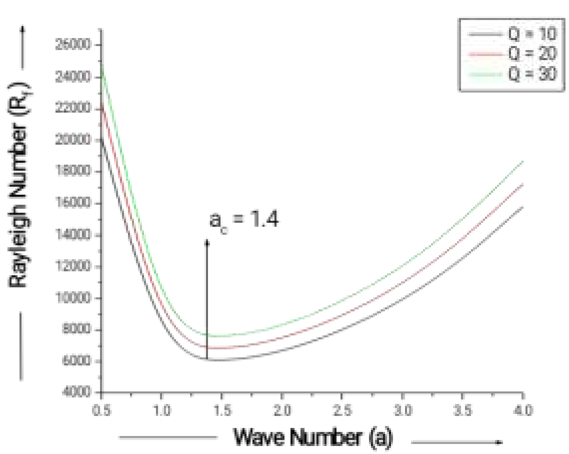

Figure 6: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (Stationary) versus wave number **a** for three different values of the magnetic field

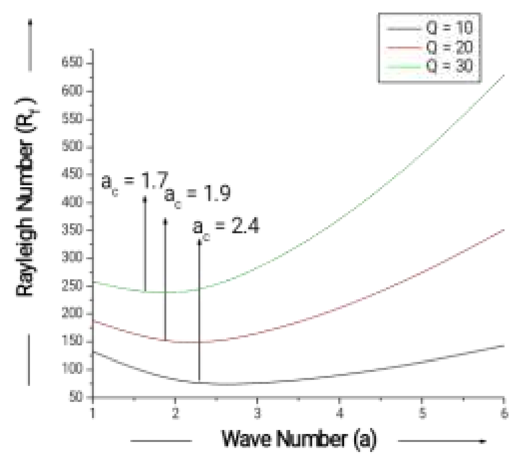

Figure 7: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the magnetic field parameter **Q** = 10, 20, 30.

Figures 6 and 7 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of magnetic field parameter $Q=10, 20, 30$. The Rayleigh number increase with the increase in the parameter Q showing thereby the stabilizing effect of magnetic field parameter

on the system.

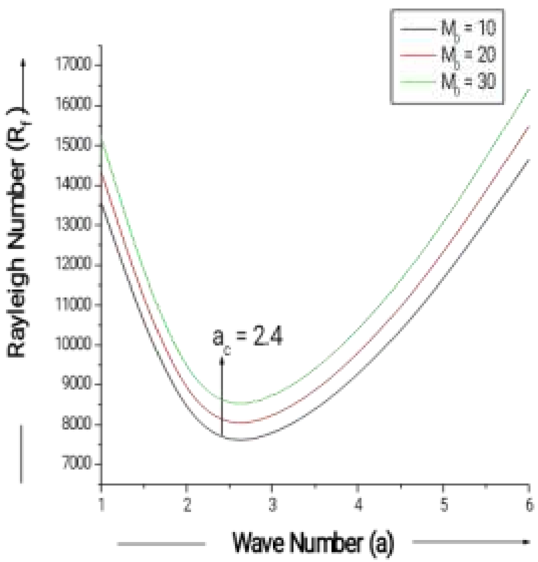

Figure 8: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (Stationary) versus wave number **a** for three different values of the magnetization parameter M_0 = 10, 20, 30.

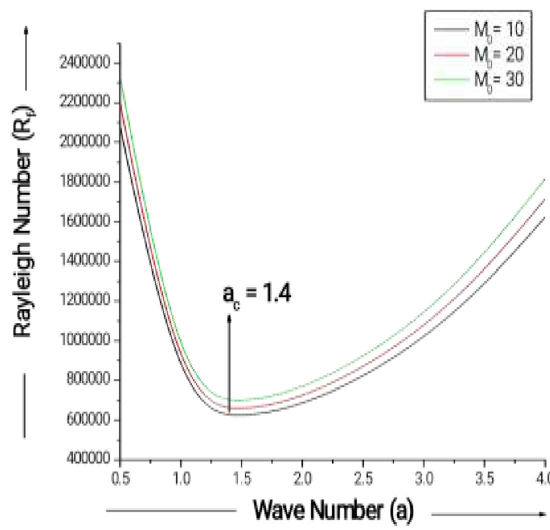

Figure 9: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the magnetization parameter M_0 = 10, 20, 30.

Figures 8 and 9 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of magnetization parameter $M_0 = 10, 20, 30$. The Rayleigh number increase with the increase in the parameter M_0 showing thereby the stabilizing effect of magnetization parameter on the system.

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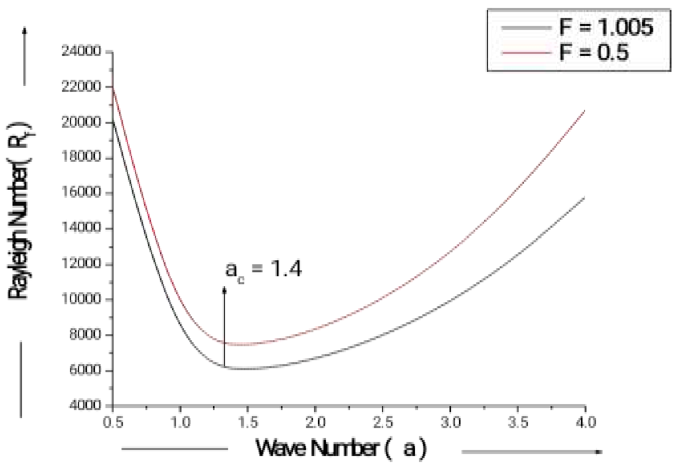

Figure 10: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for different values of the viscoelastic number parameter F = 0.5, 1.005.

Figure 10 represents the variation of thermal Rayleigh number R_f for the overstability convection with the wave number a for various value of viscoelastic number parameter $F = 0.5, 1.005$. The Rayleigh number increase with increase in the parameter F showing thereby the stabilizing effect of the viscoelastic number parameter on the system. It is also clear from the figure 2- 10 that overstability dominant mode of stability.

9. CONCLUSION

In the present paper, the combined effect of medium permeability, horizontal magnetic field, Hall effect, viscoelastic number and magnetization has been considered on the thermal stability of an elastic-viscous ferromagnetic fluid. The effect of various parameters such as medium permeability, horizontal magnetic field, Hall effect, viscoelastic number and magnetization has been investigated analytically as well as numerically. The main results from the analysis of paper are as follows.

- In order to investigate the effects of medium permeability, horizontal magnetic field, Hall effect and magnetization, we examine the behavior of $\frac{dR_f}{dP_1}, \frac{dR_f}{dQ}, \frac{dR_f}{dM}$ and $\frac{dR_f}{dM_0}$ analytically.
- It is found that magnetic field and magnetization have a stabilizing effect whereas medium permeability and Hall effect have a destabilizing effect on the system. Figures 2, 4, 6 and 8 support the analytic results graphically. The reasons for stabilizing effect of magnetic field and destabilizing effect of Hall currents are accounted by Chandrashekhar(Chandrashekhar, 1961). This is also valid for second- order fluids as well.
- It is found that for overstability convection magnetic field and magnetization as well as viscoelastic number parameter have a stabilizing effect whereas medium permeability and Hall current have a destabilizing effect on the system but for wavenumber $a \geq 2$ Hall

current as well as medium permeability parameter both shows stabilizing effect on the system.

- The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of Hall current (and hence magnetic field), it is valid under certain conditions.

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PROPAGATION OF WAVES IN FIBRE-REINFORCED ANISOTROPIC THERMOELASTIC PLATES

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Abstract
The propagation of waves in a homogeneous fibre-reinforced linear thermoelastic isotropic plate is investigated in the context of couple theory of thermoelasticity. Secular equations for symmetric and antisymmetric wave modes in completely separate terms are derived in closed form and isolated mathematical conditions. It is shown that the purely transverse motion (SH mode), which is not affected by thermal variations, gets decoupled from rest of the motion of wave propagation. The results for isotropic materials theory of thermoelasticity have been obtained as particular case. The special cases, such as short wavelength waves and thin plate waves of the secular equations are also discussed. Finally, the numerical solution is carried out for fibre-reinforced isotropic plate. The dispersion curves for symmetric and antisymmetric wave modes and attenuation coefficient are presented in order to illustrate and compare the theoretical results. The theory and numerical computations are found to be in close agreement.

1. Introduction

A fibre reinforced composite can be designed for all conditions of stresses that may occur and in accordance with the principle of mechanics. The characteristic property of a reinforced composite is that its components act together as single anisotropic units as long as they remain in the elastic condition. The wave propagation in a reinforced medium plays a very interesting role in civil engineering and geophysics. The studies of propagation, reflection and transmission of waves are of a great interest to seismologists. The propagation of waves depends upon the ground vibration and the physical properties of the structure. Thus the role of the propagation of surface waves in a fibre reinforced medium cannot be neglected in any way.

The dynamical problem of propagation of Surface waves in elastic media are of considerable importance in earth quake, engineering and seismology on account of occurrence in the earth's crust as the earth is made up of different layers. Surface waves have been well recognized in the study of earthquake waves, seismology, geophysics and geodynamics. A good amount of literature is to be found in the standard books of Bullen [1], Ewing *et al.* [2], Rayleigh [3], Love [4], Stoneley [5] and Jeffreys [6] regarding surface waves in classical elasticity.

Most of the previous investigations have been studied the effect of reinforcement. The idea of introducing a continuous self reinforcement at every point of an elastic solid was introduced by Belfied *et al.* [7]. The problem of surface waves in fibre reinforced anisotropic elastic media was discussed by Sengupta and Nath [8]. The elastic moduli for fibre reinforced materials were discussed by Hoshin and Rosen [9]. The propagation of plane waves in a fibre reinforced media is investigated by Chattapadhyay *et al.* [10]. The problem of wave propagation in thermally conducting linear fibre reinforced composite materials is discussed by Singh [11]. Othman and Abbas [12] investigated the effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half space.

The coupling between the strain and temperature fields was first studied by Duhamel [13] who derived equations for the distribution of strains in an elastic medium subjected to temperature gradients. Biot [14] gave a satisfactory derivation of the equation of thermal conductivity – which includes the dilation term – based on thermodynamics of irreversible processes. Sharma [15], Sharma et al. [16], investigated the propagation of thermoelastic waves in homogeneous isotropic plate subjected to stress free and rigid insulated and isothermal conditions in the conventional coupled and generalized theories of thermoelasticity. Sharma and Singh [17] studied the propagation of circularly crested thermoelastic waves in a homogeneous isotropic cylindrical plate subjected to stress free and isothermal conditions. Sharma and Pathania [18] studied the propagation of waves in a homogeneous, transversely isotropic, thermally conducting plate bordered with layers of inviscid liquid or half-space of inviscid liquid on both sides.

For wave propagation in a homogenous medium, the introduction of displacement potentials leads to the decoupling of P, SV and SH motions. On close examination of Sengupta and Nath [8], it is found by Singh [19] that their expressions for the potentials do not satisfy one of the equations of motion. Consequently, most of the result and conclusions of Sengupta and Nath [8] are unacceptable. The method of potentials is not suitable for studying wave propagation in fibre reinforced anisotropic elastic media. Sengupta and Nath [8] apply this method incorrectly.

In the present paper, we have discussed the propagation of waves in a homogeneous fibre-reinforced linear thermoelastic isotropic plate and coupled thermoelastic plate. Secular equations for symmetric and skew-symmetric wave modes are derived in closed form and isolated mathematical conditions. The special cases, such as short wavelength waves and thin plate waves of the secular equations are also discussed. The dispersion curves for symmetric and antisymmetric wave modes and attenuation coefficient of symmetric and skew-symmetric modes.

2. Basic Equations

The constitutive equations for a fibre-reinforced linearly thermoelastic isotropic medium with respect to the reinforcement direction $\vec{a}$ as discussed by (Belfield *et al.* [7])

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) + 2(\mu_L - \mu_T)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta a_k a_m e_{km} a_i a_j - \gamma T\delta_{ij} \tag{1}$$

where σ_{ij} are components of stress; $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ are components of strain; λ, μ_T are elastic parameters; $\alpha, \beta, \mu_L - \mu_T$ are reinforced anisotropic elastic parameters; $\gamma = (3\lambda + 2\mu)\alpha_t$; α_t is thermal expansion coefficient; T is the temperature change; u_i are the displacement vectors

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components and $\vec{a} = (a_1, a_2, a_3)$, where $a_1^{\,2} + a_2^{\,2} + a_3^{\,2} = 1$. If $\vec{a}$ has components that are $(1,0,0)$ so that the preferred direction is the x_1 axis, Eq. (1) simplifies, as given below

$$\begin{aligned}
&\sigma_{11} = A_{11}e_{11} + A_{12}e_{22} + A_{12}e_{33} - \gamma T\,;\\
&\sigma_{22} = A_{12}e_{11} + A_{22}e_{22} + \lambda e_{33} - \gamma T\,;\\
&\sigma_{33} = A_{12}e_{11} + \lambda e_{22} + A_{22}e_{33} - \gamma T\,;\\
&\sigma_{23} = 2\mu_T e_{23}\,;\\
&\sigma_{13} = 2\mu_L e_{13}\,;\\
&\sigma_{12} = 2\mu_L e_{12}\,;
\end{aligned} \tag{2}$$

where $A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta\,;$

$A_{12} = \alpha + \lambda\,;$

$A_{22} = \lambda + 2\mu_T\,.$

The equations of motion in the absence of body forces are

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{3}$$

and equation of heat condition in absence of heat source

$$KT_{,ii} = \rho c_e \dot{T} + \gamma T_0 \dot{u}_{ii} \tag{4}$$

where ρ is mass density; T_0 is the reference temperature; c_e is the specific heat at constant strain; K is thermal conductivity.

3. Formulation of the problem

We consider a homogeneous fibre-reinforced linear thermoelastic isotropic plate of thickness $2d$ initially at uniform temperature T_0. We take origin of the co-ordinate system (x, y, z) on the middle surface of the plate. The $x - z$ plane is chosen to coincide with the middle surface and the y-axis normal to it along the thickness. The surface $y = \pm d$ is subjected to different boundary conditions. We take $x - y$ as the plane of incidence and assume that the solutions are explicitly independent of z but implicit dependence is there so that component w of displacement is non-vanishing.

The basic governing equations for homogeneous fibre-reinforced linear thermoelastic isotropic medium, in the absence of body forces and heat sources(on using Eqs. 1-4), are given by

$$\left(A_{11}\frac{\partial^2}{\partial x^2} + \mu_L \frac{\partial^2}{\partial y^2}\right)u + A_{21}\frac{\partial^2 v}{\partial x \partial y} - \gamma \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$A_{21}\frac{\partial^2 u}{\partial x \partial y} + \left(\mu_L \frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2}\right)v - \gamma \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \tag{6}$$

$$\left(\mu_L \frac{\partial^2}{\partial x^2} + \mu_T \frac{\partial^2}{\partial y^2}\right) w = \rho \frac{\partial^2 w}{\partial t^2} \tag{7}$$

$$K\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T = \rho c_e \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{8}$$

where $A_{21} = \mu_L + A_{12}$;

For convenience, we define the quantities

$(x^/, y^/, z^/) = c_1\eta(x,\ y,\ z)$, $(u^/, v^/, w^/) = c_1\eta(u,\ v,\ w)$,

$$t^/ = c_1^2 \eta\, t\ ,\ \theta = \frac{\gamma}{\rho c_1^2} T\ ,\ \sigma_{ij}^/ = \frac{\sigma_{ij}}{\mu_T}\ ,\ \eta = \frac{\rho c_e}{K},\ c_1^2 = \frac{\mu}{\rho}. \tag{9}$$

Upon introducing quantities (9) in governing Eqs. (5) to (8) we obtain (on suppressing dashes for convenience)

$$\left(h_{11} \frac{\partial^2}{\partial x^2} + h_1 \frac{\partial^2}{\partial y^2}\right) u + h_2 \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{10}$$

$$h_2 \frac{\partial^2 u}{\partial x \partial y} + \left(h_1 \frac{\partial^2}{\partial x^2} + h_{22} \frac{\partial^2}{\partial y^2}\right) v - \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2} \tag{11}$$

$$\varepsilon_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta = \frac{\partial \theta}{\partial t} + \varepsilon_1 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{12}$$

$$\left(h_1 \frac{\partial^2}{\partial x^2} + h_3 \frac{\partial^2}{\partial y^2}\right) w = \frac{\partial^2 w}{\partial t^2} \tag{13}$$

where $h_1 = \frac{\mu_L}{\rho c_1^2}$; $h_2 = \frac{A_{21}}{\rho c_1^2}$; $h_3 = \frac{\mu_T}{\rho c_1^2}$; $h_{11} = \frac{A_{11}}{\rho c_1^2}$; $h_{22} = \frac{A_{22}}{\rho c_1^2}$.

4. Boundary conditions

The surfaces $y = \pm d$ of the plate are assumed to be stress free and thermally insulated or isothermal. Therefore, the non-dimensional boundary conditions to be satisfied are given as

$$\sigma_{xy} = g_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\sigma_{yy} = g_1 \frac{\partial u}{\partial x} + g_2 \frac{\partial v}{\partial y} - g_3\theta$$

$$\theta_{,y} + g_4 H\theta = 0 \tag{14}$$

where H is Biot's heat transfer coefficient, $g_0 = \frac{\mu_L}{\mu_T}, g_1 = \frac{A_{12}}{\mu_T}, g_2 = \frac{A_{22}}{\mu_T}, g_3 = \frac{\rho c_1^{\,2}}{\mu_T}$, $g_4 = \frac{1}{c_1\eta}$.

5. Solution of the problem

We assume solutions of the form

$$(u,\quad v,\quad w,\quad \theta) = (1,\quad V,\quad W,\quad \Theta)U\exp\{i\xi(x\sin\phi + my - ct)\} \tag{15}$$

where $c = \omega/\xi$ is the non-dimensional phase velocity, ω is the frequency and ξ is the wave number. Here ϕ is the angle of inclination of wave normal with y-axis; m is still unknown parameter. Here V, W and Θ respectively, the amplitude ratios of displacements v, w and temperature θ to that of displacement u.

Upon using solutions (15) in Eqs. (10)–(13), we obtain

$$\left(h_1 s^2 + h_3 m^2 - c^2\right)W = 0 \tag{16a}$$

$$h_{11}s^2 + h_1 m^2 - c^2 + smh_2 V + \tau_1 s\Theta = 0 \tag{16b}$$

$$h_2 sm + (h_1 s^2 + h_{22}m^2 - c^2)V + \tau_1 m\Theta = 0 \tag{16c}$$

$$\varepsilon_1 sc + \varepsilon_1 mcV + (\varepsilon_2 s^2 + m^2 - \tau_1 c)\Theta = 0 \tag{16d}$$

where $\tau_1 = i\xi^{-1},\ s = \sin\phi$.

Eq. (16a) in the above system corresponds to purely transverse wave mode (SH) that decoupled from rest of the motion and is not affected by the thermal variations. The characteristic roots corresponding to Eq. (16a) are given by

$$m_7, m_8 = \pm\sqrt{\frac{c^2 - h_1 s^2}{h_3}}\,. \tag{17}$$

From the rest of Eqs. (16), which corresponds to the coupled longitudinal, shear vertical (SV) and thermal (T-mode) motion, one can obtain

$$\left(m^2 - m_1^{\,2}\right)\left(m^2 - m_3^{\,2}\right)\left(m^2 - m_5^{\,2}\right) = 0 \tag{18}$$

where $m_k^{\,2},\ k = 1, 3, 5$ are the roots of the equation

$$m^6 + Am^4 + Bm^2 + C = 0 \tag{19}$$

Here the coefficients A, B, C are given by

$$A = -\frac{Jc^2 - Ps^2 + \varepsilon_1\tau_1 h_1 c + (c\tau_1 - \varepsilon_2)h_1 h_{22}}{h_1 h_{22}};$$

$$B=\frac{\left(c^2-h_{11}s^2\right)\left(c^2-h_1s^2\right)+\left(Jc^2-Ps^2\right)\left(c\tau_1-s^2\varepsilon_2\right)+\varepsilon_1\tau_1c\left[c^2-(h_{11}+h_{22}-2h_2)s^2\right]}{h_1h_{22}}$$

$$C=-\frac{\left(c^2-h_1s^2\right)\left[\left(c\tau_1-s^2\varepsilon_2\right)\left(c^2-h_{11}s^2\right)-\varepsilon_1\tau_1cs^2\right]}{h_1h_{22}}$$

where

$J=h_1+h_{22}$, $P=h_1^{\,2}-h_2^{\,2}+h_{11}h_{22}$

Eq. (19) being cubic in m^2 admits six solutions for m; which also have the property $m_2=-m_1$, $m_4=-m_3$, $m_6=-m_5$. For each m_q; $q=1,2,\cdots,6$ the amplitude ratios V and Θ can be expressed as

$$V_q=\begin{cases}\dfrac{m_qa_q}{s}, & q=1,\ 2,\ 3,\ 4\\[2ex] \dfrac{c^2-h_{11}s^2-h_1m_q^{\,2}-\tau_1s\Theta_q}{sh_2m_q}, & q=5,\ 6\end{cases}\tag{20}$$

$$\Theta_q=\begin{cases}\dfrac{c^2-h_{11}s^2-(h_1+h_2a_q)m_q^{\,2}}{s\tau_1}, & q=1,\ 2,\ 3,\ 4\\[2ex] \dfrac{h_1h_{22}m_q^{\,4}-Pm_q^{\,2}+(c^2-h_1s^2)(c^2-h_{11}s^2)}{s\tau_1\left(c^2-h_1s^2+(h_2-h_{22})m_q^{\,2}\right)}, & q=5,\ 6\end{cases}\tag{21}$$

where

$$a_q=\frac{c^2-h_{11}m_q^{\,2}+(h_2-h_{11})s^2}{c^2-h_1s^2+(h_2-h_{22})m_q^{\,2}},\qquad q=1,2,\cdots,6.\tag{22}$$

Combining Eqs. (20) and (21) with stress–strain–temperature relations we rewrite the formal solution for the displacements, temperature, stresses and temperature gradient as

$$\left(u,\quad v,\quad \theta\right)=\sum_{q=1}^{6}\left(1,\quad V_q,\quad \Theta_q\right)U_q\exp\left\{i\xi\left(xs+m_qy-ct\right)\right\},\qquad -d<y<d\tag{23}$$

$$\left(\sigma_{xy},\quad \sigma_{yy},\quad \theta_{,y}\right)=\sum_{q=1}^{6}i\xi\left(D_{1q},\quad D_{2q},\quad D_{3q}\right)U_q\exp\left\{i\xi\left(xs+m_qy-ct\right)\right\},\tag{24}$$

Here

$D_{1q}=g_0\left(m_q+sV_q\right),\ q=1,2,\cdots,6$

$D_{2q}=g_1s+g_2m_qV_q-g_3\Theta_q,\ \ q=1,2,\cdots,6$

$$D_{3q} = m_q\,,\ q = 1,2,\cdots,6 \tag{25}$$

6. Derivation of the secular equations

By invoking stress free and thermal boundary conditions at plate surfaces $y = \pm d$; we obtain a system of six simultaneous linear equations in amplitudes U_q; $q = 1,2,\cdots,6$ as

$$\sum_{q=1}^{6} D_{1q}E_qU_q = 0; \qquad \sum_{q=1}^{6} D_{2q}E_qU_q = 0, \qquad \sum_{q=1}^{6} D_{3q}E_qU_q = 0 \tag{26}$$

where $E_q = \exp(\pm i\xi m_q d)$; $q = 1,2,\cdots,6$.

System of Eqs. (26) have a non-trivial solution if the determinant of the coefficient of U_q; $q = 1,2,\cdots,6$ vanishes, which leads to a characteristic equation for the propagation of modified guided thermoelastic waves in the plate. We refer such waves as plate waves rather than Lamb waves whose properties were originally derived by Lamb in 1917 for isotropic elastic solids. The characteristic equation for the thermoelastic plate waves in this case, after applying lengthy algebraic reduction and manipulations of the determinant leads to the following secular equations

$$\left(\frac{T_1}{T_5}\right)^{\pm 1} - \frac{D_{13}G_3}{D_{11}G_1}\left(\frac{T_3}{T_5}\right)^{\pm 1} = -\frac{D_{15}G_5}{D_{11}G_1} \tag{27}$$

Here the superscript $+1$ corresponds to skew-symmetric and -1 refers to symmetric modes and $T_k = \tan(\xi\, d\, m_k), k = 1,\ 3,\ 5$

$$G_1 = D_{23}\left(D'_{45} + iD''_{45}T_5^{\pm 1}\right) - D_{25}\left(D'_{43} + iD''_{43}T_3^{\pm 1}\right)$$
$$G_3 = D_{21}\left(D'_{45} + iD''_{45}T_5^{\pm 1}\right) - D_{25}\left(D'_{41} + iD''_{41}T_1^{\pm 1}\right)$$
$$G_5 = D_{21}\left(D'_{43} + iD''_{43}T_3^{\pm 1}\right) - D_{23}\left(D'_{41} + iD''_{41}T_1^{\pm 1}\right)$$

where $D'_{4q} = m_q\Theta_q$, $D''_{4q} = -H\tau_1 g_4\Theta_q$.

Here

$$G_1 = D_{23}D'_{45} - D_{25}D'_{43}$$
$$G_3 = D_{21}D'_{45} - D_{25}D'_{41}$$
$$G_5 = D_{21}D'_{43} - D_{23}D'_{41} \tag{28}$$

for a stress-free thermally insulated $(H \to 0)$ plate and

$$G_1 = D_{23}D''_{45}T_5^{\pm 1} - D_{25}D''_{43}T_3^{\pm 1}$$
$$G_3 = D_{21}D''_{45}T_5^{\pm 1} - D_{25}D''_{41}T_1^{\pm 1}$$
$$G_5 = D_{21}D''_{43}T_3^{\pm 1} - D_{23}D''_{41}T_1^{\pm 1} \tag{29}$$

for stress-free isothermal $(H \to \infty)$ plate.

The secular Eq. (27) is the transcendental equation, which contains complete information about the phase velocity, wave number and attenuation coefficient of the plate waves. In general, wave number and hence the phase velocity of the waves is complex quantity, therefore the waves are attenuated in space. If we write

$$c^{-1} = V_P{}^{-1} + i\omega^{-1}Q \tag{30}$$

where V_P and Q are real, the exponent $e^{i\xi(x\sin\phi - ct)}$ in the plane wave solution (15) becomes

$$\frac{i\omega}{V_P}\{x\sin\phi - V_P t\} - Qx\sin\phi .$$

This shows that V_P is the propagation speed and Q the attenuation coefficient of the waves. Upon using Eq. (29) in Eq. (27) the values of V_P and Q for different modes can be obtained.

Also all the results reduce to the theory of coupled thermoelasticity of isotropic materials when the anisotropic parameters $\alpha = 0$, $\beta = 0$, $|\mu_L - \mu_T| = 0$ for the fibre-reinforced medium tend to zero (if necessary writing $\mu_L = (\mu_L - \mu_T) + \mu_T$ and considering $|\mu_L - \mu_T| \to 0$).

7. Discussion of the secular equation

1. Regions of the secular equation

Here depending on whether m_1, m_2, m_3 being purely imaginary or complex, the frequency Eq. (27) is correspondingly altered as follows.

1.1 Region I

When the roots of characteristic Eq. (19) are of type $m_k{}^2 = -m_k'{}^2, k = 1,3,5$; $m_k = im_k'$ is purely imaginary or complex number. This ensures that the superposition of partial waves has the property of ‘‘exponential decay’’. In this case the secular equation is written from Eq. (27) by replacing circular tangent functions of m_k, $k = 1,3,5$ with hyperbolic tangent functions of m_k', $k = 1,3,5$:

$$\left(\frac{\tanh(\xi d m_1')}{\tanh(\xi d m_5')}\right)^{\pm 1} - \frac{D_{13}G_3'}{D_{11}G_1'}\left(\frac{\tanh(\xi d m_3')}{\tanh(\xi d m_5')}\right)^{\pm 1} = -\frac{D_{15}G_5'}{D_{11}G_1'} \tag{31}$$

where

$$\begin{aligned}
G_1' &= D_{23}\left\{D_{45}' + iD_{45}''\left(\tanh(\xi d m_5')\right)^{\pm 1}\right\} - D_{25}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\}\\
G_3' &= D_{21}\left\{D_{45}' + iD_{45}''\left(\tanh(\xi d m_5')\right)^{\pm 1}\right\} - D_{25}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\}\\
G_5' &= D_{21}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\} - D_{23}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\}
\end{aligned} \tag{32}$$

Here $D_{1q}, D_{2q}, D_{3q}, D_{4q}$; $q = 1,2,\cdots,5$ can be obtained on replacing m_k by im_k'; $k = 1,3,5$ in the corresponding expressions.

Region II

In case two of the roots of Eq. (19) are of the type $m_k{}^2 = -m_k'{}^2, k = 1,3$; then the frequency equation can be obtained from Eq. (27) by replacing circular tangent functions of m_k; $k = 1,3$ with hyperbolic tangent functions of $m_k', k = 1,3$:

$$\left(\frac{\tanh(\xi d m_1')}{\tan(\xi d m_5)}\right)^{\pm 1} - \frac{D_{13}G_3}{D_{11}G_1}\left(\frac{\tanh(\xi d m_3')}{\tan(\xi d m_5)}\right)^{\pm 1} = -\frac{D_{15}G_5}{D_{11}G_1} \qquad (33)$$

where

$$G_1' = D_{23}\left\{D_{45}' + iD_{45}''\left(\tan(\xi d m_5)\right)^{\pm 1}\right\} - D_{25}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\}$$
$$G_3' = D_{21}\left\{D_{45}' + iD_{45}''\left(\tan(\xi d m_5)\right)^{\pm 1}\right\} - D_{25}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\}$$
$$G_5' = D_{21}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\} - D_{23}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\} \qquad (34)$$

Here $D_{1q}, D_{2q}, D_{3q}, D_{4q}$; $q = 1`,2,\cdots,5$ can be obtained on replacing $m_k$ by $im_k'$; $k = 1`,3$ in the corresponding expressions.

1.2 Region III

In the general case the roots $m_k{}^2$; $k = 1,3,5$ are complex numbers, and then the frequency equation is given by Eq. (27).

Waves of short wavelength

Some information on the asymptotic behavior is obtainable by letting $\xi \to \infty$: If we take $\xi > \frac{\omega}{\sqrt{\mu_L / \rho}}$; it follows that $c < \sqrt{\mu_L / \rho}$ and the roots of characteristic equation lies in region I in this case. Then we replace m_1, m_3 and m_5 in the secular equation by im_1', im_3' and im_5'. Here for

$$\xi \to \infty,\ \frac{\tanh(\xi d m_1')}{\tanh(\xi d m_5')} \to 1 \text{ and } \frac{\tanh(\xi d m_3')}{\tanh(\xi d m_5')} \to 1$$

so that secular Eq. (27) reduces to

$$D_{11}G_1'' - D_{13}G_3'' + D_{15}G_5'' = 0 \qquad (35)$$

Here

$$G_1'' = D_{23}\left(D_{45}' - D_{45}''\right) - D_{25}\left(D_{43}' - iD_{43}''\right)$$
$$G_3'' = D_{21}\left(D_{45}' - D_{45}''\right) - D_{25}\left(D_{41}' - D_{41}''\right)$$
$$G_5'' = D_{21}\left(D_{43}' - D_{43}''\right) - D_{23}\left(D_{41}' - D_{41}''\right)$$

Eq.(35) is the similar to as obtained and discussed by Sharma et al.[20] in case of homogeneous transversely isotropic thermoelastic plate with stress-free thermally insulated plate. It is merely Rayleigh surface wave equation. The Rayleigh results enter here since for such small wavelengths, the finite thickness plate appears as semi-infinite medium. Hence vibration energy is transmitted along the surface of the plate.

8. Thin plate results

The thin plate limits are specified by $\xi d << 1$ when the transverse wavelength of the plate is quite large as compared to the thickness of the plate. So for $\xi > \frac{\omega}{\sqrt{\mu_L / \rho}}$; we have $m_k = im'_k$; $k = 1,3,5$. In this case the secular equation can be obtained from Eq. (27) just by replacing circular tangent functions with hyperbolic tangent functions.

9. Numerical result and Discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results for the physical constants as discussed in Singh and Singh [21].

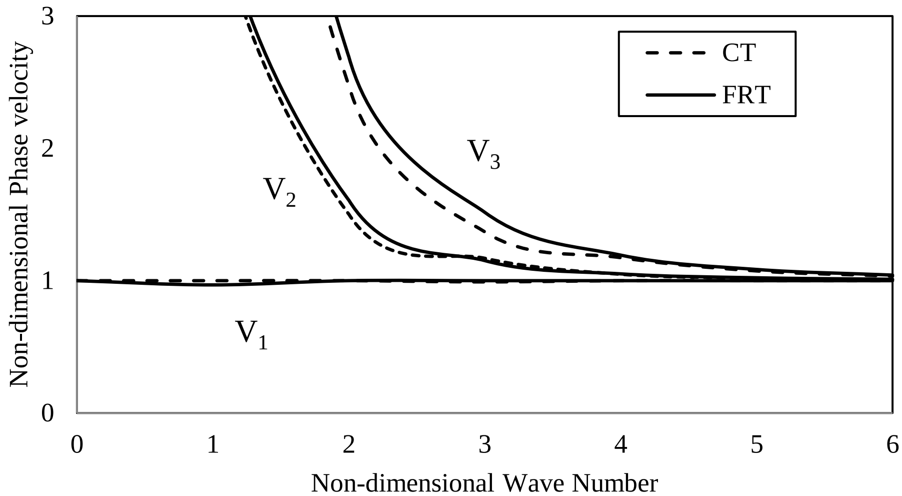
$\lambda = 7.59\times10^7\ N\cdot m^{-2}, \mu = 3.86\times10^7\ kg\cdot m^{-1}\cdot s^{-2}, \mu_T = 1.89\times10^9\ N\cdot m^{-2}$,

$\mu_L = 2.45\times10^9\ N\cdot m^{-2}, T_0 = 200K, \alpha = -1.28\times10^9\ N\cdot m^{-2}, \beta = 0.32\times10^9\ N\cdot m^{-2}$,

$\rho = 7800 kg\cdot m^{-3}, \alpha_t = 1.78\times10^{-4}\ K^{-1}, c_e = 50\,J\cdot kg^{-1}\cdot K^{-1}, K = 10^7\ w\cdot m^{-1}\cdot K^{-1}$.

The complex roots of characteristic equation (19) have been computed with the help of reduced Cardano's method, which are then used in various relevant relations. The secular equation (27) is solved for the phase velocity by using iteration method. The sequence of iteration is made to converge after sampling it over about 100 sample values in order to achieve the desired level of accuracy. The phase velocity, group velocity and attenuation coefficient profiles of first three symmetric and skew-symmetric modes of fibre-reinforced thermoelastic (FRT) and coupled thermoelastic (CT) plate has been computed and corresponding dispersion curves are represented graphically in Fig.1-6 for wave normal inclination $\phi = 90^\circ$ with the axis of symmetry.

Fig. 1: Variations of phase velocity of symmetric modes versus wave number.



Figures 1-2 show the variations of non-dimensional phase velocities of symmetric and skew-symmetric modes $V_i\ (i=1, 2, 3)$ respectively, versus wave number in the context of FRT and CT theories. The phase velocity of fundamental mode (V_1) of symmetric motion has been observed to be almost dispersionless and whereas it found to be zero at vanishing wave number in case skew symmetric motion of both FRT and CT theories. The phase velocities of higher modes $V_i(i=2, 3)$ attain quite large values at vanishing wave number which sharply slash down to become steady and asymptotically closer to shear wave velocity at extremely large wave numbers. It is observed that the reinforcement has a significant effect on the distribution of the field quantities. It is noticed that the reinforcement effect has negligibly small effect on the phase velocities at higher wave numbers and the phase velocities of various modes become asymptotic and tend to the Rayleigh wave speed. In such situation a finite thickness plate appears to be half-space and the vibration energy is mainly transmitted through the surface of the plate. Figs. 3-4 present the variations of group velocities $(V_{g_i}, i=1, 2, 3)$ of symmetric and skew symmetric modes in FRT and CT plates versus non-dimensional wave number respectively. The values of group velocities of various modes are found to be zero at vanishing wave numbers which correspond to the condition of zero energy transmission in horizontal direction. The profiles of group velocity, both in FRT and CT, increase monotonically in the interval $[0, 1]$ and it tends to phase velocity profiles of respective modes at high frequency limits for all modes. Figs. 5-6 show the variations of attenuation coefficient $(Q_i, i=1, 2, 3)$ for symmetric and skew symmetric modes in FRT and CT plates versus non-dimensional wave number respectively. It is noticed that the attenuation coefficient is small for the mode which have large magnitude of the phase velocity and vice versa. It is also noticed that curves of attenuation coefficient with wave number in each mode is almost reflection (mirror image) of the graph of phase velocity with wave number.

Fig. 2: Variations of phase velocity of skew-symmtric modes versus wave number**.**

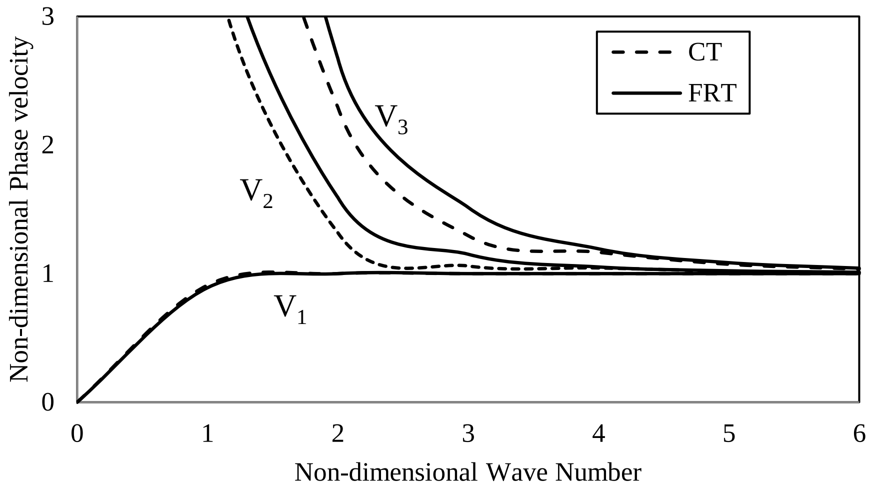


Fig. 3: Variations of group velocity of symmetric modes versus wave number**.**

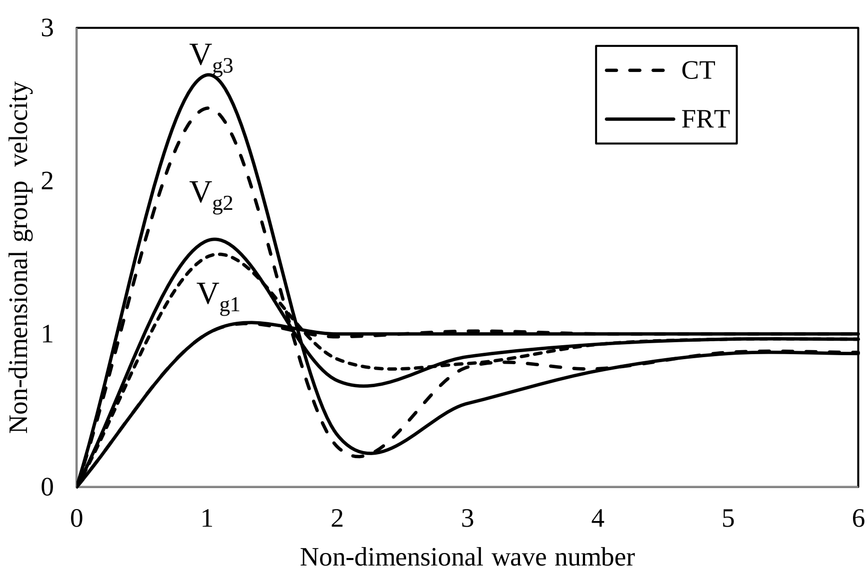


Fig. 4: Variations of group velocity of skew-symmetric modes versus wave number.

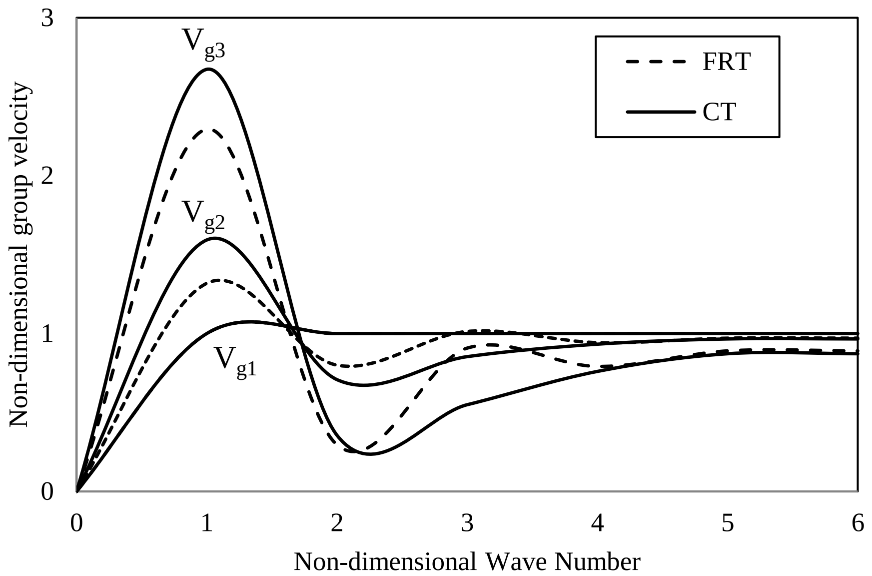


Fig. 5: Variations of attenuation coefficient of symmetric modes versus wave number.

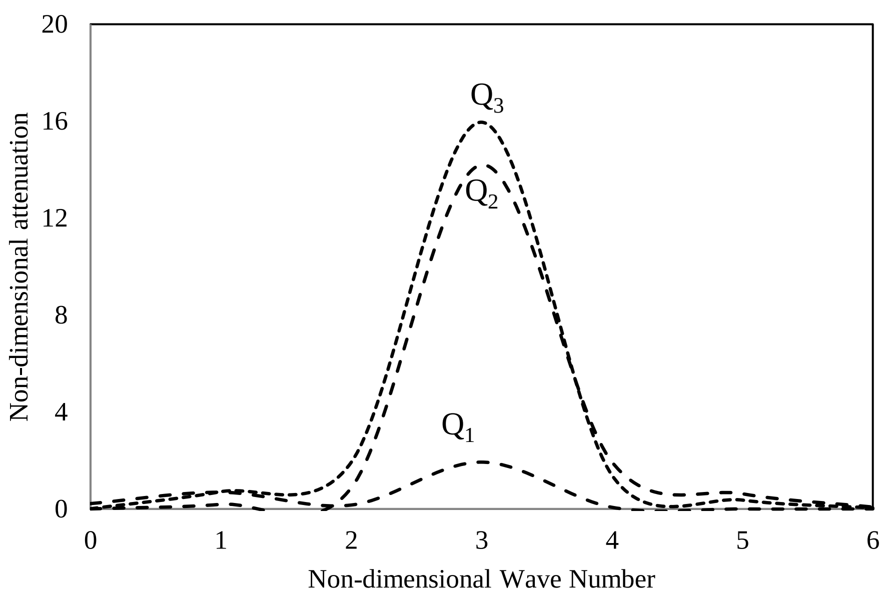
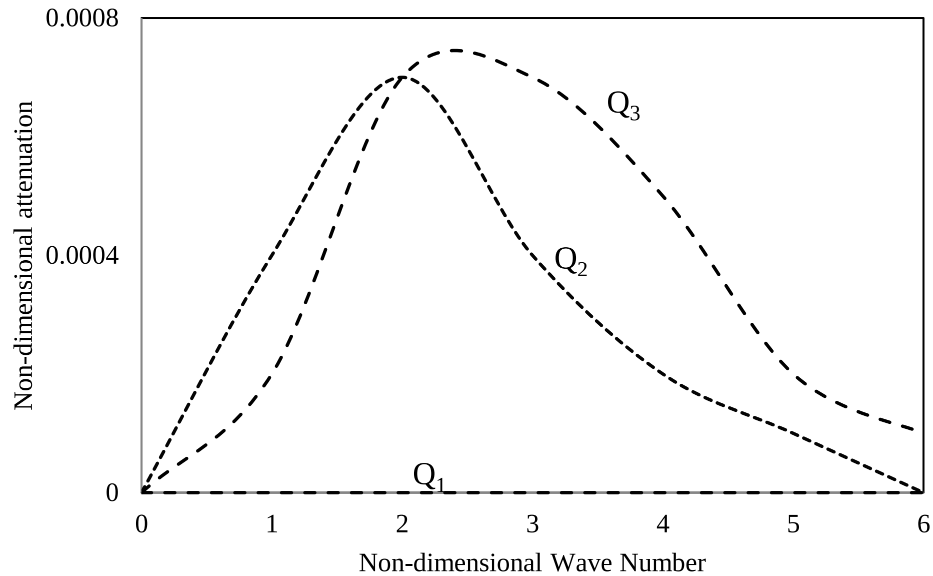


Fig. 6: Variations of attenuation coefficient of skew-symmertic modes versus wave number.



10. CONCLUSIONS:

It is observed that the phase velocity of the acoustic symmetric mode become dispersionless and asymptotic. For increasing wave number this remains closer to the velocity of thermoelastic Lamb waves because in this case the energy transmission takes place mainly along the surface (interface) of the plate. The acoustic skewsymmetric mode has zero velocity at vanishing wave number, which increases to become closer to the velocity of thermoelastic Lamb wave with increasing wave number. The phase velocities of optical modes of propagation attain large values at vanishing wave number, which slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number. The hypothetical results in case of FRT theory here observed to resemble closely to that of CT theory of thermoelasticity for initial two modes namely acoustic and first modes of wave propagation. It is clear from the above investigation that the surface waves in the fibre-reinforced medium are affected by the reinforced parameters. Also all the results reduce to the CT results when the anisotropic parameters for the fibre-reinforced medium tend to zero. We conclude that the Rayleigh wave velocity in a fibre-reinforced elastic medium is considerably higher than the Rayleigh wave velocity in isotropic media.

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EFFECT OF ROTATION ON ELECTROTHERMAL - CONVECTION IN A DIELECTRIC MAXWELLIAN NANOFLUID LAYER SATURATING A DARCY POROUS MEDIUM

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Abstract

In this paper, effect of rotation on electrothermal-convection in a horizontal layer of dielectric rheological nanofluid saturating a porous medium is investigated. The rheological behaviour of nanofluid is described by Maxwell model and for porous medium, Darcy model is employed. The used Boungiorno model for nanofluid incorporates the effects of thermophoresis and Brownian diffusion. The Navier-Stokes equations of motion are modified due to the presence of applied AC electric field and rotation by the inclusion of dielectrophoretic force, Coulomb force and Coriolis force, respectively. By applying linear stability analysis based upon perturbation theory and one-term Galerkin method, we derive the expressions for thermal Rayleigh number for both the cases of stationary convection and the oscillatory motion are derived. The effects of Taylor number, AC electric Rayleigh number, Lewis number, modified diffusivity ratio and nanoparticles Rayleigh number on the onset of stationary and oscillatory modes.

Keywords: Electrohydrodynamics, Nanofluid, Maxwell model, Porous Medium, Rotation.

1. Introduction

Electro-thermo-hydrodynamics (ETH) is an interdisciplinary area of composite communications among the thermal gradient and electric field applied to a horizontal dielectric nanofluid layer. Due to its practical importance, numerous studies have been approved to evaluate the consequence of electric field on natural convection. A temperature gradient applied to a dielectric fluid produces a gradient in the dielectric constant and electrical conductivity. The applied electric force of fluid motions is effective method in getting very helpful interesting results in the cooling of lap-tops and devices of the flight in space, usage to a variety of applications ranging from electro-kinematic assays to electro spray ionization made on nanoscale being used at a large scale in the present era. In dielectric liquids characterized by low values of conductivity, Maxwell's equations reduce to the electro-quasistatic limit. Keeping this information in mind, many envils of the onset of convection instability in a horizontal layer and a vertical temperature gradient have been investigated in the history.

Maxwell (1891) was the first who coined the idea of dispersing solids in fluids. Nanofluids has been formed with a completely different thermophysical properties such as density, specific heat capacity, thermal conductivity, convective heat transfer, thermal difusivity, and viscosity. Commercializing nanofluid is facing challenges due to their poor stability which occurred by

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the interaction between the particles themselves and between the particles and the surrounding liquid. Convection of non-Newtonian fluids in a porous medium is of considerable importance in several applied fields such as oil recovery , food processing, and the spread of contaminants in the environment, and in various processes the chemical and materials industries. Rotation plays important role in the thermal instability of nanofluid layer and has applications in rotating machineries such as nuclear reactors, petroleum industry, biomechanics, automotive industries, power plant cooling systems, computers etc. The effect of rotation on viscous and viscoelastic fluids has been studied by different authors [Chandrasekhar (1961), Vadasz, (1996) , Malashetty et al. (2007), Dhannajay et al.(2011), Agarwal and Bhadauria (2014). Chand et al. (2015), Rana and Agarwal (2015), Govender (2016a, b). Mahajan and Sharma (2018) studied the penetrative convection stimulated by internal heating in magnetic nanofluid saturating a rotating porous medium while detailed review on the instability and convection in rotating porous media has been explained by Vadasz (2019).They found that the rotation parameter has a stabilizing effect on thermal instability of viscous and viscoelastic fluid. Takashima (1976) studied the convective instability in a dielectric fluid under the simultaneous action of rotation and the *AC* electric field whereas the electrohydrodynamic instability of a rotating layer of viscoelastic dielectric regular fluid was studied by Othman (2004). Shivakumara et al. (2013) studied electrohydrodynamic instability of a rotating couple stress dielectric fluid layer while Rana et al. (2016) studied the instability of a viscoelastic fluid saturating a porous medium in electrohydrodynamics. The onset of electroconvection in a dielectric nanofluid saturating a rotating Darcy porous medium has been studied analytically and numerically by Wakif et al. (2018) using realistic boundary conditions and they have shown that the onset of electroconvection can be accelerated on increasing AC electric Rayleigh –Darcy number, the Lewis number and the modified diffusivity ratio.

Recently, Rana et al. (2019) investigated electrohydrodynamic thermal instability of an elastico-viscous nanofluid saturating a porous in the presence of vertical AC electric field and found that couple-stress parameter, medium porosity and Brinkman-Darcy number have stabilizing effect whereas AC electric Raleigh number, Lewis number and modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effect on the system. Sharma et al. (2019) have examined the rheological behavior of electro-hydrodynamics convection in dielectric rotating maxwellian nanofluid layer with vertical AC electric field and have shown that applied electric field has a stabilizing effect on the system. Here, effect of rotation on electrothermal-convection in a dielectric Maxwellian nanofluid layer saturating a Darcy porous medium is studied which include one additional parameter, namely, the Taylor number.

2 Formulation of the Problem and Mathematical Model

An infinitely extending electrically horizontal layer of an incompressible dielectric Shear-Thinning viscoelastic nanofluid for bottom/top-heavy distribution of nanoparticles, heated from below is considered. It is confined between two parallel planes $\tilde{z}=0$ and $\tilde{z}=\tilde{d}$, where temperature and volumetric fraction are kept constant : $\tilde{T}=\tilde{T}_0$ and $\tilde{\varphi}=\tilde{\varphi}_0$ at $\tilde{z}=0$ and $\tilde{T}=\tilde{T}_1$ and $\tilde{\varphi}=\tilde{\varphi}_1$ at $\tilde{z}=\tilde{d}$, $\left(\tilde{T}_0>\tilde{T}_1\right)$. Both the bounding surfaces are assumed to be stress free. The thermophysical properties of nanofluids (viscosity, density, thermal conductivity and specific heat) are taken as constants for the analytical formulation, but these quantities are not constant and strongly depend on the volume fraction of nanoparticles. The nanofluid layer is rotating uniformly about vertical axis with a uniform angular velocity $\tilde{\mathbf{\Omega}}(\mathbf{0,0,\Omega})$ and the acceleration due to gravity $\tilde{\mathbf{g}}(\mathbf{0,0,-g})$ pervade the system, z-axis being taken as vertical.

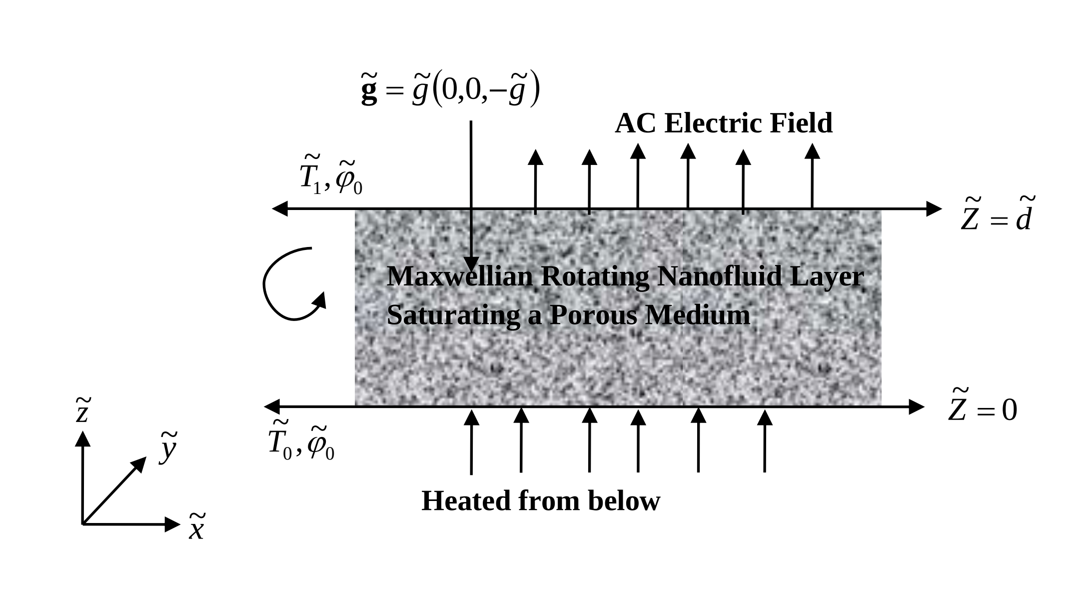

Figure 1: Physical Configuration of the Problem

2.2 Governing Equations

The basic hydrodynamic equations that govern the physical problem using Lapwood (1948), Chandrasekhar (1961), Buongiorno model (2006) are

$$\nabla.\tilde{\mathbf{q}}_{\mathbf{D}} = 0, \tag{1}$$

$$\tilde{\rho}_{nf} = \tilde{\varphi}\tilde{\rho}_{np} + (1-\tilde{\varphi})\tilde{\rho}_{bf}, \tag{2}$$

where $\tilde{\rho}_{np}$ is the density of nanoparticles and $\tilde{\rho}_{bf}$ is the base fluid, respectively. Taking the density of the nanofluid as that of the base fluid, as adopted by Tzou (2008a, 2008b), the specific weight thus becomes

$$\tilde{\rho}_{nf}\tilde{\mathbf{g}} = \left[\tilde{\varphi}\tilde{\rho}_{np} + (1-\tilde{\varphi})\left\{\tilde{\rho}_{nf}\left(1-\tilde{\alpha}\left(\tilde{T}-\tilde{T}_0\right)\right)\right\}\right]\tilde{\mathbf{g}}. \tag{3}$$

where $\tilde{\alpha}$ is the coefficient of thermal expansion.

Due to the Brownian motion and thermophoresis mass flux of the nanoparticles in the base fluid, $\tilde{\mathbf{j}}_{\mathbf{p}} = -\rho_{np}\tilde{D}_B\nabla\tilde{\varphi} - \tilde{\rho}_{np}\dfrac{\tilde{D}_T}{\tilde{T}}\nabla\tilde{T}$. (4)

where $\tilde{D}_B$ is the Brownian diffusion coefficient, given by Einstein-Stokes equation and $\tilde{D}_T$ is the thermophoretic diffusion coefficient of the nanoparticles and are given as

$$\tilde{D}_B = \frac{\tilde{k}_B\tilde{T}}{3\pi\tilde{\mu}_{bf}\tilde{d}_{np}}, \quad \tilde{D}_T = \frac{\tilde{\mu}_{bf}\,0.26\tilde{k}_{bf}}{\tilde{\rho}_{nf}\left(2\tilde{k}_{bf}+\tilde{k}_{np}\right)}\tilde{\varphi}, \tag{5}$$

where $\tilde{k}_B$ is Boltzmans constant, $\tilde{\mu}_{bf}$ is the base fluid viscosity, $\tilde{d}_{np}$ is the diameter of the nanoparticle, $\tilde{\rho}_{bf}$ is the base fluid density , $\tilde{k}_{bf}$ and $\tilde{k}_{np}$ are the thermal conductivities of the base fluid and nanoparticles respectively.

Equations of motion for an incompressible rotating viscoelastic Maxwellian nanofluid saturating a porous medium are

$$\frac{\tilde{\rho}_{bf}}{\tilde{\varepsilon}}\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\left[\frac{\partial}{\partial\tilde{t}}+\frac{\tilde{q}_D}{\tilde{\varepsilon}}.\nabla\right]\tilde{\mathbf{q}}_{\mathbf{D}}=\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\left[-\nabla\tilde{P}-\tilde{f}_e+\frac{2\tilde{\rho}_{nf}}{\tilde{\varepsilon}}\left(\tilde{\mathbf{q}}_{\mathbf{D}}\times\tilde{\mathbf{\Omega}}\right)+\tilde{\varphi}\tilde{\rho}_{np}+(1-\tilde{\varphi})\tilde{\rho}_{nf}\left\{1-\tilde{\alpha}\left(\tilde{T}-\tilde{T}_1\right)\right\}\tilde{\mathbf{g}}\right]-\frac{\tilde{\mu}}{\tilde{k}_1}\tilde{\mathbf{q}}_{\mathbf{D}}. \tag{6}$$

Where, $\tilde{P}=\tilde{p}-\frac{\tilde{\rho}_{bf}}{2}\left|\tilde{\mathbf{\Omega}}\times\vec{\mathbf{r}}\right|^2$

$$\tilde{\mathbf{f}}_e=\tilde{\rho}_e\tilde{\mathbf{E}}-\frac{1}{2}\tilde{\mathbf{E}}^2\nabla\tilde{K}+\frac{1}{2}\left(\tilde{\rho}_{nf}\frac{\partial\tilde{K}}{\partial\tilde{t}}\tilde{\mathbf{E}}^2\right), \tag{7}$$

The term $\tilde{\rho}_e\tilde{\mathbf{E}}$ is neglected as compared to the di-electrophoretic force term $-\frac{1}{2}\tilde{\mathbf{E}}^2\nabla\tilde{K}$ for dielectric fluids. It is assumed that the dielectric constant , $\tilde{K}$ can be expressed as [Yadav et al. (2015)]

$$\tilde{K}=\tilde{K}_0\left[1-\tilde{\gamma}_0\left(\tilde{T}-\tilde{T}_0\right)\right], \tag{8}$$

$\tilde{\gamma}_0>0$, is the coefficient of the dielectric constant with temperature relative variations, which is assumed to be small $0<\tilde{\gamma}_0\Delta\tilde{T}<<1$.

The modified pressure term using equation (6) is

$$\tilde{P}=\tilde{p}-\frac{1}{2}\left(\tilde{\rho}_{nf}\frac{\partial\tilde{K}}{\partial\tilde{t}}\tilde{\mathbf{E}}^2\right), \tag{9}$$

Assuming free charge density to be very small, the relevant Maxwell equations [Roberts (1969)] are

$$\nabla.\left(\tilde{K}\tilde{\mathbf{E}}\right)=0, \tag{10}$$

$$\nabla\times\tilde{\mathbf{E}}=0, \tag{11}$$

In view of equation (11), $\tilde{\mathbf{E}}$ can be expressed as $\tilde{\mathbf{E}}=-\nabla\tilde{\varphi}$.

The conservation equation for the nanoparticles is

$$\left[\frac{\partial}{\partial\tilde{t}}+\frac{\tilde{\mathbf{q}}_{\mathbf{D}}\cdot\nabla}{\tilde{\varepsilon}}\right]\tilde{\varphi}=-\frac{1}{\tilde{\rho}_{np}}\nabla.\tilde{\mathbf{j}}_{\mathbf{p}}. \tag{12}$$

Here $\tilde{\varphi}$ is the nanoparticle volumetric fraction, $\tilde{\rho}_{np}$ is the density of nanoparticles and $\tilde{\mathbf{j}}_{\mathbf{p}}$, the nanoparticles diffusion mass flux is given by

$$\tilde{\mathbf{j}}_{\mathbf{p}}=-\tilde{\rho}_{np}\tilde{D}_B\nabla\tilde{\varphi}-\tilde{\rho}_{np}\frac{\tilde{D}_T}{\tilde{T}_1}\nabla\tilde{T}, \tag{13}$$

Using the value of $\tilde{\mathbf{j}}_{\mathbf{p}}$ from equation (13) into equations (12), the conservation equation of nanoparticles yields that

$$\left[\frac{\partial}{\partial\tilde{t}}+\frac{\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla}{\tilde{\varepsilon}}\right]\tilde{\varphi}=\tilde{D}_B\nabla^2\tilde{\varphi}+\frac{\tilde{D}_T}{\tilde{T}_1}\nabla^2\tilde{T}. \tag{14}$$

The heat energy equation is

$$\left(\tilde{\rho}_{nf}\tilde{c}\right)_{bm}\left\{\frac{\partial\tilde{T}}{\partial\tilde{t}}+\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{T}\right\}=\tilde{k}_m\nabla^2\tilde{T}+\tilde{\varepsilon}\left(\tilde{\rho}_{nf}\tilde{c}\right)_{np}\left\{\tilde{D}_B\nabla\tilde{\varphi}.\nabla\tilde{T}+\frac{\tilde{D}_T}{\tilde{T}_0}\nabla\tilde{T}.\nabla\tilde{T}\right\}, \tag{15}$$

where $\tilde{c}_{np}$ is the specific heat of the material constituting the nanoparticles. $\left(\tilde{\rho}_{nf}\tilde{c}\right)_{bm}$ is the effective capacity, $\left(\tilde{\rho}_{nf}\tilde{c}\right)_{bm}$ is the heat capacity of nanofluid.

We introduce non-dimensional variables as

$$\left(\tilde{x}', \tilde{y}', \tilde{z}'\right) = \left(\frac{\tilde{x}, \tilde{y}, \tilde{z}}{\tilde{d}}\right), \left(\tilde{u}', \tilde{v}', \tilde{w}'\right) = \left(\frac{\tilde{u}, \tilde{v}, \tilde{w}}{\tilde{\kappa}_{bm}}\right)\tilde{d}, \tilde{t}' = \frac{\tilde{t}\,\tilde{\kappa}_{bm}}{\tilde{\sigma}\tilde{d}^{2}}, \tilde{p}' = \frac{\tilde{p}\tilde{k}_1}{\tilde{\mu}\tilde{\kappa}_{bm}}\tilde{d}^{2},$$
$$\tilde{\varphi}' = \frac{\tilde{\varphi} - \tilde{\varphi}_0}{\tilde{\varphi}_1 - \tilde{\varphi}_0}, \tilde{T}' = \frac{\tilde{T} - \tilde{T}_1}{\tilde{T}_0 - \tilde{T}_1}, \tilde{K}' = \frac{\tilde{K}}{\tilde{\gamma}_0 \tilde{\mathbf{E}}_{\mathbf{0}} \Delta\tilde{T}\tilde{d}}, \tilde{V}' = \tilde{\gamma}_0 \tilde{\mathbf{E}}_{\mathbf{0}} \tilde{\beta}\tilde{d}\tilde{V}. \tag{16}$$

where $\tilde{\kappa}_{bm} = \frac{\tilde{k}_m}{(\tilde{\rho}_{nf}\tilde{c}_{np})_{bf}}$ is thermal diffusivity of the base fluid, $\tilde{\sigma} = \frac{\left(\tilde{\rho}_{nf}\tilde{c}_{np}\right)_{bm}}{(\tilde{\rho}_{nf}\tilde{c}_{np})_{bf}}$ is the thermal capacity ratio, $\tilde{\beta} = \frac{\Delta\tilde{T}}{\tilde{d}}$ or $\tilde{\beta} = \frac{\tilde{T}_0 - \tilde{T}_1}{\tilde{d}}$ is the adverse temperature gradient, $\tilde{E}_0$ is the root mean square value of the electric field at $\tilde{z} = 0$ and $\nabla\tilde{T}$ is the temperature difference between the horizontal planes, $\tilde{\gamma}_0$ is the dielectric constant at reference temperature $\tilde{T}_0$ and $\tilde{\varphi}_0$ is a reference value for the nanoparticles volume fraction.

The equations (2)-(15) equations in non-dimensional form after dropping the dashes (') for convenience can be written as

$$\nabla.\tilde{\mathbf{q}}_{\mathbf{D}} = 0, \tag{17}$$

$$0 = \left(1 + \tilde{\lambda}_0 \frac{\partial}{\partial \tilde{t}}\right)\left[-\nabla\tilde{P} - \tilde{\mathbf{f}}_{\mathbf{e}} + \frac{2\tilde{\rho}_{nf}}{\tilde{\varepsilon}}\left(\tilde{\mathbf{q}}_{\mathbf{D}} \times \tilde{\mathbf{\Omega}}\right) + \tilde{\varphi}\tilde{\rho}_{np} + (1 - \tilde{\varphi})\tilde{\rho}_{nf}\left\{1 - \tilde{\alpha}\left(\tilde{T} - \tilde{T}_1\right)\right\}\tilde{\mathbf{g}}\right] - \frac{\tilde{\mu}}{\tilde{k}_1}\tilde{\mathbf{q}}_{\mathbf{D}} \tag{18}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial\tilde{\varphi}}{\partial\tilde{t}} + \frac{1}{\tilde{\varepsilon}}\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{\varphi} = \frac{1}{\tilde{L}_e}\nabla^2\tilde{\varphi} + \frac{\tilde{N}_A}{\tilde{L}_e}\nabla^2\tilde{T}, \tag{19}$$

$$\frac{\partial\tilde{T}}{\partial\tilde{t}} + \tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{T} = \nabla^2\tilde{T} + \frac{\tilde{N}_A}{\tilde{L}_e}\nabla\tilde{\varphi}.\nabla\tilde{T} + \frac{\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}\nabla\tilde{T}.\nabla\tilde{T}, \tag{20}$$

$$\nabla^2\tilde{V} = \frac{\partial\tilde{T}}{\partial\tilde{z}}, \tag{21}$$

where the non-dimensional parameters are defined as $\tilde{p}_1 = \frac{\tilde{\mu}}{\tilde{\rho}_0\tilde{\alpha}}$ (Prandtl number), $\tilde{D}_a = \frac{\tilde{k}_1}{\tilde{d}^2}$ (Darcy number), $\tilde{V}_a = \frac{\tilde{\varepsilon}\tilde{P}_r}{\tilde{D}_a}$ (Vadasz number), $\tilde{R}_a = \frac{\tilde{\rho}_{nf}\tilde{\mathbf{g}}\tilde{\alpha}\tilde{d}\tilde{k}\left(\tilde{T}_0 - \tilde{T}_1\right)}{\tilde{\mu}_{bf}\tilde{\kappa}_{bm}}$ (Rayleigh number), $\tilde{R}_{ea} = \frac{\tilde{\gamma}_0^{\,2}\tilde{K}\tilde{\mathbf{E}}_{\mathbf{0}}^{\,2}\tilde{d}^{\,2}\left(\Delta\tilde{T}\right)^2}{\tilde{\mu}\tilde{\kappa}_{bm}}$ (electric Rayleigh number), $\tilde{R}_n = \frac{\left(\tilde{\rho}_{np} - \tilde{\rho}_{bf}\right)\left(\tilde{\varphi}_1 - \tilde{\varphi}_0\right)\tilde{\mathbf{g}}\tilde{k}_1\tilde{d}}{\tilde{\mu}\tilde{\alpha}}$ (nano particles Rayleigh number), $\tilde{N}_B = \frac{\tilde{\rho}_{np}\tilde{c}_{np}}{\left(\tilde{\rho}\tilde{c}\right)_{bf}}\left(\tilde{\varphi}_1 - \tilde{\varphi}_0\right)$ (modified particle density increment), $\tilde{L}_e = \frac{\tilde{\kappa}_{bm}}{\tilde{D}_B}$ (Lewis number of the nanofluid), $\tilde{N}_A = \frac{\tilde{D}_T\left(\tilde{T}_0 - \tilde{T}_1\right)}{\tilde{D}_B\tilde{T}_1\left(\tilde{\varphi}_1 - \tilde{\varphi}_0\right)}$ (modified diffusivity ratio), $\tilde{R}_m = \frac{\left\{\tilde{\rho}_{np}\tilde{\varphi}_0 + \tilde{\rho}_{bf} + \left(1 - \tilde{\varphi}_0\right)\right\}\tilde{\mathbf{g}}\tilde{k}_1\tilde{d}}{\tilde{\mu}\tilde{\alpha}}$ (basic density Rayleigh number), $\tilde{\lambda}_0 = \frac{\tilde{\lambda}\tilde{\kappa}_{bm}}{\tilde{d}^2}$ (parameter accounting for stress-relaxation time) and $\tilde{T}_a = \frac{4\tilde{\mathbf{\Omega}}^2\tilde{d}^4}{\tilde{\nu}^2}$ (the Taylor number). (22)

Here both the bounding surfaces of the fluid are assumed to be stress-free and the medium adjoining the nanofluid is a perfect conductor, the appropriate boundary conditions are

$$\tilde{w} = \frac{\partial^2 \tilde{w}}{\partial z^2} = \tilde{T} = \tilde{\varphi} = \frac{\partial \tilde{\varphi}}{\partial \tilde{z}} = 0 \text{ at } \tilde{z} = 0 \text{ and } \tilde{z} = \tilde{d}\,. \tag{23}$$

2.3 Basic Solutions

The primary flow representing the basic state is assumed to be quiscent [Kuznetsov and Nield (2009), Nield and Kuznetsov (2009), Sheu (2011), Chand and Rana (2012b), Nield and Kuznetsov (2014), Chand et al. (2014) and Yadav and Kim (2015)], no settling of suspended nanoparticles and is assumed to be stationary . Initially, no motions are present in the nanofluid flow and the physical quantities vary in the vertical direction z-axis only. Therefore, the velocity, pressure, temperature, dielectric constant, electric field, electric potential and nanoparticle volume fraction are given by

$$\tilde{\mathbf{q}}_{\mathbf{D}} = \tilde{\mathbf{q}}_{\mathbf{b}} = 0, \tilde{P} = \tilde{P}_b(\tilde{z}), \tilde{K} = \tilde{K}_b(\tilde{z}), \tilde{T} = \tilde{T}_b(\tilde{z}), \tilde{\varphi} = \tilde{\varphi}_b(\tilde{z}), \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\mathbf{b}}(\tilde{z}), \tilde{V} = \tilde{V}_b(\tilde{z})$$

$$\tilde{T}_b = \tilde{T}_0 - \frac{\Delta\tilde{T}}{\tilde{d}}\tilde{z}, \tilde{\varphi}_b = \tilde{\varphi}_0 + \left(\frac{\tilde{D}_T\Delta\tilde{T}}{\tilde{D}_B\tilde{T}_1\tilde{d}}\right)\tilde{z}, \tilde{K}_b = \tilde{K}_0\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\tilde{z}\right)\hat{k}, \tilde{\mathbf{E}}_{\mathbf{b}} = \frac{\tilde{\mathbf{E}}_{\mathbf{0}}}{\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\tilde{z}\right)}\hat{k}, \tag{24}$$

where subscript 'b' denotes the basic state and $\hat{k}$ is the unit vector along z-axis.
Also we have

$$\tilde{V}_b(\tilde{z}) = -\frac{\tilde{\mathbf{E}}_{\mathbf{0}}\tilde{d}}{\tilde{\gamma}_0\Delta\tilde{T}}\log\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\right)\hat{k} \quad ,$$

where $\tilde{\mathbf{E}}_{\mathbf{0}} = -\frac{\tilde{V}\left(\frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\right)}{\log(1+\tilde{\gamma}_0\Delta\tilde{T})}$ is the root mean square value of the intensity of electric field at $\tilde{z} = 0$.

2.4 Perturbation Solutions

Let the primary flow be slightly disturbed from the equilibrium position so as to examine the stability of the perturbed modes with respect to the involved, physical variables by superimposing infinitesimal disturbances to the basic state flow. It is assumed that

$$\begin{aligned} &\tilde{\mathbf{q}}_{\mathbf{D}} = \tilde{\mathbf{q}}^*_{\mathbf{D}}, \tilde{T} = \tilde{T}_b + \tilde{T}^*, \tilde{K} = \tilde{K}_b + \tilde{K}^*, \tilde{P} = \tilde{P}_b + \tilde{P}^*, \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\mathbf{b}} + \tilde{\mathbf{E}}^*, \\ &\tilde{V} = \tilde{V}_b + \tilde{V}^*, \tilde{\varphi} = \tilde{\varphi}_b + \tilde{\varphi}^*, \tilde{\varepsilon} = \tilde{\varepsilon}_b + \tilde{\varepsilon}^* \end{aligned}, \tag{25}$$

where $\tilde{\mathbf{q}}_{\mathbf{D}}{}^*, \tilde{T}^*, \tilde{K}^*, \tilde{P}^*, \tilde{\mathbf{E}}^*, \tilde{V}^*, \tilde{\varphi}^*$ and $\tilde{\varepsilon}^*$ are the perturbations superimposed into the physical quantities of the equilibrium state.

On substituting these perturbations and using the solutions of primary flow (24) the equations (17)-(21) in the non-dimensional linearized perturbed form using linear theory (neglecting the products and higher orders of perturbed quantities) and Boussinesq approximation yield

$$\left[1+\frac{1}{\tilde{V}_a^*}\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\frac{\partial}{\partial\tilde{t}}\right]\nabla^2\tilde{w}^*-\left\{\begin{array}{l}\tilde{R}_a\nabla^2\tilde{T}^*-\tilde{R}_{ea}\nabla^2\tilde{T}^*+\tilde{R}_n\nabla^2\tilde{\varphi}+\sqrt{\tilde{T}_a^*}\left(\tilde{v}\hat{e}_x-\tilde{u}\hat{e}_y\right)\\+\tilde{R}_{ea}\nabla^2\dfrac{\partial\tilde{V}^*}{\partial\tilde{z}}\left(1+\tilde{\lambda}_0\dfrac{\partial}{\partial\tilde{t}}\right)\end{array}\right\}=0, \quad (26)$$

$$\frac{\partial\tilde{T}^*}{\partial\tilde{t}}-\tilde{w}^*=\nabla^2\tilde{T}^*+\frac{\tilde{N}_B}{\tilde{L}_e}\left(\frac{\partial\tilde{T}^*}{\partial\tilde{z}}-\frac{\partial\tilde{\varphi}^*}{\partial\tilde{t}}\right)-\frac{2\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}\frac{\partial\tilde{T}^*}{\partial\tilde{z}}, \quad (27)$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial\tilde{\varphi}^*}{\partial\tilde{t}}+\frac{\tilde{w}^*}{\tilde{\varepsilon}}=\frac{1}{\tilde{L}_e}\nabla^2\tilde{\varphi}^*+\frac{\tilde{N}_A}{\tilde{L}_e}\nabla^2\tilde{T}^*, \quad (28)$$

$$\frac{\partial\tilde{T}^*}{\partial z}-\nabla^2\tilde{V}^*=0, \quad (29)$$

Also

$\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$ is a Laplacian operator,

$\nabla_1{}^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

Equation (18) is derived by the use of the identity curlcurl = graddiv - ∇^2.

The boundary conditions (23) , in the non-dimensional form becomes

$$\tilde{w}^*=\frac{\partial^2\tilde{w}^*}{\partial\tilde{z}^2}=\tilde{T}^*=\tilde{\varphi}^*=\frac{\partial\tilde{\phi}^*}{\partial\tilde{z}}=0 \text{ at } \tilde{z}^*=0 \text{ and } \tilde{z}^*=1. \quad (30)$$

3 Normal Mode Analysis

Now an arbitrary perturbation is analyzed into a complete set of normal modes and then the stability of each of these modes is examined individually. For the system of equations (26)-(29), the analysis can be made in terms of two–dimensional periodic wave numbers. Thus, we ascribe to the quantities describing the dependence on $\tilde{x},\tilde{y}$ and $\tilde{t}$ of the form $\exp(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t})$, where $\tilde{l}$, $\tilde{m}$ are the wave numbers in the x and y-direction, respectively, and $\tilde{\omega}$ is the growth rate of the disturbances, which in general is a complex constant.

Above consideration allows to suppose that the perturbations quantities $\tilde{w}^*,\tilde{T}^*,\tilde{\varphi}^*$ and $\tilde{V}^*$ are of the form

$$\tilde{w}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=W(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),$$
$$\tilde{T}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Theta(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),$$
$$\tilde{\varphi}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Phi(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),$$
$$\tilde{V}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Psi(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right). \quad (31)$$

Using expression (31) and using linear theory, equations (26)-(29), in the linearized form become

$$\left[1+\frac{\tilde{\sigma}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right]^2\left(\tilde{D}^2-a^2\right)W-\left[1+\tilde{\lambda}_0\tilde{\omega}\right]\left[1+\frac{\tilde{\sigma}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right]\left[a^2\tilde{R}_n\Phi-a^2\left(\tilde{R}_a-\tilde{R}_{ea}\right)\Theta-a^2\tilde{R}_{ea}\tilde{D}\Psi\right]$$
$$+\left[1+\tilde{\lambda}_0\tilde{\omega}\right]^2\tilde{T}_a\tilde{D}^2W=0 \tag{32}$$

$$W+\left\{\frac{\tilde{N}_B}{\tilde{L}_e}\tilde{D}+\left(\tilde{D}^2-a^2\right)-2\frac{\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}-\tilde{\omega}\right\}\Theta-\frac{\tilde{N}_B}{\tilde{L}_e}\tilde{D}\Phi=0, \tag{33}$$

$$\frac{W}{\tilde{\varepsilon}}-\frac{\tilde{N}_A}{\tilde{L}_e}\left(\tilde{D}^2-a^2\right)\Theta-\left\{\frac{1}{\tilde{L}_e}\left(D^2-a^2\right)-\frac{\tilde{\omega}}{\tilde{\sigma}}\right\}\Phi=0, \tag{34}$$

$$\left(\tilde{D}^2-a^2\right)\psi=\tilde{D}\Theta \tag{35}$$

where $\tilde{D}\equiv\frac{d}{d\tilde{z}}$ and $a^2=\tilde{l}^2+\tilde{m}^2$ is the dimensionless horizontal wave number.

Using the expression (31), the boundary conditions (30) transform to

$$W=\tilde{D}^2W=\Theta=\Phi=\tilde{D}\Psi=0 \text{ at } \tilde{z}=0 \text{ and } \tilde{z}=1. \tag{36}$$

The set of differential equations (32)–(35) together with the boundary conditions (36) constitute a characteristic value problem for Rayleigh number $\tilde{R}_a$ and given values of the other parameters $\tilde{T}_a,\tilde{\lambda}_0,\tilde{R}_n,\tilde{R}_{ea},\tilde{\varepsilon},\tilde{L}_e,\tilde{N}_A.$., whose solutions ought to be obtained.

4 Linear Stability Convection

Using term Galerkin-technique of first approximation, the trial functions of lowest mode satisfying the boundary conditions (36) are chosen as

$$W=\tilde{A}_1\sin\pi\tilde{z}\,,\Theta=\tilde{B}_1\sin\pi\tilde{z}\,,\Phi=\tilde{C}_1\sin\pi\tilde{z}\,,\Psi=\tilde{D}_1\cos\pi\tilde{z}\,. \tag{37}$$

Substituting the trial functions given by (37) into equations (32)-(35) and using the orthogonality of trial functions and boundary conditions (36), a system of linear homogeneous equations can be written in matrix form as

$$\begin{bmatrix} \tilde{T}_a\tilde{M}^2\pi^2+\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)^2\left(\pi^2+a^2\right) & -\tilde{M}\left[1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right]a^2\left(\tilde{R}_a+\tilde{R}_{ea}\right) & \tilde{M}\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)a^2\tilde{R}_n & -\tilde{M}\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)\pi a^2\tilde{R}_{ea} \\ 1 & -\left(a^2+\pi^2\right)-\tilde{\omega} & 0 & 0 \\ \frac{1}{\tilde{\varepsilon}} & \frac{\tilde{N}_A\left(a^2+\pi^2\right)}{\tilde{L}_e} & \frac{\left(a^2+\pi^2\right)}{\tilde{L}_e}+\frac{\tilde{\omega}}{\tilde{\varepsilon}} & 0 \\ 0 & -\pi & 0 & -\left(a^2+\pi^2\right) \end{bmatrix}\begin{bmatrix} W \\ \Theta \\ \Phi \\ \Psi \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{38}$$

where $\tilde{M}=\left(1+\tilde{\lambda}_0\tilde{\omega}\right)$.

Using orthogonality, the non-trivial solution of the above matrix requires the vanishing of the determinant of the coefficient matrix in equation (38) which yields eigen –value equation as

$$\tilde{R}_a = -\left(\frac{a^2}{a^2+\pi^2}\right)\tilde{R}_{ea} + \frac{\tilde{T}_a\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\pi^2\left(a^2+\pi^2+\tilde{\omega}\right)}{\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right)a^2} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e}{\tilde{\sigma}\left(a^2+\pi^2\right)+\tilde{\omega}\tilde{L}_e}\left[\frac{a^2+\pi^2+\tilde{\omega}}{\tilde{\varepsilon}} + \frac{\left(a^2+\pi^2\right)\tilde{N}_A}{\tilde{L}_e}\right]$$
$$+\frac{\left(a^2+\pi^2+\tilde{\omega}\right)}{a^2\left(1+\tilde{\lambda}_0\tilde{\omega}\right)}\left[\left(a^2+\pi^2\right)+\frac{\left(a^2+\pi^2\right)\tilde{\omega}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)}{\tilde{V}_a}\right] \tag{39}$$

Equation (39) is the required dispersion relation accounting for the effect of Lewis number, kinematic visco-elasticity parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio on thermal electro instability in a layer of Maxwell elastico-viscous nanofluid saturating a porous medium under vertical AC electric field. Let us write $\tilde{\omega} = \tilde{\omega}_r + i\tilde{\omega}_i$, where $\tilde{\omega}_r$ and $\tilde{\omega}_i$ are real. For oscillatory motion, the real part of $\tilde{\omega}$ is zero. i.e., $\tilde{\omega}_r = 0, \therefore \tilde{\omega} = i\tilde{\omega}_i \neq 0$.

4.1 Stationary Convection

The stationary motion is characterized by putting $\tilde{\omega} = 0$ in equation (39), one gets

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}}+\tilde{N}_A\right), \tag{40}$$

which expresses the nanofluid Darcy Rayleigh number $\tilde{R}_a$ for stationary convection as a function of the dimensionless wave number a, electric Rayleigh number $\tilde{R}_{ea}$, nanofluid Lewis number $\tilde{L}_e$, modified diffusivity ratio $\tilde{N}_A$, concentration Rayleigh number $\tilde{R}_n$ and medium porosity $\tilde{\varepsilon}$. It is clear from the equation (39) that $\tilde{R}_a$ is independent of stress-relaxation time $\tilde{\lambda}_0$, Vadasz number $\tilde{V}_a$ and ratio of specific heat $\tilde{\sigma}$ for stationary modes, since these vanish with the vanishing of $\tilde{\omega}$.

In the absence of rotation parameter the thermal Rayleigh number is

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}}+\tilde{N}_A\right) = 0, \tag{41}$$

The Darcy Rayleigh number, $\tilde{R}_a^S$ given by equation (41) takes its minimum value by putting $\frac{\partial\tilde{R}_a^S}{\partial a^2} = 0$, we get

$$\frac{a^2.2\left(a^2+\pi^2\right)-\left(a^2+\pi^2\right)^2}{a^4} + \frac{a^2+\pi^2-\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^4} - \left[\frac{\left(a^2+\pi^2\right)-a^2}{\left(a^2+\pi^2\right)^2}\right]\tilde{R}_{ea} = 0, \tag{42}$$

It is clear from equation (42) that the values of critical wave number does not depend on the parameters accounting for nanoparticles, however, depends on $\tilde{R}_{ea}$ only. The effects of various non-dimensional parameters, namely, the electric field $\tilde{R}_{ea}$, the nanofluid Lewis number $\tilde{L}_e$, the modified diffusivity ratio $\tilde{N}_A$, and the concentration Rayleigh number $\tilde{R}_n$ on the stability of stationary modes have been investigated analytically by examining the behavior of $\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_{ea}}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{L}_e}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{N}_A}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_n}$ and $\frac{\partial \tilde{R}_a^S}{\partial \tilde{\varepsilon}}$.

It is depicted from equation (41) that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_{ea}} = -\frac{a^2}{\left(a^2+\pi^2\right)}, \tag{43}$$

which is always negative for all wave numbers, thereby decreasing the Darcy Rayleigh number decreases with increment in $\tilde{R}_{ea}$ (electric Rayleigh number). Thus $\tilde{R}_{ea}$ has always a destabilizing effect on the system.

Equation (41) further yields that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{L}_e} = -\frac{\tilde{R}_n}{\tilde{\varepsilon}}, \tag{44}$$

and

$$\frac{\partial \tilde{R}_a^S}{\partial N_A} = -\tilde{R}_n, \tag{45}$$

It is clear from equation (41) for the bottom-heavy particles (for negative value of $\tilde{R}_n$) both the nanofluid Lewis number $\tilde{L}_e$ and the modified diffusivity ratio $\tilde{N}_A$ stabilize the system for negative value of $\tilde{R}_n$.

Equation (41) also gives that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_n} = -\left(\tilde{N}_A + \frac{\tilde{L}_e}{\tilde{\varepsilon}}\right), \tag{46}$$

Which is always negative for $\left(\tilde{N}_A + \frac{\tilde{L}_e}{\tilde{\varepsilon}}\right) > 0$, since the value of $\tilde{N}_A$ is taken in the range of -1 to -25 and $\tilde{L}_e$ in the range of 100-400. Here a negative $\tilde{N}_A$, (modified diffusivity ratio), indicates that the density of nanoparticles is larger than that of the base fluid. An increase in negative values of $\tilde{N}_A$ reduces the thermophoresis to push the heavier nanoparticles upwards, which enhances the stabilizing effects of particle distributions.

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{\varepsilon}} = \frac{\tilde{R}_n \tilde{L}_e}{\tilde{\varepsilon}^2}. \tag{47}$$

The right hand sides of equations (42) and (45) are negative implying there by the AC electric Rayleigh number and nanoparticle Rayleigh number have destabilizing effect of stationary convection. The right hand sides of equations (43) and (44) are negative if $\tilde{R}_n > 0$. Thus, the Lewis number and the diffusivity ratio have destabilizing effect. But if $\tilde{R}_n < 0$, then Lewis number and the diffusivity ratio have stabilizing effect. The right hand side of equation (46) is positive if $\tilde{R}_n > 0$ and is negative if $\tilde{R}_n < 0$. Thus, medium porosity has stabilizing/destabilizing effect on the convection. These results are in good agreement with the results derived by Nield and Kuznetsov (2014), Rana et al. (2015,2016) and Chand et al. (2016).

4.2 Oscillatory Motion

Since $\tilde{R}_a$ is a physical quantity , it must be a real value . Hence it follows from equation that either $\tilde{\omega}_i = 0$ (exchange of stability , steady onset) or $\Delta_2 = 0$ ($\tilde{\omega}_i \neq 0$,overstability, oscillatory onset). Since for overstability, the critical Darcy Rayleigh number for the onset of instability via a state of pure oscillations of increasing amplitude is determined by putting $\tilde{\omega} = i\tilde{\omega}_i$ in equation (39) and after some algebraic simplifications, we get

$$\tilde{R}_a = \Delta_1 + i\tilde{\omega}_i\Delta_2 . \tag{48}$$

where Δ_1 and Δ_2 are given as follows :

$$\Delta_1 = -\left\{\frac{a^2}{(a^2+\pi^2)}\right\}\tilde{R}_{ea} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left[\tilde{\sigma}(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e\right]}{\tilde{\varepsilon}\left[\tilde{\sigma}^2(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e^{\,2}\right]} - \frac{\tilde{\sigma}^2\tilde{R}_n(a^2+\pi^2)^2\tilde{N}_A}{\tilde{\sigma}^2(a^2+\pi^2)^2 + \omega^2 L_e^{\,2}}$$
$$+\frac{\left[(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right]}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}\left[(a^2+\pi^2) - \frac{\tilde{\omega}^2\tilde{\lambda}_0}{\tilde{V}_a}(a^2+\pi^2)\right] - \frac{\tilde{\omega}}{\tilde{V}_a}\left[\frac{\tilde{\omega}^2 - \tilde{\omega}\tilde{\lambda}_0(a^2+\pi^2)}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}\right](a^2+\pi^2)$$
$$+\frac{1}{a^2\left[\left(1-\frac{\tilde{\lambda}_0\tilde{\omega}^2}{\tilde{V}_a}\right)^2 + \frac{\tilde{\omega}^2}{\tilde{V}_a^{\,2}}\right]}\tilde{T}_a\pi^2\left[\left\{(a^2+\pi^2) - \tilde{\lambda}_0\tilde{\omega}^2\right\}\left(1-\frac{\tilde{\lambda}_0\tilde{\omega}^2}{\tilde{V}_a}\right) + \frac{\tilde{\omega}}{\tilde{V}_a}\left\{\tilde{\omega} + \tilde{\lambda}_0\tilde{\omega}(a^2+\pi^2)\right\}\right]. \tag{49}$$

and

$$\Delta_2 = \frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{(a^2+\pi^2)\tilde{V}_a - \tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a + \tilde{\lambda}_0^{\,2}\tilde{\omega}^4 + \tilde{\omega}^2\right\}}{a^2\left\{(\tilde{V}_a - \tilde{\lambda}_0\tilde{\omega}^2)^2 + \tilde{\omega}^2\right\}} + \frac{(a^2+\pi^2)\left\{(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}$$
$$-\frac{a^2\tilde{\sigma}^2(a^2+\pi^2)^2\tilde{R}_{ea} - \tilde{\sigma}^2\tilde{R}_n(a^2+\pi^2)^3\tilde{N}_A}{\tilde{\sigma}^2(a^2+\pi^2)^3 + \tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e^{\,2}\right\}} .$$
$$-\frac{a^2\tilde{\omega}^2\tilde{L}_e^{\,2}\tilde{R}_{ea}}{\tilde{\sigma}^2(a^2+\pi^2)^3 + \tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) \tag{50}$$

Equating the real and imaginary parts of equation (48),we have $\tilde{R}_a = \Delta_1$,which gives on simplification, the Darcy Rayleigh number as

$$\tilde{R}_a^{osc} = \frac{(a^2+\pi^2)\left\{(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) - \frac{a^2\tilde{\omega}^2\tilde{L}_e\tilde{R}_{ea}}{\tilde{\sigma}^2(a^2+\pi^2)^3 + \tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)}$$
$$+\frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{(a^2+\pi^2)\tilde{V}_a - \tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a + \tilde{\lambda}_0^{\,2}\tilde{\omega}^4 + \tilde{\omega}^2\right\}}{a^2\left\{(\tilde{V}_a - \tilde{\lambda}_0\tilde{\omega}^2)^2 + \tilde{\omega}^2\right\}} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2(a^2+\pi^2)^2 + \tilde{\omega}^2\tilde{L}_e^{\,2}\right\}}$$
$$-\frac{(a^2+\pi^2)\tilde{\sigma}^2\left\{a^2\tilde{R}_{ea} + (a^2+\pi^2)\tilde{N}_A\tilde{R}_n\right\}}{(a^2+\pi^2)^2\tilde{\sigma}^2 + \tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{51}$$

and

$$i\tilde{\omega}_i\Delta_2 = 0 \tag{52}$$

Since for oscillatory modes , $\tilde{\omega}_i \neq 0$, therefore equation (52) implies that $\Delta_2 = 0$, which yields a dispersion relation (relation between $\tilde{\omega}$ and a) as

$$a_1\left(\tilde{\omega}^2\right)^4 + a_2\left(\tilde{\omega}^2\right)^3 + a_3\left(\tilde{\omega}^2\right)^2 + a_4\left(\tilde{\omega}^2\right) + a_5 = 0, \tag{53}$$

where,

$$a_1 = \tilde{\varepsilon}a^2\tilde{\lambda}_0^{\,4}\tilde{L}_e^{\,2}\left(a^2+\pi^2\right), \tag{54}$$

$$\begin{aligned} a_2 = &-\tilde{T}_a\pi^2\tilde{\varepsilon}\tilde{V}_a^{\,2}\tilde{L}_e^{\,2}\lambda_0^{\,4}\left(a^2+\pi^2\right) + a^2\tilde{V}_a\tilde{\lambda}_0^{\,4}\tilde{R}_n\tilde{L}_e\tilde{\sigma}\left(a^2+\pi^2\right)\left[-\tilde{\sigma}+\tilde{L}_e+\tilde{\varepsilon}\tilde{N}_A\right] \\ &+2\tilde{\varepsilon}a^2\tilde{L}_e^{\,2}\tilde{\lambda}_0\left(a^2+\pi^2\right)^2\left(1-\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{\varepsilon}a^2\tilde{L}_e^{\,2}\tilde{\lambda}_0^{\,2}\tilde{V}_a\left(a^2+\pi^2\right)\left(1-\tilde{\lambda}_0\right) + \tilde{\varepsilon}a^2\tilde{\lambda}_0^{\,4}\tilde{\sigma}^2\left(a^2+\pi^2\right)^4, \end{aligned} \tag{55}$$

$$\begin{aligned} a_3 = &\,\tilde{T}_a\pi^2\tilde{V}_a^{\,3}\tilde{\varepsilon}\tilde{L}_e^{\,2}\tilde{\lambda}_0^{\,2}\left\{1+\tilde{\lambda}_0\left(a^2+\pi^2\right)\right\} - \tilde{T}_a\pi^2\tilde{V}_a^{\,2}\tilde{\varepsilon}\tilde{\lambda}_0^{\,2}\left(a^2+\pi^2\right)\left\{2\tilde{L}_e^{\,2}+\tilde{\lambda}_0^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right)\right\} \\ &+2a^2\tilde{\lambda}_0^{\,2}\tilde{V}_a\tilde{R}_n\tilde{L}_e\tilde{\sigma}\left(a^2+\pi^2\right)\left\{-\tilde{\sigma}+\tilde{\lambda}_0\tilde{V}_a\tilde{\sigma}+\tilde{L}_e-\tilde{\lambda}_0\tilde{V}_a\tilde{L}_e+\tilde{\varepsilon}\tilde{N}_A\left(1-\tilde{\lambda}_0\tilde{V}_a\right)\right\} \\ &+2\tilde{\varepsilon}a^2\tilde{\lambda}_0^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right)^4\left(1-\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{\varepsilon}a^2\tilde{L}_e^{\,2}\left(a^2+\pi^2\right)\left(1-3\tilde{\lambda}_0-2\tilde{\lambda}_0\tilde{V}_a^{\,2}+\tilde{V}_a+2\tilde{\lambda}_0\tilde{V}_a^{\,2}\right), \\ &+\tilde{\varepsilon}a^2\tilde{\lambda}_0^{\,2}\tilde{V}_a\left(a^2+\pi^2\right)^2\left(\tilde{V}_a\tilde{L}_e^{\,2}-\tilde{\lambda}_0\tilde{\sigma}^2\right) + \tilde{\varepsilon}a^2\tilde{\lambda}_0^{\,2}\tilde{V}_a\tilde{\sigma}^2\left(a^2+\pi^2\right)^3 \end{aligned} \tag{56}$$

$$\begin{aligned} a_4 = &\,\tilde{T}_a\pi^2\tilde{V}_a^{\,2}\tilde{\varepsilon}\tilde{\lambda}_0^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left(-2+\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{T}_a\pi^2\tilde{V}_a^{\,2}\tilde{\varepsilon}\tilde{L}_e^{\,2}\left(a^2+\pi^2\right)\left(-1+\tilde{\lambda}_0\tilde{V}_a\right) \\ &+\tilde{T}_a\pi^2\tilde{V}_a\tilde{\varepsilon}\tilde{\lambda}_0^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right) + a^2\tilde{V}_a\tilde{R}_n\tilde{L}_e\tilde{\sigma}\left(a^2+\pi^2\right)\begin{pmatrix}2\tilde{\lambda}_0\tilde{V}_a\tilde{\sigma}-\tilde{\lambda}_0\tilde{V}_a\tilde{\sigma}-\tilde{\sigma}-2\tilde{\lambda}_0\tilde{V}_a\tilde{L}_e \\ +\tilde{\lambda}_0^{\,2}\tilde{V}_a\tilde{L}_e+\tilde{L}_e\end{pmatrix} \\ &+\tilde{\varepsilon}a^2\tilde{V}_a\tilde{R}_n\tilde{N}_A\tilde{L}_e\tilde{\sigma}\left(a^2+\pi^2\right)\left(-2\tilde{\lambda}_0\tilde{V}_a+\tilde{V}_a^{\,2}+1\right) + \tilde{\varepsilon}a^2\tilde{V}_a\tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left(-3\tilde{\lambda}_0+2\tilde{V}_a\tilde{\lambda}+1-2\tilde{\lambda}_0^{\,2}\tilde{V}_a\right) \\ &+\tilde{\varepsilon}a^2\tilde{L}_e^{\,2}\tilde{V}_a^{\,2}\left(a^2+\pi^2\right)^2 + \tilde{\varepsilon}a^2\tilde{\sigma}^2\left(a^2+\pi^2\right)^4\left(1+\tilde{\lambda}_0^{\,2}\tilde{V}_a^{\,2}\right) \end{aligned} \tag{57}$$

$$\begin{aligned} a_5 = &\,\tilde{T}_a\pi^2\tilde{V}_a^{\,2}\tilde{\varepsilon}\tilde{\sigma}^2\left(a^2+\pi^2\right)^2\left\{-\left(a^2+\pi^2\right)+\tilde{V}_a+\tilde{\lambda}_0\tilde{V}_a\left(a^2+\pi^2\right)\right\} + a^2\tilde{V}_a^{\,3}\tilde{R}_n\tilde{L}_e\tilde{\sigma}\left(a^2+\pi^2\right) \\ &\left(-\tilde{\sigma}+\tilde{L}_e+\tilde{\varepsilon}\tilde{N}_A\right) + \tilde{\varepsilon}a^2\tilde{V}_a^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left\{\left(a^2+\pi^2\right)+\tilde{V}_a-\tilde{\lambda}_0\tilde{V}_a\right\}. \end{aligned} \tag{58}$$

Equations (51) and (53) are the equations which must be satisfied for the occurrence of overstability for a wave number corresponding to various non-dimensional parameters $\tilde{L}_e, \tilde{V}_a, \tilde{\lambda}_0, \tilde{R}_n, \tilde{\varepsilon}$.

As $\tilde{\omega}$ is real for overstability and at most there must be one change of sign in equation (53) implying thereby at most one positive root of equation (53) for which the critical Darcy Rayleigh number for oscillatory modes is obtained for various values of non-dimensional wave number from equation (51).

It is noteworthy from equation (53) that the existence of oscillatory modes is not (uninfluenced) due to the presence of vertical AC electric field. However, these modes depend on the other non-dimensional parameters accounting for nanoparticles, porous medium and viscoelasticity.

4.3 Validation of Results

In the absence of nanoparticles, rheological parameter and electric field, the equation (53) reduces

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2}, \tag{59}$$

which is confirmation with the earlier result by Lapwood (1948) for classical Newtonian fluids in porous medium. Therefore, due to immersion on nanoparticles in fluid, the parameters $\tilde{N}_A, \tilde{R}_n, \tilde{R}_{ea}, \tilde{V}_a, \tilde{\lambda}_0$ and $\tilde{\sigma}$ are introduced in the expression for the Darcy Rayleigh number, which strongly affect the convection of porous Maxwellian nanofluid layer.

In the absence of electric field that is, $\tilde{R}_{ea}=0$, the equations (51) and (40) reduce to

$$\tilde{R}_a^{osc} = \frac{\left(a^2+\pi^2\right)\left\{\left(a^2+\pi^2\right)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2\left(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2}\right)} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) + \frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{\left(a^2+\pi^2\right)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{\left(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\right)^2+\tilde{\omega}^2\right\}}$$
$$-\frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}\left(a^2+\pi^2\right)^2+\tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2\left(a^2+\pi^2\right)^2+\tilde{\omega}^2\tilde{L}_e^{\,2}\right\}} - \frac{\left(a^2+\pi^2\right)\tilde{\sigma}^2\left\{\left(a^2+\pi^2\right)\tilde{N}_A\tilde{R}_n\right\}}{\left(a^2+\pi^2\right)^2\tilde{\sigma}^2+\tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{60}$$

and

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}}+\tilde{N}_A\right) = 0, \tag{61}$$

which are in good agreement with the earlier results given by Umavathi et al. (2016) (for the limiting case (i.e. $\tilde{\upsilon},\tilde{\eta}\to 1$) for both stress-free boundaries.

When the nanoparticles are not embedded i.e. $\tilde{R}_n=0, \tilde{N}_A=0$, the equation (51) and (40) gives

$$\tilde{R}_a^{osc} = \frac{\left(a^2+\pi^2\right)\left\{\left(a^2+\pi^2\right)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2\left(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2}\right)} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) - \frac{a^2\tilde{\omega}^2\tilde{L}_e\tilde{R}_{ea}}{\tilde{\sigma}^2\left(a^2+\pi^2\right)^3+\tilde{\omega}^2\tilde{L}_e^{\,2}\left(a^2+\pi^2\right)}$$
$$+\frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{\left(a^2+\pi^2\right)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{\left(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\right)^2+\tilde{\omega}^2\right\}} - \frac{\left(a^2+\pi^2\right)\tilde{\sigma}^2\left\{a^2\tilde{R}_{ea}\right\}}{\left(a^2+\pi^2\right)^2\tilde{\sigma}^2+\tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{62}$$

and

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea} = 0, \tag{63}$$

which agrees with the earlier results of Roberts (1969) in the limiting case.

5 Numerical Results and Discussions

The expressions of thermal Rayleigh number for both the oscillatory and stationary motions are encapsulated in equations (51) and (40) , respectively. The variations of $\tilde{R}_a$ along x-axis , with respect to wave-number a along y-axis have been plotted graphically in figures (1)-(7) and (8)-(14) using equations (40) and (51) for both the cases of stationary and oscillatory modes, respectively. The experimental values and the fixed permissible values of the dimensionless parameters $\tilde{R}_n = -0.1, \tilde{R}_{ea} = 20, \tilde{L}_e = 500, \tilde{N}_A = -5, \tilde{V}_a = 0.1, \tilde{\lambda}_0 = 0.5, \tilde{T}_a = 10$ for bottom heavy distribution and $\tilde{R}_n = 0.1, \tilde{R}_{ea} = 20, \tilde{L}_e = 500, \tilde{N}_A = 5, \tilde{V}_a = 0.1, \tilde{\lambda}_0 = 0.5, \tilde{T}_a = 10$ for top-heavy distribution are used to investigate the effects of various involved parameters on the system numerically by using MATHEMATICA software version - 5.2.

The effects of different values of $\tilde{R}_n$ are illustrated in figures 2 and 3 and stationary convection w.r. t. wave number a. It is depicted from the graphs that $\tilde{R}_n$ destabilizing the stationary modes and stabilizing the oscillatory modes for both bottom/top-heavy distribution of nanoparticles. The effect of $\tilde{N}_A$ for both bottom/top-heavy distribution of nanoparticles on the onset of stationary modes is displayed in figure 4 and 8 respectively.. The figures show that the $\tilde{N}_A$ does not affect the stability of the system both type of distribution of nanoparticles. In figure 5, the variation of $\tilde{R}_a$ has been plotted versus wave number a for three different values of $\tilde{R}_{ea}$. It is observed from the figure that $\tilde{R}_{ea}$ has slight destabilizing effect for both stationary and oscillatory modes for bottom-heavy nanoparticles distributions. Figure 6: illustrates the effect of Lewis number on the stationary modes. It is depicted from the graphs that $\tilde{L}_e$ number has stabilizing effect implying thereby postponement on the onset of stationary convection. The influence of the Lewis number on neutral curves is shown on the graphs. It is observed that the effect of $\tilde{L}_e$ is stabilizing for stationary modes ; whereas this effect is slightly destabilizing for oscillatory mode. The critical wave number for stationary and oscillatory modes are 5.9 and 3.1, respectively.Negative / positive values of $\tilde{R}_n$ and $\tilde{N}_A$ represents a bottom/top-heavy distribution of nanoparticles.

Figure 7: shows the variation of the thermal Rayleigh-Darcy number with respect to wave number for different values of the Taylor number $\tilde{T}_a$. It is observed from the graphs that there is an increment in $\tilde{R}_a^S$ with an increment in $\tilde{T}_a$ for stationary modes, while a reverse effect is observed for oscillatory modes. This happens so because the presence of the Coriolis forces due to rotation of nanofluids allows to minimize the effect of buoyancy forces on the onset of electroconvection is dielectric nanofluids. Hence, $\tilde{T}_a$ has a stabilizing influence on stationary modes for the dielectric viscoelastic nanofluids for bottom-heavy distribution of nanoparticles. The effect of $\tilde{R}_n$ for both bottom-/top-heavy nanoparticles distribution are illustrated in figures 9 and 10 repectively. It is observed from the figure 9 that $\tilde{R}_a$ takes very large value in the range of $0 < a \le 25$ and there is uninfluenced effect of $\tilde{R}_n$ on $\tilde{R}_a$ in this regime. For $a > 2.5$, $\tilde{R}_a$ decreases with increase in $\tilde{R}_n$ implying thereby a destabilizing effect of $\tilde{R}_n$ on $\tilde{R}_a$. Figure 10 shows that $\tilde{R}_n$ has no significant effect on the oscillatory modes in the range $0 < a \le 3.1$ and for $a > 3.1$, $\tilde{R}_n$ destabilize the system.

Uninfluenced effects of $\tilde{N}_A$ for both bottom/top- heavy nanoparticles distributions are clear from figure 11. Vadasz number is shown in the figure 12. From figure 12 we see that there is a destabilizing effect on the physical system. The effect of $\tilde{T}_a$ accounting for angular velocity of the viscoelastic nanofluids is shown in figure 13. It is depicted from the figure 13 that there is a decrement in $\tilde{R}_a^{OSC}$ with increment in $\tilde{T}_a$, thereby destabilizing the oscillatory modes which is contrary to that of the stationary mode. The effect of $\tilde{L}_e$ on the oscillatory modes is displayed in figure 14. It is again found that Lewis number $\tilde{L}_e$ does not show significant effect in the range $0 < a \leq 3.1$. For $a > 3.1$, Lewis number $\tilde{L}_e$ shows slight destabilizing effect.

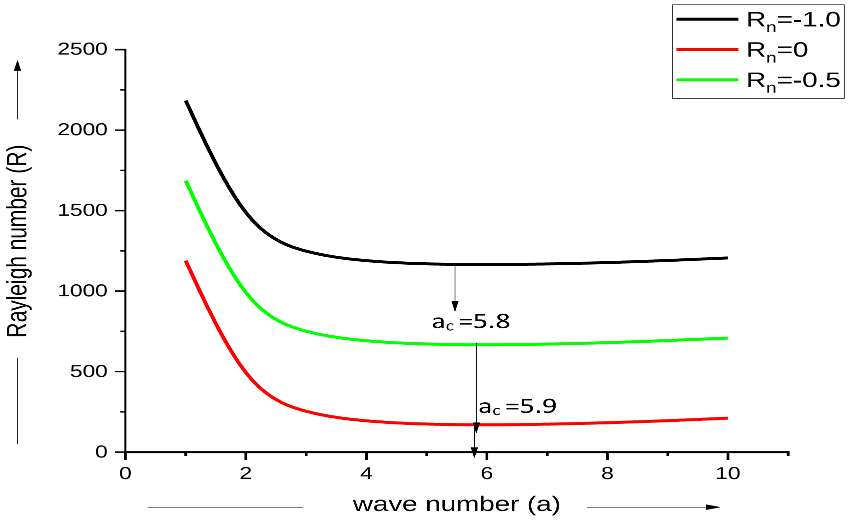

Figure 2: $\tilde{R}_a$ as a function of a for different negative values of $\tilde{R}_n$.

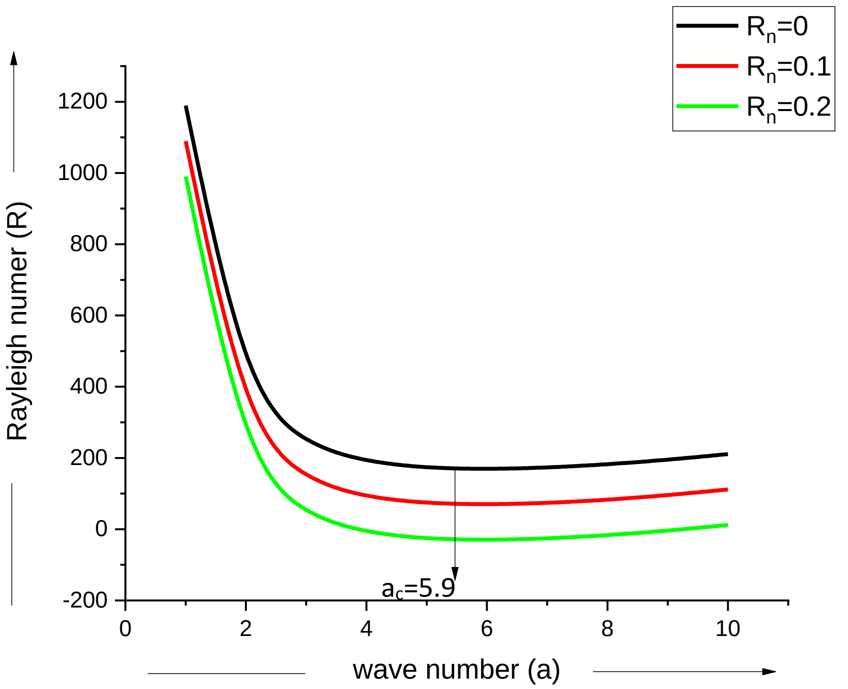

Figure 3: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{R}_n$.

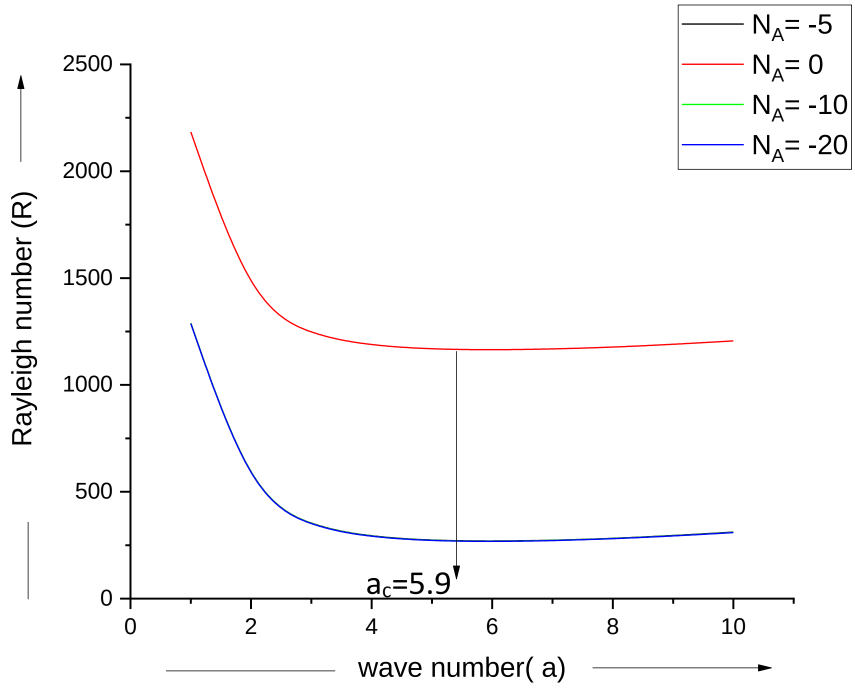

Figure 4: $\tilde{R}_a$ as a function of a for different negative values of $\tilde{N}_A$.

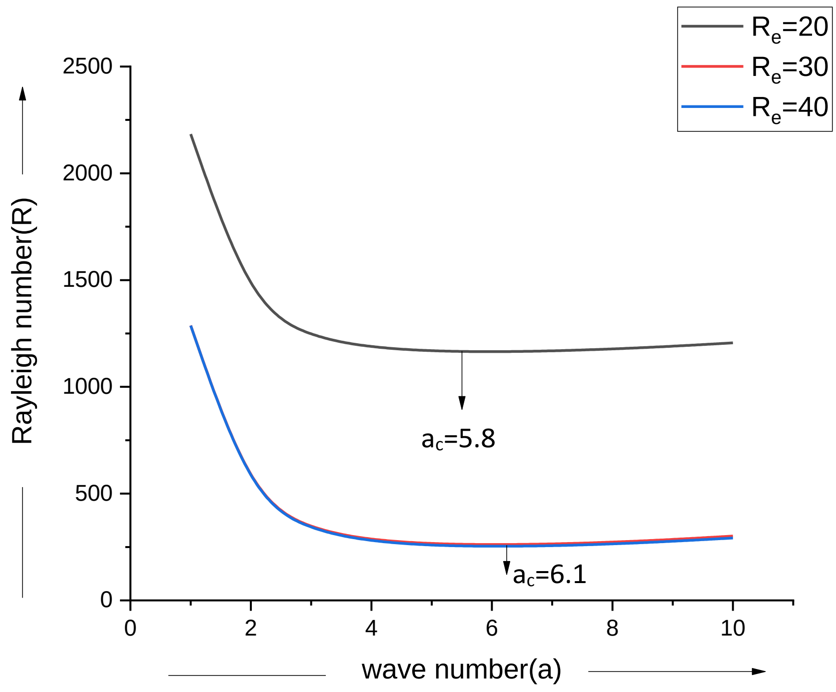

Figure 5: $\tilde{R}_a$ as a function of a for positive values of $\tilde{R}_{ea}$.

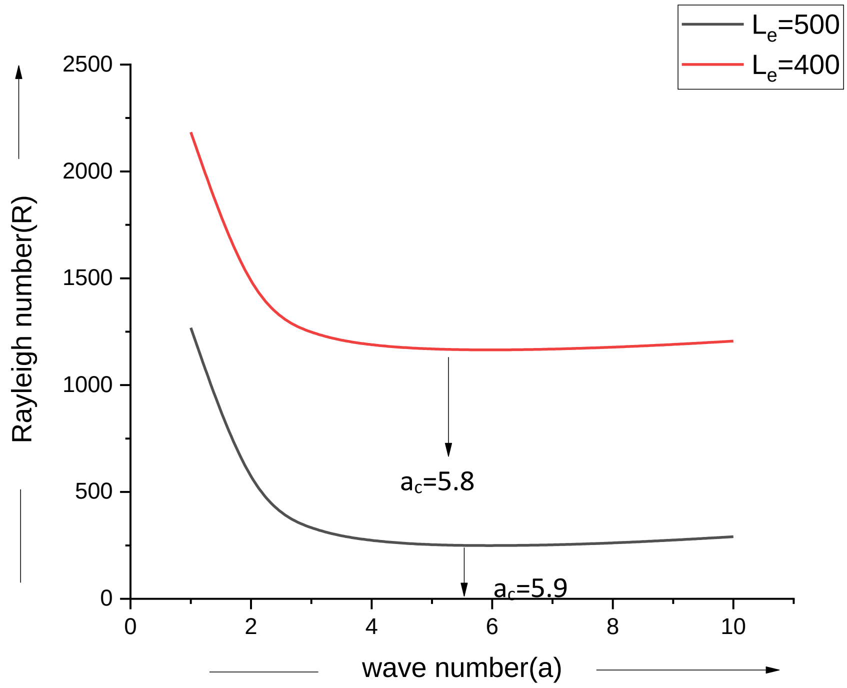

FIGURE: 6 $\tilde{R}_a$ as a function of a for different positive values of $\tilde{L}_e$.

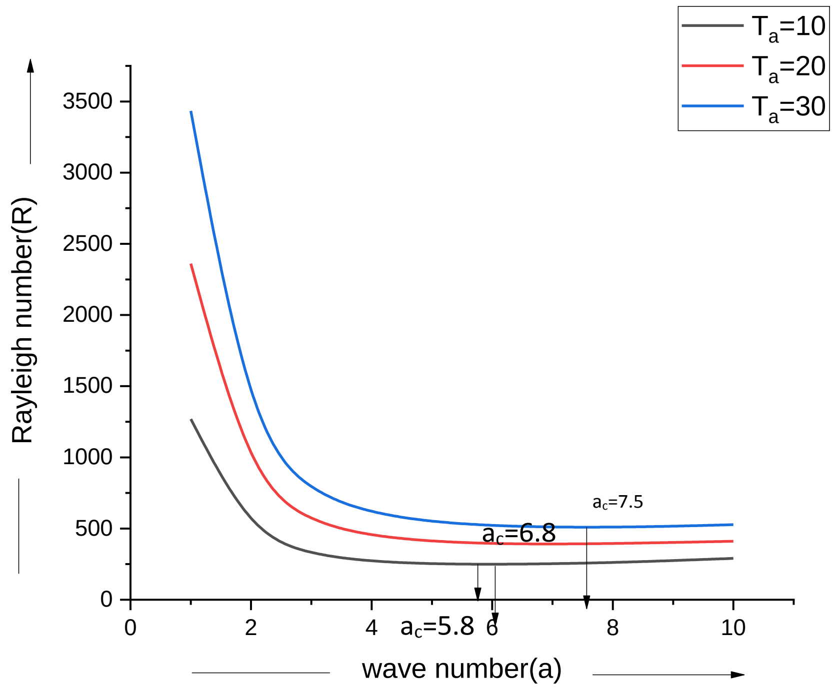

Figure 7: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{T}_a$.

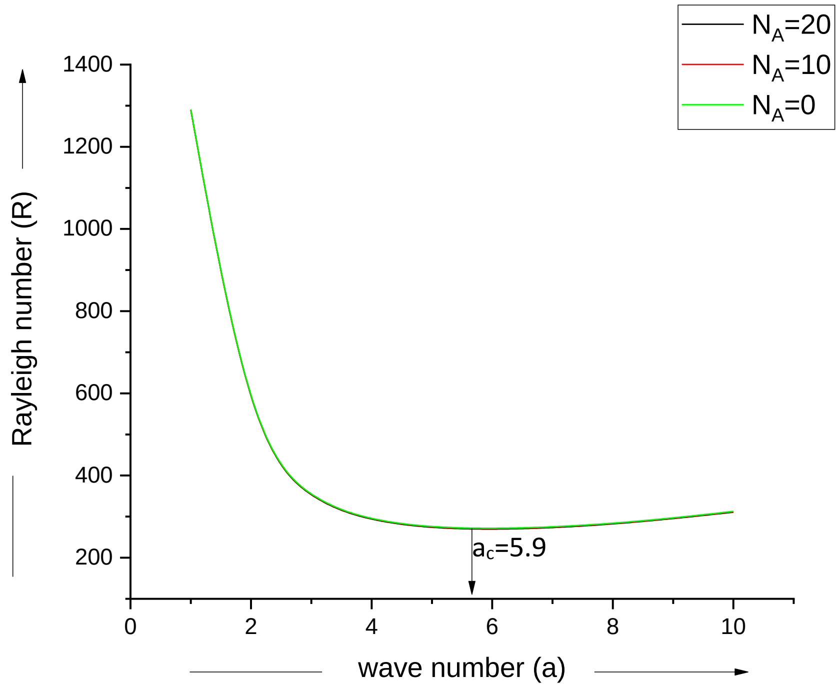

Figure 8: $\tilde{R}_a$ as a function of a for different values of $\tilde{N}_A$.

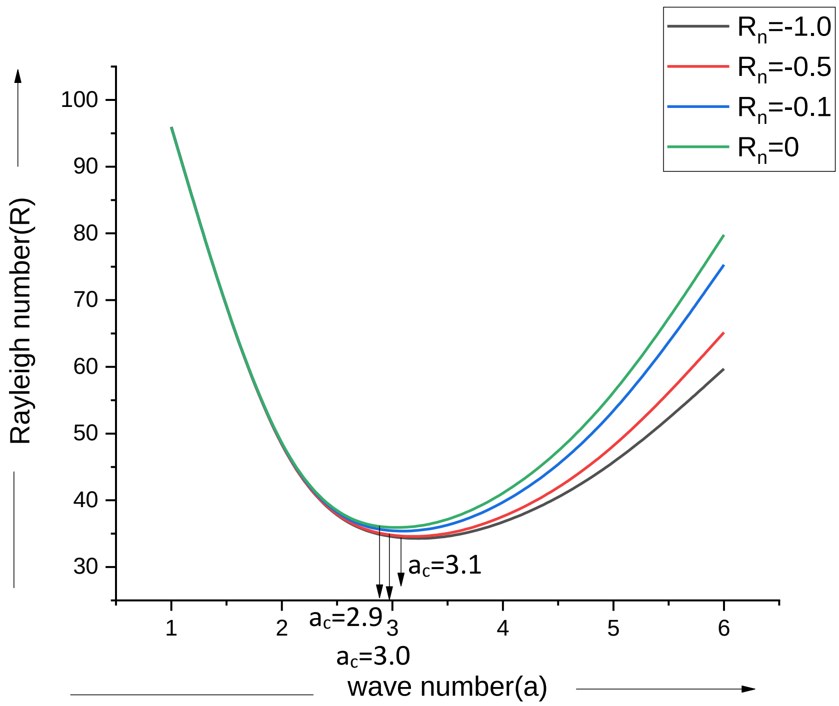

Figure 9: $\tilde{R}_a^{osc}$ as a function of a for different values of $\tilde{R}_n$.

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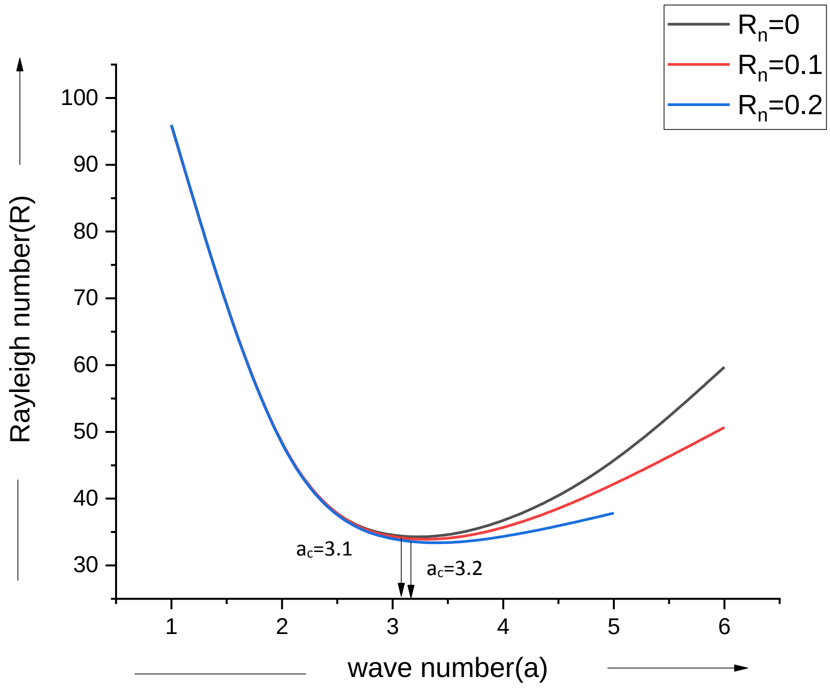

Figure 10: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{R}_n$.

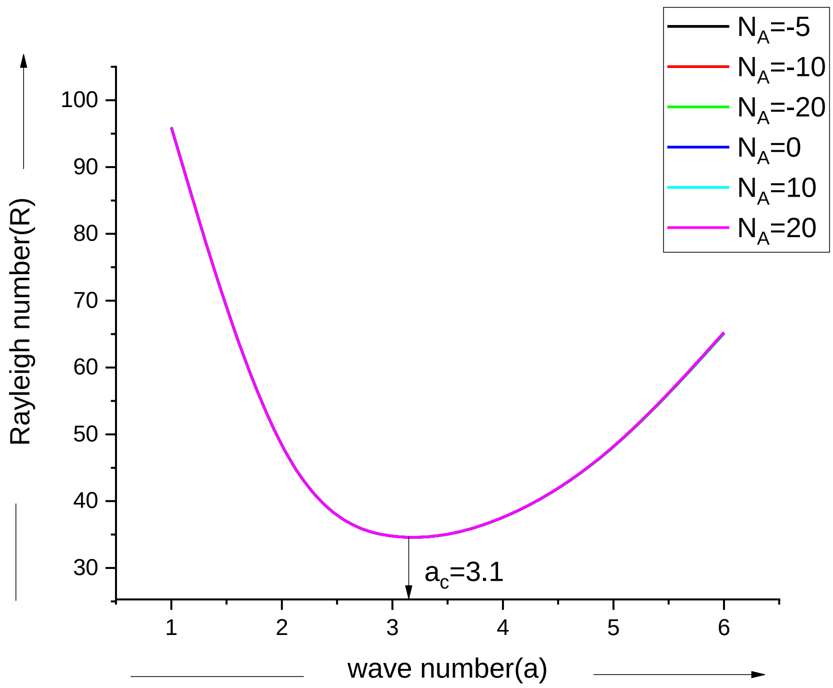

Figure 11: $\tilde{R}_a$ as a function of a for different values of $\tilde{N}_A$.

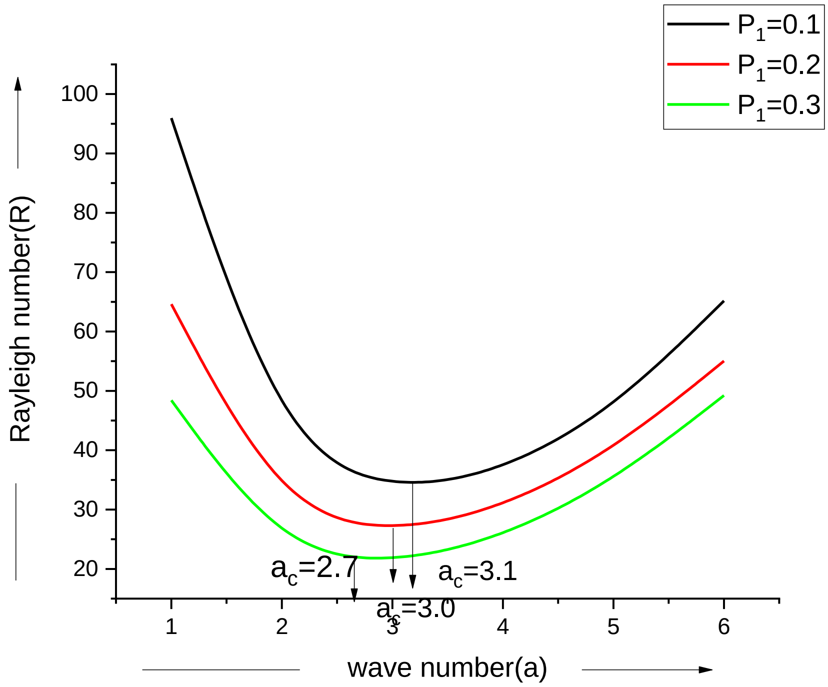

Figure 12: $\tilde{R}_a$ as a function a of for different values of $\tilde{V}_a$.

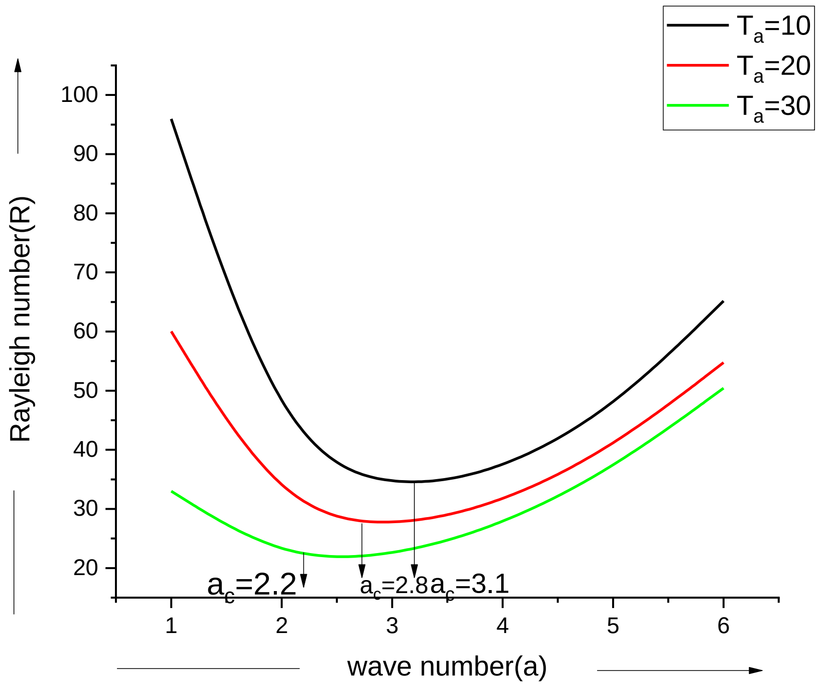

Figure 13: $\tilde{R}_a$ as a function of a for different values of $\tilde{T}_a$

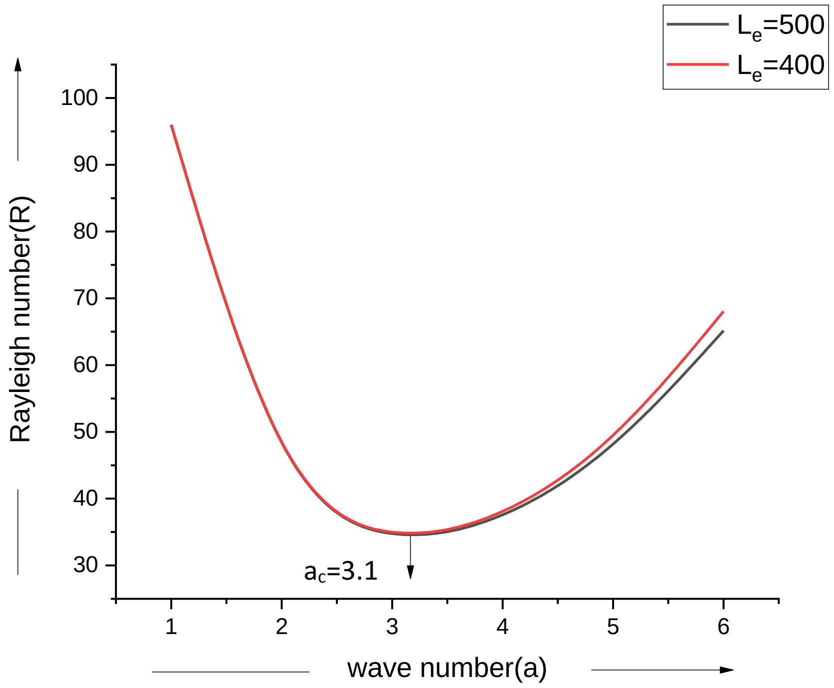

Figure 14: $\tilde{R}_a$ as a function of a for different values of $\tilde{L}_e$.

6 CONCLUSIONS:

The effect of rotation on electrothermal-convection in a dielectric Maxwellian nanofluid layer saturating a Darcy porous medium is studied. The main important results are as under:

- The concentration Rayleigh number $\tilde{R}_n$ destabilize the stationary modes and stabilize the oscillatory modes for both bottom / top - heavy nanoparticles distribution.
- The effect of modified diffusivity ratio $\tilde{N}_A$ is uninfluenced for both the stationary and oscillatory modes.
- Taylor number $\tilde{T}_a$ and Lewis number $\tilde{L}_e$ stabilizes/destabilizes the stationary / oscillatory modes.
- The effect of electric Rayleigh number $\tilde{R}_{ea}$ is very slight on both the stationary and oscillatory modes.
- The Vadasz number accounting for Darcy number destabilizes the oscillatory modes.

Interestingly, the occurrence of oscillatory modes is possible for top-heavy distribution of nanoparticles in the simultaneous presence of electric field and rotation, which was not the case in the absence of rotation and for Newtonian nanofluids.

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CONTENT

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EFFECT OF INTERNAL HEAT SOURCE ON THE ONSET OF CONVECTION IN A LAYER OF OLDROYDIAN VISCO-ELASTIC NANOFLUID IN A POROUS MEDIUM

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ABSTRACT:
Analytical investigation for study of the thermal instability of Oldroydian visco-elastic nanofluid in a porous medium with realistic boundary condition is presented. For porous medium the Brinkman model is considered. The used model incorporates the effect of Brownian diffusion and thermophoresis. The eigen value problem is solved by employing the Galerkin weighted residuals method. The influence of the Lewis number, Brinkman-Darcy number, nanoparticle Rayleigh number, modified diffusivity ratio and porosity parameter on the stationary convection studied analytically and found that Lewis number, modified diffusivity ratio and nanoparticles Rayleigh number destabilizes while Brinkman-Darcy number and porosity parameter stabilize the stationary convection.

1. INTRODUCTION

Thermal instability of nanofluids in a porous medium is an important phenomenon because of its importance in many fields of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. The natural convection for flow in a porous medium based upon Buongiorno's model [Buongiorno (2006)] was studied by Nield and Kuznetsov (2009) using the Darcy model. The extension to the Brinkman model was made by Kuznetsov and Nield (2010, 2011) and Chand and Rana (2012a). In studying these convective instability problems, the volume fraction of nanoparticles was prescribed at the boundaries. Nield and Kuznetsov (2014) suggested that the value of the temperature can be imposed on the boundaries, but the nanoparticle fraction adjusts so that the nanoparticle flux is zero on the boundaries. With these new boundary conditions, Chand and Rana (2014, 2015a, 2015b) and Yadav et al. (2016) studied thermal instability in a layer of nanofluid based upon Buongiorno's model. The above literature deals with the study of nanofluids as Newtonian fluids. The onset of convection in a horizontal layer of nanofluid as Newtonian fluids uniformly heated from below (Bénard convection) has been extensively investigated but a little attention has been made to study the thermal convection of non-Newtonian fluids. With the growing importance of non-Newtonian fluids in technology and industries, the investigations of such fluids are desirable. In the category of non- Newtonian fluids Oldroydian nanofluids have distinct features and well represented by the Oldroydian constitutive model as given by Oldroyd (1958). The Oldroydian constitutive model is adopted widely to examine the influence of elasticity on thermal convective instability. Thermal convection in a layer of Oldroydian visco-elastic fluid saturated by Brinkman–

Darcy porous medium was investigated by Chand (2015) and found that Brinkman stabilize the fluid layer. Thermal instability problems in non-Newtonian nanofluid problems were investigated theoretically by Sheu (2011), Shivakumara et al. (2015), Rana and Chand (2015), Chand and Rana (2012b) by taking different type of non-Newtonian fluid as base fluid.

Due to importance of Oldroydian visco-elastic nanofluids in porous medium an attempt has been made to study the thermal instability of a horizontal layer of Oldroydian visco-elastic nanofluids for more realistic boundary conditions in Brinkman porous medium.

2. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of Oldroydian visco-elastic nanofluid of thickness ‘d’ bounded by horizontal boundaries z = 0 and z = d. Fluid layer is acted upon by a gravity force **g**(0,0,-g) and is heated from below in such a way that horizontal boundaries z = 0 and z = d respectively maintained at a uniform temperature T_0 and T_1 ($T_0 > T_1$) as shown is Fig.1. The normal component of the nanoparticles flux has to vanish at an impermeable boundaries and the reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively.

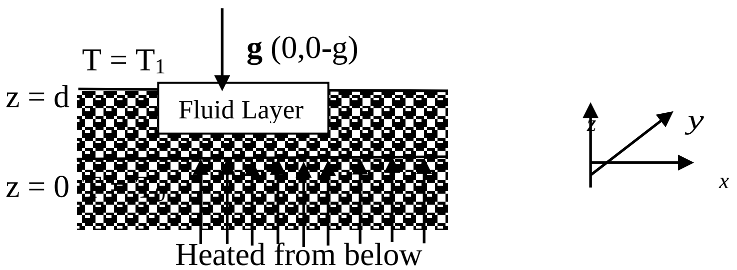

Fig.1 Physical configuration of the problem

The equation of continuity and motion for Oldroydian visco-elastic nanofluid in Brinkman porous medium under the Boussinesq approximation are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{d\mathbf{q}}{dt} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p + \left(\varphi\rho_p + (1-\varphi)\{\rho_f(1-\alpha(T-T_0))\}\right)\mathbf{g}\right) - \left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\tilde{\mu}\nabla^2 - \frac{\mu}{k_1}\right)\mathbf{q}, \tag{2}$$

where **q(u, v, w)** is the Darcy velocity vector, p is the hydrostatic pressure, μ is viscosity, $\tilde{\mu}$ effective viscosity, α is the coefficient of thermal expansion, λ the relaxation time, λ_0 the retardation time, T the temperature of the nanofluid, φ is the volume fraction of the nanoparticles, ρ_p density of nanoparticles, ρ_f density of base fluid and $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)$ is stands for convection derivative.

The equation of energy for Oldroydian visco-elastic nanofluid in porous medium is

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f\, \mathbf{q}\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p\left(D_B \nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right) + Q_0, \tag{3}$$

where $(\rho c)_m$ is effective heat capacity of fluid, $(\rho c)_p$ is heat capacity of nanoparticles, Q_0 is the overall uniformly distributed effective volumetric internal heat source, T_1 is the temperature of the fluid layer at z = d and k_m is effective thermal conductivity of the porous medium.

The continuity equation for the nanoparticles is

$$\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_1}\nabla^2 T\,, \tag{4}$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation and D_T is the thermoporetic diffusion coefficient of the nanoparticles.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus boundary conditions [Chandrasekhar (1981), Nield and Kuznetsov (2014)] are

$$w=0, T=T_0,\ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_0}\frac{\partial T}{\partial z}=0 \text{ at } z=0 \text{ and } w=0, T=T_1,\ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \text{ at } z=d. \tag{5}$$

Introducing non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right),\ v'(u',v',w')=v\left(\frac{u,v,w}{\kappa}\right)d,\ t'=\frac{t\kappa}{\sigma d^2},\ p'=\frac{pk_1}{\mu\kappa},\ \varphi'=\frac{(\varphi-\varphi_0)}{\varphi_0},$$

$T'=\frac{(T-T_1)}{(T_0-T_1)}$, where $\kappa=\frac{k_m}{(\rho c_p)_f}$ is thermal diffusivity of the fluid.

Equations (1) - (5) in non-dimensional form can be written as

$$\nabla'\cdot\mathbf{q}'=0, \tag{6}$$

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t'}\right)\frac{\partial\mathbf{q}'}{\partial t'}=\left(1+F\frac{\partial}{\partial t'}\right)\left(-\nabla' p' - Rm\hat{e}_z+RaT'\hat{e}_z-Rn\varphi'\hat{e}_z\right)-\left(1+F_0\frac{\partial}{\partial t'}\right)\left(\tilde{D}a\nabla'^2\mathbf{q}'-\mathbf{q}'\right),$$

(7)

$$\frac{\partial T'}{\partial t'}+\mathbf{q}'\cdot\nabla'T'=\nabla'^2T'+\frac{N_B}{Le}\nabla'\varphi'\cdot\nabla'T'+\frac{N_AN_B}{Le}\nabla'T'\cdot\nabla'T'+Hs\,, \tag{8}$$

$$\frac{1}{\sigma}\frac{\partial\varphi'}{\partial t'}+\frac{1}{\varepsilon}\mathbf{q}'\cdot\nabla'\varphi'=\frac{1}{Le}\nabla'^2\varphi'+\frac{N_A}{Le}\nabla'^2T', \tag{9}$$

Here the non-dimensional parameters are given as

$Le=\frac{\kappa}{D_B}$ is the Lewis number,

$Pr=\frac{\mu}{\rho\kappa}$ is the Prandtl number,

$Da=\frac{k_1}{d^2}$ is the Darcy number,

$\tilde{D}a=\frac{\tilde{\mu}k_1}{\mu d^2}$ is the Brinkman-Darcy number,

$Hs = \dfrac{Q_0 d^2}{k_m\left(T_0 - T_1\right)}$ is the dimensionless constant of heat source strength,

$Va = \dfrac{\varepsilon \Pr}{Da}$ is the Prandtl-Darcy number,

$F = \dfrac{\lambda\kappa}{d^2}$ is the stress relaxation parameter,

$F_0 = \dfrac{\lambda_0\kappa}{d^2}$ is the strain retardation parameter,

$Ra = \dfrac{\rho\alpha d k_1 g\left(T_0 - T_1\right)}{\mu\kappa}$ is the Rayleigh Darcy number,

$Rm = \dfrac{\left(\rho_p\varphi_0 + \rho\left(1-\varphi_0\right)\right) g d k_1}{\mu\kappa}$ is the density Rayleigh Darcy number,

$Rn = \dfrac{\left(\rho_p - \rho\right)\varphi_0 g d k_1}{\mu\kappa}$ is the nanoparticles Rayleigh Darcy number,

$N_A = \dfrac{D_T\left(T_0 - T_1\right)}{D_B T_1 \varphi_0}$ is the modified diffusivity ratio,

$N_B = \dfrac{(\rho c)_p \varphi_0}{(\rho c)_f}$ is the modified particle-density increment,

$\mathbf{e}_z$ is the unit vector along z-axis.

In spirit of Oberbeck-Boussinesq approximation, equation (7) has been linearized by the neglect of a term proportional to the product of φ_0 and T. This approximation is valid in the case of small temperature gradients in a dilute suspension of nanoparticles.

The dimensionless boundary conditions are

$$w' = 0,\quad T' = 1,\quad \frac{\partial\varphi'}{\partial z'} + N_A\frac{\partial T'}{\partial z'} = 0 \text{ at } z' = 0 \text{ and } w' = 0,\ T' = 0,\ \frac{\partial\varphi'}{\partial z'} + N_A\frac{\partial T'}{\partial z'} = 0 \text{ at } z' = 1. \tag{10}$$

2.3 THE BASIC STATE AND ITS SOLUTIONS

The basic state was assumed to be quiescent and is given by

$$\mathbf{q}'\left(u', v', w'\right) = 0,\quad p' = p_b(z),\ T' = T_b(z),\ \varphi' = \varphi_b(z), \tag{11}$$

Equations (6) – (9) reduce to

$$0 = -\frac{dp_b}{dz'} - Rm + RaT_b - Rn\varphi_b, \tag{12}$$

$$\frac{d^2T_b}{dz'^2} + \frac{N_B}{Le}\frac{d\varphi_b}{dz'}\frac{dT_b}{dz'} + \frac{N_A N_B}{Le}\left(\frac{dT_b}{dz'}\right)^2 + Hs = 0, \tag{13}$$

$$\frac{d^2\varphi_b}{dz'^2} + N_A\frac{d^2T_b}{dz'^2} = 0. \tag{14}$$

Using boundary conditions in (10), equation (14) gives

$$\varphi_b = -N_A T_b + \varphi_0 + N_A. \tag{15}$$

On substituting the value of the φ_b from equation (15) in equation (13), we get

$$\frac{d^2T_b}{dz^2} + \frac{(1-N_A)N_B}{Le}\frac{dT_b}{dz^2} + Hs = 0. \tag{16}$$

On integrating equation (16) with respect to z and using boundary conditions (10), we get

$$T_b = \frac{e^{\frac{-(1-N_A)N_B}{Le}z}\left[-N_B(-1+N_A) - LeHs\right] - LeHs(-1+z) + e^{\frac{-(1-N_A)N_B}{Le}[(-1+N_A)N_B + LeHsz]}}{-1+e^{\frac{-(1-N_A)N_B}{Le}(-1+N_A)N_B}}. \tag{17}$$

$$\varphi_b = z + \frac{N_A\left[(-1+N_A)N_B - LeHs\right]\left(-1+e^{\frac{-(1-N_A)N_B}{Le}z} + z - ze^{\frac{-(1-N_A)N_B}{Le}}\right)}{-1+e^{\frac{-(1-N_A)N_B}{Le}(-1+N_A)N_B}}. \tag{18}$$

According to Buongiorno (2006), for most nanofluid investigated so far Le is large, is of order 10^2-10^3, while N_A is no greater than about 10, N_B is of order 10^{-4} to 10^{-2}. Then, the exponents $\frac{-(1-N_A)N_B}{Le}$ in equations (17)–(18) are very small. By expanding the exponential function into the power series and retaining up to the first order is negligible and so to a good approximation for the solution of basic state is given by

$$T_b = \frac{1}{2}\left(2 - 2z + Hsz - Hsz^2\right)$$

and

$$\varphi_b = \varphi_0 + N_A\left(z - \frac{Hsz}{2} + \frac{Hsz^2}{2}\right).$$

In the absence of internal heat generation i.e. Hs = 0, then basic flow distributions for temperature and nanoparticles volume fraction are:

$T_b = 1 - z$ and

$$\varphi_b = \varphi_0 + N_A z. \tag{19}$$

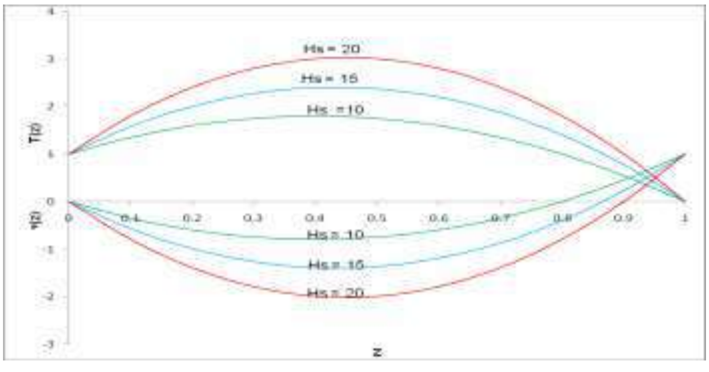

Fig.10.2 Basic state temperature and basic state nanoparticles volumetric distributions for different values of internal heat source strength parameter Hs

To see the effect of internal heat source strength Hs on the criterion for the onset of thermal convection in nanofluids, the distributions of dimensionless basic temperature and basic nanoparticles volumetric fraction are drawn in the Fig. 10.2 for different values of Hs. The discrete values of Hs are purposely taken to see the behavior of both distributions. This plot shows the behavior of basic temperature distribution which is parabolic in positive direction and same behavior in negative direction for the basic nanoparticles distribution as internal heat source strength Hs increases. That is increase in the internal heat source strength Hs amounts to increase in energy supply to the system. This gives large deviations in these distributions which in turn improve the disturbances in the layer and thus system is more unstable.

2.3. PERTURBATION EQUATIONS

Let the initial basic state described by (11) is slightly perturbed so that perturbed state is given by

$$\mathbf{q}'(u',v',w') = 0 + \mathbf{q}''(u'',v'',w''),\ T' = T_b + T'',\ \varphi' = \varphi_b + \varphi'',\ p' = p_b + p'', \tag{20}$$

where $T_b = 1 - z'$, $\varphi_b = \varphi_0 + N_A z'$ and (u'',v'',w''), T'', φ'' and p'' respectively the perturbations in initial velocity, temperature, volume fraction of the nanoparticles and pressure.

By substituting (20) in equations (6) – (9) and linearize by neglecting the product of the prime quantities, we obtained following equations

$$\nabla \cdot \mathbf{q} = 0, \tag{21}$$

$$\frac{1}{Va}\left(1 + F\frac{\partial}{\partial t}\right)\frac{\partial \mathbf{q}}{\partial t} = \left(1 + F\frac{\partial}{\partial t}\right)\left(-\nabla p + RaT\hat{e}_z - Rn\varphi\hat{e}_z\right) - \left(1 + F_0\frac{\partial}{\partial t}\right)\left(\tilde{D}a\nabla^2\mathbf{q} - \mathbf{q}\right), \tag{22}$$

$$\begin{aligned}\frac{\partial T}{\partial t} - w(-2 + Hs - 2Hsz) &= \nabla^2 T + \frac{2N_A N_B}{Le}(-2 + Hs - 2Hsz)\frac{\partial T}{\partial z}\\ &+ \frac{N_B N_A}{Le}\left(1 - \frac{Hs}{2} + 2Hsz\right)\frac{\partial T}{\partial z} + \frac{2N_B}{Le}(-2 + Hs - 2Hsz)\frac{\partial \varphi}{\partial z},\end{aligned} \tag{23}$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}wN_A = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T. \tag{24}$$

Boundary conditions for the infinitesimal perturbation are given by

$$w = 0, T = 0,\ \frac{\partial \varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, 1. \tag{25}$$

[Dashes (") have been suppressed for convenience]

Applying Curl operator twice to the equation (22) under the assumption of linear theory, the resulting equations are given by

$$\frac{1}{Va}\left(1 + F\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 w = \left(1 + F\frac{\partial}{\partial t}\right)\left(Ra\nabla_H^2 T - Rn\nabla_H^2\varphi\right) + \left(1 + F_0\frac{\partial}{\partial t}\right)\left(\tilde{D}a\nabla^4 w - \nabla^2 w\right), \tag{26}$$

where ∇_H^2 is two-dimensional Laplacian operator.

4. NORMAL MODES ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)] \exp(ik_x x + ik_y y + nt), \tag{27}$$

where k_x, k_y are wave numbers in x and y direction and n is growth rate of disturbances.
Using equation (27), equations (26), (23) and (24) become

$$\left[\left(\frac{n(1+Fn)}{Va}(D^2-a^2) - (1+F_0 n)\left(\tilde{D}a(D^2-a^2)^2 - (D^2-a^2)\right)\right)D^2\right]W - (1+nF)(a^2 Ra\Theta - a^2 Rn\Phi) = 0, \tag{28}$$

$$-\frac{1}{2}(-2+Hs-2Hsz)W + \begin{pmatrix} D^2 - a^2 - n + \dfrac{N_A N_B}{Le}(-2+Hs-2Hsz)D \\ -\dfrac{N_A N_B}{Le}\left(1 - \dfrac{Hs}{2} + Hsz\right)D \end{pmatrix}\Theta \tag{29}$$

$$-\frac{N_B}{2Le}(-2+Hs-2Hsz)D\Phi = 0,$$

$$\frac{1}{\varepsilon}N_A W - \frac{N_A}{Le}(D^2-a^2)\Theta - \left(\frac{1}{Le}(D^2-a^2) - \frac{n}{\sigma}\right)\Phi = 0, \tag{30}$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the dimensionless resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W = 0, D^2W = 0, \Theta = 0, D\Phi + N_A D\Theta = 0 \text{ at } z = 0,1. \tag{31}$$

5. METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (28) – (30) with boundary conditions (31). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{N} A_p W_p, \Theta = \sum_{p=1}^{N} B_p \Theta_p, \Phi = \sum_{p=1}^{N} C_p \Phi_p, \tag{32}$$

where $W_p = \Theta_p = \sin p\pi z, \Phi_p = -N_A \sin p\pi z$, A_p, B_p and C_p are unknown coefficients, p = 1, 2, 3, ..., N and the base functions W_p, Θ_p, and Φ_p satisfying the boundary conditions (31). Using expression for W, Θ and Φ in equations (28) – (30) and multiplying the first equation by W_p the second equation by Θ_p and third equation by Φ_p and then integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations with 3N unknown A_p, B_p and C_p; p = 1, 2, 3, ..., N. For existing of nontrivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number Ra.

6. LINEAR STABILITY ANALYSIS

For the present formulation, we have considered the which system of equations (28) – (30) together with the boundary conditions (31) constitute a linear eigen value problem with variable coefficient for the growth rate of disturbance n of the system. Substituting equation (32 into the system of equations (28) – (30) and multiplying the first equation by

W_p the second equation by Θ_p and third equation by Φ_p and then integrating in the limits from zero to unity and performing some integration by parts, one obtains the following matrix equation

$$\begin{bmatrix} \frac{(1+nF_0)\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)}{(1+nF)}+\frac{n}{Va}(\pi^2+a^2) & -a^2Ra & -a^2N_ARn \\ 1-\frac{Hs}{2}+Hsz & -(\pi^2+a^2+n) & 0 \\ \frac{N_A}{\varepsilon} & \frac{N_A}{Le}(\pi^2+a^2) & -N_A\left(\frac{1}{Le}(\pi^2+a^2)+\frac{n}{\sigma}\right) \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The non-trivial solution of the above matrix requires that

$$Ra=\frac{1}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{(1+nF_0)}{(1+nF)}\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)+\frac{n}{Va}(\pi^2+a^2)\right)(\pi^2+a^2+n)$$
$$-\frac{\frac{(\pi^2+a^2)}{Le}+\frac{(\pi^2+a^2+n)}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}}{\frac{(\pi^2+a^2)}{Le}+\frac{n}{\sigma}}N_ARn. \tag{26}$$

The growth rate n is in general a complex quantity such that $n = \omega_r + i\omega$, the system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability, the real part of n is zero, thus on putting $n = i\omega$, (where ω is real and is a dimensionless frequency) in equation (26), we have

$$Ra=\Delta_1+i\omega\Delta_2, \tag{27}$$

where

$$\Delta_1=\frac{(\pi^2+a^2)}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{(1+\omega^2FF_0)\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)-\omega^2\left(\tilde{D}a(\pi^2+a^2)^2+(\pi^2+a^2)\right)(F_0-F)-\frac{(\pi^2+a^2)}{Va}}{1+\omega^2F^2}\right)$$
$$-\frac{\left(\frac{(\pi^2+a^2)}{Le}\right)^2\left(1+\frac{Le}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)+\frac{\omega^2}{\varepsilon\sigma\left(1-\frac{Hs}{2}+Hsz\right)}}{\left(\frac{(\pi^2+a^2)}{Le}\right)^2+\left(\frac{\omega}{\sigma}\right)^2}N_ARn \tag{28}$$

and

$$\Delta_2 = \frac{\left(\tilde{D}a\left(\pi^2+a^2\right)^2+\left(\pi^2+a^2\right)\right)}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}\left(\frac{\left(1+\omega^2FF_0\right)+\left(\pi^2+a^2\right)\left(F_0-F\right)}{1+\omega^2F^2}\right)+\frac{\left(\pi^2+a^2\right)^2}{Va}$$

$$-\frac{\dfrac{1}{Le\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}-\dfrac{1}{\sigma}\left(\dfrac{\left(\pi^2+a^2\right)}{Le}+\dfrac{\left(\pi^2+a^2\right)}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)}{\left(\dfrac{\left(\pi^2+a^2\right)}{Le}\right)^2+\left(\dfrac{\omega}{\sigma}\right)^2}N_ARn \qquad (29)$$

Since Ra is a physical quantity, so it must be real. Hence, it follows from the equation (27) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset).

7. STATIONARY CONVECTION

For the case of stationary convection $\omega = 0$, equation (27) reduces to

$$\left(Ra\right)_s = \frac{\tilde{D}a\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2}{a^2\left(1-\frac{Hs}{2}+Hsz\right)}-\left(1+\frac{Le}{\varepsilon\left(1-\frac{Hs}{2}+Hsz\right)}\right)N_ARn. \qquad (30)$$

We find that for the stationary convection, the stress relaxation time parameter F and strain retardation time parameter F_0 vanishes with n and the Oldroydian visco-elastic nanofluid behaves like an ordinary Newtonian nanofluid.

It is observed that stationary Rayleigh number Ra is function of the Lewis number, the modified diffusivity ratio, Brinkman-Darcy number, nanoparticles Rayleigh and porosity parameter but independent of the stress relaxation time parameter F and strain retardation time parameter F_0, Vadasz number Va and modified particle- density increment N_B. Thus Oldroydian nanofluid behaves like an ordinary Newtonian nanofluid and instability is purely a phenomenon due to buoyancy coupled with the conservation of nanoparticles.

$$\left(Ra\right)_s = \frac{\tilde{D}a\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2}{a^2}-\left(1+\frac{Le}{\varepsilon}\right)N_ARn. \qquad (30)$$

If $\tilde{D}a = 0$ then the critical value of the wave number is attained at $a_c = \pi$ and corresponding critical Rayleigh number given by

$$\left(Ra\right)_c = 4\pi^2-\left(1+\frac{Le}{\varepsilon}\right)N_ARn\,. \qquad (31)$$

In the absence of nanoparticles ($Rn = Le = N_A = 0$) i.e. for ordinary Newtonian fluid, one recovers the well-known results that the critical Rayleigh-Darcy number is equal to $\left(Ra\right)_c = 4\pi^2$.

This is good agreement of the result obtained by Nield and Kuznetsov (2009).

If $\tilde{D}a$ is very large as compared to the unity, then critical value of wave number is attained at $a = \pi/\sqrt{2}$, critical value of the Rayleigh Darcy number is

$$\left(\mathrm{Ra}\right)_c = \frac{27\pi^4}{4} - \left(1 + \frac{\mathrm{Le}}{\varepsilon}\right) \mathrm{N_A Rn}\,. \tag{32}$$

In the absence of nanoparticles (Rn = Le = N_A = 0), one recovers the well- known results that the critical Rayleigh-Darcy number is equal to $\left(\mathrm{Ra}\right)_c = \frac{27\pi^4}{4}$.

8. RESULTS AND DISCUSSION

To study the effect of Lewis number Le, Brinkman-Darcy number $\tilde{\mathrm{D}}\mathrm{a}$, nanoparticles Rayleigh Rn, modified diffusivity ratio N_A and porosity parameter ε on stationary convection, we examine the behavior of $\frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Le}}, \frac{\partial(\mathrm{Ra})_s}{\partial \tilde{\mathrm{D}}\mathrm{a}}, \frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Rn}}, \frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{N_A}}$ and $\frac{\partial(\mathrm{Ra})_s}{\partial \varepsilon}$ analytically.

$$\left(\mathrm{Ra}\right)_s = \frac{\tilde{\mathrm{D}}\mathrm{a}\left(\pi^2 + a^2\right)^3 + \left(\pi^2 + a^2\right)^2}{a^2\left(1 - \frac{\mathrm{Hs}}{2} + \mathrm{Hsz}\right)} - \left(1 + \frac{\mathrm{Le}}{\varepsilon\left(1 - \frac{\mathrm{Hs}}{2} + \mathrm{Hsz}\right)}\right) \mathrm{N_A Rn}.$$

From equation (30), we have

(i) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{Le}} < 0$,

(ii) $\frac{(\partial \mathrm{Ra})_s}{\partial \tilde{\mathrm{D}}\mathrm{a}} > 0$

(iii) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{Rn}} < 0$,

(iv) $\frac{(\partial \mathrm{Ra})_s}{\partial \mathrm{N_A}} < 0$,

(v) $\frac{(\partial \mathrm{Ra})_s}{\partial \varepsilon} > 0$.

These inequalities shows that Lewis number Le, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn have destabilizing effect while Brinkman-Darcy number $\tilde{\mathrm{D}}\mathrm{a}$ and porosity parameter ε have stabilizing effect on the stationary convection.

9. CONCLUSIONS

Thermal instability in a horizontal layer of Oldroydian visco- elastic nanofluid in a porous medium is investigated theoretically. Brinkman model is used for porous medium. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries and the eigen value problem is solved using the Galerkin residual method. The results have been obtained analytically.

The main conclusions derived from the present analysis are as follows:

(i) The instability purely phenomenon due to buoyancy coupled with the conservation of nanoparticle and is independent of the contribution of Brownian motion and thermophoresis.

(ii) For stationary convection Oldroydian visco- elastic nanofluid behaves like an ordinary Newtonian nanofluid.

(iii) Lewis number Le, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn destabilizes while Brinkman-Darcy number $\tilde{D}a$ and porosity parameter ε stabilizes the stationary convection.

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Propagation of Rayleigh Waves in Semi-infinite Thermoelastic Porous Material under an Inviscid Liquid Half-space

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Abstract

In the present paper, an attempt is made to investigate the propagation of Rayleigh waves in a semi-infinite homogeneous isotropic porous generalized thermoelastic medium underlying a homogeneous inviscid liquid half-space in the context of Lord Shulman theory. Helmholtz decomposition technique is employed to simplify the problem. The governing equations are derived and solved to show the existence of four waves in a generalized thermoelastic porous solid half-space and one wave in the liquid half-space. By invoking appropriate boundary conditions at the interface, secular equations are derived for stress-free isothermal $(H \to 0)$ and stress-free insulated $(H \to \infty)$ porous thermoelastic medium. The secular equation is also obtained in the context of uncoupled thermoelasticity. The frequency equation is found to agree with the standard result. Some special cases are also discussed in the paper. The study may find various applications in the field of acoustics and seismology.

INTRODUCTION

The theory of thermoelasticity has become very important due to its practical and theoretical relevance. Beneath the earth's surface are lying many valuable materials like oils, minerals, water, hydrocarbons, etc. To know the exact nature of the layers under the Earth's surface, we need to consider many appropriate models for the theoretical investigation. These models help in the exploration of valuable materials as well as provides better information regarding the internal composition of the earth. Surface waves provide a lot of information about the crust of the earth and dispersion analysis of surface waves is concerning with the phase velocity and wave number.

Rayleigh waves are a type of surface acoustic wave that travels along the surface of solids. They include both longitudinal and transverse motions that decrease exponentially in amplitude as the distance from the surface increases. In isotropic solids, these waves cause the surface particles to move in ellipses. Surfaces wave propagation in an elastic medium is of great practical importance. To detect deposits of oil and other valuable organic and inorganic material under the earth's surface, the most economic technique is wave propagation as it is a difficult and tedious process to drill randomly.

For the present paper, the previous works of Cowin and Nunziato [4] is found helpful. They developed a theory of linear elastic material with voids and discussed several applications like response to homogeneous deformation. Puri and Cowin [8] studied the behavior of plane harmonic waves in linear elastic material with voids. They derived two dilatational waves in the paper. Both waves are found to attenuate in the direction of their propagation, to be dissipative and dispersive. The nature of surface waves in an elastic half-space with voids is studied by Chandrasekharaiah [3]. Iesan [6] gave a theory of thermoelastic material with voids. He studied acceleration waves and some problems of equilibrium. Rayleigh surface waves problem in linear thermoviscoelasticity with voids is studied by Bucur [2]. In his paper the dissipative character of the porous thermoviscoelastic model was taken into consideration and dampness in time wave solutions was studied.

Biot [1] studied the interaction of Stoneley and Rayleigh waves in the ocean bottom. They developed a theory for the two-dimensional unattenuated waves. Kumar and Kumar [7] studied the wave motion in transversely isotropic elastic material with voids under an inviscid liquid layer. The paper found useful in studying geophysical problems. Thermoelastic Lamb waves in a homogeneous isotropic plate bordered with layers of inviscid liquid discussed by Sharma and Pathania [9]. Also, Rayleigh waves in rotating thermoelastic solids with voids were studied by Sharma and Kaur [10]. They observed a significant effect of voids, thermal relaxation, and rotation in certain ranges of frequency.

In the present investigation, we consider a semi-infinite isotropic thermoelastic material with voids in contact with inviscid liquid half-space which helps in exploring geophysical problems like explosions and earthquakes. The secular equations in mathematical form for Rayleigh wave propagation are derived for stress-free insulated and isothermal boundaries, for the thermoelastic solid half-space, elastic half-space with voids in contact with inviscid liquid.

FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, thermoelastic solid half-space with voids lying under a homogeneous inviscid liquid half-space. The surface is assumed to be stress-free, thermally insulated boundaries with no fractional change across the boundary. The z-axis is taken vertically downwards and the direction of the propagation of the wave is along the x-axis so that all the particles on a line parallel to the y-axis are equally displaced. Therefore all the field quantities will be independent of the y-axis. Further, the disturbance is assumed to be confined to the neighborhood of the free surface $z = 0$ and hence vanishes as $z \to \infty$.

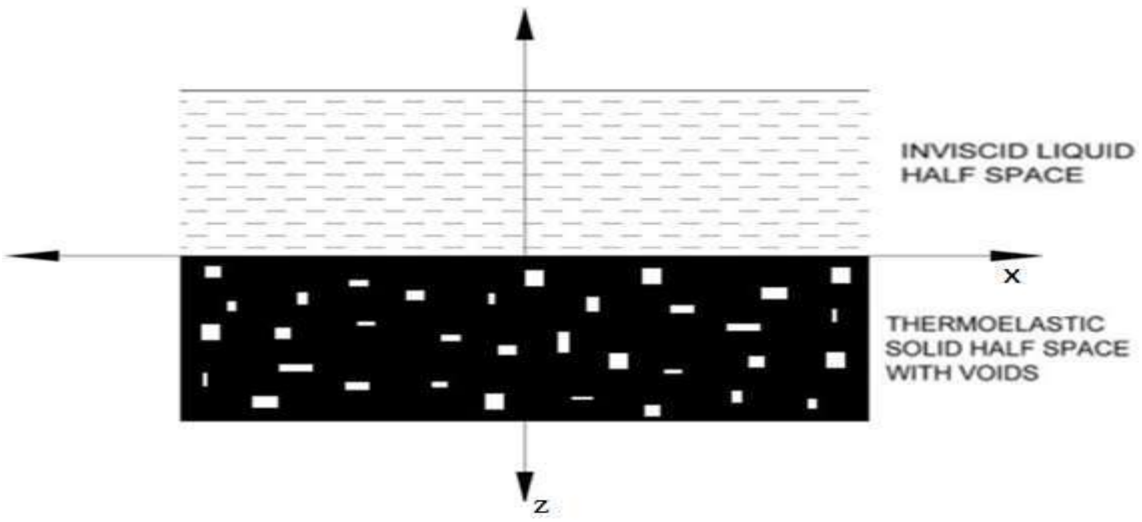

Figure 1: Geometry of the problem

The basic governing equations for stress-free thermoelastic solid with voids in contact with inviscid liquid half-space in the absence of body forces, equilibrated forces and heat sources following Iesan [6] and Sharma and Pathania [9] are given as:

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\nabla\nabla.\vec{u}-\beta\nabla T+b\nabla\phi=\rho\ddot{\vec{u}} \tag{1}$$

$$K\nabla^2 T-\rho C_e\left(\dot{T}+t_0\ddot{T}\right)=\beta T_0\nabla.\left(\dot{\vec{u}}+t_0\ddot{\vec{u}}\right)+mT_0\left(\dot{\phi}+t_0\ddot{\phi}\right) \tag{2}$$

$$\alpha\nabla^2\phi-\xi_1\phi-\xi_2\dot{\phi}-b\nabla.\vec{u}+mT=\rho\chi\ddot{\phi} \tag{3}$$

$$\nabla\nabla.\vec{u}_L=\frac{\rho_L}{\lambda_L}\ddot{\vec{u}}_L \tag{4}$$

where $\vec{u}(x,z,t)=(u,0,w)$ and $\vec{u}_L=(u_L,0,w_L)$ are the displacement vectors in the solid half-space and liquid half-space respectively, $\phi(x,z,t)$ is the volume fractional change, $T(x,z,t)$ is the temperature change in the medium, λ and μ are Lame's parameter, ρ and ρ_L are densities for solid and liquid half-space, C_e is the specific heat at constant strain, K is the thermal conductivity, α b m ξ_1 ξ_2 χ are material parameters due to the presence of voids, t_0 is thermal relaxation time and $\beta=\left(3\lambda+2\mu\right)\alpha_T$, α_T being coefficient of linear thermal expansion. Defining the dimensionless quantities as

$$x'=\frac{\omega^* x}{c_1},\ z'=\frac{\omega^* z}{c_1},\ u'=\frac{\rho\omega^* c_1 u}{\beta T_0},\ w'=\frac{\rho\omega^* c_1 w}{\beta T_0},\ \ \phi'=\frac{\omega^{*2}\chi}{c_1^2}\phi,\ u_L'=\frac{\rho\omega^* c_1 u_L}{\beta T_0},$$

$$w_L'=\frac{\rho\omega^* c_1 w_L}{\beta T_0},$$

$$t'=\omega^* t,\ t_0'=\omega^* t_0,\ T'=\frac{T}{T_0},\sigma_{ij}'=\frac{\sigma_{ij}}{\beta T_0},\ \varepsilon_T=\frac{\beta^2 T_0}{\rho c_e\left(\lambda+2\mu\right)},a_1=\frac{c_1^{\,2}b}{\beta T_0\chi\omega^{*2}},\ a_2=\frac{\beta T_0\chi b}{\alpha\rho c_1^{\,2}},a_3=\frac{\xi_1 c_1^{\,2}}{\alpha\omega^{*2}}$$

$$a_4=\frac{mT_0\chi}{\alpha},\ a_5=\frac{mc_1^{\,4}}{K\chi\omega^{*3}},\ \ \omega^*=\frac{C_e\left(\lambda+2\mu\right)}{K}, \tag{5}$$

Introducing equation (5) and suppressing the primes, the equations (1)-(4) are obtained as:

$$u_{,xx}+\left(1-\delta^2\right)w_{,xz}+\delta^2 u_{,zz}-T_{,x}+a_1\phi_{,x}=\ddot{u} \tag{6}$$

$$\left(1-\delta^2\right)u_{,xz}+\delta^2 w_{,xx}+w_{,zz}-T_{,z}+a_1\phi_{,z}=\ddot{w} \tag{7}$$

$$\left(T_{,xx}+T_{,zz}\right)-\left(\dot{T}+t_0\ddot{T}\right)-\varepsilon_T\left[\left(\dot{u}_{,x}+\dot{w}_{,z}\right)+t_0\left(\ddot{u}_{,x}+\ddot{w}_{,z}\right)\right]-a_5\left(\dot{\phi}+t_0\ddot{\phi}\right)=0 \tag{8}$$

$$\left(\phi_{,xx}+\phi_{,zz}\right)-a_2\left(u_{,x}+w_{,z}\right)-a_3(\phi+\bar{\xi}\dot{\phi})+a_4 T=\frac{\ddot{\phi}}{\delta_1^2} \tag{9}$$

$$u_{L,_{xx}}+w_{L,_{xz}}=\frac{1}{\delta_L^2}\ddot{u}_L \tag{10}$$

where $\delta^2=\frac{c_2^2}{c_1^2},\ \delta_1^2=\frac{c_3^2}{c_1^2},\ \bar{\xi}=\frac{\xi_2}{\xi_1}\omega^*,\ c_1^{\,2}=\frac{\left(\lambda+2\mu\right)}{\rho},c_2^{\,2}=\frac{\mu}{\rho},\ c_3^{\,2}=\frac{\alpha}{\rho\chi},\delta_L^2=\frac{c_L^2}{c_1^2},c_L^2=\frac{\lambda_L}{\rho_L}.$

Here ω^{*} is the characteristic frequency of solid half-space, ε_T is the thermoelastic coupling parameter, c_L is the velocity of sound in the liquid, λ_L is the bulk modulus, c_1, c_2, and c_3 are velocities of longitudinal, transverse and volume fractional fields respectively.The coefficient ξ and equilibrated inertia χ must be positive and satisfy the dissipation inequality resulting from the second law of thermodynamics. The comma denotes the spatial derivatives and superposed dot is used for time differentiation.

FORMAL SOLUTION OF THE PROBLEM

Introducing the scalar point function G, G_L and vector point function ψ through the relations:

$$u = G_{,x} + \psi_{,z}, \quad , w = G_{,z} - \psi_{,x}, \quad u_L = G_{L,x}, \; w_L = G_{L,z} \tag{11}$$

where G , G_L and ψ represent the displacement potentials of longitudinal and transverse waves respectively for solid and liquid half-spaces. Substituting the values from equation (11) in equations (6)-(10), the equations become

$$G_{,xx} + G_{,zz} - T + a_1\phi = \ddot{G} \tag{12}$$

$$\delta^2\psi_{,xx} + \delta^2\psi_{,zz} = \ddot{\psi} \tag{13}$$

$$\left(T_{,xx} + T_{,zz}\right) - \left(\dot{T} + t_0\ddot{T}\right) - \varepsilon_T\left[\left(\dot{G}_{,xx} + \dot{G}_{,zz}\right) + t_0\left(\ddot{G}_{,xx} + \ddot{G}_{,zz}\right)\right] - a_5\left(\dot{\phi} + t_0\ddot{\phi}\right) = 0 \tag{14}$$

$$-a_2\left(G_{,xx} + G_{,zz}\right) + \left[\left(\phi_{,xx} + \phi_{,zz}\right) - a_3\left(\phi + \overline{\xi}\dot{\phi}\right)\right] + a_4 T = \frac{\ddot{\phi}}{\delta_1^2} \tag{15}$$

$$\mathrm{G}_{L,xx} + \mathrm{G}_{L,zz} - \frac{1}{\delta_L^2}\ddot{\mathrm{G}}_L = 0 \tag{16}$$

It can be seen that equations (12), (14) and (15) are coupled through the quantities G, ϕ and T while (13) and (16) are uncoupled in potentials ψ and G_L respectively.

BOUNDARY CONDITIONS

The boundary conditions at the solid–liquid interface $z = 0$ to be satisfied are as follows:

(i) $\sigma_{zz} = \left(\sigma_{zz}\right)_L$.

(ii). $\sigma_{xz} = \left(\sigma_{xz}\right)_L$.

(iii) $w = w_L$.

(iv) $\phi_{,z} = 0$.

(v) $T_{,z} + HT = 0$, where H is the Biot's heat transfer coefficient.

SOLUTION OF THE PROBLEM

To discuss the wave propagation in a linear homogeneous thermoelastic material with voids of infinite extent underlying over an inviscid liquid, consider the form of plane waves traveling in the positive direction of x-axis, which are given by

$$\left(G, \psi, \phi, T, G_L\right) = \left[1, V, W, S, R\right] U \exp\left\{ik\left(x - ct\right) - rz\right\} \tag{17}$$

where k is the wavenumber, ω is the angular frequency of the plane wave propagating in $x-z$ plane. The phase velocity c is connected with angular frequency ω and wavenumber k through the relation $c=\frac{\omega}{k}$. U is the scalar constant and V,W,S,R are amplitude ratios. Invoking equation (17) in equations (12)-(16), we have

$$(r^2-\alpha_1^2)+a_1W-S=0 \tag{18}$$

$$\left(r^2-\beta_1^2\right)V=0 \tag{19}$$

$$\varepsilon_T\tau_0\omega^2\left(r^2-k^2\right)+a_5\tau_0\omega^2W+\left[r^2-k^2\left(1-\tau_0c^2\right)\right]S=0 \tag{20}$$

$$-a_2\left(r^2-k^2\right)+\left(r^2-\gamma^2\right)W+a_4S=0 \tag{21}$$

$$(r^2-\beta_2^2)R=0 \tag{22}$$

where

$$\alpha_1^2=k^2\left(1-c^2\right),\ \ \beta_1^2=k^2\left(1-\frac{c^2}{\delta^2}\right),\ \beta_2^2=k^2\left(1-\frac{c^2}{\delta_L^2}\right),\ \tau_0=t_0+i\omega^{-1},\ \ \bar{\xi}_0=1-i\bar{\xi}\omega,$$

$$\gamma^2=k^2\left[1-c^2\left(\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_0}{\omega^2}\right)\right]. \tag{23}$$

For the requirement of non-trivial solution of the system of equations (18)-(22) the determinant of their coefficients vanish. On simplifying the determinant, after some algebraic manipulations which leads to the determination of characteristics roots as

$$r_j^2=k^2\left(1-\lambda_j^2c^2\right);\quad j=1,2,3,4,5 \tag{24}$$

where, λ_j^2; $j=1,2,3,4,5$ are the roots of the polynomial.

$$\sum\lambda_1^2=1+\tau_0\left(1+\varepsilon_T\right)+\frac{1}{\delta_1^2}-\frac{a_3}{\omega^2}\left(\bar{\xi}_o-\frac{\varepsilon_\phi\varepsilon_b^2}{\varepsilon_T}\right),$$

$$\sum\lambda_1^2\lambda_2^2=\tau_0+\frac{1+\tau_0\left(1+\varepsilon_T\right)}{\delta_1^2}-\frac{a_3}{\omega^2}\left[\bar{\xi}_o\left\{1+\tau_0\left(1+\varepsilon_T\right)\right\}+\frac{\tau_0\varepsilon_\phi}{\varepsilon_T}\left\{\varepsilon_T-\varepsilon_b\left(2\varepsilon_T+\varepsilon_b\right)\right\}\right],$$

$$\lambda_1^2\lambda_2^2\lambda_3^2=\tau_0\left[\frac{1}{\delta_1^2}-\frac{a_3}{\omega^2}\left(\bar{\xi}_o+\varepsilon_\phi\right)\right],\ \lambda_4^2=\frac{1}{\delta^2},\ \lambda_5^2=\frac{1}{\delta_L^2}. \tag{25}$$

where $\varepsilon_\phi=\dfrac{a_4a_5}{a_3}$, $\varepsilon_b=\dfrac{a_2}{a_4}$, $\dfrac{\varepsilon_\phi\varepsilon_b}{\varepsilon_T}=\dfrac{a_4a_1}{a_3}$ (26)

Dealing with surface wave motion must be confined to the free surface $z=0$ of the half-space. Consequently, the formal solutions which satisfy the radiations condition $\mathrm{Re}\left(r_j\right)\geq 0$ is given by

$$\left(G,\psi,\phi,T,G_L\right)=\sum_{k=1}^{5}\left(1,V_k,W_k,S_k,R_k\right)U_k\exp\left\{ik\left(x-ct\right)-r_kz\right\} \tag{27}$$

where the amplitude ratios are given as

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$$V_k = \begin{cases} 0 \ , \ k=1,2,3,5 \\ \infty \ , \ k=4 \end{cases}, \quad S_k = \begin{cases} \omega^2\left(1-\lambda_k^2\right)+a_1W_k \ , \ k=1,2,3 \\ 0 \ , \ k=4,5 \end{cases}$$

$$W_k = \begin{cases} \dfrac{a_4\left[\left(\varepsilon_b-1\right)\lambda_k^2+1\right]}{\lambda_k^2-\dfrac{1}{\delta_1^2}+\dfrac{a_3}{\omega^2}\left(\bar{\xi}_o-\dfrac{\varepsilon_b\varepsilon_\phi}{\varepsilon_T}\right)} \ , \ k=1,2,3 \\ 0 \ , \ k=4,5 \end{cases}, \quad \mathrm{R}_k = \begin{cases} 0, k=1,2,3,4 \\ \infty, k=5 \end{cases} \tag{28}$$

PARTICULAR CASES OF FORMAL SOLUTIONS

The particular cases in the absence of inviscid liquid half-space are discussed below.

(i) Thermoelasticity without voids

In the absence of voids $b=0$ which leads to $\varepsilon_\phi=\varepsilon_b=0$ one can obtain the characteristic roots (25) and amplitude ratios (28) as

$$\lambda_1^2+\lambda_3^2=1+\tau_0\ \left(1+\varepsilon_T\right)\ ,\ \lambda_2^2=\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_o}{\omega^2}\ ,\ \lambda_1^2\lambda_3^2=\tau_0\ ,\ \lambda_4^2=\frac{1}{\delta^2} \tag{29}$$

$$W_k = \begin{cases} 1 \ , \ k=2 \\ 0 \ , \ k=1,3 \end{cases}, \quad S_k = \begin{cases} \omega^2\left(1-\lambda_k^2\right) \ , \ k=1,3 \\ 0 \ , \ k=2 \end{cases} \tag{30}$$

(ii) Elasticity with voids

If we neglect the thermal effect, then the elastic field and thermal field are independent of each other $m=\beta=0$ which implies that $\varepsilon_T=\varepsilon_\phi=a_4=a_5=0$. The characteristic roots (25) and amplitude ratios (28) are thus reduced to

$$\lambda_1^2+\lambda_2^2=1+\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_0}{\omega^2}+\frac{a_1a_2}{\omega^2}\ ,\ \lambda_1^2\lambda_2^2=\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_0}{\omega^2}\ ,\ \lambda_3^2=\tau_0\ ,\ \lambda_4^2=\frac{1}{\delta^2} \tag{31}$$

$$W_k = \begin{cases} \dfrac{a_2\lambda_k^2}{\lambda_k^2-\dfrac{1}{\delta_1^2}+\dfrac{a_3\bar{\xi}_o}{\omega^2}} \ , \ k=1,2 \\ 0 \ , \ k=3 \end{cases}, \quad S_k = \begin{cases} 0 \ , \ k=1,2 \\ 1 \ , \ k=3 \end{cases} \tag{32}$$

(iii) Elasticity without voids

The characteristic roots (25) and the amplitude ratios (28), in the absence of voids and thermomechanical coupling parameters, $m=b=0=\beta$ lead to $\varepsilon_T=\varepsilon_\phi=\varepsilon_b=0$ and are obtained as

$$\lambda_1^2=1,\ \ \lambda_2^2=\frac{1}{\delta_1^2}-\frac{a_3\bar{\xi}_o}{\omega^2},\quad \lambda_3^2=\tau_0\ ,\ \ \lambda_4^2=\frac{1}{\delta^2} \tag{33}$$

$$W_k = \begin{cases} 1 \ , \ k=2 \\ 0 \ , \ k=1,3 \end{cases}, \quad S_k = \begin{cases} 0 \ , \ k=1,2 \\ 1 \ , \ k=3 \end{cases} \tag{34}$$

The above results are verified and same as obtained by Sharma and Kaur [10].

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DERIVATION OF THE SECULAR EQUATIONS

Invoking the boundary conditions and equation (27) at the stress-free surface i.e. $z=0$, the equations of Rayleigh waves for insulated and isothermal surface of solid half-space in contact with inviscid liquid half-space are obtained as

$$1+QV_kr_k=\mathrm{MR}_k,\ \ V_k-Qr_k=0,\ \ W_kr_k=0,\ \ r_k+ikV_k=R_kr_k,\ \ S_k=0,\ \ S_kr_k=0. \tag{35}$$

where

$$Q=\frac{2ik}{P},\ P=k^2\left(2-\frac{c^2}{\delta^2}\right),\ M=\frac{1}{P}\frac{\rho_L\omega^2}{\rho\delta^2} \tag{36}$$

The system ofequations (35) have a non-trivial solution if the determinant of the coefficients vanishes and it leads to following secular equations for insulated and isothermal surface respectively.

$$r_1S_1F_1-r_2S_2F_2+r_3S_3F_3-r_4S_4F_4-r_5S_5F_5=0,\ S_1F_1-S_2F_2+S_3F_3-S_4F_4+S_5F_5=0. \tag{37}$$

where

$$F_1=\begin{vmatrix}1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_4-MR_4 & 1+QV_5-MR_5\\ V_2-Qr_2 & V_3-Qr_3 & V_4-Qr_4 & V_5-Qr_5\\ W_2r_2 & W_3r_3 & W_4r_4 & W_5r_5\\ r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5\end{vmatrix}$$

$$F_2=\begin{vmatrix}1+QV_1-MR_1 & 1+QV_3-MR_3 & 1+QV_4-MR_4 & 1+QV_5-MR_5\\ V_1-Qr_1 & V_3-Qr_3 & V_4-Qr_4 & V_5-Qr_5\\ W_1r_1 & W_3r_3 & W_4r_4 & W_5r_5\\ r_1+ikV_1-R_1r_1 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5\end{vmatrix}$$

$$F_3=\begin{vmatrix}1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_4-MR_4 & 1+QV_5-MR_5\\ V_1-Qr_1 & V_2-Qr_2 & V_4-Qr_4 & V_5-Qr_5\\ W_1r_1 & W_2r_2 & W_4r_4 & W_5r_5\\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_4+ikV_4-R_4r_4 & r_5+ikV_5-R_5r_5\end{vmatrix}$$

$$F_4=\begin{vmatrix}1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_5-MR_5\\ V_1-Qr_1 & V_2-Qr_2 & V_3-Qr_3 & V_5-Qr_5\\ W_1r_1 & W_2r_2 & W_3r_3 & W_5r_5\\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_5+ikV_5-R_5r_5\end{vmatrix}$$

$$F_5=\begin{vmatrix}1+QV_1-MR_1 & 1+QV_2-MR_2 & 1+QV_3-MR_3 & 1+QV_4-MR_4\\ V_1-Qr_1 & V_2-Qr_2 & V_3-Qr_3 & V_4-Qr_4\\ W_1r_1 & W_2r_2 & W_3r_3 & W_4r_4\\ r_1+ikV_1-R_1r_1 & r_2+ikV_2-R_2r_2 & r_3+ikV_3-R_3r_3 & r_4+ikV_4-R_4r_4\end{vmatrix} \tag{38}$$

The complete information regarding wave number, phase velocity and attenuation coefficient of Rayleigh waves in the considered medium in case of thermally insulated and stress-free, isothermal boundaries, can be obtained from equations (37). In the absence of voids, inviscid liquid and thermomechanical coupling,both the secular equations (37) with the help of equation (33) lead to

$$\left(2-\frac{c^2}{\delta^2}\right)^2 = 4\sqrt{1-c^2}\sqrt{1-\frac{c^2}{\delta^2}} \qquad (39)$$

Equation (39) represents the classical Rayleigh wave velocity equation in an elastic solid half-space which is the same as given by Graff [5], Kumar and Kumar [7], Sharma and Pathania [9] and Sharma and Kaur [10].

CONCLUSIONS

The propagation of thermoelastic waves in a semi-infinite homogeneous isotropic thermoelastic material with voids in contact with inviscid liquid half-space is investigated and the results concluded from the above analysis can be summarized as:

1. There exist four waves in a generalized thermoelastic porous solid and one wave in inviscid liquid half-space. In the solid half-space, one of the waves is a transverse wave and the remaining three are set of coupled longitudinal waves.
2. Each set of longitudinal waves is found to be influenced by void volume fraction and the thermal properties.
3. The transverse wave remains independent of the presence of voids and thermal properties of the medium.
4. The Rayleigh wave propagation has importance in geophysical problems as this field studies the propagation of progressive elastic waves. The surface wave propagation in a thermoelastic material containing voids finds many applications in the industrial world in the manufacturing of resonators, filters, and sensors, detectors, electronic components, etc.
5. The study may find applications in the development and designs of surface acoustic wave devices.

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FINITE DIFFERENCE TECHNIQUE FOR THREE DIMENSIONAL MHD FREE CONVECTIVE FLOWS PAST A POROUS MEDIUM WITH VARIABLE SUCTION AND PERIODIC PERMEABILITY

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ABSTRACT:
The free convection three dimensional steady viscous incompressible flow in porous medium is investigated. The effect of transverse magnetic on three dimensional flow and mass transfer in the viscous dissipative heat when porous plate is subjected to a slightly sinusoidal transverse suction velocity distribution. The aim of this study is to know the effect of periodic permeability on the flow in the presence of magnetic field and heat transfer. The effect of magnetic field, Schmidt and Grashof number and Reynolds number on velocity, concentration and temperature explained with the help of graphs. The governing equations are solved numerically with finite difference method.

KEYWORDS: Free Convection, Periodic Permeability, Grashof number, magnetic field porous medium.

1. INTRODUCTION:

Magneto-Hydrodynamics (MHD) is the branch of continuum mechanics deals with the study of magnetic properties and behaviour of electrically conducting fluids. Many engineering problems and natural phenomenon are based upon the concept of magneto-hydrodynamics. MHD is related to engineering problems such as plasma confinement, liquid metal cooling into nuclear reactors and electromagnetic casting. The concept of MHD is used in Geo-physics to study the flow pattern in the core of the earth. Some examples of magneto fluids are plasma, salt water, electrolytes and liquid metals. Hannes Alfen in 1942 introduced the concept of Magneto-hydrodynamics for which he received the Nobel prize in 1970. The word Magneto-Hydrodynamics is derived from the word '*magne*t' which means magnetic field , '*hydro*' means liquid and '*dynamics'* means in motion.

The movement of conducting fluid in magnetic field produces an induced electric current in fluid. The magnetic field exerts a force known as Lorentz force. The set of equations for MHD are combination of Navier-Stokes equations for fluid dynamics and Maxwell's equations for electromagnetism. An ideal MHD, assumes that the fluid has very low resistivity that it can be treated as perfect conductor. The problems of MHD laminar flow through a porous medium have become very important in recent years due to its applications in many branches of Science and Technology.

Free convection flows have number of industrial applications such as fibre, granular insulation and geo-thermal energy recover, oil extraction and flow through filtering devices. Soundalgekar (1981) studied MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field. Bejan & Khaair (1985) discussed heat and mass transfer by natural convection. Hossain & Mandal (1985) have investigated the effects of magnetic field on natural convection flow past a vertical surface. Hydromagnetic free convective flow past a porous plate studied by Singh (1991). Singh, Chand & Rana(1993) studied the effect of heat transfer in three dimensional flow past a porous plate. Later on Gupta & Johari (2000) have investigated the MHD three dimensional flow past a porous plate. Singh & Sharma (2002) have discussed three dimensional free convective flow and heat transfer in rotating elastic-viscous fluid past an infinite vertical porous flat plate with constant suction. Cookey et.al. (2003) have studied the problems which includes thermal radiation, heat source, thermal diffusion on unsteady MHD free convection flows past an infinite heated vertical plate in porous medium with time-dependent suction. A numerical solution of unsteady free convection and mass transfer flow is presented by Alam & Rahman (2005). Sharma & Yadav (2006) have discussed steady MHD boundary layer flow and heat transfer between two long vertical wavy walls. R.C.Choudhary & Arpita Jain (2010) investigated an exact solution of MHD convection flow past an accelerated surface embedded in porous. S. Siviach et.al. (2012) have studied finite element analysis of chemical reaction and radiation effects on isothermal vertical oscillating plate with variable mass diffusion. Ravi kumar & Raju (2012) have discussed MHD three dimensional couette flow past a porous plate with heat transfer.

Ahmed, Batin & Chamkha (2013) have investigated finite difference approach in porous media transport modelling for MHD unsteady flow over a vertical plate using Darcian Model. Finite difference solutions of MHD free convective flow with constant suction and variable thermal conductivity in a Darcy- Forchheimer porous medium studied by Uwanta & Usman (2015). Reddy & Raju (2018) have studied MHD free convective flow past a porous plate.

In this paper, we have investigated the MHD free convection flow through a highly porous medium. Due to three dimensional flow the permeability of the porous medium is considered to be periodic. The flow is moving sinusoidal with variable suction velocity. The governing partial differential equations are transformed into a set of ordinary equations and solved numerically by finite difference method. We discussed the effect of parameter involved on the flow; computed skin friction and rate of heat transfer.

2. MATHEMATICAL FORMULATION:

Consider the flow of electrical conducting and incompressible viscous fluid with heat and mass transfer along an infinite vertical non-conducting porous plate with variable suction velocity. The plate is lying vertically on x' -z' plane with x'-axis is taken along the plate in the upward direction. The y'-axis is taken normal to the plate and directed along the fluid with free stream velocity U . All physical quantities will be independent of x', flow remains three dimensional due to variable suction. The sinusoidal suction velocity at plate is considered to be of the form

$$v'(z') = -V\left[1+\varepsilon\cos\frac{\pi z'}{L}\right] \tag{1}$$

which consists of basic steady distribution V > 0 superimposed with a very weak transversally direction $\varepsilon\cos\frac{\pi z'}{L}$. Here L is half wave length of periodic suction velocity. The negative sign in (1) indicates that suction is towards the plate. The magnetic field B_0 is applied along y'-axis. The permeability of the porous medium is assumed to be of the form

$$k'(z') = \frac{K'_0}{\left[1 + \varepsilon \cos \frac{\pi z'}{L}\right]} \tag{2}$$

where K'_0 is the mean permeability of the porous medium. The ε (<<1) is amplitude of permeability variation. For the flow problem under consideration the following assumptions are made:

(i) The flow is steady, laminar and applied magnetic field B_0 perpendicular to the plate.
(ii) Molecular transport properties are constant.
(iii)The fluid under consideration is incompressible and viscous.
(iv)The density variation due to temperature and concentration are approximated by Boussinesq approximation.
(v) Hall effect, polarization and chemical reactions are neglected.

Denoting velocities components u', v', w' in the direction of x', y', z' respectively and temperature T' and concentration C', the flow through highly porous medium is governed by the following equations:

Continuity Equation

$$\frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0 \tag{3}$$

Momentum Equations

$$v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'} = g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) + \upsilon\left(\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right) - \frac{\upsilon}{k'}(u' - U) - \frac{\sigma B_0^2(u' - U)}{\rho} \tag{4}$$

$$v'\frac{\partial v'}{\partial y'} + w'\frac{\partial v'}{\partial z'} = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} + \upsilon\left(\frac{\partial^2 v'}{\partial y'^2} + \frac{\partial^2 v'}{\partial z'^2}\right) - \frac{\upsilon}{k'}(v') \tag{5}$$

$$v'\frac{\partial w'}{\partial y'} + w'\frac{\partial w'}{\partial z'} = -\frac{1}{\rho}\frac{\partial p'}{\partial z'} + \upsilon\left(\frac{\partial^2 w'}{\partial y'^2} + \frac{\partial^2 w'}{\partial z'^2}\right) - \frac{\upsilon}{k'}(w') - \frac{\sigma B_0^2 w'}{\rho} \tag{6}$$

Energy Equation

$$\rho c_p\left(v'\frac{\partial T'}{\partial y'} + w'\frac{\partial T'}{\partial z'}\right) = k\left(\frac{\partial T'}{\partial y'^2} + \frac{\partial^2 T'}{\partial z'^2}\right) \tag{7}$$

Concentration Equation

$$\left(v'\frac{\partial C'}{\partial y'} + w'\frac{\partial C'}{\partial z'}\right) = D\left(\frac{\partial^2 C'}{\partial y'^2} + \frac{\partial^2 C'}{\partial z'^2}\right) \tag{8}$$

where g- acceleration due to gravity, β- coefficient of volume expansion, β'- coefficient of mass expansion, p'- pressure, ρ- density, ν- kinematic viscosity, μ- dynamic viscosity, k- thermal conductivity, c_p- specific heat at constant pressure, σ- electrical conductivity and D- concentration diffusivity.

The boundary conditions are

$$y' = 0 \;;\; u' = 0,\ w' = 0,\ T' = T'_w,\ C' = C'_w \quad v' = -V\left[1+\varepsilon\cos\frac{\pi z'}{L}\right]$$
$$y' \to \infty \;;\; u' = U,\ w' = 0,\ T' = T'_\infty,\ p' = p^*_\infty,\ v' = -V,\ C' = C'_\infty \tag{9}$$

The subscripts ' w' and '∞' denote the physical quantities at the wall and in free stream respectively.
Introducing the following non-dimensional quantities

$$y = \frac{y'}{L}, z = \frac{z'}{L}, u = \frac{u'}{U}, v = \frac{v'}{U}, w = \frac{w'}{U}, p = \frac{p'}{\rho U^2}, T = \left(\frac{T'-T'_\infty}{T'_w - T'_\infty}\right), C = \left(\frac{C'-C'_\infty}{C'_w - C'_\infty}\right) \tag{10}$$

With the help of above (10) non-dimensional variables equations (3) to (8) reduces:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{11}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = GrRT + GmRC + \frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{(u-1)(1+\varepsilon\cos\pi z)}{R\,K'_0} - \frac{M^2(u-1)}{R} \tag{12}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)v}{R\,K'_0} \tag{13}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)w}{R\,K'_0} - \frac{M^2 w}{R} \tag{14}$$

$$v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{R\Pr}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{15}$$

$$v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \frac{1}{RSc}\left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{16}$$

Where

$Gr = \dfrac{\nu g\beta(T'_w - T'_\infty)}{U\ V^2}$ - Grashof number , $Gm = \dfrac{\nu g\beta(C'_w - C'_\infty)}{U\ V^2}$ - Modified Grashof number

$R = \dfrac{UL}{\upsilon}$ - Reynolds number , $\Pr = \dfrac{\mu c_p}{k}$ - Prandlt number

$K = \dfrac{K'_0}{L^2}$ - Permeability parameter, $Sc = \dfrac{\nu}{D}$ - Schmidt number

$M = \left(\dfrac{\sigma}{\mu}\right)^{1/2} B_0 L$ - Hartmann number or magnetic parameter and $\alpha = \dfrac{V}{U}$ -Suction parameter

The corresponding boundary conditions become:

$$y = 0; \quad u = 0,\ v = -\alpha(1+\cos\pi z),\ w = 0,\ T = 1 \text{ and } C = 1$$
$$y \to \infty: \quad u = 1,\ w = 0,\ p = p_\infty,\ T = 0 \text{ and } C = 0. \tag{17}$$

3. Method of Solution

We assume that the solutions of following form because ε(<<1) amplitude permeability variation is very small.

$u(y, z) = u_0(y) + \varepsilon\, u_1(y, z) + \varepsilon^2 u_2(y, z) +$ ....

$v(y, z) = v_0(y) + \varepsilon\, v_1(y, z) + \varepsilon^2 v_2(y, z) + \ldots$
$w(y, z) = w_0(y) + \varepsilon\, w_1(y, z) + \varepsilon^2 w_2(y, z) + \ldots$
$p(y, z) = p_0(y) + \varepsilon\, p_1(y, z) + \varepsilon^2 p_2(y, z) + \ldots$

$T(y, z) = T_0(y) + \varepsilon\, T_1(y, z) + \varepsilon^2 T_2(y, z) + \ldots$
$C(y, z) = C_0(y) + \varepsilon\, C_1(y, z) + \varepsilon^2 C_2(y, z) +$ (18)

When $\varepsilon = 0$, the using equations (18) into equations (11) – (16), we get the following equations.

$$\frac{dv_0}{dy} = 0 \tag{19}$$

$$\frac{d^2u_0}{d\ y^2} - v_0 R\frac{du_0}{dy} - \frac{1}{K}u_0 - M^2u_0 = -GrR^2T_0 - GmR^2C_0 - \left(\frac{1}{K} + M^2\right) \tag{20}$$

$$\frac{d^2T_0}{dy^2} - v_0 R\Pr\frac{dT_0}{dy} = \quad 0 \tag{21}$$

$$\frac{dC_0}{dy^2} - v_0 RSc\frac{dC_0}{dy} = \quad 0 \tag{22}$$

The corresponding boundary conditions are:
$y = 0$; $u_0 = 0$, $v_0 = -\alpha$, $w_0 = 0$, $T_0 = 1$ and $C_0 = 1$
$y \to \infty$: $u_0 = 1$, $w_0 = 0$, $p_0 = p_\infty$, $T_0 = 0$ and $C_0 = 0$. (23)

The solutions of equations (19) – (22) with boundary conditions (23) are

$$v_0 = -\alpha \tag{24}$$

$$u_0 = 1 + \left(Gr\lambda_0 + Gm\lambda_1 - 1\right)e^{\alpha\lambda y} - Gr\lambda_0 e^{\alpha R\Pr y} - Gm\lambda_1 e^{\alpha RScy} \tag{25}$$

$$T_0 = e^{-\alpha R\Pr y} \tag{26}$$

$$C_0 = e^{-\alpha RScy} \tag{27}$$

with $v_0 = -\alpha$, $w_0 = 0$ and $p_0 = p_\infty$, (28)

where $\lambda = \dfrac{\alpha R}{2} + \sqrt{\dfrac{\alpha^2R^2}{4} + \dfrac{1}{K} + M^2}$ and $\lambda_0 = \dfrac{R^2}{\alpha^2R^2\Pr(\Pr - 1) - \left(\dfrac{1}{K} + M^2\right)}$

$$\lambda_1 = \frac{R^2}{\alpha^2R^2Sc(Sc-1) - \left(\dfrac{1}{K} + M^2\right)}$$

When $\epsilon \neq 0$, the periodic permeability enters the equations (11) – (16) and comparing the coefficients of like power of ϵ and neglecting higher power of ϵ^2, we get the following with equations using equation (28):

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{29}$$

$$v_1\frac{\partial u_0}{\partial y} - \alpha\frac{\partial u_1}{\partial y} = GrRT_1 + GmRC_1 + \frac{1}{R}\left(\frac{\partial^2u_1}{\partial y^2} + \frac{\partial^2u_1}{\partial\ z^2}\right) - \frac{(u_0-1)\cos\pi z + u_1}{R\ K} - \frac{M^2u_1}{R} \tag{30}$$

$$-\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right)-\frac{(v_1-\cos\pi z)}{R\ K} \tag{31}$$

$$-\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-\frac{w_1}{R\ K}-\frac{M^2 w_1}{R} \tag{32}$$

$$v_1\frac{\partial T_0}{\partial y}-\alpha\frac{\partial T_1}{\partial y}=\frac{1}{R\Pr}\left(\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right) \tag{33}$$

$$v_1\frac{\partial C_0}{\partial y}-\alpha\frac{\partial C_1}{\partial y}=\frac{1}{RSc}\left(\frac{\partial^2 C_1}{\partial y^2}+\frac{\partial^2 C_1}{\partial z^2}\right) \tag{34}$$

The corresponding boundary conditions are:
$y = 0$; $u_1 = 0$, $v_1 = -\alpha\cos\pi z$, $w_1 = 0$, $T_1 = 0$ and $C_1 = 0$
$y\to\infty$: $u_1 = 0$, $w_1 = 0$, $p_1 = 0$, $T_1 = 0$ and $C_1 = 0$. (35)

The equations (29) – (34) are the linear partial differential equations, which describes free convective, three dimensional flow. We assume the solution of equations (29) – (34) of the form:

$v_1(y, z) = v_{11}(y)\cos\pi z$ (36)

$w_1(y, z) = -\frac{1}{\pi}v'_{11}(y)\sin\pi z$ (37)

$p_1(y, z) = p_{11}(y)\cos\pi z$ (38)

$u_1(y, z) = u_{11}(y)\cos\pi z$ (39)

$T_1(y, z) = T_{11}(y)\cos\pi z$ (40)

$C_1(y, z) = C_{11}(y)\cos\pi z$ (41)

Where prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expressions for $v_1(y, z)$ and $w_1(y, z)$ are chosen in such a way that equation of continuity (29) is satisfied. Substituting the expressions (36)-(38) into equations (31)-(32) we have

$$v''_{11}+\alpha R v'_{11}-\left(\pi^2+\frac{1}{K}\right)v_{11}=Rp'_{11}-\frac{1}{K} \tag{42}$$

$$v'''_{11}+\alpha R v''_{11}-\left(\pi^2+M^2+\frac{1}{K}\right)v'_{11}=R\pi^2 p_{11} \tag{43}$$

Eliminating the terms p_{11} and p'_{11} in equations (42) and (43), we get the following,

$$v^{iv}_{11}+\alpha R v'''_{11}-\left(M^2+\frac{1}{K}+2\pi^2\right)v''_{11}-R\pi^2 v'_{11}+\left(\pi^4+\frac{\pi^2}{K}\right)v_{11}+\frac{\pi^2}{K}=0 \tag{44}$$

The corresponding boundary conditions are:
$y = 0$; $v_{11} = -\alpha$, $v'_{11} = 0$
$y\to\infty$; $v_{11} = 0$ (45)

Substituting the equations (39), (40) and (41) in equations (30), (33) and (34), we get following:

$$u''_{11}+\alpha R u'_{11}-\left(M^2+\frac{1}{K}+\pi^2\right)u_{11}=Rv_{11}u'_0-GrR^2T_{11}-GmR^2C_{11}+\frac{u_0-1}{K} \tag{46}$$

$$T_{11}'' + \alpha RprT_{11}' - \pi^2 T_{11} = -R\Pr v_{11}T_0' \tag{47}$$

$$C_{11}'' + \alpha RScC_{11}' - \pi^2 C_{11} = -RScv_{11}C_0' \tag{48}$$

The corresponding boundary conditions are:

y = 0 ; $u_{11} = 0$, $T_{11} = 0$, $C_{11} = 0$

y→∞; $u_{11} \to 0$, $T_{11} \to 0$, $C_{11} \to 0$ (49)

Using Finite difference formulae

$$v_{11}'(i) = \frac{v_{11}(i+1) - v_{11}(i-1)}{2h}$$

$$v_{11}''(i) = \frac{v_{11}(i+1) - 2v_{11}(i) + v_{11}(i-1)}{h^2}$$

$$v_{11}'''(i) = \frac{v_{11}(i+2) - 2v_{11}(i+1) + 2v_{11}(i-1) - v_{11}(i-2)}{2h^3}$$

$$v_{11}''''(i) = \frac{v_{11}(i+2) - 4v_{11}(i+1) + 6v_{11}(i) - 4v_{11}(i-1) + v_{11}(i-2)}{h^4} \tag{50}$$

Equation (44) using finite difference form (50) becomes

$$D_1 v_{11}(i+2) - D_2 v_{11}(i+1) + D_3 v_{11}(i) - D_4 v_{11}(i-1) - D_5 v_{11}(i-2) + \frac{2h^4\pi^2}{K} = 0 \tag{51}$$

$$D_1 = 2 + \alpha Rh$$

,

$$D_2 = 8 + 2\alpha Rh + 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + Rh^3\pi^2$$

$$D_3 = 12 + 4h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + 2h^4\left(\pi^4 + \frac{\pi^2}{K}\right)$$

$$D_4 = 8 - 2\alpha Rh + 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) - R\pi^2 h^3$$

$$D_5 = 2 - \alpha Rh$$

Similarly solving equations (46), (47) and (48) using (50), we get the following:

$$D_1 u_{11}(i+1) - B_1 u_{11}(i) + D_5 u_{11}(i-1) = B(i) \tag{52}$$

$$K_1 T_{11}(i+1) - K_2 T_{11}(i) + K_3 T_{11}(i-1) = D(i) \tag{53}$$

$$F_1 C_{11}(i+1) - K_2 C_{11}(i) + F_2 C_{11}(i-1) = E(i) \tag{54}$$

Where $B_1 = 4 + 2h^2\left(M^2 + \frac{1}{K} + \pi^2\right)$

$$B(i) = -2h^2 Rv_{11}(i)B_2(i) - 2h^2R^2(GrT_{11}(i) - GmC_{11}(i) + \frac{2h^2}{K}B_3(i)$$

$$B_2(i) = [\alpha\lambda(Gr\lambda_0 + Gm\lambda_1 - 1)]e^{\alpha\lambda ih} - \alpha R\Pr Gr\lambda_0 e^{\alpha R\Pr ih} - \alpha RScGm\lambda_1 e^{\alpha RScih}$$

$$B_3(i) = (Gr\lambda_0 + Gm\lambda_1 - 1)e^{\alpha\lambda ih} - Gr\lambda_0 e^{\alpha R\Pr ih} - Gm\lambda_1 e^{\alpha RScih}$$

$$K_1 = 2 + \alpha R \Pr h$$
$$K_2 = 4 + 2\pi^2 h^2 \qquad K_3 = 2 - \alpha R \Pr h$$
$$D(i) = 2\alpha R^2 \Pr^2 h^2 v_{11}(i) e^{\alpha R \Pr ih}$$
$$F_1 = 2 + \alpha RSch$$
$$F_2 = 2 - \alpha RSch$$
$$E(i) = 2\alpha R^2 Sc^2 h^2 v_{11}(i) e^{\alpha RScih}$$

Equations (51) – (54) have been solved by Gauss-Seidal iteration method for velocity and temperature. To prove the convergence of finite difference scheme, the calculations are carried out slightly changed value of h .Smaller change is observed in the value of velocity and temperture.Thus, it is concluded that finite difference scheme is convergent and stable.

5.4 Skin –friction coefficient:

We know discuss the skin friction components in the x' – direction in the non- dimensional form.

$$\tau_x = \frac{\tau'_x}{\rho UV} = \frac{\nu}{VL}\left(\frac{du_0}{dy}\right)_{y=0} \quad \tau_x = \frac{1}{R}\left(\frac{du_0}{dy}\right)_{y=0} + \frac{1}{R}\varepsilon\left(\frac{du_{11}}{dy}\cos\pi z\right)_{y=0} \tag{55}$$

5.5 Nusselt number

The rate of heat transfer i.e heat flux at the surface in terms of Nusselt number(Nu) is given by

$$Nu = \frac{q_w}{\rho V c_p (T_w - T_\infty)} = \frac{k}{\rho V c_p L}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{1}{Rpr}\left[\frac{dT}{dy} + \varepsilon \frac{dT_{11}}{dy}\cos\pi z\right]_{y=0} \tag{56}$$

5.6 Results and discussion

In view of physical nature of the problem, numerical calculations are carried out for different parameters such as Grashof number(Gr), Modified Grashof number(Gm), Hartmann number(M) , Suction velocity(α), Permeability(K) , Reynolds number(R), Prandtl number (Pr) and Schmidt number(Sc) are studied.

Figure 1 depicts the effect of suction parameter(α) on velocity field when R=10, K=1, M=2,ϵ=0.1 and z=0. As the suction parameter increases there is increase in velocity of the fluid.

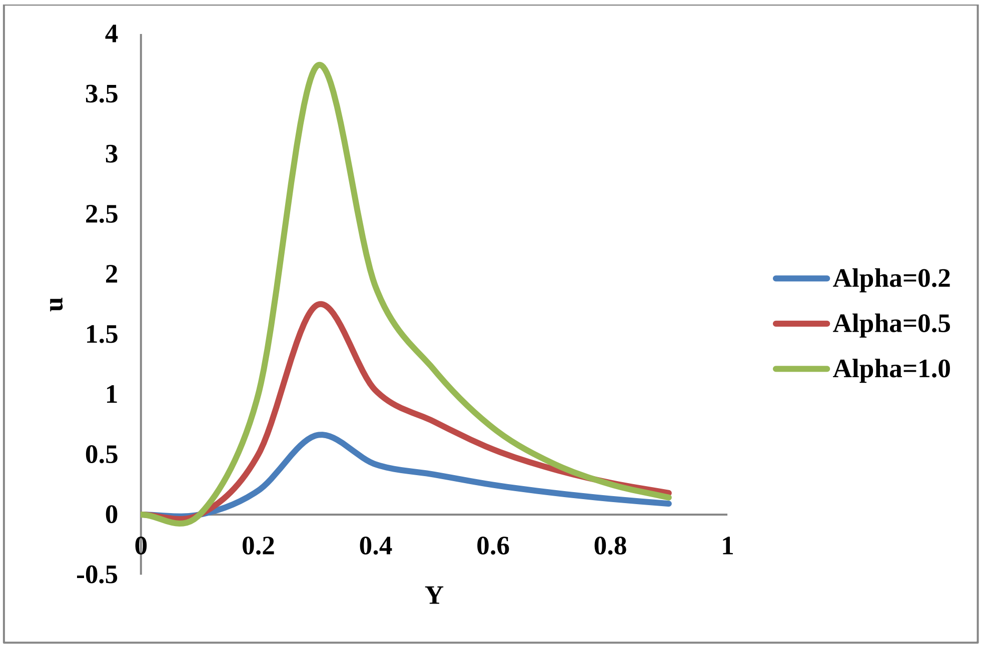

Figure 1. Effect of suction parameter (α) on velocity field when R = 10, K=1, M=2,ϵ=0.1 and z=0.

Figure 2 shows the effect of Hartmann number(M) on velocity field when α = 0.2, R= 10, K=1, ϵ= 0.1 and z=0. This indicates that as magnetic parameter increases there is decrease in velocity field.

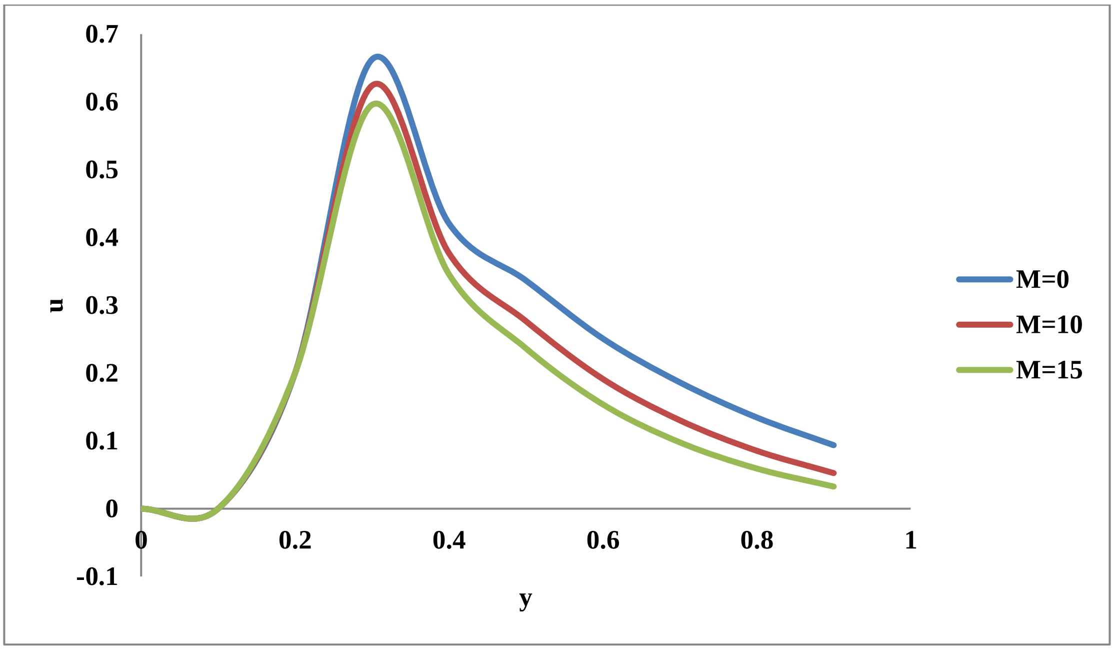

Figure 2. Effect of magnetic parameter (M) on velocity field when α =0.2,R = 10, K=1,,ϵ=0.1 and z=0.

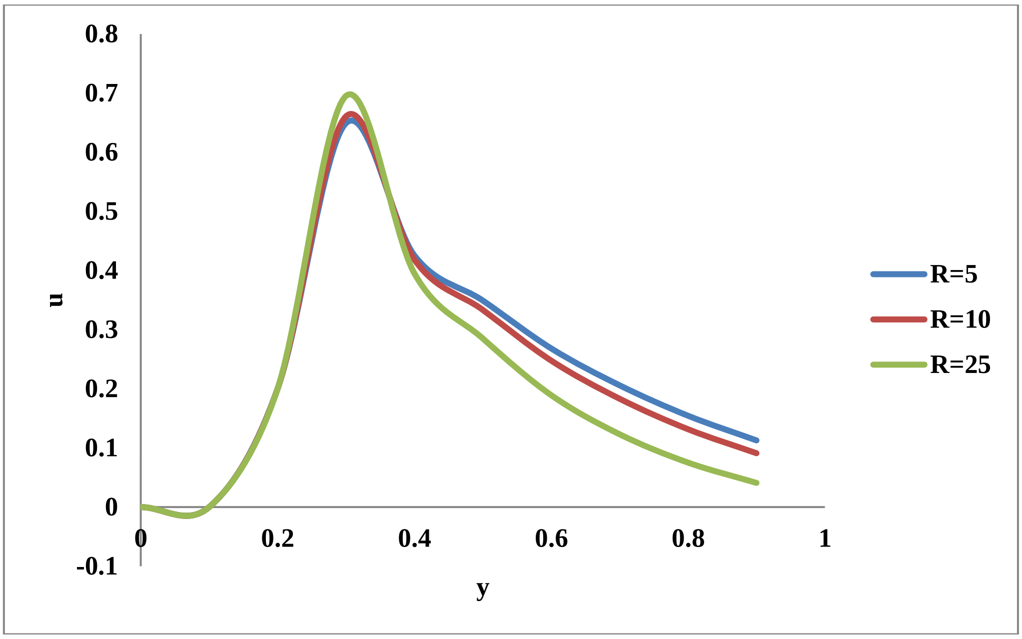

Figure 3. Effect of Reynolds number (R) on velocity field when α=0.2, K=1, M=2,ϵ=0.1 and z=0.

Figures 3 and 4 depicts the Reynolds number and permeability effect on velocity field.

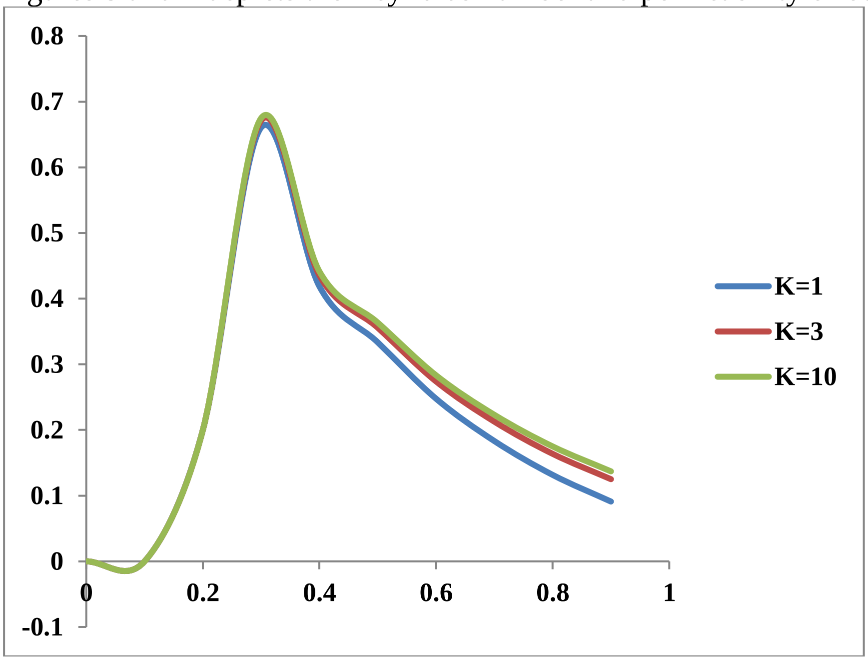

Figure 4. Effect of Permeability parameter (K) on velocity field when α=0.2, R=1, M=2,ϵ=0.1 and z=0.

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STABILITY OF STRATIFIED ELASTICO-VISCOUS WALTERS' (MODEL B') FLUID IN THE PRESENCE OF VARIABLE MAGNETIC FIELD AND ROTATION SATURATING POROUS MEDIUM

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ABSTRACT:
The influence of viscosity, viscoelasticity, medium permeability and medium porosity on the stability of a stratified elastic-viscous fluid is examined for viscoelastic polymeric solutions in the simultaneous presence of a variable horizontal magnetic field $H(H_0(z),0,0)$ and uniform horizontal rotation $\Omega(\Omega,0,0)$ in porous medium. These solutions are known as Walters' (model B') fluid and their rheology is approximated by the Walters' (model B') fluid constitutive relations, proposed by Walters'. The effects of coriolis force on the stability are chosen along the direction of the magnetic field and transverse to that of the gravitational field $g(0, 0, -g)$. Assuming the exponential stratifications in density, viscosity and viscoelasticity, the appropriate solution for the case of free boundaries is obtained using a linearized stability theory and normal mode analysis method. The dispersion relation is obtained and the behaviour of growth rates with respect to kinematic viscosity, kinematic viscoelasticity, medium permeability and medium porosity is examined numerically using Newton-Raphson method through the software Fortran-90 and Mathcad. In contrast to the Newtonian fluids, the system is found to be unstable, for stable stratifications, for small wavelength perturbations. It is found that the magnetic field stabilizes the certain wave number band, for unstable stratification in the presence of rotation and this wave number range increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity implying thereby the stabilizing effect of magnetic field and kinematic viscoelasticity and the kinematic viscosity has a stabilizing effect on the system for the low wave number range. The medium permeability has enhancing effect on the growth rates with its increase for a fixed wave number. These results are shown graphically.

KEY WORDS: Walters' (model B') fluid; magnetic field; rotation; viscosity; viscoelasticity, medium permeability, medium porosity.

1. INTRODUCTION:
The flow through porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamics. When we consider flow in porous medium, some additional complexities arise which are due to the interaction between the fluids and the porous medium. Here we consider those fluid flows for which Darcy's law is applicable. This law is empirical in nature and is usually considered valid for creeping flows where the Reynolds's number as defined for a porous medium is less than one. Darcy's law states that the gross effect, as the fluid slowly percolated through the pores of rock, is that usual viscous term in the equation of elastic-viscous fluid motion will be replaced by the resistance

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terms $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity, of Walters' (model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity (seepage) of the fluid. The stability of flow of a single component fluid through porous medium taking into account the Darcy's resistance has been studied by Lapwood [1] and Wooding [2]. The effect of the Earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The physical properties of comets and meteorites strongly suggest importance of porosity in astrophysical context (McDonnell [3]).

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid) was investigated by Rayleigh [4]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases everywhere. An experimental demonstration of the development of the Rayleigh–Taylor instability was performed by Taylor [5]. The effect of a vertical magnetic field on the development of Rayleigh–Taylor instability was considered by Hide[6]. Reid [7] studied the effect of surface tension and viscosity on the stability of two superposed fluids. The Rayleigh–Taylor instability of a Newtonian fluid has been studied by several authors accepting varying assumptions of hydrodynamics and hydromagnetics and Chandrasekhar [8] in his celebrated monograph has given a detailed account of these investigations. Bellman and Pennington [9] further investigated in detail illustrating the combined effects of viscosity and surface tension. Gupta [10] again studied the stability of a horizontal layer of a perfectly conducting fluid with continuous density and viscosity stratifications in the presence of a horizontal magnetic field. The Rayleigh–Taylor instability problems arise in oceanography, limnology and engineering.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [11] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role on the stability of the system. In all the above studies the fluid has been assumed to be Newtonian.

With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. Fredricksen [12] has given a good review of non–Newtonian fluids whereas Joseph [13] has also considered the stability of viscoelastic fluids. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One of such viscoelastic fluids is Walters' (model B') fluid. Walters' [14] has proposed a constitutive equation for such type of elastico–viscous fluids. Many other research workers have paid their attention towards the study of Walters' (model B') fluid. The mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 grams of polymers per litre behaves very nearly as the Walters' (model B') viscoelastic fluid and which is proposed by Walters'[15]. This class of fluids is used in the manufacture of parts of space cafts, aeroplane, tyres, beltconveyors, rops, cushions, seats, foams, plastics, engineering equipments etc. Sharma and Kumar [16] have studied the steady flow and heat transfer of Walters' fluid (model B') through a porous pipe of uniform circular–cross section with small suction. Sharma and Kumar [17] have studied the stability of two superposed Walters' (model B') viscoelastic fluid. The magnetic field stabilizes the system. The viscoelasticity of the medium has damping effects on the growth rates but has enhancing effects for certain ranges of the wave-numbers. Sharma *et al.* [18] have studied the stability of stratified Walters' (model B') fluid in the presence of horizontal magnetic field and rotation in porous medium. Yadav and Sharma [19] have studied the effects of porous medium on MHD fluid flow along a stretching cylinder. Sharma and Gupta[20] have also studied the stability of stratified elastico–viscous Walters' (model B') fluid in the presence of variable magnetic field and rotation.

Keeping in mind the importance of non–Newtonian fluids, medium permeability in modern technology and their various applications mentioned above, the present paper is devoted to consider the stability of rotating stratified elastico–viscous Walters' (model B') fluid in the presence of variable magnetic field and rotation in porous medium.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:

The initial stationary state whose stability we wish to examine is that of an incompressible, heterogeneous infinitely extending and conducting $(\sigma \to \propto)$ elastico–viscous Walters' (model B') fluid of thickness d bounded by the planes $z = 0, d$ and of variable density, kinematic viscosity and viscoelasticity, arranged in horizontal strata in a porous medium of variable porosity and medium permeability so that the free surface is almost horizontal and the electrical conductivity $\eta = \frac{1}{4\pi\mu_e\sigma}$ is zero. The fluid is acted on by gravity force $\boldsymbol{g}(0,0,-g)$, a uniform horizontal rotation $\boldsymbol{\Omega}(\Omega,0,0)$ and a variable horizontal magnetic field $\boldsymbol{H}(H_0(z),0,0)$.The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution..

The equations expressing conservation of momentum, mass, incompressibility and Maxwell's equations for the elastico–viscous Walters' (model B') fluid are

$$\rho\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\in}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right] = -\nabla p + \rho\,\boldsymbol{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{q} + \frac{2\rho}{\in}(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\mu_e}{4\pi}[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}], \tag{1}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{2}$$

$$\in\frac{\partial\rho}{\partial t} + (\boldsymbol{q}\cdot\nabla)\rho = 0, \tag{3}$$

$$\nabla\cdot\boldsymbol{H} = 0, \tag{4}$$

$$\in\frac{\partial\boldsymbol{H}}{\partial t} = \nabla\times(\boldsymbol{q}\times\boldsymbol{H}), \tag{5}$$

where μ_e, the medium permeability, is assumed to be constant. Equation (3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.

Let $\delta\rho$, δp, $\boldsymbol{q}(u,v,w)$ and $\boldsymbol{h}(h_x,h_y,h_z)$ denote, respectively, the perturbations in density $\rho(z)$, pressure $p(z)$, velocity $\boldsymbol{v}(0,0,0)$ and horizontal magnetic field $\boldsymbol{H}(H,0,0)$. Then the equations (1)–(5) after perturbations in the cartesian form become

$$\frac{\rho}{\in}\frac{\partial u}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial}{\partial x}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 u+\frac{\mu_e}{4\pi}\left(h_z\frac{\partial}{\partial z}H_0\right)+\frac{2}{\in}\rho\, v\,\Omega, \tag{6}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial}{\partial y}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 v+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y-\frac{\partial}{\partial y}h_x\right)-\frac{2}{\in}\rho\, u\,\Omega, \tag{7}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t}+\frac{\rho}{\in}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial}{\partial z}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_z-\frac{\partial}{\partial z}h_x-\frac{h_z}{H_0}\frac{\partial}{\partial z}H_0\right)-g\delta\rho, \tag{8}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{9}$$

$$\in\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial\rho}{\partial z}=0, \tag{10}$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \tag{11}$$

$$\in\frac{\partial}{\partial t}h_x=\frac{\partial}{\partial x}\left\{u\left(H_0+h_x\right)-uh_y\right\}-\frac{\partial}{\partial z}\left\{w\left(H_0+h_x\right)-uh_z\right\}, \tag{12}$$

$$\in\frac{\partial}{\partial t}h_y=\frac{\partial}{\partial z}\left\{vh_z-wh_y\right\}-\frac{\partial}{\partial x}\left\{uh_y-v\left(H_0+h_x\right)\right\}, \tag{13}$$

$$\in\frac{\partial}{\partial t}h_z=\frac{\partial}{\partial x}\left\{w\left(H_0+h_x\right)-uh_z\right\}-\frac{\partial}{\partial y}\left\{vh_z-wh_y\right\}, \tag{14}$$

$$\frac{\rho}{\in}\frac{\partial u}{\partial t}=-\frac{\partial}{\partial x}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 u+\frac{\mu_e}{4\pi}h_z\frac{\partial}{\partial z}H_0+\frac{2}{\in}\rho\, v\,\Omega, \tag{15}$$

$$\frac{\rho}{\in}\frac{\partial v}{\partial t}=-\frac{\partial}{\partial y}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 v+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y-\frac{\partial}{\partial y}h_x\right)-\frac{2}{\in}\rho\, u\Omega, \tag{16}$$

$$\frac{\rho}{\in}\frac{\partial w}{\partial t}=-\frac{\partial}{\partial z}\delta p-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}-\frac{h_x}{H_0}\frac{\partial H_0}{\partial z}\right)-g\delta\rho, \tag{17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{18}$$

$$\in\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial\rho}{\partial z}=0, \tag{19}$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \tag{20}$$

$$\in\frac{\partial}{\partial t}h_x=H_0\frac{\partial}{\partial x}u-w\frac{\partial}{\partial z}H_0, \tag{21}$$

$$\in\frac{\partial}{\partial t}h_y=H_0\frac{\partial}{\partial x}v, \tag{22}$$

$$\in\frac{\partial}{\partial t}h_z=H_0\frac{\partial}{\partial x}w\cdot \tag{23}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, z and time t is given by

$$f(z)\exp\left(ik_x x+ik_y y+nt\right), \tag{24}$$

where $f(z)$ is the some function of z–only; k_x, k_y are the wave-numbers in the x– and y–directions, respectively, $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave-number and n is the growth rate of the disturbance which is, in general, a complex constant.

Equations (15)–(23) using expression (24) become

$$\frac{\rho}{\in} n\,u = -ik_x\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)u + \frac{\mu_e}{4\pi} h_z\, DH_0 + \frac{2}{\in}\rho\, v\,\Omega, \qquad (25)$$

$$\frac{\rho}{\in}\, n\,v = -ik_y\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)v + \frac{\mu_e H_0}{4\pi}\left(ik_x h_y - ik_y h_x\right) + \frac{2}{\in}\rho u\Omega, \qquad (26)$$

$$\frac{\rho}{\in}\, n\,w = -D\,\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)w + \frac{\mu_e H_0}{4\pi}\left(ik_x h_z - Dh_x - \frac{h_x DH_0}{H_0}\right) - g\delta\rho, \qquad (27)$$

$$ik_x\, u + ik_y v + Dw = 0, \qquad (28)$$

$$\in n\delta\rho + wD\rho = 0, \qquad (29)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \qquad (30)$$

$$\in nh_x = ik_x H_0\, u - w\, DH_0, \qquad (31)$$

$$\in nh_y = ik_x H_0 v, \qquad (32)$$

$$\in nh_z = ik_x H_0 w, \qquad (33)$$

Now substituting the values of h_x, h_y and h_z from equations (31)–(33) in equations (25)–(27), we get

$$\frac{\rho}{\in} n\,u = -ik_x\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)u + \frac{\mu_e}{4\pi}\left(\frac{ik_x H_0 w}{n}\right)DH_0 + \frac{2}{\in}\rho\, v\Omega, \qquad (34)$$

$$\frac{\rho}{\in} n\,v = -ik_y\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)v + \frac{\mu_e}{4\pi} H_0\left(\frac{ik_x H_0\zeta_z}{n} + \frac{ik_y w\, DH_0}{n}\right) - \frac{2}{\in}\rho\, u\Omega, \qquad (35)$$

$$\frac{\rho}{\in} n\,w = -D\,\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)w + \frac{\mu_e H_0}{4\pi \in}\left[-\frac{k_x^2 H_0 w}{n} - D\left(\frac{ik_x H_0\, u}{n} - \frac{w\, DH_0}{n}\right)\right.$$
$$\left. - \left(\frac{ik_x H_0 u}{n} - \frac{w\, DH_0}{n}\right)\frac{DH_0}{H_0}\right] + \frac{g\,(D\rho)\,w}{\in}, \quad (36)$$

where $\zeta_z = ik_x v - ik_y u$, is the z–component of vorticity.

Multiplying equations (34) and (35) by $-ik_y$ and ik_x, respectively, and then adding we get

$$\frac{\rho\, n}{\in}\zeta_z = -\frac{\rho}{k_1}(\upsilon + \upsilon' n)\left(D^2 - k^2\right)\zeta_z - \frac{\mu_e\, k_x^2 H_0^2}{\in 4\pi\, n}\zeta_z + \frac{2}{\in}\Omega\, Dw,$$

or

$$\zeta_z = \frac{2\, n\,\Omega\, Dw}{n^2 - \frac{\in n}{k_1}(\upsilon - \upsilon' n)(D^2 - k^2) + k_x^2 V_A^{\,2}}, \qquad (37)$$

where $\upsilon = \dfrac{\mu}{\rho}$, $\upsilon' = \dfrac{\mu'}{\rho}$ and $V_A^2 = \dfrac{\mu_e H_0^2}{4\pi\rho}$ (square of the Alfvén's velocity).

Substituting the value of ζ_z in equation (35), we get

$$\frac{\rho}{\in} n\,v = -ik_y\delta p - \frac{1}{k_1}(\mu - \mu' n)\left(D^2 - k^2\right)v - \frac{\mu_e H_0}{4\pi \in n}\left(\frac{2\,\Omega\, n\, Dw\;\, ik_x}{n^2 - n(\mu - \mu' n)\left(D^2 - k^2\right) + k_x^2 V_A^2}\right) + \frac{\mu_e H_0}{4\pi\, n} ik_y w\, D(H_0) - 2\frac{\rho}{\in}u\Omega. \quad (38)$$

Multiplying equations (34) and (36) by $-ik_x$ and $-ik_y$, respectively, and then adding and using (28), we obtain

$$\frac{\rho}{\in} n\, Dw = -k^2\delta p - \frac{\rho}{k_1}(\upsilon - \upsilon' n)\left(D^2 - k^2\right)Dw + \left(\frac{2n\Omega}{n^2 - \frac{\in n}{k_1}(\upsilon - \upsilon' n)\left(D^2 - k^2\right) + V_A^{\,2} k_x^2}\right)\left(\frac{\mu_e H_0^2}{4\pi \in n} k_x^2 k_y - \frac{2}{\in}\rho\right)Dw \qquad (39)$$

Eliminating u, v and δp from equations (35)–(39) using equations (29), after little algebra, we get

$$\in\frac{n\rho}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)^2 w-\frac{n\rho}{k_1}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)w-\left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2-\frac{n\in}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)+k_x{}^2V_A^2}\right)-\frac{\mu_e k_x^2 D\left(H_0^2\right)}{4\pi\in n}\right]$$

$$Dw+gk^2(D\rho)w=0. \qquad (40)$$

Equation (40) is the general equation formulating the effect of variable magnetic field and uniform rotation on the stability of stratified Walters' (model B′) fluid saturating porous medium.

3. THE CASE OF EXPONENTIALLY VARYING STRATIFICATIONS:

In order to obtain the solution of the stability problem of a layer of Walters' (model B′) fluid, we suppose that the density ρ, viscosity μ, viscoelasticity μ' medium porosity $\in$ and medium permeability μ' vary exponentially along the vertical direction i.e.

$$\rho=\rho_0\, e^{\beta_1 z},\quad \mu=\mu_0\, e^{\beta_1 z},\quad \mu'=\mu_0'\, e^{\beta_1 z},\quad \in=\in_0\, e^{\beta_1 z}, k_1=k_{10}e^{\beta_1 z} \qquad (41)$$

where $\rho_0,\ \mu_0,\ \mu_0',\ H_1,\ \in_0,\ k_{10}$ and β_1 are constants and so the kinematic viscosity $\upsilon\left(=\frac{\mu}{\rho}=\frac{\mu_0}{\rho_0}\right)$, the kinematic viscoelasticity $\upsilon'\left(=\frac{\mu'}{\rho}=\frac{\mu_0'}{\rho_0}\right)$ and the Alfvén velocity $V_A=\left(\frac{\mu_e H_0^2}{4\pi\rho}\right)^{1/2}=\left(\frac{\mu_e H_1{}^2}{4\pi\rho}\right)^{1/2}$ are constant everywhere.

Using the stratifications of the form (41), equation (40) transforms to

$$\left(D^2-k^2\right)^3 w-\frac{2}{\frac{\in_0\, n}{k_{10}}(\upsilon_0-\upsilon_0' n)}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)^2 w+\frac{1}{n^2\frac{\in_0}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[n^4+k_x^2V_A^2\left(2n^2+k_x^2V_A^2\right)-V_A^2\, k_x^2\, \beta_1\, n\,(\upsilon_0-\upsilon_0' n)\right.$$

$$\left.-\,g\,k^2\beta_1\, n\,(\upsilon_0-\upsilon_0' n)\right]\left(D^2-k^2\right)w-\frac{1}{\frac{\in_0\, n}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[4\Omega^2 n^2+V_A^2k_x^2\beta_1\left(n^2+k_x^2V_A^2\right)-g\,k^2\beta_1\left(n^2+k_x^2V_A^2\right)w\right]=0.$$

$$(42)$$

Considering the case of two free boundaries, we must have

$$w=D^2w=0 \text{ at } z=0 \text{ and } z=d. \qquad (43)$$

The appropriate solution of equation (42) satisfying the above boundary condition is

$$w=A_0\sin\frac{m\pi z}{d}, \qquad (44)$$

where m is an integer and A_0 is a constant.

Substituting the value of w from equation (44) in equation (42) we obtain dispersion relation

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\,\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\,\upsilon_0^2+\left(2k_x^2\,V_A^2-\frac{g\,k^2\beta_1}{L_3}\right)\left(1-\upsilon_0' L_3\right)+\frac{4\in_0\,\Omega^2k_x^2}{L_3k_{10}{}^2}-\right.$$

$$\left.\frac{1}{L_3}V_A^2\,k_x^2\,\beta_1\left(1-\upsilon_0' L_3\right)\right]+n\left[v_0L_3\left(2\,k_x^2\,V_A^2-\frac{\in_0{}^2\,g\beta_1 k^2}{k_{10}{}^2L_3}\right)-\frac{1}{L_3}V_A^2\,k_x^2\,\beta_1\left(1-\upsilon_0' L_3\right)\right]+k_x^2V_A^2\left[k_x^2\,V_A^2-\frac{\beta_1}{L_3}\left(g\,k^2+V_A^2\,k_x^2\right)\right]=0, \qquad (45)$$

where $L_3=\left[k^2+\frac{m_1^2\,\pi^2}{d^2}\right]$.

Equation (45) is biquadratic in n and is the dispersion relation governing the effects of uniform rotation, variable horizontal magnetic field, viscosity, viscoelasticity medium permeability and medium porosity on the stability of stratified Walters' (model B') fluid.

4. RESULTS AND DISCUSSIONS:

(a) Case of stable stratifications (i.e. $\beta_1 < 0$) and $\left(k_{10} > 4 \in_0 v'_o\right)$, Equation (45) does not admit any positive real root or complex root with positive real part using Routh–Hurwitz criterion; therefore, the system is always stable for disturbances of all wave-number.

(b) Case of unstable stratifications (i.e. $\beta_1 > 0$) and $\left(k_{10} < 4 \in_0 v'_o\right)$, If $\beta_1 > 0$, $\frac{k_x^2 V_A^2}{k^2}\left(1-\frac{\beta_1}{L_3}\right) < \frac{\beta_1}{L_3} g$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part using Routh–Hurwitz criterion; so the system is unstable for all wave-numbers satisfying the inequality

$$k^2 < \frac{\beta_1 d^2 g \sec^2\theta - V_A^2\left(m_1^2\pi^2 - \beta_1 d^2\right)}{V_A^2 d^2}, \tag{46}$$

where θ is the angle between k_x and k i.e. $\left(k_x = k\cos\theta\right)$.

If $\beta_1 > 0$, (unstable stratifications) $1 > \frac{\beta_1}{L_3}$ and $V_A^2 > \frac{\beta_1\, g\, k^2}{L_3\, k_x^2\left(1-\frac{\beta_1}{L_3}\right)}$, equation (45) does not admit of any positive real root or complex root with positive real part, therefore, the system is stable. The system is clearly unstable in the absence of magnetic field, rotation and for non–viscoelastic fluid.

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon'_0 L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon'_0 L_3\right)\right]+n^2\left[L_3^2\,\upsilon_0^2-\frac{g\,k^2\beta_1}{L_3}\left(1-\upsilon'_0 L_3\right)+\frac{1}{L_3}V_A^2\,k_x^2\,\beta_1\left(1-\upsilon'_0 L_3\right)\right]-n\left[v_0 L_3\,\frac{{\in_0}^2\, g\beta_1 k^2}{L_3 {k_{10}}^2}\right]=0. \tag{47}$$

For $\beta_1 > 0$, the constant term in the equation (45) is negative and therefore has at least one root with positive real part therefore the system is clearly unstable. The magnetic field, therefore, stabilizes potentially unstable stratifications for small wave-length perturbations

$$k^2 > \frac{\beta_1\, d^2 g \sec^2\theta - V_A^2\left(m_1^2\pi^2 - \beta_1 d^2\right)}{V_A^2 d^2}. \tag{48}$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wave-length perturbations satisfying equation (48) remain unstable and are not stabilized by magnetic field.

The behaviour of growth rates with respect to kinematic viscosity υ_0, kinematic viscoelasticity υ'_0 and square of the Alfvén velocity V_A^2 satisfying equation (45) has been examined numerically using Newton–Raphson method through the software Mathcad. Figure (1) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible value of $\beta_1 = 2$, $\in_0 = 0.5, k_{10} = 6, m_1 = 1$, $d = 6$ cm, $\Omega = 1$revolution/minute, $\upsilon'_0 = 1$, $g = 980\text{cm/s}^2$, $k_x = k\cos 45^0$, $V_A^2 = 55$ for three values of $\upsilon'_0 = 2$, 3 and 4 respectively. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [8] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscoelasticity υ'_0, which indicates the destabilizing effect of viscoelasticity whereas the

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growth rate decreases for certain wave numbers implying thereby the stabilizing effect of kinematic viscoelasticity on the system in the presence of medium permeability and medium porosity for low wave numbers range.

Figure (2) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0' = 1$, $g = 980\text{cm/s}^2,\ k_x = k\cos 45^0,\ V_A^2 = 55, \in_0 = 0.5, k_{10} = 6,$ for three values of $\upsilon_0 = 2,$ 4 and 6 respectively. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscosity υ_0 which indicates the destabilizing influence of kinematic viscosity, whereas the growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of kinematic viscosity on the system in the presence of medium permeability and medium porosity.

Figure (3) shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0 = 4,\ \upsilon_0' = 2,\ g = 980\,\text{cm/s}^2,\ k_x = k\cos 45^0, \in_0 = 0.5, k_{10} = 6$ for two values of $V_A^2 = 15$ and 55 respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in the square of the Alfvén velocity V_A^2 for certain wave number which indicates the destabilizing influence of the square of the Alfvén velocity, whereas growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of the square of the Alfvén velocity on the system in the presence of medium permeability and medium porosity.

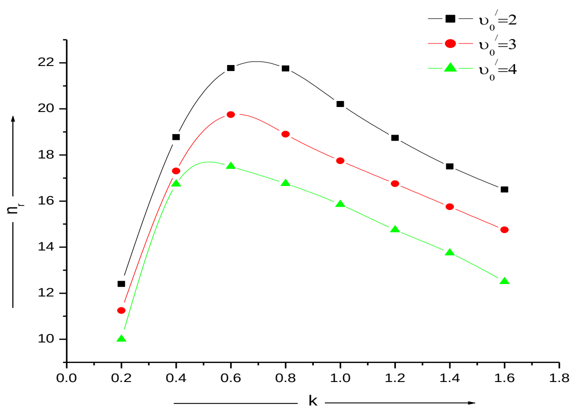

Figure 1: The variation of n_r with wave-number k for three values of $\upsilon_0' = 2,3,4$.

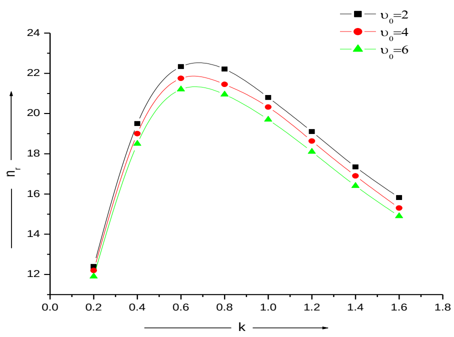

Figure 2: The variation of n_r with wave-number k for three values of $\upsilon_0 = 2, 4, 6$.

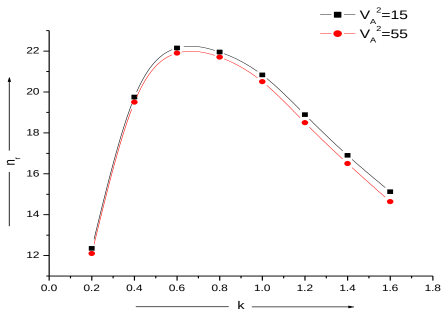

Figure 3: The variation of n_r with wave-number k for two values of $V_A^2 = 15, 55$.

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ON TRIPLY DIFFUSIVE CONVECTION IN A DENSELY PACKED POROUS MEDIUM: DARCY MODEL

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ABSTRACT:

It is mathematically established that thetriply diffusive convection in a densely packed porous medium with one of the components as heat, with diffusivity κ, cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively(with no loss of generality $\kappa > \kappa_1 > \kappa_2$), the porous parameterA and a constant P_1satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$. The result thus obtained is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection, Rayleigh number, Concentration Rayleigh number, Porous medium, Darcy Model.

INTRODUCTION:

Thermohaline convection has received much attention during some decades. It has outstanding applications in various fields of modern science like chemical engineering, nuclear engineering, astrophysics, oceanography, geophysics etc. The conditions under which thermohaline convection occurs has been extensively studied by Brandt and Fernando[1], Turner [2], Lapwood [3], Nield and Bejan [4], Radko [5], Wooding [6], Vafai [7], Abdullah and Alkazmi [8], Straughan [9] etc.

Later it is observed thatthere are many situations in nature wherein density of fluid systems depends upon more than two independently stratifying agencies having different diffusivities.Examples of such triply diffusive fluid systems include the Earth’s molten core, solidification of molten alloys, ocean water, geothermally heated lakes etc. Various researchers have contributed towards the theoretical as well as analyticalstudy of the onset of convection in triply diffusive fluid systems. For a broad view of the subject one may referred to Griffiths [10], Pearlstein et al. [11], Terrones [12], Lopez et al. [13], Rionero [14], Prakash et al. [15,16,17], Ghalambaz et al. [18] etc.

Banerjee et al. [19] derived a characterization theorem for thermohaline convection problem which proves the non-existence of oscillatory motions of growing amplitude in an initially bottom heavy configuration in a certain parameter regime and paved the way for further theoretical and experimental investigations in this field of enquiry. Later, Prakash et al. [20,21] further extended these results of Banerjee et al. [19] to more general triply diffusive configurations in the domain of astrophysics, wherein liquid concern has the property of magnetic field and rotation prevalent. As a further step, in the present paper, we have derived such characterization theorem for the triply diffusive convection in a densely packed porous medium.

MATHEMATICAL FORMULATION:

Consider an infinite horizontal densely packed porous layer filled with a viscous and Boussinesq fluid, statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11} (< S_{10}), S_{21} (< S_{20})$ in the force field of gravity (seeFig.1). It is assumed that the porous medium is of constant porosity. The Darcy model has been utilized to study the triply diffusive convection in a densely packed porous medium.

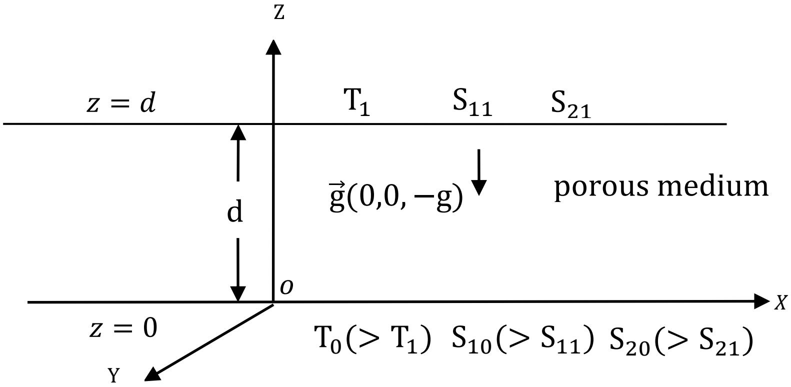

Fig.1. Physical configuration

The governing hydrodynamic equations and boundary conditions for the present problem in their non-dimensional form, are given by (Wankat and Schowalter [22], Vafai [7] and Prakash et al. [23]):

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R\, a^2\theta + R_1 a^2 \phi_1 + R_2 a^2 \phi_2, \quad (1)$$

$$(D^2 - a^2 - Ap)\theta = -w\,, \quad (2)$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}\,, \quad (3)$$

$$\left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}\,, \quad (4)$$

The Eqs. (1) -(4) are to be solved by using the following boundary conditions:

$w = 0 = \theta = \phi_1 = \phi_2 = Dw$ at $z = 0$ and at $z = 1$,
(when both the boundaries are rigid) (5)

or $w = 0 = \theta = \phi_1 = \phi_2 = D^2 w$ at $z = 0$ and at $z = 1$,
(when both the boundaries are free) (6)

where z is the real independent variable such that $0 \leq z \leq 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number, $\tau_1, \tau_2 > 0$ are the Lewis numbers for two concentration components respectively, $R > 0$ is the Rayleigh number, $R_1 > 0$ and $R_2 > 0$ are the two concentration Rayleigh numbers, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the two concentrations, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z), \theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi_1(z) = \phi_{1r}(z) + i\phi_{1i}(z)$ and $\phi_2(z) = \phi_{2r}(z) + i\phi_{2i}(z)$ are complex valued functions of the real variable z. The governing equations also involve two more positive constants namely$P_1 = \frac{k_1}{\epsilon d^2}$ and $A = 1 + \frac{\rho_s c_s}{\rho_0 c_0}\frac{(1-\epsilon)}{\epsilon}$, where$k_1$ is the permeability,ϵ is the porosity of the porous medium, d is the depth of the fluid layer;$\rho_s$is the solid density, c_s is the heat capacity of the solid. The suffix $'0'$ denotes the values of various parameters involved in the governing equations at some properly chosen reference temperature T_0.

It may further be noted that the Eqs.(1) -(6) describe an eigen value problem for p and govern triply diffusive convectionin a densely packed porous medium with constant porosity for any combination of rigid and free boundaries.

MATHEMATICAL ANALYSIS:

Now we prove the following theorem:

Theorem 1: If $R > 0, R_1 > 0, R_2 > 0, p_r \geq 0, p_i \neq 0$, and$R_s = R_1 + R_2 \leq \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$then a necessary condition for the existence of non-trivial solution $(w, \theta, \phi_1, \phi_2, p)$ of Eqs.(1)-(4) together with boundary conditions (5) or (6) is that$R_s = R_1 + R_2 < R$.

Proof: Multiplying Eq. (1) by w* (the superscript * henceforth denotes complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\, dz = -R\, a^2 \int_0^1 w^*\, \theta\, dz + R_1 a^2 \int_0^1 w^*\, \phi_1\, dz + R_2 a^2 \int_0^1 w^*\, \phi_2\, dz. \quad (7)$$

Using Eqs. (2)-(4) and the fact, that w(0) = 0 = w(1), we can write

$$R\, a^2 \int_0^1 w^*\, \theta\, dz = -R a^2 \int_0^1 \theta\, (D^2 - a^2 - Ap^*)\theta^* dz, \quad (8)$$

$$R_1 a^2 \int_0^1 w^*\, \phi_1 dz = -R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz, \quad (9)$$

$$R_2 a^2 \int_0^1 w^*\, \phi_2 dz = -R_2 a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz. \quad (10)$$

Combining Eqs. (7) -(10), we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\, dz = Ra^2 \int_0^1 \theta\, (D^2 - a^2 - Ap^*)\theta^* dz - R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz - R_2 a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz \quad (11)$$

Integrating various terms of Eq. (11), by parts, for an appropriate number of times and making use of either of the boundary conditions (5)-(6), it follows that

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz = Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + Ap^*|\theta|^2)\, dz - R_1 a^2 \tau_1 \int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{p^*}{\tau_1}|\phi_1|^2\right) dz - R_2 a^2 \tau_2 \int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{p^*}{\tau_2}|\phi_2|^2\right) dz \quad (12)$$

Equating the real and imaginary parts of both sides of Eq. (12) and cancelling p_i ($\neq 0$)throughout from the imaginary part, we get

$\left(\frac{p_r}{\sigma}+\frac{1}{P_1}\right)\int_0^1(|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1(|D\theta|^2+\ a^2|\theta|^2+Ap_r|\theta|^2)\,dz - R_1a^2\tau_1\int_0^1\left(|D\phi_1|^2+\ a^2|\phi_1|^2+\frac{p_r}{\tau_1}|\phi_1|^2\right)dz - R_2a^2\tau_2\int_0^1\left(|D\phi_2|^2+\ a^2|\phi_2|^2+\frac{p_r}{\tau_2}|\phi_2|^2\right)dz$ v (13)

and

$\int_0^1(|Dw|^2+\ a^2|w|^2)dz = -Ra^2A\sigma\int_0^1|\theta|^2dz + R_1a^2\sigma\int_0^1|\phi_1|^2\,dz + R_2a^2\sigma\int_0^1|\phi_2|^2\,dz$
(14)

We write Eq. (13) in an alternative form as

$$p_r\int_0^1(|Dw|^2+\ a^2|w|^2)\,dz+\frac{\sigma}{P_1}\int_0^1(|Dw|^2+\ a^2|w|^2)\,dz$$
$$= Ra^2\sigma\int_0^1(|D\theta|^2+\ a^2|\theta|^2)dz - R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+\ a^2|\phi_1|^2)\,dz$$
$$- R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+\ a^2|\phi_2|^2)dz$$
$$+p_r\left[Ra^2A\sigma\int_0^1|\theta|^2dz - R_1a^2\sigma\int_0^1|\phi_1|^2dz - R_2a^2\sigma\int_0^1|\phi_2|^2\,dz\right]$$
(15)

and derive the desired result from the resulting inequality obtained by replacing each term of this equation by its appropriate estimate.

We first note that since w, θ, ϕ_1, and ϕ_2 satisfy $w(0) = 0 = w(1)$, $\theta(0) = 0 = \theta(1)$, $\phi_1(0) = 0 = \phi_1(1)$ and $\phi_2(0) = 0 = \phi_2(1)$, we have by Rayleigh-Ritz inequality (Schultz[24])

$\int_0^1|Dw|^2dz \ge \pi^2\int_0^1|w|^2dz,$
(16)

$\int_0^1|D\theta|^2dz \ge \pi^2\int_0^1|\theta|^2dz,$
(17)

$\int_0^1|D\phi_1|^2dz \ge \pi^2\int_0^1|\phi_1|^2dz,$
(18)

$\int_0^1|D\phi_2|^2dz \ge \pi^2\int_0^1|\phi_2|^2dz,$ (19) Further, since $p_r \ge 0$, we note that

$p_r\int_0^1(|Dw|^2+a^2|w|^2)dz \ge 0.$ (20)

Now multiplying Eq. (2) by θ^* and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely $\theta(0) = 0 = \theta(1)$, we have from the real part of the final equation

$\int_0^1(|D\theta|^2+\ a^2|\theta|^2+Ap_r|\theta|^2)dz = \text{Real part of}\int_0^1\theta^*w\,dz,$

$\le\left|\int_0^1\theta^*w\,dz\right|,$

$\le\int_0^1|\theta^*w|dz,$

$\le\left(\int_0^1|\theta|^2dz\right)^{1/2}\left(\int_0^1|w|^2dz\right)^{1/2}$(using Schwartz inequality). (21)

Combining the above inequality with inequality(17) and the fact that $p_r \ge 0$, we have

$$(\pi^2 + a^2)\int_0^1 |\theta|^2 dz \leq \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2},$$

which gives that

$$\left(\int_0^1 |\theta|^2 dz\right)^{1/2} \leq \frac{1}{(\pi^2 + a^2)}\left(\int_0^1 |w|^2 dz\right)^{1/2},$$

and thus inequality (21) gives

$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz \leq \frac{1}{(\pi^2+a^2)}\int_0^1 |w|^2 dz$.(22) Further, combining the above inequality with inequality(17), we get

$\int_0^1 |\theta|^2 dz \leq \frac{1}{(\pi^2+a^2)^2}\int_0^1 |w|^2 dz$.(23)

Now using inequalities (18) and (19) it follows that

$$R_1 a^2\sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\, dz + R_2 a^2\sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz$$
$$\geq (\pi^2 + a^2)\left(R_1 a^2\sigma\tau_1 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_2 \int_0^1 |\phi_2|^2 dz\right)$$

$\geq (\pi^2 + a^2)\left\{\tau_1 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz + \tau_2 \int_0^1 (|Dw|^2 + a^2|w|^2)dz - R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz\right\}$ (utilizing Eq. (14)) (24)

and thus

$$-R_1 a^2\sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\, dz - R_2 a^2\sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz$$

$\leq (\pi^2 + a^2)\left[R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz\right] - (\pi^2 + a^2)(\tau_1 + \tau_2)\int_0^1 (|Dw|^2 + a^2|w|^2)dz.$ (25)

Again, using Eq.(14), we obtain

$R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz \leq \tau_2 \int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2A\sigma\tau_2 \int_0^1 |\theta|^2 dz,$ (26)

and

$R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz \leq \tau_1 \int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2A\sigma\tau_1 \int_0^1 |\theta|^2 dz.$ (27)

Now using inequalities (26) and (27), we can write

$R_1 a^2\sigma\tau_2 \int_0^1 |\phi_1|^2 dz + R_2 a^2\sigma\tau_1 \int_0^1 |\phi_2|^2 dz \leq (\tau_1 + \tau_2)\int_0^1 (|Dw|^2 + a^2|w|^2)dz + Ra^2A\sigma(\tau_1 + \tau_2)\int_0^1 |\theta|^2 dz.$ (28)

Using this inequality (28) in inequality (25), we get

$$-R_1 a^2\sigma\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)\, dz - R_2 a^2\sigma\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz$$

$\leq (\pi^2 + a^2)Ra^2A\sigma(\tau_1 + \tau_2)\int_0^1 |\theta|^2 dz,$

$\leq \frac{Ra^2A\sigma}{(\pi^2+a^2)}(\tau_1 + \tau_2)\int_0^1 |w|^2 dz.$ (using inequality (23)) (29)

Also from Eq. (14) and the fact that$p_r \geq 0$, we obtain

$p_r\left[Ra^2A\sigma\int_0^1 |\theta|^2 dz - R_1 a^2\sigma\int_0^1 |\phi_1|^2 dz - R_2 a^2\sigma\int_0^1 |\phi_2|^2\, dz\right] < 0$ (30)

Now, if permissible, let $R_s = R_1 + R_2 \geq R$. Then, in that case, we derive from Eq. (15) and the inequalities(16), (20),(22), (29)and (30) that

$$\left[\frac{\sigma(\pi^2+a^2)}{P_1} - \frac{R_s a^2\sigma}{(\pi^2+a^2)}\left(1 + A(\tau_1+\tau_2)\right)\right]\int_0^1 |w|^2 dz < 0\ , \tag{31}$$

which clearly implies that

$$R_s > \frac{\frac{1}{P_1}\left[\frac{(\pi^2+a^2)^2}{a^2}\right]}{[1+A(\tau_1+\tau_2)]}\ ,$$

so that we necessarily have

$$R_s > \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}, \tag{32}$$

since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ (for $a^2 = \pi^2$) is $4\pi^2$.

Hence if $R_s = R_1 + R_2 \le \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$,then we must have

$$R_s = R_1 + R_2 < R. \tag{33}$$

This establishes the theorem.

The essential content of the theorem from the physical point of view are that the triply diffusive convection in porous medium with one of the components as heat with diffusivity κ cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively (with no loss of generality $\kappa > \kappa_1 > \kappa_2$),the Porous parameter A and a constant P_1 satisfy the inequality $R_1 + R_2 \le \frac{4\pi^2}{P_1[1+A(\tau_1+\tau_2)]}$. It is further established that this result is uniformly valid for the quite general nature of the bounding surfaces.

CONCLUSION:

The linear stability theory and Darcy model is used to characterize the onset of triply diffusive convection in a densely packed porous medium. The layer is considered to be heated and soluted from below. The porous medium is taken as isotropic and homogeneous. A mathematical theorem is derived which provides a classification of the neutral or unstable triply diffusive configuration into two classes namely the bottom-heavy class and the top-heavy class and then strikes a distinction between them by means of a characterization theorem which disallow the existence of oscillatory instability in the former class. It is further established that this result is uniformly valid for the quite general nature of the bounding surfaces.

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A MATHEMATICAL THEOREM IN MAGNETOHYDRODYNAMIC TRIPLY DIFFUSIVE CONVECTION IN A DENSELY PACKED POROUS MEDIUM

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ABSTRACT:
In the Present paper, it has been mathematically established that the magnetohydrodynamic triply diffusive convection in a densely packed porous medium with one of the components as heat, with diffusivity κ, cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivities κ_1 and κ_2 respectively, the Porous parameter A and a constant P_1 satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$. It is further proved that the above result is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection, Oscillatory motions,the Lewis numbers, Concentration Rayleigh number, Porous medium, Darcy Model.

INTRODUCTION:

The problem of thermohaline instability in porous medium has been extensively studied in the recent past by many researchers on account of its importance in geophysics, in the fields of oceanography, chemical process industry, food processing industry, solidification, in the prediction of ground water movement in aquifers, in assessing the effectiveness of fibrous insulations, in the energy extraction process from the geothermal reservoirs, in nuclear engineering and due to influence of porous material (like fibre materials used for insulating purposes or metallic foams in heat transfer devices) on human life (Abdullah et al. [1], Deepika and Narayana [4], Lapwood [8], Nield and Bejan [11], Radko [18], Straughan [23],Taunton et al.[24], Vafai [25], Wooding [26], Yadav et al. [27]).

These researchers have studied the physical configurations in whichthe density depends on two stratifying agencies having different diffusivities. However, it has been found later that there are many physical configurations, in whichthe density depends on three or more stratifying agencies having different diffusivities. For example,the Earth's core, saline waters of geothermally heated lakes, sea and the oceans which contain many salts having concentrations less than a few percent of the sodium chloride concentration. For a broad view of the subject one may referred to Ghalambaz et al.[5], Griffiths [6, 7], Lopez [9], Moroz[10], Prakash et al. [15,16, 17], Rionero[19], Rudraiah and Vortmeyer [20], Ryzhkov and Shevtsova [21] andZhao et al. [28]. These researchers have theoretically studied the onset of convection in a triply diffusive fluid layer (in porous and non porous medium, where density depends on three independently diffusing agencies with different diffusivities).

Banerjee et al. [2] derived a mathematical theorem in thermohaline convection for the nonexistence of oscillatory motions of growing amplitude in an initially bottom heavy configuration of the Veronis type and brought a fresh outlook to the subject matter of double diffusive convection which pave the way for further theoretical and experimental investigations in this field of enquiry. Later Prakash and Kumar [14] extended their result to thermohaline convection configuration in porous medium.The work of Banerjee et al. [2] have been further extended to double and triply diffusive convection configurations in a porous and non porous medium in the presence of magnetic field by Prakash et al. [12,13]. As a further step in the present problem an analogous mathematical theorem for the certain parameter regime, is derived for magnetohydrodynamic triply diffusive convection in densely packed porous medium which states that oscillatory motions (neutral or unstable) of growing amplitude cannot manifest in an initially bottom heavy configurations of magnetohydrodynamic triply diffusive convection whenever the sum of the concentration Rayleigh numbers is less than a critical value.

MATHEMATICAL FORMULATION:

Let us consider an infinite horizontal densely packed porous layer filled with a viscous and Boussinesq fluid, statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11} (< S_{10})$, $S_{21} (< S_{20})$ in the force field of gravity and in the presence of uniform vertical magnetic field, $\vec{H}$. (seeFig.1). It is assumed that the saturating fluid and the porous layer are incompressible and that the porous medium is a constant porosity medium. The Darcy's [3] modelhas been utilized to study themagnetohydrodynamic triply diffusive convection in a densely packed porous medium.

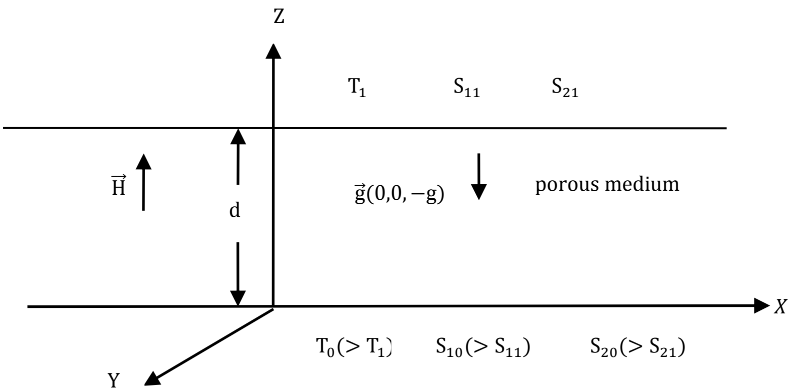

Fig.1. Physical configuration.

The equations that govern the motion of magnetohydrodynamictriply diffusive fluid layer in a densely packed porous medium (Darcy Model), in the non-dimensional form,are given by (Rudraiah and Vortmeyer [20], Prakash et al.[12,16,17]):

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R\,a^2\theta + R_1 a^2 \phi_1 + R_2 a^2 \phi_2 + Q\,D(D^2 - a^2)h_z, \tag{1}$$

$$(D^2 - a^2 - Ap)\theta = -w\,, \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}\,, \tag{3}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}\,, \tag{4}$$

$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right) h_z = -Dw,$ (5)

The Eqs. (1)-(5) are to be solved by using the following boundary conditions:
$w = 0 = \theta = \phi_1 = \phi_2 = D^2 w = h_z$ at $z = 0$ and at $z = 1$,
(when both the boundaries are free and perfectly conducting) (6)
or $w = 0 = \theta = \phi_1 = \phi_2 = Dw = h_z$ at $z = 0$ and at $z = 1$,
(when both the boundaries are rigid and perfectly conducting) (7)

where z is the real independent variable such that $0 \leq z \leq 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number,$\sigma_1 > 0$ is the magnetic Prandtl number, $\tau_1, \tau_2 > 0$ are the Lewis numbers for two components respectively, R > 0 is the Rayleigh number, $R_1 > 0$ and $R_2 > 0$ are the two concentration Rayleigh numbers, Q > 0 is the Chandrasekhar number, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the two concentrations,h_z is the vertical component of the perturbation in the initially external imposed magnetic field,$p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z), \theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi_1(z) = \phi_{1r}(z) + i\phi_{1i}(z), \phi_2(z) = \phi_{2r}(z) + i\phi_{2i}(z)$ and $h_z(z) = h_{z,r}(z) + ih_{z,i}(z)$ are complex valued functions of the real variable z.The governing equations also involve two more positive constants namely$P_1 = \frac{k_1}{\epsilon d^2}$ and $A = 1 + \frac{\rho_s c_s}{\rho_0 c_0}\frac{(1-\epsilon)}{\epsilon}$, where$k_1$ is the permeability,ϵ is the porosity of the porous medium, d is the depth of the fluid layer;$\rho_s$is the solid density, c_s is the heat capacity of the solid. The suffix '0' denotes the values of various parameters involved in the governing equations at some properly chosen reference temperature T_0.

It may further be noted that Eqs.(1)-(7) describe an eigen value problem for p and govern magnetohydrodynamic triply diffusive convectionin a densely packed porous mediumfor any combination of dynamically free and rigid perfectly conducting boundaries.

MATHEMATICAL ANALYSIS:

Now we prove the following theorem:

Theorem: If $R > 0, R_1 > 0, R_2 > 0, Q > 0, p_r \geq 0, p_i \neq 0$, and$R_s = R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$then a necessary condition for the existence of nontrivial solution $(w, \theta, \phi_1, \phi_2, h_z, p)$ ofEqs.(1)-(5) together with boundary conditions (6) or (7) is that$R_s = R_1 + R_2 < R$.

Proof: Multiplying Eq. (1) by w* (the superscript * henceforth denotes complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\left(\frac{p}{\sigma} + \frac{1}{P_1}\right)\int_0^1 w^* (D^2 - a^2)w\,dz = -R\,a^2 \int_0^1 w^*\,\theta\,dz + R_1 a^2 \int_0^1 w^*\,\phi_1\,dz + R_2 a^2 \int_0^1 w^*\,\phi_2\,dz$$

$+Q\int_0^1 w^* D\,(D^2 - a^2)h_z dz.$ (8)

Using Eqs. (2)-(5) and the fact, that w(0) = 0 = w(1), we can write

$R\,a^2 \int_0^1 w^*\,\theta\,dz = -Ra^2 \int_0^1 \theta\,(D^2 - a^2 - Ap^*)\theta^* dz,$ (9)

$R_1 a^2 \int_0^1 w^*\,\phi_1 dz = -R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{p^*}{\tau_1}\right)\phi_1^* dz,$ (10)

$R_2 a^2 \int_0^1 w^*\,\phi_2 dz = -R_2 a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{p^*}{\tau_2}\right)\phi_2^* dz,$(11)

$$Q\int_0^1 w^* D\,(D^2-a^2)h_z dz = -Q\int_0^1 Dw^*\,(D^2-a^2)h_z dz =$$
$$Q\int_0^1 \left(D^2-a^2-\frac{p^*\sigma_1}{\sigma}\right)h_z^*\,(D^2-a^2)h_z dz. \qquad (12)$$

Making use of Eqs. (9) – (12), from Eq. (8)we obtain

$$\left(\frac{p}{\sigma}+\frac{1}{P_1}\right)\int_0^1 w^*\,(D^2-a^2)w\,dz = Ra^2\int_0^1 \theta\,(D^2-a^2-\ Ap^*)\theta^* dz$$
$$-R_1a^2\tau_1\int_0^1 \phi_1\left(D^2-a^2-\ \frac{p^*}{\tau_1}\right)\phi_1^* dz - R_2a^2\tau_2\int_0^1 \phi_2\left(D^2-a^2-\ \frac{p^*}{\tau_2}\right)\phi_2^* dz$$
$$+Q\int_0^1 \left(D^2-a^2-\frac{p^*\sigma_1}{\sigma}\right)h_z^*\,(D^2-a^2)h_z dz. \qquad (13)$$

Integrating various terms of Eq. (13), by parts, for a suitable number of times and making use of either of the boundary conditions (6)-(7), it follows that

$$\left(\frac{p}{\sigma}+\frac{1}{P_1}\right)\int_0^1 (|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1 (|D\theta|^2+a^2|\theta|^2+Ap^*|\theta|^2)\,dz$$
$$-R_1a^2\tau_1\int_0^1 \left(|D\phi_1|^2+\ a^2|\phi_1|^2+\frac{p^*}{\tau_1}|\phi_1|^2\right)dz - R_2a^2\tau_2\int_0^1 \left(|D\phi_2|^2+\ a^2|\phi_2|^2\right.$$
$$\left.+\ \frac{p^*}{\tau_2}|\phi_2|^2\right)dz$$
$$-Q\int_0^1 |(D^2-a^2)h_z|^2 dz - Q\frac{p^*\sigma_1}{\sigma}\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz. \qquad (14)$$

Equating the real and imaginary parts of Eq. (14) form both sides and cancelling the term $p_i\ (\neq 0)$ throughout from the imaginary part, we get

$$\left(\frac{p_r}{\sigma}+\frac{1}{P_1}\right)\int_0^1 (|Dw|^2+a^2|w|^2)\,dz = Ra^2\int_0^1 (|D\theta|^2+\ a^2|\theta|^2+Ap_r|\theta|^2)\,dz$$
$$-R_1a^2\tau_1\int_0^1 \left(|D\phi_1|^2+\ a^2|\phi_1|^2+\frac{p_r}{\tau_1}|\phi_1|^2\right)dz$$
$$-R_2a^2\tau_2\int_0^1 \left(|D\phi_2|^2+\ a^2|\phi_2|^2+\frac{p_r}{\tau_2}|\phi_2|^2\right)dz$$
$$-Q\int_0^1 |(D^2-a^2)h_z|^2 dz - Q\frac{p_r\sigma_1}{\sigma}\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz, \qquad (15)$$

and

$$\int_0^1 (|Dw|^2+\ a^2|w|^2)dz = -Ra^2A\sigma\int_0^1 |\theta|^2 dz + R_1a^2\sigma\int_0^1 |\phi_1|^2\,dz + R_2a^2\sigma\int_0^1 |\phi_2|^2\,dz +$$
$$Q\sigma_1\int_0^1 (|Dh_z|^2+a^2|h_z|^2)dz. \qquad (16)$$

Alternatively Eq. (15) can be written as

$$p_r\int_0^1 (|Dw|^2+\ a^2|w|^2)\,dz + \frac{\sigma}{P_1}\int_0^1 (|Dw|^2+\ a^2|w|^2)\,dz =$$
$$Ra^2\sigma\int_0^1 (|D\theta|^2+\ a^2|\theta|^2)dz - R_1a^2\sigma\tau_1\int_0^1 (|D\phi_1|^2+\ a^2|\phi_1|^2)\,dz$$
$$-R_2a^2\sigma\tau_2\int_0^1 (|D\phi_2|^2+\ a^2|\phi_2|^2)dz - Q\sigma\int_0^1 |(D^2-a^2)h_z|^2 dz + p_r\Big[Ra^2A\sigma\int_0^1 |\theta|^2 dz -$$

$R_1a^2\sigma\int_0^1|\phi_1|^2dz - R_2a^2\sigma\int_0^1|\phi_2|^2\,dz - Q\sigma_1\int_0^1(|Dh_z|^2 + a^2|h_z|^2)dz\Big]$, (17)

andderive the desired result from the resulting inequality obtained by replacing each terms of this equation by its appropriate estimate.

As w, θ, ϕ_1, ϕ_2 and $h_z$satisfy$w(0) = 0 = w(1)$, $\theta(0) = 0 = \theta(1)$,$\phi_1(0) = 0 = \phi_1(1)$,$\phi_2(0) = 0 = \phi_2(1)$and $h_z(0) = 0 = h_z(1)$,therefore, we haveby Rayleigh-Ritz inequality (Schultz[22])

$\int_0^1|Dw|^2dz \geq \pi^2\int_0^1|w|^2dz,$ (18)

$\int_0^1|D\theta|^2dz \geq \pi^2\int_0^1|\theta|^2dz,$ (19)

$\int_0^1|D\phi_1|^2dz \geq \pi^2\int_0^1|\phi_1|^2dz,$ (20)

$\int_0^1|D\phi_2|^2dz \geq \pi^2\int_0^1|\phi_2|^2dz,$(21)

$\int_0^1|Dh_z|^2dz \geq \pi^2\int_0^1|h_z|^2dz.$ (22)

Since $p_r \geq 0$, thus we can write

$p_r\int_0^1(|Dw|^2 + a^2|w|^2)dz \geq 0.$ (23)

Now multiplying Eq. (2) by θ^* and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on θ, namely$\theta(0) = 0 = \theta(1)$, we have from the real part of the final equation

$\int_0^1(|D\theta|^2 + a^2|\theta|^2 + Ap_r|\theta|^2)dz = \text{Real part of}\int_0^1\theta^*wdz,$

$\leq \left|\int_0^1\theta^*wdz\right|,$

$\leq \int_0^1|\theta^*w|dz,$

$\leq \left(\int_0^1|\theta|^2dz\right)^{1/2}\left(\int_0^1|w|^2dz\right)^{1/2}$(using Schwartz inequality) . (24)

Making use of inequality (19) and the fact that $p_r \geq 0$, the above inequality (24) reduces to

$(\pi^2 + a^2)\int_0^1|\theta|^2dz \leq \left(\int_0^1|\theta|^2dz\right)^{1/2}\left(\int_0^1|w|^2dz\right)^{1/2},$

which gives that

$\left(\int_0^1|\theta|^2dz\right)^{1/2} \leq \frac{1}{(\pi^2+a^2)}\left(\int_0^1|w|^2dz\right)^{1/2},$

and thus inequality (24) gives

$\int_0^1(|D\theta|^2 + a^2|\theta|^2)dz \leq \frac{1}{(\pi^2+a^2)}\int_0^1|w|^2dz.$(25) Further, combining the above inequality with inequality(19), we get

$\int_0^1|\theta|^2dz \leq \frac{1}{(\pi^2+a^2)^2}\int_0^1|w|^2dz\,.$ (26)

Now using inequalities (20) and (21) it follows that

$$R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2 + a^2|\phi_1|^2)\,dz + R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2 + a^2|\phi_2|^2)dz$$

$$\geq (\pi^2 + a^2)\left(R_1a^2\sigma\tau_1\int_0^1|\phi_1|^2dz + R_2a^2\sigma\tau_2\int_0^1|\phi_2|^2dz\right)$$

$\geq (\pi^2 + a^2)\Big\{\tau_1\int_0^1(|Dw|^2+a^2|w|^2)dz - R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz - Q\sigma_1\tau_1\int_0^1(|Dh_z|^2 + a^2|h_z|^2)dz + \tau_2\int_0^1(|Dw|^2+a^2|w|^2)dz - R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz - Q\sigma_1\tau_2\int_0^1(|Dh_z|^2 +$

$a^2|h_z|^2)dz\Big\}$, (utilizing Eq. (16)) (27)
and thus

$$-R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+a^2|\phi_1|^2)\,dz-R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+a^2|\phi_2|^2)dz$$

$\leq(\pi^2+a^2)\left[R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz+R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\right]+(\pi^2+a^2)Q\sigma_1(\tau_1+\tau_2)\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz-(\pi^2+a^2)(\tau_1+\tau_2)\int_0^1(|Dw|^2+a^2|w|^2)dz.$ (28)Further, using Eq.(16), we obtain

$R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz\leq\tau_2\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma\tau_2\int_0^1|\theta|^2dz-Q\sigma_1\tau_2\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz,$ (29)
and
$R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\leq\tau_1\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma\tau_1\int_0^1|\theta|^2dz-Q\sigma_1\tau_1\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz.$ (30)
Combining inequalities (29) and (30), we have
$R_1a^2\sigma\tau_2\int_0^1|\phi_1|^2dz+R_2a^2\sigma\tau_1\int_0^1|\phi_2|^2dz\leq(\tau_1+\tau_2)\int_0^1(|Dw|^2+a^2|w|^2)dz+Ra^2A\sigma(\tau_1+\tau_2)\int_0^1|\theta|^2dz-Q\sigma_1(\tau_1+\tau_2)\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz,$ (31)
and thus utilizing the above inequality (31) in inequality (28), we obtain

$$-R_1a^2\sigma\tau_1\int_0^1(|D\phi_1|^2+a^2|\phi_1|^2)\,dz-R_2a^2\sigma\tau_2\int_0^1(|D\phi_2|^2+a^2|\phi_2|^2)dz$$

$\leq(\pi^2+a^2)Ra^2A\sigma(\tau_1+\tau_2)\int_0^1|\theta|^2dz,$
$\leq\frac{Ra^2A\sigma}{(\pi^2+a^2)}(\tau_1+\tau_2)\int_0^1|w|^2dz.$ (using inequality (26)) (32)
Further, using the fact that$p_r\geq0$,we can writefrom Eq. (16) as
$p_r\left[Ra^2A\sigma\int_0^1|\theta|^2dz-R_1a^2\sigma\int_0^1|\phi_1|^2dz-R_2a^2\sigma\int_0^1|\phi_2|^2\,dz-Q\sigma_1\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz\right]<0.$
(33)
Now, if permissible, let $R_s=R_1+R_2\geq R$. Then, in that case, we derive from Eq. (17) and the inequalities(18), (23),(25), (32)and (33) that
$\left[\frac{\sigma(\pi^2+a^2)}{P_1}-\frac{R_sa^2\sigma}{(\pi^2+a^2)}(1+A(\tau_1+\tau_2))\right]\int_0^1|w|^2dz+Q\sigma\int_0^1|(D^2-a^2)h_z|^2dz<0$, (34)
which clearly implies that
$R_s>\frac{(\frac{(\pi^2+a^2)^2}{a^2})}{P_1(1+A(\tau_1+\tau_2)}$,
so that we necessarily have
$R_s>\frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$, (35)
sincethe minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ (for $a^2=\pi^2$) is $4\pi^2$.
Hence if$R_s=R_1+R_2\leq\frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$,then we must have
$R_s=R_1+R_2<R.$ (36)
This establishes the theorem.

The essential content of the theorem from the physical point of view are that magneto hydrodynamic triply diffusive convection in densely packed porous medium with one of the components as heat with diffusivity κ cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers R_1 and R_2, the Lewis numbers τ_1 and τ_2 for the two concentrations with diffusivitiesκ_1 and κ_2 respectively (with no loss of generality $\kappa > \kappa_1 > \kappa_2$),the Porous parameterA and a constant P_1satisfy the inequality $R_1 + R_2 \leq \frac{4\pi^2}{P_1(1+A(\tau_1+\tau_2))}$. It is further established that this result is uniformly valid for any combination of free and/or rigid perfectly boundaries.

CONCLUSION:

The onset of magnetohydrodynamic triply diffusive convectionin a horizontal layer of fluid heated and soluted from below is investigated usinglinear stability analysis. The Darcy model has been utilized to derive amathematical theorem which disapprove the existence of oscillatory motions of growing amplitude in an initially bottom heavymagnetohydrodynamic triply diffusive configurations in a densely packed porous medium.The work done in the present paper will certainly pave the way for further theoretical and experimental investigations in this field of enquiry.

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STABILITY ANALYSIS OF THERMOHALINE CONVECTION PROBLEM IN A ROTATING VISCOELASTIC FLUID LAYER

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ABSTRACT:
The eigenvalue problem governing the Veronis type thermohaline convection in a viscoelastic fluid layer in the presence of uniform rotation is mathematically investigated for both linear and nonlinear stability. The expressions for Rayleigh numbers are derived for various orders of stability problems using linear and nonlinear stability analysis, for the cases of infinite and finite amplitude of disturbances, respectively. The effects of various parameters namely; rotation, stress relaxation time, strain retardation time and concentration gradient on steady and oscillatory thermal Rayleigh numbers have been discussed numerically for infinite amplitude of distubances and the results are depicted graphically. It is observed that the stress relaxation time has destabilizing effect, however the strain retardation time, concentration gradient and rotation have stabilizing effect on the onset of oscillatory thermohaline convection in a viscoelastic fluid. Also, it is observed from expression for Rayleigh number for steady second order nonlinear stability problem for finite disturbances that the viscoelastic parameters have no effects on the onset of instability. However, the concentration gradient and rotation reinforce each other and have destabilizing effect on the onset of instability.

KEYWORDS: Thermohaline convection; Oscillatory convection; Viscoelastic fluid; Rotation; Finite amplitude; Critical Rayleigh number.

1. INTRODUCTION:

Thermohaline convection or, more generally, double diffusive convection is an important phenomenon that involves the motions driven by two different density gradients diffusing at different rates, namely; thermal and concentration gradients. Thermohaline convection has its importance in the fields of oceanography, modelling of solar ponds, geophysics, astrophysics, chemical engineering, atmospheric pollution, etc. and various applications of the problem have aroused the interest of many research workers in this subject.

Stern [1] was the first to study the thermohaline (double diffusive) convection in the context of ocean by considering stable temperature and unstable concentration gradients, now known as Stern’s type configuration and derived some results concerning the instability of this configuration. He found that both steady as well as oscillatory motions of growing amplitudes occur in this configuration and steady motion is preferred one. Veronis [2] also studied the thermohaline configuration heated from below and salted from above creating destabilizing temperature gradient and stabilizing concentration gradient and derived the expressions for Rayleigh numbers for both stationary as well as oscillatory convection, when disturbances are infinitesimal small. This configuration is known as Veronis’ type configuration. The thermohaline convection problems of Stern and Veronis types have been revisited by Gupta et al[3] in the context of initially bottom/top heavy configurations and derived certain conditions for the stability and the bounds for the growth rate of oscillatory convection for both bottom/top heavy configurations, which are valid for all combinations of dynamically free rigid boundaries.

Veronis [2] in his analysis also discussed the two dimensional convective rolls in thermohaline convection by considering finite amplitude disturbances and predicted that the system becomes unstable to finite amplitude steady disturbances before it becomes unstable to disturbances of infinitesimal amplitude. Gupta and Gupta [4] extended the analysis of Veronis to include the effect of uniform rotation on thermohaline convective rolls and studied the effect of Taylor's number on the onset of steady as well as oscillatory convection with both finite and infinite disturbances.

For a broad view of the thermohaline convection, one may be referred to Turner and Stommel[5], Bains and Gill [6], Turner [7], Huppert and Turner [8], Brandt and Fernando [9], Radko[10].

Fluids are classified as Newtonian or non-Newtonian on the basis of the linear or non-linear relation between shearing stress to the rate of strain, respectively. A non-Newtonian fluid may not have well defined viscosity, therefore it is inadequate to describe the behaviour of the non-Newtonian fluids and is best studied through other rheological properties. Viscoelastic fluids fall in the category of non-Newtonian fluids. Idealized viscoelastic fluids are those fluids whose behaviour at sufficiently small variable shear stresses can be characterized by three constants i.e. coefficient of viscosity, relaxation time and retardation time. A non-linear theory of class of isotropic incompressible viscoelastic fluid with the stress relaxation time ($\bar{\lambda}_1$) and strain retardation time ($\bar{\lambda}_2$) are known as Oldroydian fluid (Oldroyd [11, 12]) and the viscoelastic fluid is termed as Maxwellian fluid when strain retardation time in the constitutive relation is absent ($\text{i.e. } \bar{\lambda}_2 = 0$). However, when both stress relaxation time and strain retardation time are zero ($\text{i.e. } \bar{\lambda}_1 = \bar{\lambda}_2 = 0$), the above constitutive relation reduced to a relation applicable for Newtonian viscous fluid. Industrial fluid are basically non-Newtonian; in particular viscoelastic fluids have been relevance in industrial applications. The characteristics of heat transfer in a viscoelastic fluid layer are important in chemical processing industries. Viscosity and elasticity of the fluid have certain effects on the heat transfer rate and thus can alter the instability of the system in comparison to Newtonian fluid (*cf.* Basu and Layek [13]).

The present study is motivated by the above analysis of Veronis [2] and Gupta and Gupta [4] for thermohaline convection for Newtonian fluids. Many authors including Chandrasekhar [14], Gupta and Gupta [4] and Murthy and Ram [15] have shown that the uniform rotation have stabilizing effect on the onset of convection for Newtonian fluids. Also viscosity and the elastic properties of non-Newtonian fluids together have destabilizing effect on the stability (Rosenblat [16]). So, our aim is to study the combined effect of rotation and concentration gradient on thermohaline convection on the Oldroydian viscoelastic fluid layer heated from below. Linear stability analysis for infinitesimal amplitude disturbances and weakly non-linear stability analysis for finite amplitude disturbances are carried out and the effects of various parameters on the convection have been computed numerically.

2. Mathematical Formulation of the Physical Problem

An infinite horizontal layer of viscoelastic incompressible fluid is statically confined between two horizontal boundaries in the force field of gravity, which are respectively maintained at uniform temperatures T_0 and $T_0 - \Delta T$ and uniform concentrations S_0 and $S_0 - \Delta S$ in the presence of uniform rotation with angular velocity $\vec{\Omega}$ along vertical z-axis. We consider two-dimensional rolls, so that all the physical quantities are independent of the y-coordinate. The boundaries are taken to be *isothermal* and *dynamically free* (on which tangential stresses vanish).

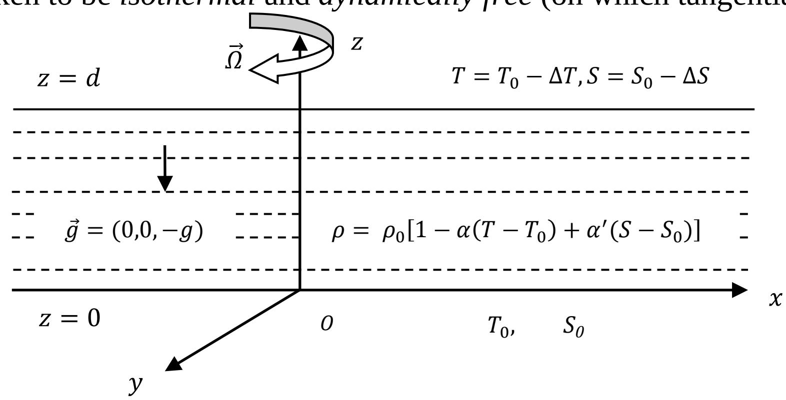

Figure 1. Schematic representation of the physical problem

The basic hydrodynamical equations governing the thermohaline convection problem in rotating viscoelastic fluid, in view of Boussinesq approximation, representing equation of continuity, equations of motion, equations of heat and mass transfer and equation of state, are respectively given by (*cf.* Veronis [2], Gupta and Gupta [4] and Sengupta and Kumar [17]);

$$\nabla.\vec{v} = 0 \tag{1}$$

$$\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}.\nabla\vec{v}\right) = -\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\nabla\left(\frac{P}{\rho_0} - \frac{1}{2}\left|\vec{\Omega} \times \vec{r}\right|^2\right) + \left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\frac{\rho}{\rho_0}\vec{g} - \left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right)\left(2\vec{\Omega} \times \vec{v}\right) + \left(1 + \bar{\lambda}_2 \frac{\partial}{\partial t}\right)\nu\nabla^2\vec{v} \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{v}.\nabla T = \kappa_T \nabla^2 T \tag{3}$$

$$\frac{\partial S}{\partial t} + \vec{v}.\nabla S = \kappa_s \nabla^2 S \tag{4}$$

and $$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha^{'}(S - S_0)] \tag{5}$$

In the above equations; ρ is density, $\vec{g} = (0,0,-g)$ is the external force, $\vec{v} = (u, v, w)$ are the components of velocity, T and S are respectively the temperature and the concentration at any point on the layer, P is the pressure, ν is the kinematic viscosity, c_v is the specific heat at constant volume, $\kappa_T \left(= \frac{K_0}{\rho_0 c_{v_0}}\right)$ is the thermometric diffusivity, κ_s is the coefficient of salt diffusion, $\bar{\lambda}_1$ is stress relaxation time, $\bar{\lambda}_2$ is strain retardation time, α and $\alpha^{'}$ are respectively the thermal and solutal coefficients of volume expansion, ρ_0 is the reference density of the fluid at

lower boundary.
Initially, when there are no motions, the basic state is assumed to be quiescent state. So, the initial state is represented by;

$$\vec{v} \equiv 0, \qquad T \equiv T(z), \qquad S \equiv S(z), \qquad \rho \equiv \rho(z), \qquad P \equiv P(z) \tag{6}$$

and hence the initial stationary state solutions are given by;

$$\vec{v} = (u, v, w) = (0, 0, 0), \quad T = T_0 - \beta z, \quad S = S_0 - \beta' z, \quad \rho = \rho_0[1 + \alpha\beta z - \alpha'\beta' z],$$
$$P = P_0 - g\,\rho_0 \left[z + (\alpha\beta - \alpha'\beta')\frac{z^2}{2}\right], \tag{7}$$

where, P_0 is the pressure at the lower boundary, $\beta = \frac{\Delta T}{d}$ and $\beta' = \frac{\Delta S}{d}$ are respectively the maintained uniform adverse temperature gradient and the non-adverse concentration gradient.
Imparting the *infinite amplitude* perturbations in the basic state, the perturbed state is given by;

$$\vec{\bar{v}} = 0 + \overrightarrow{v'}, \qquad \bar{S} = S_0 - \beta' z + \phi', \qquad \bar{T} = T_0 - \beta z + \theta',$$
$$\overline{P} = P_0 - g\left[z + (\alpha\beta - \alpha'\beta')\frac{z^2}{2}\right] + \delta P', \quad \rho = \rho_0[1 + \alpha(T_0 - T - \theta') - \alpha'(S_0 - S - \phi')] \tag{8}$$

where, $\overrightarrow{v'}$, ϕ', θ' and $\delta P'$ are perturbed variables.

Now, substituting the perturbed quantities given by (8) in equations (1)-(4), using the basic state solution given by equations (7) and using the stream function ψ (for two dimensional flow) defined by $u' = \partial\psi/\partial z$, $w' = -\partial\psi/\partial x$, and then non-dimensionalizing the resulting equations upon using the following non-dimensional quantities;

$$\psi = \kappa_T\hat{\psi}, \quad t = \frac{d^2}{\kappa_T}\hat{t}, \quad (x, y, z) = d(\hat{x}, \hat{y}, \hat{z}), \quad \theta' = \Delta T.\hat{\theta}, \quad \phi' = \Delta S.\hat{\phi},$$
$$\delta P' = \frac{\hat{P}\rho_0\nu\kappa_T}{d^2}, \qquad \mathrm{v} = \frac{\kappa_T}{d}\hat{\mathrm{v}}, \qquad \hat{\lambda}_1 = \bar{\lambda}_1\frac{\kappa_T}{d^2}, \qquad \hat{\lambda}_2 = \bar{\lambda}_2\frac{\kappa_T}{d^2},$$

we have the following dimensionless system of equations, after eliminating $\hat{P}$ from the resulting equations of motions and dropping the caps for the convenience in writing;

$$\left[\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} - \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\psi = \left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[-R\frac{\partial\theta}{\partial x} + R_s\frac{\partial\phi}{\partial x}\right] + Y\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial z} +$$
$$\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)J(\psi, \nabla^2\psi) \tag{9}$$

$$\left[\frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} - \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\mathrm{v} = -Y\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial z} + \frac{1}{\sigma}\left(1 + \lambda_1\frac{\partial}{\partial t}\right)J(\psi, \mathrm{v}) \tag{10}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = J(\psi, \theta) - \frac{\partial\psi}{\partial x} \tag{11}$$

$$\left(\frac{\partial}{\partial t} - \tau\nabla^2\right)\phi = J(\psi, \phi) - \frac{\partial\psi}{\partial x} \tag{12}$$

where, $\tau = \frac{\kappa_s}{\kappa_T}$, $\sigma = \frac{\nu}{\kappa_T}$, $R = \frac{g\alpha\Delta T d^3}{\nu\kappa_T}$, $R_s = \frac{g\alpha'\Delta S d^3}{\nu\kappa_T}$ are respectively, the Lewis number, the Prandtl number, the thermal Rayleigh number and the salinity Rayleigh number. Also $Y^2 = 4\Omega^2\mathrm{d}^4/\nu^2$ is the Taylor number and v is the azimuthal component of velocity induced by rotation and $J(.,.)$ are the Jacobians which represent the nonlinear terms.

The above system of equations is to be solved under the following *stress free* and *isothermal* non-dimensional boundary conditions;

$$\psi = \frac{\partial^2\psi}{\partial z^2} = \phi = \theta = \frac{\partial\mathrm{v}}{\partial z} = 0 \text{ at } z = 0 \text{ and } 1. \tag{13}$$

3. Linearized Stability Analysis:

In order to study the effects of various parameters on the onset of convection; when the disturbances are infinite, we use the linear stability analysis theory. So, linearizing the system of equations (9)-(12) by dropping the nonlinear terms, we have the following linearized perturbation equations governing the problem;

$$\left[\frac{1}{\sigma}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\psi=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[-R\frac{\partial\theta}{\partial x}+R_s\frac{\partial\phi}{\partial x}\right]+Y\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial z} \quad (14)$$

$$\left[\frac{1}{\sigma}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\mathrm{v}=-Y\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial z} \quad (15)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta=-\frac{\partial\psi}{\partial x} \quad (16)$$

$$\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)\phi=-\frac{\partial\psi}{\partial x} \quad (17)$$

Consider the solutions of the above linearized equations (14)-(17) satisfying the boundary conditions (13) of the following form;

$$\left.\begin{array}{ll}\psi=Ae^{pt}\sin\pi ax\sin n\pi z, & \phi=Be^{pt}\cos\pi ax\sin n\pi z\,,\\ \mathrm{v}=Ce^{pt}\sin\pi ax\cos n\pi z, & \theta=De^{pt}\cos\pi ax\sin n\pi z\,.\end{array}\right\} \quad (18)$$

where, a is the wave number, $p=p_r+ip_i$ is the complex constant, n is an integer and A,B,C,D are real constants.

Substituting the above periodic solutions in equations (14)-(17), eliminating the constants A,B,C and D amongst the resulting equations after taking $p=0$ for the steady marginal state solution, we have the following equation;

$$\sigma\pi^3(n^2+a^2)^2+\frac{\sigma a^2R_s}{\tau\pi(n^2+a^2)}+\frac{n^2\sigma Y^2}{\pi(n^2+a^2)}-\frac{\sigma a^2R}{\pi(n^2+a^2)}=0 \quad (19)$$

For the lowest mode, $n=1$, we have the following expression for the Rayleigh number for steady convection;

$$R^{Steady}=\frac{\pi^4(1+a^2)^3}{a^2}+\frac{R_s}{\tau}+\frac{Y^2}{a^2}. \quad (20)$$

The critical value of above Rayleigh number for corresponding minimum value of α for steady case can be easily obtained following (Chandrasekhar [14]);

$$R_c^{Steady}=3\pi^4\left(\frac{Y^2}{2\pi^4}\right)^{\frac{2}{3}}+\frac{R_s}{\tau} \quad (21)$$

and $a_{min}=\left(\frac{1}{2}\pi^2Y^2\right)^{\frac{1}{6}}$. (22)

When $R_s=0$, the value of critical Rayleigh number R_c^{Steady} coincides with the value of critical Rayleigh number obtained by Chandrasekhar [14] for rotatory thermal convection.

Further, for the case when marginal state is oscillatory, we have $p=ip_i$, $p_i$being real. Substituting solutions (18) in equations (14)-(17) with $p=ip_i$ and eliminating constants A,B,C and D amongst the resulting equations, we obtain a single expression, the real and imaginary parts of which for $n=1$ (the lowest mode), respectively yield the following expressions for the thermal Rayleigh number for overstable case and for frequency of oscillations;

$$R^{Ov} = \left(\frac{1+a^2}{\sigma a^2}\right)\frac{[\sigma\pi^2(1+a^2)\{\pi^2(1+a^2)(1+\lambda_1\lambda_2 p_i^2)-p_i^2(\lambda_2-\lambda_1)\}-p_i^2(1+{\lambda_1}^2p_i^2)]}{(1+{\lambda_1}^2p_i^2)} + R_s\frac{\left[\tau\pi^4\left(1+a^2\right)^2+p_i^2\right]}{[\tau^2\pi^4(1+a^2)^2+p_i^2]} + \left(\frac{\sigma Y^2}{a^2}\right)\frac{\left[\left\{\sigma\pi^4\left(1+a^2\right)^2-\lambda_1\pi^2(1+a^2)p_i^2(1+\sigma)+{\lambda_1}^2p_i^4\right\}+p_i^2\left\{1+\lambda_1\lambda_2\sigma\pi^4\left(1+a^2\right)^2+\lambda_1\pi^2\left(1+a^2\right)+\lambda_2\sigma\pi^2\left(1+a^2\right)\right\}\right]}{\left[\left\{\sigma\pi^2(1+a^2)-\lambda_1p_i^2\right\}^2+p_i^2\{1+\lambda_2\sigma\pi^2(1+a^2)\}^2\right]} \quad (23)$$

and $\quad Ex^4 + Fx^3 + Gx^2 + Hx + I = 0 \quad (24)$

where, $x = p_i^2$ is the square of frequency, and the constants occurring in (24) are given by;

$$E = [\lambda_1^2(1+a^2)\{\pi^2\lambda_1^2(1+a^2) + B_1\lambda_1\lambda_2\}], \quad (25)$$

$$F = [\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}\{\lambda_1^2\pi^2(1+\alpha^2) + \lambda_1\lambda_2B_1\}(1+a^2) + \lambda_1^2\pi^2(1+a^2)^2 + \lambda_1^2B_1\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + A_1^2\lambda_1^2\{\lambda_1^2\pi^2(1+a^2) + \lambda_1\lambda_2B_1\}(1+a^2) + \sigma^2Y^2\lambda_1^2\{\lambda_1\lambda_2B_1 - \lambda_1^2\pi^2(1+a^2)\} + R_s(\tau-1)B_1a^2\lambda_1^4], \quad (26)$$

$$G = {B_1}^2\{\pi^2\lambda_1^2(1+a^2) + B_1\lambda_1\lambda_2\}(1+a^2) + \{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}\pi^2(1+a^2)^2 + B_1\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + {A_1}^2\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}\{\lambda_1^2\pi^2(1+a^2) + B_1\lambda_1\lambda_2\}(1+a^2) + A_1^2\lambda_1^2\pi^2(1+a^2)^2 + A_1^2B_1\lambda_1^2\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + \sigma^2Y^2{\lambda_1}^2\{B_1 + \lambda_1\pi^2(1+a^2)B_1 - \pi^2(1+a^2) - \pi^2\lambda_2(1+a^2)B_1\} + \sigma^2Y^2(1+A_1^2\lambda_1^2)\{\lambda_1\lambda_2B_1 - \lambda_1^2\pi^2(1+a^2)\} + R_s(\tau-1)B_1a^2\lambda_1^2\{1+(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}, \quad (27)$$

$$H = {B_1}^2\pi^2(1+a^2)^2 + {B_1}^3\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + A_1^2B_1^2\{\lambda_1^2\pi^2(1+a^2) + B_1\lambda_1\lambda_2\}(1+a^2) + A_1^2\pi^2(1+a^2)^2\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\} + A_1^2B_1\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1\}\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + \sigma^2Y^2(1+A_1^2\lambda_1^2)\{B_1 + \lambda_1\pi^2(1+a^2)B_1 - \pi^2(1+a^2) - \pi^2\lambda_2(1+a^2)B_1\} + \sigma^2Y^2{A_1}^2\{\lambda_1\lambda_2B_1 - \lambda_1^2\pi^2(1+a^2)\} + R_s(\tau-1)B_1a^2\{(1+\lambda_2B_1)^2 - 2\lambda_1B_1 + {B_1}^2\lambda_1^2\}, \quad (28)$$

$$I = [A_1^2B_1^2\pi^2(1+a^2)^2 + A_1^2{B_1}^3\{1+(\lambda_2-\lambda_1)(1+a^2)\pi^2\}(1+a^2) + \sigma Y^2{A_1}^2\{B_1 + \lambda_1\pi^2(1+a^2)B_1 - \pi^2(1+a^2) - \pi^2\lambda_2(1+a^2)B_1\} + R_s(\tau-1){B_1}^3a^2\ ] \quad (29)$$

$$A_1 = \tau\pi^2(1+a^2) \quad \text{and} \quad B_1 = \sigma\pi^2(1+a^2) \quad (30)$$

4. Weakly Nonlinear Stability Analysis

Following Veronis [2] and Gupta and Gupta [4], for the steady convection (with $\partial/\partial t = 0$), when the disturbances are of *finite amplitude*, expressing all dependent variables in powers of small perturbation parameter ε subjected to boundary conditions (13), we have;

$f = \varepsilon f_0 + \varepsilon^2 f_1 + \varepsilon^3 f_2 + \cdots,$ where $f \equiv (\psi, \mathrm{v}, \phi, R)$.

Substituting these expansions for $\psi, \mathrm{v}, \theta, \phi$ and R in governing equations (9)-(12), we obtain the following equations;

$$(-\nabla^4\psi_0)\varepsilon + (-\nabla^4\psi_1)\varepsilon^2 + (-\nabla^4\psi_2)\varepsilon^3 = [R_s\frac{\partial\phi_0}{\partial x} - R_0\frac{\partial\theta_0}{\partial x} + Y\frac{\partial \mathrm{v}_0}{\partial z}]\varepsilon + [R_s\frac{\partial\phi_1}{\partial x} - R_0\frac{\partial\theta_1}{\partial x} - R_1\frac{\partial\theta_0}{\partial x} + Y\frac{\partial \mathrm{v}_1}{\partial z} + \frac{1}{\sigma}J(\psi_0, \nabla^2\psi_0)]\varepsilon^2 + [R_s\frac{\partial\phi_2}{\partial x} - R_0\frac{\partial\theta_2}{\partial x} - R_1\frac{\partial\theta_1}{\partial x} - R_2\frac{\partial\theta_0}{\partial x} + Y\frac{\partial \mathrm{v}_2}{\partial z} + \frac{1}{\sigma}\{J(\psi_0, \nabla^2\psi_1) + J(\psi_1, \nabla^2\psi_0)\}]\varepsilon^3 \quad (31)$$

$$(-\nabla^2\mathrm{v}_0)\varepsilon + (-\nabla^2\mathrm{v}_1)\varepsilon^2 + (-\nabla^2\mathrm{v}_2)\varepsilon^3 = [-Y\frac{\partial\psi_0}{\partial z}]\varepsilon + [-Y\frac{\partial\psi_1}{\partial z} + \frac{1}{\sigma}J(\psi_0, \mathrm{v}_0)]\varepsilon^2 + [-Y\frac{\partial\psi_2}{\partial z} + \frac{1}{\sigma}\{J(\psi_0, \mathrm{v}_1) + J(\psi_1, \mathrm{v}_0)\}\ ]\varepsilon^3 \quad (32)$$

$$(-\nabla^2\theta_0)\varepsilon + (-\nabla^2\theta_1)\varepsilon^2 + (-\nabla^2\theta_2)\varepsilon^3 = [-\frac{\partial\psi_0}{\partial x}]\varepsilon + [J(\psi_0, \theta_0) - \frac{\partial\psi_1}{\partial x}]\varepsilon^2 + [J(\psi_0, \theta_1) + J(\psi_1, \theta_0) - \frac{\partial\psi_2}{\partial x}]\varepsilon^3 \quad (33)$$

$$(-\tau\nabla^2\phi_0)\varepsilon + (-\tau\nabla^2\phi_1)\varepsilon^2 + (-\tau\nabla^2\phi_2)\varepsilon^3 = [-\frac{\partial\psi_0}{\partial x}]\varepsilon + [J(\psi_0,\phi_0) - \frac{\partial\psi_1}{\partial x}]\varepsilon^2 + [J(\psi_0,\phi_1) + J(\psi_1,\phi_0) - \frac{\partial\psi_2}{\partial x}]\varepsilon^3 \quad (34)$$

To obtain the first (lowest), second and third order system of equations, comparing the coefficients of ε, ε^2 and ε^3 respectively, we can have three sets of four equations each.

Now eliminating v_0, θ_0, ϕ_0 from the lowest order system obtained from equations (31)-(34), we get

$$\mathcal{L}\psi_0 = -\left[\nabla^6 + Y^2\frac{\partial^2}{\partial z^2} + \left(\frac{R_s}{\tau} - R_0\right)\frac{\partial^2}{\partial x^2}\right]\psi_0 = 0. \quad (35)$$

Similarly, eliminating v_1, θ_1, ϕ_1 from the second order and v_2, θ_2, ϕ_2 from the third order system of equations, we have the following equations;

$$\mathcal{L}\psi_1 = -R_1\frac{\partial^2\psi_0}{\partial x^2} - \frac{Y}{\sigma}\frac{\partial}{\partial z}[J(\psi_0,\mathrm{v}_0)] + R_0\frac{\partial}{\partial x}[J(\psi_0,\theta_0)] - \frac{R_s}{\tau}\frac{\partial}{\partial x}[J(\psi_0,\phi_0)] + \frac{1}{\sigma}J(\nabla^2\psi_0,\nabla^4\psi_0) \quad (36)$$

$$\mathcal{L}\psi_2 = -R_2\frac{\partial^2\psi_0}{\partial x^2} - R_1\frac{\partial}{\partial x}\left[\frac{\partial\psi_1}{\partial x} - J(\psi_0,\theta_0)\right] + R_0\frac{\partial}{\partial x}[J(\psi_0,\theta_1) + J(\psi_1,\theta_0)] - \frac{Y}{\sigma}\frac{\partial}{\partial z}[J(\psi_0,\mathrm{v}_1) + J(\psi_1,\mathrm{v}_0)] - \frac{R_s}{\tau}\frac{\partial}{\partial x}[J(\psi_0,\phi_1) + J(\psi_1,\phi_0)] + \frac{1}{\sigma}[J(\nabla^2\psi_0,\nabla^4\psi_1) + J(\nabla^2\psi_1,\nabla^4\psi_0)] \quad (37)$$

Let us consider the following periodic solution of the linearized stability problem governed by the above first order system of equations satisfying the boundary conditions (13);

$$\psi_0 = -\frac{2}{\pi a}\sin\pi ax\,.\sin n\pi z, \qquad \mathrm{v}_0 = \frac{2Y}{a\pi^2(a^2+1)}\sin\pi ax\,.\cos n\pi z,$$
$$\theta_0 = \frac{2}{\pi^2(a^2+1)}\cos\pi ax\,.\sin n\pi z, \qquad \phi_0 = \frac{2}{\tau\pi^2(a^2+1)}\cos\pi ax\,.\sin n\pi z. \quad (38)$$

Substituting solution (38), with $n = 1$ (for lowest mode), in equation (36), we get

$$\mathcal{L}\psi_1 = -R_1(2\pi a\sin\pi ax\,.\sin\pi z) = \pi^2a^2R_1\psi_0 \quad (39)$$

where, all the nonlinear terms represented by Jacobians are zero.

Now, equation (39) implies that $R_1 = 0$ when R_1 is to be evaluated to cancel the form of ψ_0 as it is a secular term and shall affect the periodicity of the above solution. Therefore, we must have $\mathcal{L}\psi_1 = 0$ and its solution which satisfy the boundary conditions (13) is now given by;

$$\psi_1 = 0 \quad (40)$$

Using the above value of ψ_1 in the equations obtained for second order system by comparing the coefficients of ε^2, we can deduce the following solutions satisfying the boundary conditions (13);

$$\left.\begin{aligned} \mathrm{v}_1 &= \frac{Y}{2\pi^3a^3\sigma(a^2+1)}\sin 2\pi ax \\ \theta_1 &= -\frac{1}{2\pi^3(a^2+1)}\sin 2\pi z \\ \phi_1 &= -\frac{1}{2\tau^2\pi^3(a^2+1)}\sin 2\pi z \end{aligned}\right\} \quad (41)$$

Hence, using equation (41) in equation (37), we get

$$\mathcal{L}\psi_2 = \left[-\frac{Y^2}{\pi a^3\sigma^2(a^2+1)} + \frac{aR_c^{Steady}}{\pi(a^2+1)} - 2\pi aR_2 - \frac{aR_s}{\pi\tau^3(a^2+1)}\right]\sin\pi ax\,.\sin\pi z + \frac{Y^2}{\pi a^3\sigma^2(a^2+1)}\sin 3\pi ax\,.\sin\pi z + \left[\frac{aR_s}{\pi\tau^3(a^2+1)} - \frac{aR_0}{\pi(a^2+1)}\right]\sin 3\pi z\,.\sin\pi ax. \quad (42)$$

On comparing the value of ψ_0 given in (38), one observes that the first term on the right hand side of (42) is of the form of ψ_0 and hence must vanish. Therefore, we have

$$R_2 = \frac{R_c^{Steady}}{2\pi^2(a^2+1)} - \frac{Y^2}{2\pi^2 a^4 \sigma^2 (a^2+1)} - \frac{R_s}{2\tau^3 \pi^2 (a^2+1)}, \tag{43}$$

where R_c^{Steady} is the critical Rayleigh number at the onset of steady convection with respect to infinitesimal disturbances. From the above expression for Rayleigh number for second order, we observe that viscoelastic parameters have no effects on the finite amplitude steady convection, as have also been observed for the infinite amplitude by linear stability analysis for the steady case.

5. RESULTS AND DISCUSSIONS:

To study the effects of various parameters on the thermal Rayleigh number for steady and oscillatory cases, we have carried out the numerical computations of the respective expressions given by (20) and (23) for some fixed values of $\tau = 0.01$, $\sigma = 7$, $R_s = 10$, $\lambda_1 = 0.75$ and $\lambda_2 = 0.40$ (*cf.* Basu and Layek [13], Chandrasekhar [14], Kiran [18]).

The variations of thermal Rayleigh number for steady case (R^{Steady}) and overstable case (R^{Ov}) for some fixed values of other parameters given above with respect to square of wave number (a^2) is computed numerically and the results are shown in Figure 2. It is observed from the figure that oscillatory convection is the preferred mode in the thermohaline convection problem for viscoelastic fluids in the presence of rotation, when disturbances are infinite. Also, it is evident from the figure that rotation has stabilizing effect on the onset of both steady as well as oscillatory convection. Further one can easily note that in the absence of viscoelastic behaviour of the Newtonian fluid, the value of critical Rayleigh number for oscillatory case (R_c^{Ov}) is higher than for non-Newtonian fluid. Hence, we can conclude that viscoelasticity has destabilizing effect on the onset of thermohaline instability.

Further, the effect of concentration gradient represented by R_s on the Rayleigh number for same fixed values of other parameters with $Y^2 = 100$, has been studied numerically and is depicted graphically in Figure 3 and Figure 4 respectively, for R^{Steady} and R^{Ov}. It is clear from the figures that non-adverse concentration gradient postpones the onset of steady as well as oscillatory convection in viscoelastic fluid layer heated from below.

Figure 5 and Figure 6 show the effects of viscoelastic parameters λ_1 (stress relaxation time) and λ_2 (strain retardation time) on the thermal Rayleigh number (R^{Ov}) for the above fixed values of other parameters and under the condition $\lambda_1 < \lambda_2$ (*cf.* Basu and Layek [13]). It is observed from the figures that stress relaxation time has destabilizing effect whereas strain retardation time has stabilizing effect on the onset of oscillatory thermohaline convection. From Figure 5, we also observed that Oldroydian fluid model is more stable than Maxwellian fluid model ($\lambda_1 > 0, \lambda_2 = 0$).

From above analysis, we concluded that the viscoelastic parameters have the same effects on the double diffusive convection as that in single diffusive (thermal) convection. Also, we concluded that salinity gradient and rotation have usual effects on the onset of convection in viscoelastic fluid layer as in the case of thermohaline convection for Newtonian fluid.

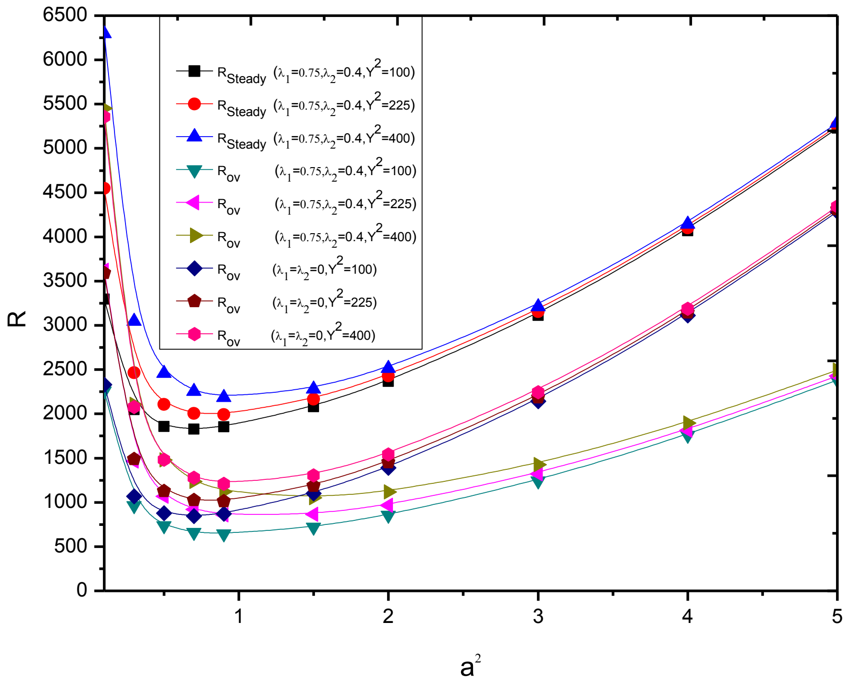

Figure 2. Variations of Rayleigh number with square of wave number for various values of Y^2 with $R_s = 10, \tau = 0.01$ and $\sigma = 7$.

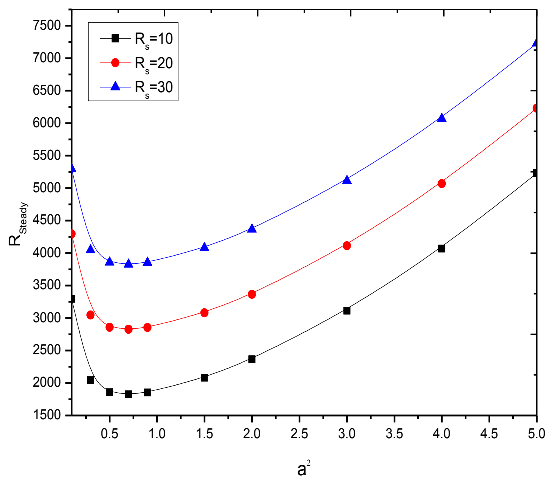

Figure 3. Variations of Rayleigh number with square of wave number for steady convection for various values of R_s with $Y^2 = 100$, $\tau = 0.01$ and $\sigma = 7$.

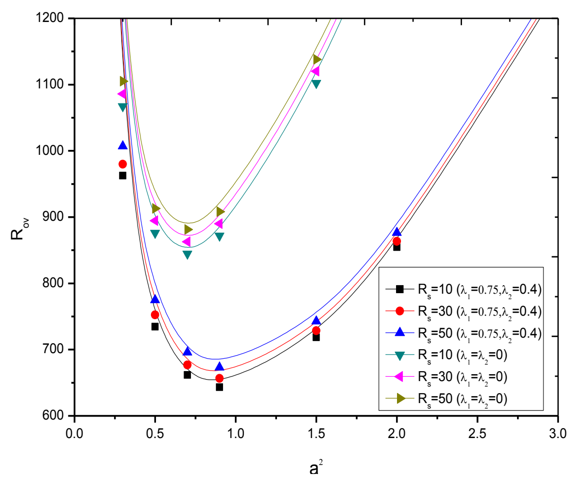

Figure 4. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of R_s with $Y^2 = 100$, $\tau = 0.01$ and $\sigma = 7$.

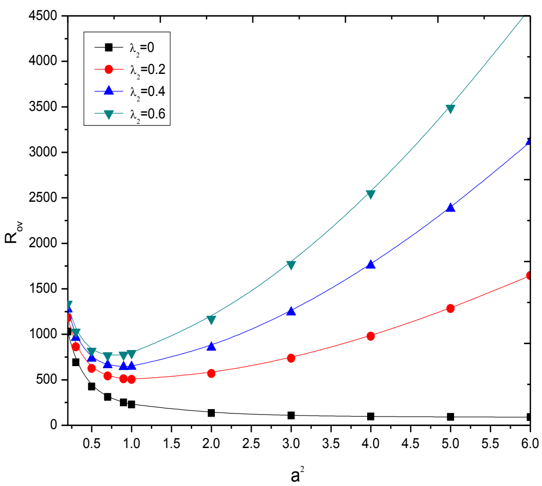

Figure 5. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of λ_2 with $Y^2 = 100$, $R_s = 10$, $\lambda_1 = 0.75$, $\tau = 0.01$ and $\sigma = 7$.

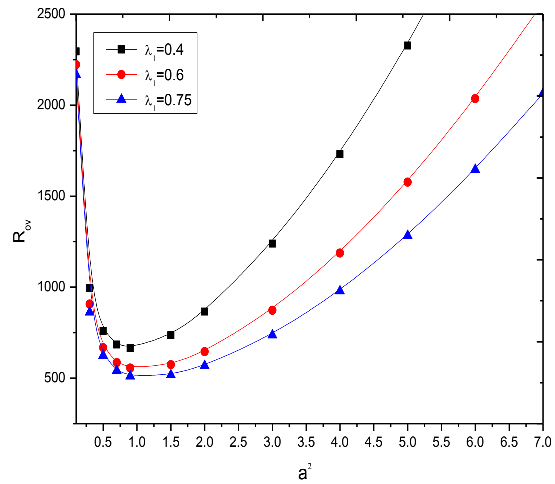

Figure 6. Variations of Rayleigh number with square of wave number for oscillatory convection for various values of λ_1 with $Y^2 = 100$, $R_s = 10$, $\lambda_2 = 0.2$, $\tau = 0.01$ and $\sigma = 7$.

6. CONCLUSIONS:

We have analysed the effects of concentration gradient, stress relaxation time, strain retardation time and rotation in thermohaline convection problem by using linear and non-linear stability analysis against infinite and finite amplitudes, respectively. Following are the main conclusions;

Using linear stability analysis, when disturbances are of infinite amplitudes, we observed from the expressions for Rayleigh numbers for stationary and oscillatory instability in two-dimensional convective rolls that overstability is preferred mode of instability. This means that the onset of instability is through oscillations, before becomes unstable through steady infinite disturbances. Also, concentration gradient (R_S) has stabilizing effect on the onset of convection. This may be due to the enhancement of density of the fluid due to mass concentration, which decreases the heat transfer rate. The stress relaxation time (λ_1) and strain retardation time (λ_2) has no effect on stationary Rayleigh number, however, for an oscillatory mode of convection, λ_1 has destabilizing effect and λ_2 has stabilizing effect on the onset of instability. These behaviour of λ_1 and λ_2 on the onset of convection can be validated from the results obtained by Basu and Layek [13] and Kiran [18]. The Taylor number (Y), which represents the coriolis force has stabilizing effect on the onset of convection, as has also been analysed by Gupta and Gupta [4].

Using non-linear stability analysis, when disturbances are of finite amplitudes, we observed from the expression for Rayleigh number for stationary mode of instability in two-dimensional convective rolls that the viscoelastic parameters λ_1 and λ_2 have no effects on the onset of instability. This behaviour of the viscoelastic parameters is expected in the lines of linear stability analysis with infinite amplitude. Also, the concentration gradient (R_S) and rotation (Y) reinforce each other and have destabilizing effect on the onset of instability. The destabilizing behaviour of R_S contrary to the linear stability case is justified by the reason provided by Kiran[18] that the effect of solutal Rayleigh number is to increase Nusselt number, which means R_S has destabilizing effect in this case. Though the presence of a stabilizing gradient (solute concentration) prevent the onset of convection, the strong finite amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. This effect of rotation on instability has also been proved by Gupta and Gupta [4] in their analysis.

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HYDROMAGNETIC FLOW NEAR AN OSCILLATING WALL WITH CONSTANT/PERIODIC SUCTION

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ABSTRACT:
An analysis of flow of a viscous incompressible and electrically conducting fluid produced by harmonically oscillating wall of infinte extent in presence of transvese magnetic field is considered. Exact solutions for velocity, induced magnetic field, electrical current density and skin friction are obtained when the magnetic pradntl number is unity. The numerical results for velocity, induced magnetic field, electrical current density and skin friction for different parameters involved in the problem are expressed through graphs. The final results are discussed in detail at the last section of the paper.

INTRODUCTION:
The effect of a transversely applied magnetic field on the flow of an electrically conducting viscous fluid has been discussed widely owing to its physical, geophysical and engineering application. Literature survey also reveals that in an electrically conducting viscous fluid subject to an externally applied magnetic field has also received significant attention. Hartmann (1937) analyzed the effect of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid exiting through parallel stationary plates that are insulated. Rassow (1957) considered the impulsive motion of an infinite plate in a viscous incompressible magnetic fluid in the presence of an external magnetic field. The flow of a magnetic field near an infinite flat wall which oscillates in its own plate has been discussed by Ong and Nicholls (1959). Romig (1964) studied the influence of an electric and magnetic field on heat transfer to electrical conducting fluids. Soundalgekar (1965) discussed the hydromagnetic flow near an accelerated plate in the presence of magnetic field. Cramer and Pai (1973) discussed the utility of magneto fluid dynamics for engineers and applied physicists. Hydromagnetic flow near an oscillating porous flat plate under the action of body force was discussed by Kishan and Sharma (1979). An attempt has been made in this paper to study the effect of constant suction and periodic suction in hydromagnetic flow near an oscillating wall. The effect of suction consists in the removal of decelerated fluid particles from the boundary layer before they are given a chance to cause separation. A new boundary layer which is again capable of overcoming a certain adverse pressure gradient is allowed to form in the region behind the slit. With a suitable arrangement of the slits and under the favorable conditions separations can be prevented completely.

MATHEMATICAL FORMULATION OF THE PROBLEM:
We take the coordinate origin o, at an arbitrary point on an infinite wall, which is taken to be an electrical insulator. Cartesian coordinate system has been used with axis ox and oy along and perpendicular to the wall, respectively. The electrostatic system of units has been used throughout, and we assume that all physical quantities as functions of the space coordinate y and time t only. An attempt has been made in this paper to study the effect of constant suction and periodic suction in hydromagnetic flow near an oscillating wall.

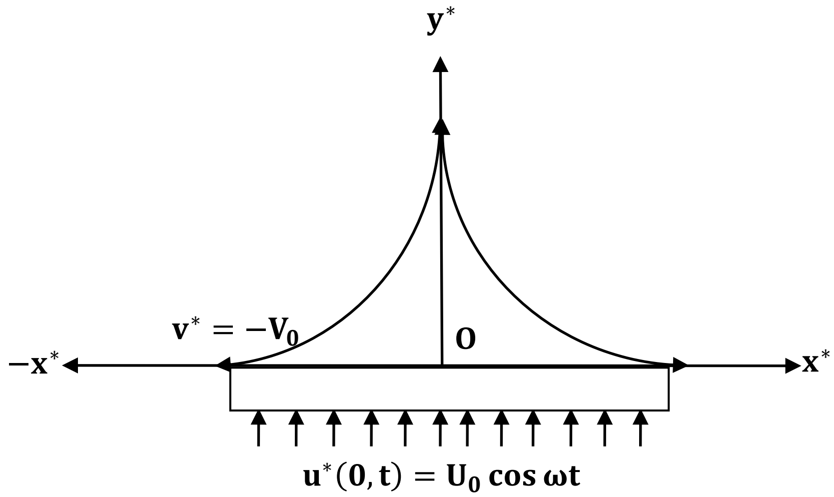

Figure 4A.1 Geometrical Interpretation of the Problem

Using Bossinesq approximation and following Pande et. al. (1975), a fluid model has been developed as follows:

Case 1: Analysis for constant suction, here we have consider the suction of the type
$v^* = -v_0$,

The basic equations governing the flows

Equation of Continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \tag{1}$$

Equations of motion

$$\frac{\partial u^*}{\partial t} - v_0 \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\mu_0}{\rho} H_y^* \frac{\partial H_x^*}{\partial y^*} \tag{2}$$

Equation of magnetic induction

$$\frac{\partial H_x^*}{\partial t} - v_0 \frac{\partial H_x^*}{\partial y^*} = \frac{1}{\sigma\mu_0} \frac{\partial^2 H_x^*}{\partial y^{*2}} + H_y^* \frac{\partial u^*}{\partial y^*} \tag{3}$$

The initial and boundary conditions for the velocity field are exactly the same as those given by Stokes (1851) for the hydrodynamic case: namely,

$$\left.\begin{array}{l} u^*(y^*, 0) = 0, \qquad \text{for } y^* \geq 0 \\ u^*(0,t) = U_0 \cos \omega t,\ u^*(\infty, t) \to 0 \end{array}\right\} \tag{4}$$

where ω denotes the frequency of oscillations of the wall and U_0 the maximum velocity of the wall.

The appropriate boundary conditions on H_x^* are

$$\left.\begin{array}{l} H_x^*(0,t) = 0, \quad H_y^*(0,t) = H_0 \\ H_x^*(\infty, t) \to 0, H_y^*(\infty, t) \to H_0 \end{array}\right\} \tag{5}$$

From Maxwell's equations the components of electrical current density are given by

$$J_x = J_y = 0 \text{ and } J_z = -\left(\frac{\partial H_y^*}{\partial y^*}\right) \tag{6}$$

And the divergence equation for the magnetic field gives

$$H_y^* = \text{constant} = H_0 \tag{7}$$

where H_0 is externally applied transverse magnetic field.

Substituting equation (7) in equation (2), (3) and (6), and introducing the following non-dimensional quantities

$y = y^* \left(\frac{\omega}{2\nu}\right)^{1/2}, H = H_x^* \left(\frac{\mu_0}{\rho}\right)^{1/2}, \lambda = \frac{V_0}{\sqrt{2\nu\omega}}, M = H_0 \left(\frac{\mu_0}{\rho}\right)^{1/2} (2\omega\nu)^{-1/2}, P_m = \nu\rho\mu_0,$

$\alpha = \left(\sigma\mu_0\right)^{-1}.$

By using above non-dimensional quantities in equations (2), (3) and (6), we get the following non-dimensional set of governing equations,

$$\frac{\partial^2 u}{\partial y^2} - \frac{2}{\omega}\frac{\partial u}{\partial t} + 2M\frac{\partial H}{\partial y} + 2\lambda\frac{\partial u}{\partial y} = 0 \tag{8}$$

$$\frac{1}{P_m}\frac{\partial^2 H}{\partial y^2} - \frac{2}{\omega}\frac{\partial H}{\partial t} + 2M\frac{\partial u}{\partial y} + 2\lambda\frac{\partial H}{\partial y} = 0 \tag{9}$$

$$J_z = -\left(\frac{\rho\omega}{2\mu_0\upsilon}\right)^{1/2}\frac{\partial H}{\partial y} \tag{10}$$

where P_m represents the magnetic prandtl number.

Following Pande et al. (1976), we seek the solution of the equation (8) and (9) as follows

$$\left.\begin{aligned} u(y,t) &= U_0\cos(\omega t)f(y) \\ H(y,t) &= U_0 os(\omega t)g(y) \end{aligned}\right\} \tag{11}$$

with periods $\frac{2\pi}{\omega}$, substituting (11) into (8) and (9), we find that f(y) and g(y) must satisfy the equation

$$\frac{\partial^2 f}{\partial y^2} - 2if + 2M\frac{\partial g}{\partial y} + 2\lambda\frac{\partial f}{\partial y} = 0 \tag{12}$$

$$\frac{1}{P_m}\frac{\partial^2 g}{\partial y^2} - 2ig + 2M\frac{\partial f}{\partial y} + 2\lambda\frac{\partial g}{\partial y} = 0 \tag{13}$$

which are to be solved under the boundary conditions

$$\left.\begin{aligned} &f(0,t) = 1, \quad g(o,t) = 0 \\ &f(\infty,t) \to 0, g(\infty,t) \to 0 \end{aligned}\right\} \tag{14}$$

If we assume the magnetic Prandtl number $P_m = 1$, then the appropriate solution of (12) and (13) satisfying the boundary conditions (14), are

$$f = \frac{1}{2}\left[e^{-(a+\lambda+M)\eta}e^{-\frac{iy}{a}} + e^{-(b+\lambda+M)\eta}e^{-\frac{iy}{b}}\right] \tag{15}$$

$$g = \frac{1}{2}\left[e^{-(a+\lambda+M)\eta}e^{-\frac{iy}{a}} - e^{-(b+\lambda+M)\eta}e^{-\frac{iy}{b}}\right] \tag{16}$$

where $2a^2 = (M+\lambda)^2 + \{(M+\lambda)^4 + 4\}^{1/2}$ and

$2b^2 = (M-\lambda)^2 + \{(M-\lambda)^4 + 4\}^{1/2}$

Thus the expression for the velocity and induced magnetic field are given by

$$\frac{u}{U_0} = \frac{1}{2}\left[e^{-(a+\lambda+M)y}\cos\left(\omega t - \frac{y}{a}\right) + e^{-(b+\lambda+M)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \tag{17}$$

$$\frac{H}{U_0} = \frac{1}{2}\left[e^{-(a+\lambda+M)y}\cos\left(\omega t - \frac{y}{a}\right) - e^{-(b+\lambda+M)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \tag{18}$$

The skin friction in presence of magnetic field

$$\tau = \mu\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = -\frac{\mu U_0}{2}\left[(a+b+2\lambda)\cos\omega t - \left(\frac{1}{a}+\frac{1}{b}\right)\sin\omega t\right] \tag{19}$$

Ratio of skin friction at the wall in the presence and absence of magnetic field is

$$\frac{\tau}{\tau_0} = \frac{\left[(a+b+2\lambda)\cos\omega t - \left(\frac{1}{a}+\frac{1}{b}\right)\sin\omega t\right]}{2\left[(a+\lambda)\cos\omega t - \frac{1}{a}\sin\omega t\right]} \tag{20}$$

where τ and τ_0 represents, the skin friction in presence and absence of the magnetic field.

On substituting the value of H from equation (18) into (10), we get the expression for electric current density as

$$\frac{J_z}{U_0} = \frac{1}{2}\left[\left\{(a+M+\lambda)e^{-(a+\lambda+M)y} - (b+M+\lambda)e^{-(b+\lambda-M)y}\right\}\cos\left(\omega t - \frac{y}{a}\right) - \left\{\frac{1}{a}e^{-(a+\lambda+M)y} - \frac{1}{b}e^{-(b+\lambda-M)y}\right\}\sin\left(\omega t - \frac{y}{b}\right)\right] \quad (21)$$

Case 2: Analysis for periodic suction, when $v = -v_0\cos(\omega t)$,
If we assume the magnetic Prandtl number $P_m = 1$, then the appropriate solution of the equation (12) and (13) are

$$f = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{a}} + e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{b}}\right] \quad (22)$$

$$g = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{a}} - e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}e^{-\frac{iy}{b}}\right] \quad (23)$$

Where $2a^2 = \left(M + V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\right)^2 + \left\{(M + V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}})^4 + 4\right\}^{1/2}$ and

$$2b^2 = \left(M - V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\right)^2 + \left\{(M - V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\lambda)^4 + 4\right\}^{1/2}$$

Thus the expression for the velocity and induced magnetic field are given by

$$\frac{u}{U_0} = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}\lambda+M\right)y}\cos\left(\omega t - \frac{y}{a}\right) + e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \quad (24)$$

$$\frac{H}{U_0} = \frac{1}{2}\left[e^{-\left(a+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{a}\right) - e^{-\left(b+V_0\cos\omega t\sqrt{\frac{2}{\nu\omega}}+M\right)y}\cos\left(\omega t - \frac{y}{b}\right)\right] \quad (25)$$

RESULTS AND DISCUSSION:

The oscillation of the wall will produce wave-like disturbances within the boundary layer. The velocity profile has the form of damped harmonic oscillations, i.e., the velocity decays exponentially as the distance from the wall increases. In the neighborhood of the wall, its amplitude decreases by the presence of the magnetic field. A fluid layer at a distance y has a phase lag given by $y/a = {y^*\left(\frac{\omega}{2\nu}\right)^{1/2}}/{a}$ with respect to the oscillations of the wall; which depends on the strength of the applied magnetic field besides the frequency of oscillation and the viscosity of the fluid. This phase lag decreases as the strength of the magnetic field increases for given ω and ν. Conversely the smaller the strength of the applied magnetic field and the viscosity of the fluid and higher the frequency, the faster the rate of decrease of velocity with the increase in y, for given U_0 . This fact is evident from figure 1 which represents the motion for several instant of time.

The induced magnetic field have phase lags given by ${y\left(\frac{\omega}{2\nu}\right)^{1/2}}/{a}$, similar to the one discussed earlier for the case of velocity profile. Also, we observe that near the wall, the velocity amplitudes increases with Hartmann number(M). Table 1 shows the periodic variation of the velocity.

It is also useful to note that, in the absence of suction parameter ($\lambda = 0$), our results become identical with the results of Pande et al. (1975) as shown in figure 6.

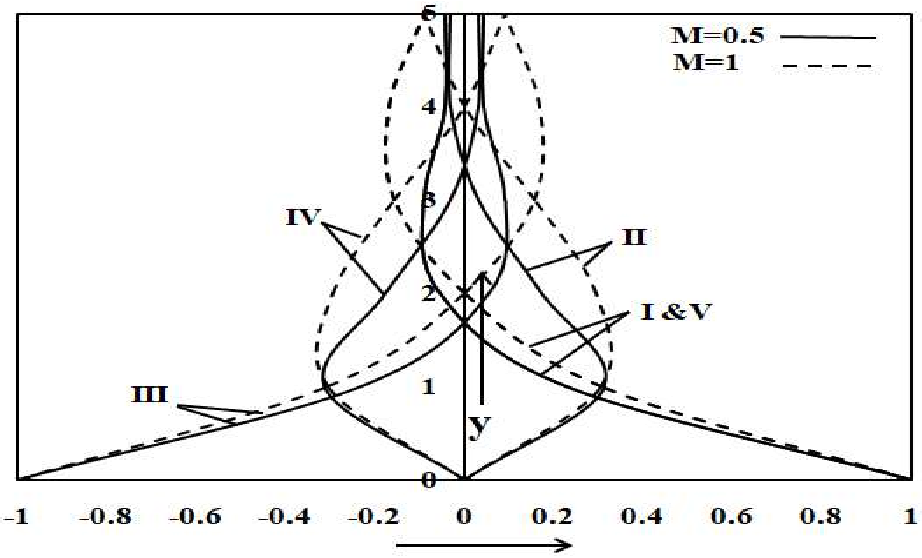

Figure 1. Velocity profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0}$.

$\mathbf{u/U_0}$

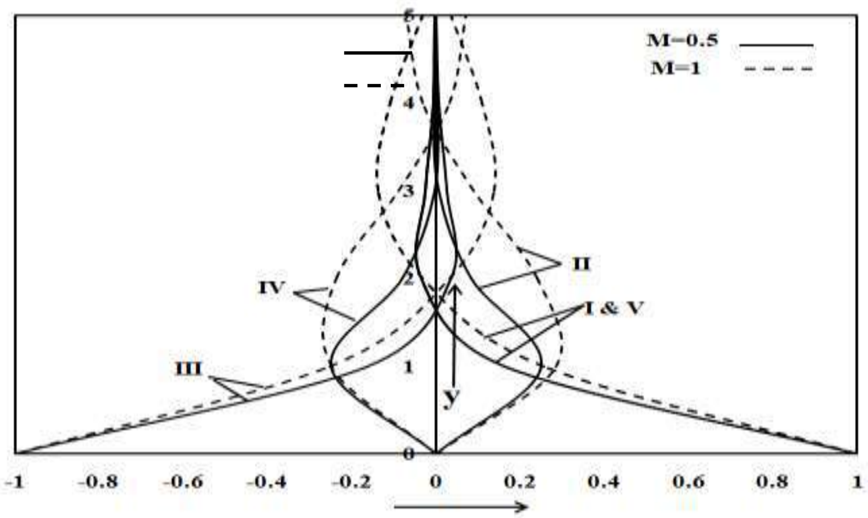

Figure 2. Velocity profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0.2}$.

$\mathbf{u/U_0}$

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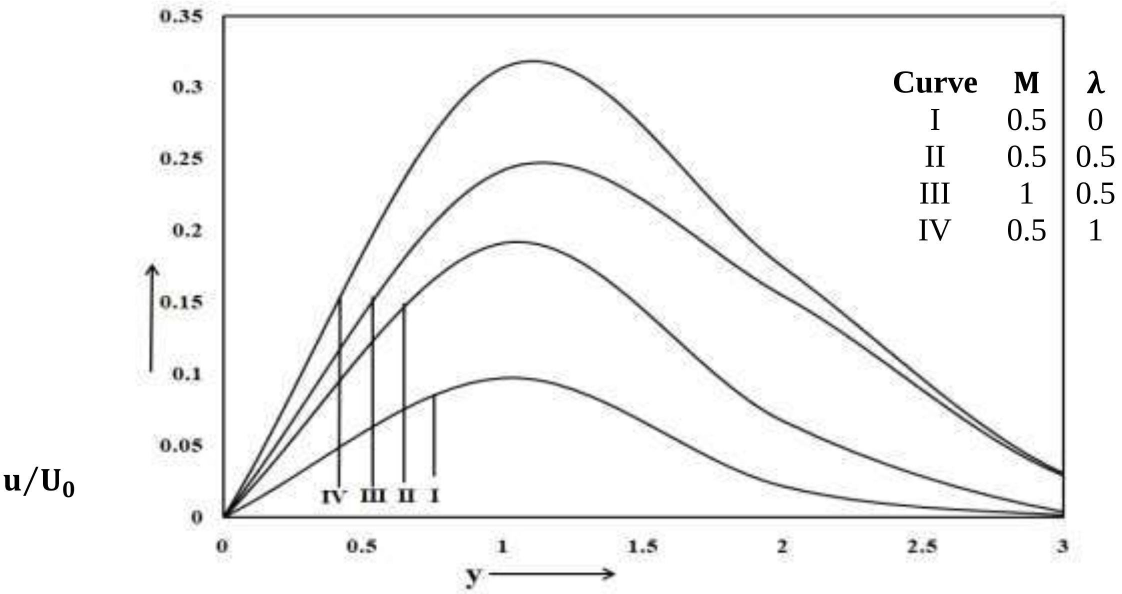

Figure 3. Velocity profiles for different values of λ and M.

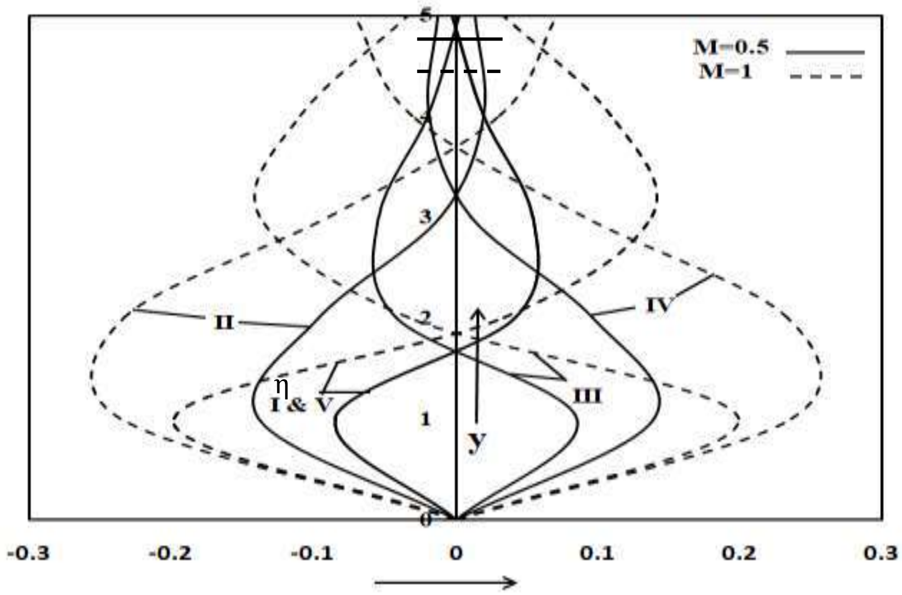

Figure 4. Induced magnetic field profiles with y for $M = 0.5, 1$ and $\lambda = 0.2$.

H/U_0

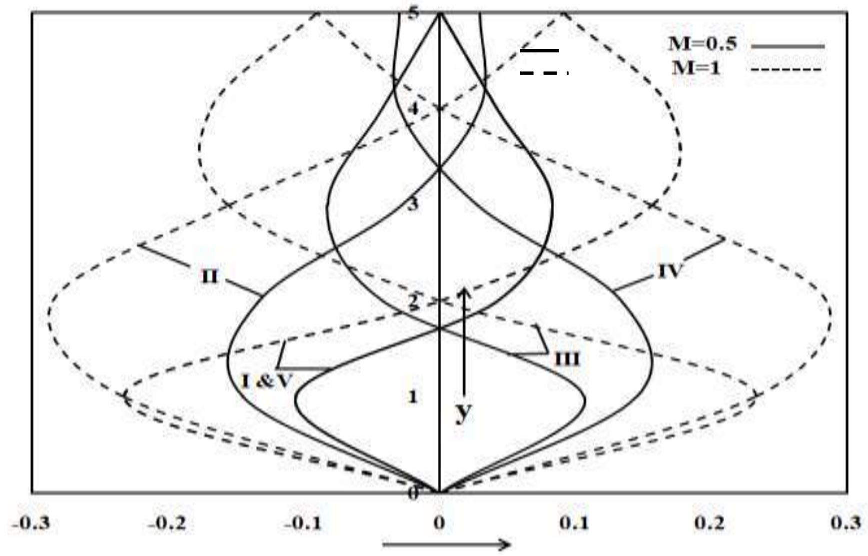

Figure 5. Induced magnetic field profiles with y for $\mathbf{M = 0.5, 1}$ and $\boldsymbol{\lambda = 0}$.

$\mathbf{H/U_0}$

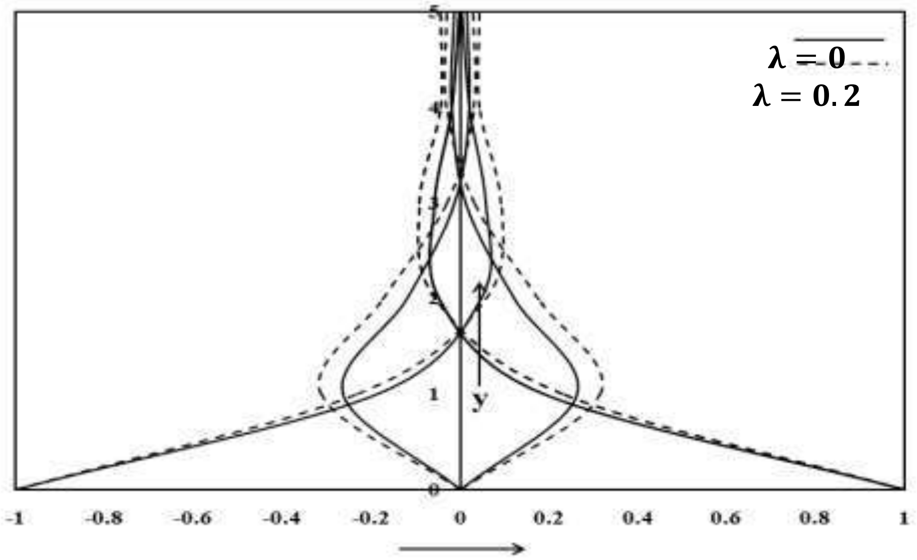

Figure 6. Velocity profiles with y for $\mathbf{M = 1}$, $\boldsymbol{\lambda = 0}$ and $\boldsymbol{\lambda = 0.2}$.

$\mathbf{u/U_0}$

Table 1. Variation of velocity for different values of t.

t/y	0	1	2	3	4	5
0	0.5	0.1274	-0.01619	-0.0689	-0.00546	-0.01673
1	0.3535	0.1984	0.08171	-0.01337	-0.05363	-0.04448
2	0	0.1530	0.07818	-0.00025	-0.04554	0.14776

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TRIPLE-DIFFUSIVE CONVECTION IN A RIVLIN-ERICKSEN FLUID LAYER IN POROUS MEDIUM

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ABSTRACT:
The onset of Triple-Diffusive convection in Rivlin-Ericksen fluid with varying gravity field is studied analytically in the presence of uniform vertical magnetic field in porous medium. For the case of stationary convection, the magnetic field, varying gravity field and the stable solute gradients have stabilizing effects whereas the medium permeability has destabilizing (or stabilizing) effect on the system under certain conditions. A linear stability analysis theory and normal mode analysis method have been carried out to study the onset convection. The kinematic viscoelasticity has no effect on the stationary convection. The solute gradients, magnetic field, varying gravity field, porosity and kinematic viscoelasticity introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained. The results are shown graphically.

KEYWORDS: Rivlin-Ericksen Fluid, Solute Gradients, Triple-Diffusive Convection, Varying Gravity Field.

1. INTRODUCTION:

The theoretical and experimental results of the onset of thermal instability in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics have been treated by Chandrasekhar[8]. Veronis [5] studied the problem of thermohaline convection in a layer of fluid heated and soluted from below.

With the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry, the investigations on such fluids are desirable. The Rivlin-Ericksen fluid [6] is one such fluid. The idealization of uniform gravity field can be hardly justified in the presence of large scale convection phenomenon occurring in atmosphere, the ocean or the mantle of the earth.

Pradhan et al [4] studied the thermal instability of the fluid layer under variable gravitational field.

A comprehensive review of the literature concerning thermosolutal convection in a fluid-saturated porous medium may be found in the book written by Nield and Bejan [3]. The thermosolutal instability in Walters' B' fluid in the presence of Hall currents in porous medium in hydromagnetics has been studied by Kango et al [9]. S. Chand [7] studied triple-diffusive convection in Walters' (Model B') fluid in porous medium in hydromagnetics.

The solution behaviour in the double-diffusive convection problem is more interesting than that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Benard problem. When temperature and two or more agencies, or two different salts, are present, the physical and mathematical situation becomes increasingly richer. Very interesting results in triply-diffusive convection have been obtained by Pearlstein et al., [1]. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker [2] derive the equation for non-Boussinesq convection in a multi-component fluid and investigate the situation analogous to that of Pearlstein et al., but allowing for a density non-linear in a temperature field. In reality the density of a fluid is never a linear function of temperature, and so the work of Straughan and Walker applies to the general situation where the equation of state is one of the density quadric in temperature. This is important, since they find that departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.

Keeping in view the recent increase in the number of non iso-thermal situations, our interest, in the present paper is to study the triple-diffusive convection in Rivlin-Ericksen Fluid with varying gravity field in porous medium.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider an infinite, horizontal, incompressible layer of thickness d of an electrically non-conducting Rivlin-Ericksen fluid heated and salted from below. The temperature T and solute concentrations C^1 and C^2 at the bottom and top surfaces $z=0,d$ are T_0 and T_1; C_0^1, C_1^1 and C_0^2, C_1^2 respectively, and a uniform temperature gradient β $(=|dT/dz|)$ and uniform solute gradients are $\beta'(=|dC^1/dz|)$ and $\beta''(=|dC^2/dz|)$ are maintained. A uniform magnetic field $\mathbf{H}=(0,0,H)$ and the gravity field $\mathbf{g}=(0,0,-g)$, where $\mathbf{g}=g_0\lambda$ $(g_0>0)$ is the value of g at z=0 and λ can be +ve or –ve according to whether gravity increases or decreases upwards from its value g_0; pervades the system. The equations relevant to the problem are:

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right]=-(\frac{1}{\rho_0})\nabla p+\mathbf{g}(1+\frac{\delta\rho}{\rho_0})-\frac{1}{k}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\mathbf{q}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\mathbf{H})\times\mathbf{H}, \tag{1}$$

$$\nabla.\mathbf{q}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T=\kappa\nabla^2 T, \tag{3}$$

$$E'\frac{\partial C^1}{\partial t}+(\mathbf{q}.\nabla)C^1=\kappa'\nabla^2 C^1, \tag{4}$$

$$E''\frac{\partial C^2}{\partial t}+(\mathbf{q}.\nabla)C^2=\kappa''\nabla^2 C^2. \tag{5}$$

The density equation of state is

$$\rho=\rho_0\left[1-\alpha(T-T_a)+\alpha'(C^1-C_a^1)+\alpha''(C^2-C_a^2)\right], \tag{6}$$

where $\rho,\rho_0,\mathbf{q},t,\mathbf{g},\nu,\nu',\kappa,\kappa',\kappa'',\alpha,\alpha',\alpha''$ are the fluid density, reference density, velocity, time, gravitational acceleration, the kinematic viscosity, the kinematic viscoelasticity, the thermal diffusivity, the solute diffusivities κ' and κ'', thermal and solvent coefficients of expansion

α' and α'' respectively. T_a is the average temperature given by $T_a = (T_0 + T_1)/2$ where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer and C_a^1, C_a^2 are the average concentrations given by $C_a^1 = (C_0^1 + C_1^1)/2$ and $C_a^2 = (C_0^2 + C_1^2)/2$, where C_0^1, C_1^1 and C_0^2, C_1^2 are the constant average concentrations of the lower and upper surfaces of the layer. Maxwell's equations yield

$$\varepsilon \frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2\mathbf{H}, \tag{7}$$

$$\nabla.\mathbf{H} = 0. \tag{8}$$

Here $E = \varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C_i}$ is a constant and E', E'' are analogous to E but corresponding to solute rather that heat. ρ_s, C_s and ρ_0, C_i stand for density and heat capacity of solid and fluid respectively. The steady state solution is

$$\mathbf{q} = (0,0,0), T = -\beta z + T_a, C^1 = -\beta' z + C_a^1, C^2 = -\beta'' z + C_a^2, \beta = (T_0 - T_1)/d,$$

$$\beta' = (C_1^1 - C_0^1)/d, \beta'' = (C_1^2 - C_0^2)/d, \rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z - \alpha''\beta'' z). \tag{9}$$

The change in density $\delta\rho$, caused by the perturbations θ in temperature and γ, γ' in concentrations, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma - \alpha''\gamma'). \tag{10}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -(\frac{1}{\rho_0})\nabla\delta p - g_0\lambda(\alpha\theta - \alpha'\gamma - \alpha''\gamma') - \frac{1}{k}(\nu + \nu'\frac{\partial}{\partial t})\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \mathbf{h}) \times \mathbf{H}, \tag{11}$$

$$\nabla.\mathbf{q} = 0, \tag{12}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{13}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma, \tag{14}$$

$$E''\frac{\partial\gamma'}{\partial t} = \beta'' w + \kappa''\nabla^2\gamma', \tag{15}$$

$$\varepsilon\frac{\partial h}{\partial t} = (\mathbf{H}.\nabla) + \varepsilon\eta\nabla^2\mathbf{h}, \tag{16}$$

$$\nabla.\mathbf{h} = 0. \tag{17}$$

3. DISPERSION RELATION:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, \gamma', h_z] = [W(z), \Theta(z), K(z), \Gamma(z), \Psi(z)]\exp i(ik_x x + ik_y y + nt), \tag{18}$$

where k_x, k_y are the wave numbers along the x and y directions, respectively and $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number and n is, in general, a complex constant. Eq. (11)-(17), using (18), in non-dimensional form become

$$[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+F\sigma)](D^2-a^2)W+\frac{g_0\lambda a^2 d^2}{\nu}g_0\lambda(\alpha\Theta-\alpha'\Gamma-\alpha''\Psi)-\frac{\mu_e H}{4\pi\rho_0\nu}(D^2-a^2)DK=0, \tag{19}$$

$$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW, \tag{20}$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \tag{21}$$

$$(D^2-a^2-E'q_1\sigma)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W, \tag{22}$$

$$(D^2-a^2-E''q_2\sigma)\Psi=-\left(\frac{\beta'' d^2}{\kappa''}\right)W, \tag{23}$$

where $P_l=\frac{k_1}{d^2}$, is the non-dimensional parameter for medium permeability; $p_1=\frac{\nu}{k_1}$, is the thermal Prandtl number;
$q_1=\frac{\nu}{\kappa'}, q_2=\frac{\nu}{\kappa''}$, are the analogous Schmdit numbers accounting for two solutes, respectively and $a=kd, \sigma=\frac{nd^2}{\nu}, z=zd, \frac{d}{dz}=D.$

The appropriate boundary conditions w.r.t. which eqns. (19)-(23) must be solved (Chandrasekhar [8]).

$W=D^2W=0, \Theta=\Gamma=\Psi=0,$ at $z=0$ and $z=1$ and $K=0$ on the perfectly conducting boundaries and h_x, h_y, h_z are continues. (24)

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres.Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and $z=1$ and hence the proper solution of W characterizing the lowest mode is

$W=W_0\sin\pi z$, where W_0 is a constant. (25)

Eliminating Θ, Γ, K, Ψ between equations (19)-(23) and substituting (25), we obtained the dispersion relation

$$R_1=\frac{1+x}{x\lambda}[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-iF\sigma_1](1+x+iEp_1\sigma_1)+\frac{Q_1}{\lambda}[\frac{(1+x)(1+x+iEp_1\sigma_1)}{x(1+x+ip_2\sigma_1)}+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q_1\sigma_1)}+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE''q_2\sigma_1)}, .. \tag{26}$$

where $R_1=\frac{g\alpha\beta d^4}{\nu\kappa\pi^4}$, is the Rayleigh number; $S_1=\frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}, S_2=\frac{g\alpha''\beta'' d^4}{\nu\kappa''\pi^4}$ are the analogous solute Rayleigh numbers; $Q_1=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}$, is the Chandrasekhar number; $x=\frac{a^2}{\pi^2}$, is the dimensionless wave number and $P_l=\frac{P}{\pi^2}$

and $i\sigma_1=\frac{\sigma}{\pi^2}$.

Equation (26) the required dispersion relation studying the effects of magnetic field, variable gravity field, medium permeability, kinematic viscoelasticity and stable solute gradients on the triple-diffusive convection of Rivlin-Ericksen fluid in porous medium.

4. STATIONARY CONVECTION

For the case of stationary convection, σ = 0 and Equation (26) reduces to

$$R_1 = \frac{(1+x)^2}{xP\lambda} + Q_1\frac{1+x}{x\lambda} + S_1\lambda + S_2\lambda, \tag{27}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters Q_1, S_1, S_2 and P. For stationary convection the parameter F accounting for the kinematic viscoelasticity effect vanishes and thus the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. Equation (27) yields

$$\frac{dR_1}{dS_1} = +1,\ \frac{dR_1}{dS_2} = +1, \tag{28}$$

which imply that the stable solute gradients have a stabilizing effect on the system.

$$\frac{dR_1}{dQ_1} = \frac{1+x}{x\lambda}, \tag{29}$$

The magnetic field, therefore, has a stabilizing effect on the system when gravity is increasingly upwards ($\lambda > 0$) and destabilizes the system when gravity is decreasing upwards.

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xP\lambda}, \tag{30}$$

which is always negative, showing destabilizing effect of medium permeability and for constant varying gravity. The dispersion relation (27) is analyzed numerically.

In Fig.1, R_1 is plotted against x for $Q_1 = 30,\ P = 1,\ \lambda = 2,\ S_2 = 100\ \&\ S_1 = 100, 200, 300$. The stabilizing role of solute gradient is clear from the increase in the Rayleigh number with increasing parameter value S_1.

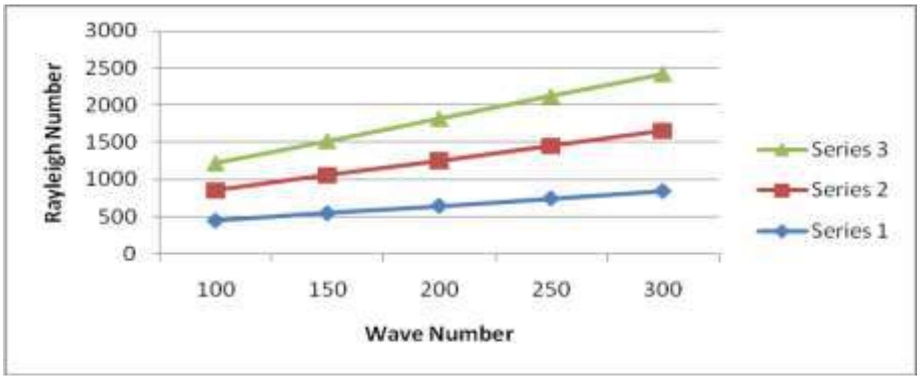

Fig. 1

In Fig.2, R_1 is plotted against x for $Q_1 = 30,\ P = 1,\ \lambda = 2,\ S_1 = 100\ \&\ S_2 = 100, 200, 300$. The stabilizing role of solute gradient is clear from the increase in the Rayleigh number with

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increasing parameter value S_2.

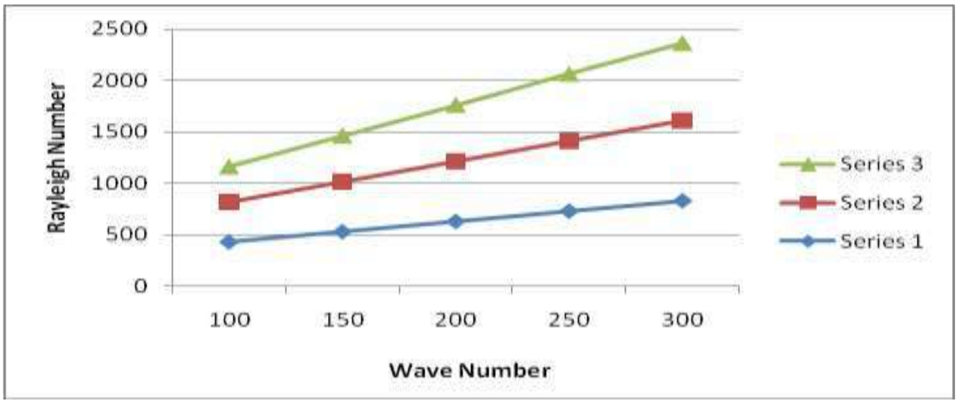

Fig. 2

In Fig.3, R_1 is plotted against x for $P=1,\ \lambda=2,\ S_1=100,\ S_2=100$ and $Q_1=30,60,90$. The stabilizing role of magnetic field is clear from the increase in the Rayleigh number with increasing parameter value Q_1.

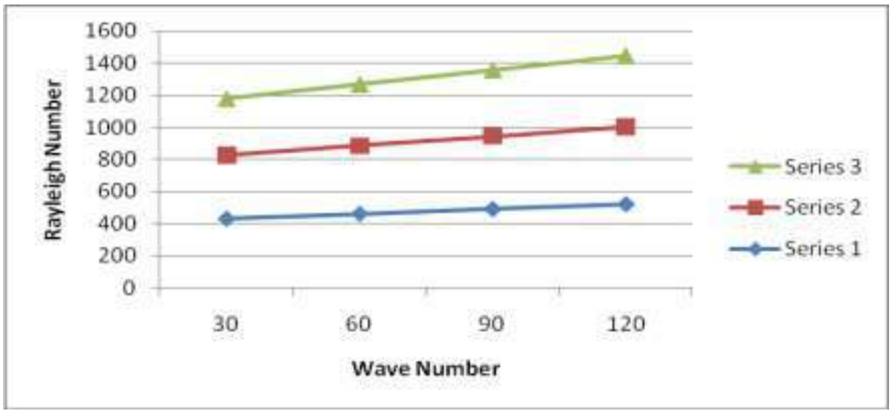

Fig. 3

In Fig.4, R_1 is plotted against x for $Q_1=30,\ \lambda=2,\ S_1=100,\ S_2=100$ and $P=2,4,6$. The destabilizing role of medium permeability is clear from the decrease in the Rayleigh number with increasing parameter value S_2.

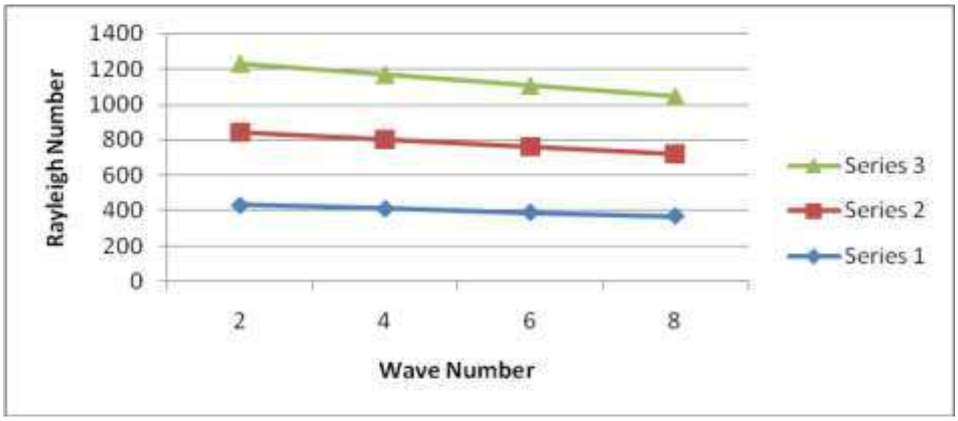

Fig. 4

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES:

Here we examine the possibility of oscillatory modes, if any, in the stability problem due to the presence of magnetic field, varying gravity field, kinematic viscoelasticity and stable solute gradients. Multiplying Eq. (19) by W*, the complex conjugate of W, integrating over the range of z and making use of Eq. (23) - (27) together with the boundary conditions (29) and putting $\sigma = \sigma_r + i\sigma_i$ & equating the real and imaginary parts, we obtain

$$[(\frac{1}{\varepsilon} - \frac{F}{P_l})I_1 - (\frac{g_0\lambda\alpha\kappa a^2}{\nu\beta})Ep_1I_3 + (\frac{g_0\lambda\alpha'\kappa' a^2}{\nu\beta'})E'q_1I_5 + (\frac{g_0\lambda\alpha''\kappa'' a^2}{\nu\beta''})E''q_2I_7 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9]\sigma_r$$
$$= -[\frac{I_1}{P_l} - \frac{g_0\lambda\alpha\kappa a^2}{\nu\beta}I_2 + \frac{g_0\lambda\alpha'\kappa' a^2}{\nu\beta'}I_4 + \frac{g_0\lambda\alpha''\kappa'' a^2}{\nu\beta''}I_6 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8], \quad (31)$$

$$[(\frac{1}{\varepsilon} - \frac{F}{P_l})I_1 + (\frac{g_0\lambda\alpha\kappa a^2}{\nu\beta})Ep_1I_3 - (\frac{g_0\lambda\alpha'\kappa' a^2}{\nu\beta'})E'q_1I_5 - (\frac{g_0\lambda\alpha''\kappa'' a^2}{\nu\beta''})E''q_2I_7 - \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9]\sigma_i = 0, \quad (32)$$

The integrals $I_1, ..., I_9$ are all positive definite, where

$$I_1 = \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz, I_2 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz, I_3 = \int_0^1 |\Theta|^2 dz, I_4 = \int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz, I_5 = \int_0^1 |\Gamma|^2 dz,$$
$$I_6 = \int_0^1 \left(|D\Psi|^2 + a^2|\Psi|^2\right)dz, I_7 = \int_0^1 |\Psi|^2 dz, I_8 = \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz, I_9 = \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz,$$

It follows from equation (31) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (32) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, varying gravity field, magnetic field and stable solute gradients which were non-existent in their absence.

6. THE CASE OF OVERSTABILITY

Here we discuss the possibility of whether instability may occur as overstability. If we equate real and imaginary parts of (26) and eliminate R_1 between them, we obtain

$$A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0, \quad (33)$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and $A_3 = b(E'q_1E''q_2)^2(\frac{1}{\varepsilon} - \frac{F}{P})^2[\frac{Ep_1}{P} + b(\frac{1}{\varepsilon} - \frac{F}{P})]$ (34)

$$A_0 = b^5[\frac{b}{P^2}(\frac{1}{\varepsilon}-\frac{F}{P})+\frac{Ep_1}{P^3}]+\frac{b^3}{P^2}(b-1)[S_1\lambda(Ep_1-E'q_1)+S_2\lambda(Ep_1-E''q_2)])^2+\frac{b^3}{P^2}Q_1(Ep_1-p_2) \quad (35)$$

As σ_1 is real for instability, the four values of $c_1\left(=\sigma_1^2\right)$ must be positive. The product of the roots of (33) is $-A_0 / A_3$, which is possible if $Ep_1 > E'q_1, Ep_1 > E''q_2, Ep_1 > p_2, \lambda > 0$ and $\frac{F}{P} < \frac{1}{\varepsilon}$ (36)

which implies that $E'\kappa < E\kappa', E''\kappa < E\kappa'', \kappa < E\eta, \lambda > 0$ and $\nu < \frac{k_1}{\varepsilon}$. (37)

Thus Equations (37) are, therefore, the necessary conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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EIGENVALUE APPROACH TO GERNELIZED PIEZOTHERMOELASTICITY IN TRANSVERSELY ISOTROPIC CONTINUUM WITH A CONTINOUS POINT SOURCE

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ABSTRACT:
The two-dimensional problem of piezothermoelasticity has been considered to investigate the disturbance in homogeneous, transversely isotropic (6mm class) generalized cylindrical piezothermoelastic continuum subjected to continuous mechanical load acting on thermally insulated and electrically shorted surface. Integral transform technique has been adopted, namely: Laplace transform for time variable and Hankel transform for space variables. These transforms technique have been employed to express the boundary conditions in the transformed domain. The formal solutions are employed to obtain the system of simultaneous linear algebraic equations. These systems of equations are solved by using Gauss elimination process for the unknowns. These values of unknowns are used to find the expressions of displacements, temperature change, electric potential, stresses and electric displacement in the transformed domain. The inverse transform integrals are evaluated by using numerical technique. Temperature, normal stress and shear stress so obtained in the physical domain, are computed numerically from the relevant expressions and relations code for PZT-5A material. Finally, the illustration of the results for classical and non-classical models of thermoelasticity has been presented graphically.

KEYWORDS: Generalized piezothermoelastic; Integral transforms; Relaxation time; Eigen value; Electrically shorted.

INTRODUCTION:
Composite structures can be exposed to variety of fields in the different environment. The knowledge of its response to the different loads is necessary to predict its capability to the elastodynamical stresses induced by sudden loading, which is essential for the proper and safe design. The general solution for transversely isotropic piezothermoelastic materials has been used by Hou et al. [1] to construct the three-dimensional solution of a steady point heat source acting on the apex of cone. Non-linear transient behavior of a piezothermoelastic laminate deviated from equilibrium state has been analyzed by Watanabe et al. [2]. The response of a functionally graded, radially polarized hollow cylinder under dynamic axisymmetric loadings has been studied by Babaei and Chen [3]. The one-dimensional problem of functionally graded medium excited by moving heat source has been investigated by Babaei and Chen [4].

Ailawalia and Khurana [5] studied the deformation of transversely isotropic piezoelectric medium with an overlying infinite viscous fluid due to moving load acting at the interface of both media by applying Fourier transforms. The bending behavior of a circularly curved, functionally graded piezoelectric cantilever actuator under an applied electric load and heat conduction has been studied by Zaman et al. [6]. Akbarzadeh et al. [7] investigated the dynamic response of a functionally graded piezoelectric medium (FGPM) subjected to a

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moving heat source. The thickness response of a piezothermoelastic panel, which is occupied by a material of hexagonal crystal class, has been studied by Montanaro [8] under quasi-static boundary conditions with one of the bounding surface subjected to prescribed temperature.

FORMULATION OF THE MODEL:

We consider a homogeneous, transversely isotropic, thermally conducting, linear generalized piezothermoelastic cylindrical medium. Initially, the medium is assumed to be undisturbed and at uniform temperature T_0. The origin of cylindrical co-ordinate system (r, θ, z) is taken at any point on the surface $(z = 0)$ and $z-$axis pointing vertically downward into the medium. It is assumed that disturbances are caused by continuous point mechanical load acting at the surface $z = 0$. Due to symmetry, all the field quantities are independent of θ co-ordinate. The governing field equations of linear piezothermoelasticity for homogeneous, transversely isotropic (6mm class) axisymmetric cylindrical body upon employing Lord and Shulman [9] and Green and Lindsay [10] models of thermoelasticity, in the absence of charge density, heat sources and body forces; are given by Thakur et al. [11]. This system of equations can be expressed in the matrix form as:

$$\left(\mathbf{A}D^2 + \mathbf{B}^{**}D + \mathbf{C}^{**}\right)\mathbf{Z} = 0 \tag{1}$$

$$\boldsymbol{\sigma}^{**} = \left(\mathbf{A}^{*}D + \mathbf{B}^{\oplus}\right)\mathbf{Z}$$

where

$$\boldsymbol{\sigma}^{**} = \begin{bmatrix}\sigma_{rr} & \sigma_{zz} & \sigma_{rz} & D_z\end{bmatrix}^T \tag{2}$$

$$\mathbf{Z} = \begin{bmatrix}u & w & \phi & T\end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} c_2 & 0 & 0 & 0 \\ 0 & c_1 & 1 & 0 \\ 0 & 1 & -\eta_3 & 0 \\ 0 & 0 & 0 & \overline{K} \end{bmatrix}$$

$$\mathbf{B}^{**} = \begin{bmatrix} 0 & c_3\dfrac{\partial}{\partial r} & e_1\dfrac{\partial}{\partial r} & 0 \\ c_3\nabla_2 & 0 & 0 & -\overline{\beta}\left(1 + t_1\delta_{2k}\dfrac{\partial}{\partial t}\right) \\ e_1\nabla_2 & 0 & 0 & p_1\left(1 + t_1\delta_{2k}\dfrac{\partial}{\partial t}\right) \\ 0 & -\varepsilon\overline{\beta}\left(\dfrac{\partial}{\partial t} + t_0\delta_{1k}\dfrac{\partial^2}{\partial t^2}\right) & \varepsilon\, p_1\left(\dfrac{\partial}{\partial t} + t_0\delta_{1k}\dfrac{\partial^2}{\partial t^2}\right) & 0 \end{bmatrix}$$

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$$\mathbf{C}^{**}=\begin{bmatrix} \left(\nabla_1^2-\frac{1}{r^2}\right)-\frac{\partial^2}{\partial t^2} & 0 & 0 & -\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial r} \\ 0 & c_2\ \nabla_1^2-\frac{\partial^2}{\partial t^2} & e_2\nabla_1^2 & 0 \\ 0 & e_2\,\nabla_1^2 & -\eta_3\bar{\varepsilon}\,\nabla_1^2 & 0 \\ -\varepsilon\left(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\nabla_2 & 0 & 0 & \nabla_1^2-\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right) \end{bmatrix}$$

$$\mathbf{A}^{*}=\begin{bmatrix} 0 & (c_3-c_2) & (e_1-e_2) & 0 \\ 0 & c_1 & 1 & 0 \\ c_2 & 0 & 0 & 0 \\ 0 & 1 & -\eta_3 & 0 \end{bmatrix}$$

$$\mathbf{B}^{\oplus}=\begin{bmatrix} \nabla_2-\frac{2c_4}{r} & 0 & 0 & -\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right) \\ (c_3-c_2)\nabla_2 & 0 & 0 & -\bar{\beta}\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right) \\ 0 & c_2\frac{\partial}{\partial r} & e_2\frac{\partial}{\partial r} & 0 \\ (e_1-e_2)\nabla_2 & 0 & 0 & p_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right) \end{bmatrix}$$

where $\nabla_1^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r},\nabla_2=\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$ (3)

Initial and Regularity Conditions:

The initial and regularity conditions are given by:

$u(r,z,0)=0=\dot{u}(r,z,0)$, $w(r,z,0)=0=\dot{w}(r,z,0)$, $T(r,z,0)=0=\dot{T}(r,z,0)$,

$\phi(r,z,0)=0=\dot{\phi}(r,z,0)$, for $t=0$, $z\geq 0$, $r\geq 0$ and (4)

$u(r,z,t)=0$, $w(r,z,t)=0$, $T(r,z,t)=0$, $\phi(r,z,t)=0$, for $t>0$, $\sqrt{r^2+z^2}\to\infty$. (5)

Boundary Conditions:

The following boundary conditions are to be satisfied in non-dimensional form at the surface $z=0$ of the solid half-space.

$\sigma_{zz}(r,0,t)=-\frac{\sigma_0^*\ \delta(r)g(t)}{2\pi r}$, $\sigma_{rz}(r,0,t)=0$, $T_{,z}(r,0,t)=0$, $\phi(r,0,t)=0$ (6)

where $\sigma_0^*=\frac{\sigma_0}{\beta_1 T_0}$ denotes intensity of the mechanical load.

FORMAL SOLUTION:

In order to solve the proposed model, we apply Laplace transform (Churchill [12]) with respect to time 't' defined by:

$$\overline{\mathbf{Z}}(r,z,\ p)=\int_0^\infty \mathbf{Z}(r,z,t)\,exp(-pt)\,dt\,,\ \overline{\boldsymbol{\sigma}}(r,z,\ p)=\int_0^\infty \boldsymbol{\sigma}(r,z,t)\,exp(-pt)\,dt \tag{7}$$

where the functions $\overline{\mathbf{Z}}(r,z,\ p)$ and $\overline{\boldsymbol{\sigma}}(r,\ z,\ p)$ represent the Laplace transforms of column vectors $\mathbf{Z}(r,z,t)$ and $\boldsymbol{\sigma}(r,z,t)$, respectively.

We define the Hankel transform with respect to 'r' as:

$$\tilde{\mathbf{Z}}(q,z,p)=\int_0^\infty r\,\overline{\mathbf{Z}}(r,z,p)J_n(qr)dr\,,\tilde{\boldsymbol{\sigma}}(q,\ z,\ p)=\int_0^\infty r\,\overline{\boldsymbol{\sigma}}\,(r,\ z,\ p)\,J_n(qr)dr \tag{8}$$

where $n=0$ for $\overline{w}(r,z,p)$, $T(r,z,p)$ and $n=1$ in the case of $\overline{u}(r,z,p)$. Here $J_n(qr)$ is a Bessel function of first kind and of order n. Applying Laplace and Hankel transform from equations (7) and (8) to equations (1) and (2), we obtain:

$$\left(\mathbf{A}D^2+\tilde{\mathbf{B}}^*D+\tilde{\mathbf{C}}^*\right)\tilde{\mathbf{Z}}=0\,, \tag{9}$$

$$\tilde{\boldsymbol{\sigma}}=\left(\mathbf{A}^*D+\tilde{\mathbf{B}}^{\oplus}\right)\tilde{\mathbf{Z}} \tag{10}$$

where $\tilde{\mathbf{B}}^*=\begin{bmatrix} 0 & -qc_3 & -q\,e_1 & 0 \\ c_3q & 0 & 0-\overline{\beta}\,p\tau_1 & \\ e_1q & 0 & 0\ p_1p\tau_1 & \\ 0 & -\varepsilon\overline{\beta}\,p^2\tau_0^* & \varepsilon\,p_1\,p^2\tau_0^* & 0 \end{bmatrix}$

$$\tilde{\mathbf{C}}^*=\begin{bmatrix} -\left(p^2+q^2\right) & 0 & 0 & \tau_1\,pq \\ 0 & \left(c_2\,q^2+p^2\right) & -e_2\,q^2 & 0 \\ 0 & -e_2\,q^2 & \eta_3\overline{\varepsilon}q^2 & 0 \\ -\varepsilon\tau_0^*\,p^2\,q & 0 & 0 & -\left(q^2+\tau_0p^2\right) \end{bmatrix}$$

$$\tilde{\mathbf{B}}^{\oplus}=\begin{bmatrix} q & 0 & 0 & -\overline{\beta}\,\tau_1\,p \\ q\,(c_3-c_2) & 0 & 0 & -\overline{\beta}\,\tau_1\,p \\ 0 & -qc_2 & -qe_2 & 0 \\ (e_1-e_2)q & 0 & 0 & p\,p_1\tau_1 \end{bmatrix} \tag{11}$$

where $\tau_0=p^{-1}+t_0$, $\tau_0^*=p^{-1}+\delta_{1k}t_0$, $\tau_1=p^{-1}+t_1\delta_{2k}$.

In order to solve the matrix ordinary differential equations (9), we assume its solution as:

$$\tilde{\mathbf{Z}}(q,\ z,\ p)=\mathbf{X}^*(q,p)exp(mz) \tag{12}$$

Upon substituting solution (12) in equation (10), we obtain:

$$\left(\mathbf{A}m^2+\tilde{\mathbf{B}}^*m+\tilde{\mathbf{C}}^*\right)\mathbf{X}^*(q,p)=0 \tag{13}$$

which is a eigenvalue problem. The system of equations (13) has a non-trivial solution if and only if:

$$det\left(\mathbf{A}m^2+\tilde{\mathbf{B}}^*m+\tilde{\mathbf{C}}^*\right)=0 \tag{14}$$

On simplification this determinantal equation leads to polynomial characteristic equation given as:

$$m^8-f_1m^6+f_2m^4-f_3m^2+f_4=0 \tag{15}$$

where

$$f_1 = \left(a_1 + \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} \right),\ f_2 = \left(a_2 + a_1 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_1 \right),$$

$$f_3 = \left(a_3 + a_2 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_2 \right),\ f_4 = \left(a_3 \frac{q^2}{\overline{K}} + \frac{p^2 \tau_0 F}{\overline{K}} A_3 \right). \tag{16}$$

Quantities F, a_k and A_k $(k = 1, 2, 3)$ used in equation (16) are defined in the Appendix. The characteristic equation (15), being biquadratic in m^2 provides four pairs of characteristic roots given by

$$\sum m_1^2 = f_1,\ \sum m_1^2 m_2^2 = f_2,\ \sum m_1^2 m_2^2\, m_3^2 = f_3,\quad m_1^2 m_2^2 m_3^2 m_4^2 = f_4 \tag{17}$$

The formal solution of equation (12) can be written as:

$$\widetilde{\mathbf{Z}}(q, z, p) = \sum_{k=1}^{4} B_k \mathbf{X}_k^*(q, p)\ exp(-m_k z) + B_{k+4}\ \mathbf{X}_{k+4}^*(q, p)\ exp(m_k z) \tag{18}$$

where $\mathbf{X}_k^*(q, p) = \left[\ 1\ \ V_k^*\ \ W_k^*\ \ S_k^* \right]^T$, $k = 1, 2, 3, 4$; $m = -m_k$

$$\mathbf{X}_{k+4}^*(q, p) = \left[\ 1\ \ -V_k^*\ -W_k^*\ \ S_k^* \right]^T,\ k = 1, 2, 3, 4;\ m = m_k \tag{19}$$

where $V_k^* = \dfrac{Q_1^*(m_k)}{Q^*(m_k)}$, $W_k^* = \dfrac{Q_2^*(m_k)}{Q^*(m_k)}$, $S_k^* = \dfrac{Q_3^*(m_k)}{Q^*(m_k)}$, $k = 1$ to 4. (20)

Here, $Q^*(m_k) = -pq\tau_1 \left\{ (c_3 \lambda_1 + e_1 \lambda_2 - \lambda_3) m_k^4 + \left(\lambda_6 q^2 - \lambda_7 p^2 \right) m_k^2 - q^2 \left(\bar{\varepsilon} \eta_3 l_4 + e_2{}^2 q^2 \right) \right\}$

$$Q_1^*(m_k) = \tau_1 p m_k \left\{ -\lambda_1 c_2 m_k^4 + \left(\lambda_1 p^2 + \lambda_8 q^2 \right) m_k^2 + q^2 \left(\lambda_4 l_2 + \lambda_9 q^2 \right) \right\}$$

$$Q_2^*(m_k) = \tau_1 p m_k \left\{ c_2 \lambda_2 m_k^4 - \left(p_1 l_3 + \overline{\beta} l_2 + \lambda_{10} q^2 \right) m_k^2 + l_2 \left(\lambda_5 q^2 + p_1 p^2 \right) + \left(\lambda_{11} q^2 + e_1 p^2 \right) q^2 \right\}$$

$$Q_3^*(m_k) = \left\{ \begin{array}{l} c_2 \lambda_3 m_k^6 - \left(\lambda_{12} q^2 + \eta_3 l_3 + l_2 \right) m_k^4 \\ + \left\{ \left(\bar{\varepsilon} \eta_3 l_3 + \lambda_{13} e_2 q^2 + 2 e_2 p^2 \right) q^2 + l_4 \left(e_1^2 q^2 + \eta_3 l_2 \right) \right\} m_k^2 - q^2 l_2 \left(\bar{\varepsilon} \eta_3 l_4 + e_2{}^2 q^2 \right) \end{array} \right\} \tag{21}$$

The quantities λ_k $(k = 1 \text{ to } 13)$ used in equation (21) are defined in the Appendix.

The formal solution of equation (12) satisfying the requirement of boundedness is given by:

$$\widetilde{\mathbf{Z}}(q, z, p) = \sum_{k=1}^{4} \mathbf{X}_k^*(q, p) B_k\ exp(-m_k z) \tag{22}$$

Upon using solution (22) in equation (10), we get:

$$\widetilde{\boldsymbol{\sigma}} = \sum_{k=1}^{4} \mathbf{Y}\, \mathbf{X}_k^*(q, p) B_k\ exp(-m_k z) \tag{23}$$

where

$$\mathbf{Y} = \widetilde{\mathbf{B}}^{\oplus} - m_k \mathbf{A}^* = \begin{bmatrix} q & -(c_3 - c_2) m_k & -(e_1 - e_2) m_k & -p\tau_1 \\ q(c_3 - c_2) & -c_1 m_k & -m_k & -\overline{\beta}\, p\tau_1 \\ -c_2 m_k & -q c_2 & -q e_2 & 0 \\ q(e_1 - e_2) & -m_k & \eta_3 m_k & p_1 p \tau_1 \end{bmatrix} \tag{24}$$

This gives the formal solution for piezothermoelastic (6mm class) cylindrical continua.

SOLUTION OF THE PROBLEM:

Upon applying integral transforms defined by equations (7) and (8) to the boundary conditions (6) and using the solution (22) and (23), we get a system of equations with four unknown arbitrary constants B_k (k = 1, 2, 3, 4) as:

$$D_{11}^* B_1 + D_{12}^* B_2 + D_{13}^* B_3 + D_{14}^* B_4 = -q\sigma_0^* \,\tilde{g}(p)\,, \quad D_{21}^* B_1 + D_{22}^* B_2 + D_{23}^* B_3 + D_{24}^* B_4 = 0$$
$$W_1\, B_1 + W_2\, B_2 + W_3\, B_3 + W_4\, B_4 = 0\,,\ m_1 S_1\, B_1 + m_2 S_2\, B_2 + m_3 S_3\, B_3 + m_4 S_4\, B_4 = 0 \quad (25)$$

where $D_{1k}^* = q\left(c_3 - c_2\right) - c_1 m_k V_k^* - m_k W_k^* - p\bar{\beta}\tau_1 S_k^*,\ \ D_{2k}^* = -c_2 m_k - qc_2 V_k^* - qe_2 W_k^*$ for $k = 1,\ 2,\ 3,\ 4$.

On solving equation (25) with the help of Gauss elimination technique, we obtain the value of B_k (k = 1, 2, 3, 4

$$B_k = (-1)^k q\,\sigma_0^* \frac{\Theta_k^{**}}{\Theta^{**}} \tilde{g}(p), \qquad k = 1,\ 2, 3, 4 \quad (26)$$

where

$$\Theta_1^{**} = \begin{vmatrix} D_{22}^* & D_{23}^* & D_{24}^* \\ m_2 S_2 & m_3 S_3 & m_4 S_4 \\ W_2 & W_3 & W_4 \end{vmatrix},\ \Theta_2^{**} = \begin{vmatrix} D_{21}^* & D_{23}^* & D_{24}^* \\ m_1 S_1 & m_2\, S_3 & m_4 S_4 \\ W_1 & W_3 & W_4 \end{vmatrix}$$

$$\Theta_3^{**} = \begin{vmatrix} D_{21}^* & D_{22}^* & D_{24}^* \\ m_1 S_1 & m_2 S_2 & m_4 S_4 \\ W_1 & W_2 & W_4 \end{vmatrix}, \qquad \Theta_4^{**} = \begin{vmatrix} D_{21}^* & D_{22}^* & D_{23}^* \\ m_1 S_1 & m_2 S_2 & m_3 S_3 \\ W_1 & W_2 & W_3 \end{vmatrix}$$

$$\Theta^{**} = D_{11}^*\, \Theta_1^{**} - D_{12}^*\, \Theta_2^{**} + D_{13}^*\, \Theta_3^{**} - D_{14}^*\, \Theta_4^{**}$$

In this case, the displacements, electric potential, temperature change, stresses and electrical displacement are obtained as:

$$\left(\tilde{u}, \tilde{w}, \tilde{\phi}, \tilde{T}\right) = \frac{q\sigma_0^*}{p} \sum_{k=1}^{4} (-1)^k \left(1,\ V_k^*,\ W_k^*,\ S_k^*\right) \frac{\Theta_k^{**}}{\Theta^{**}} exp\left(-m_k z\right)$$

which provides us the solution for this particular set of boundary conditions.

INVERSION OF THE TRANSFORMS:

According to Bradie [13], the various quadrature formulae such as Newton-Cotes, Romberg and Gaussian quadrature etc. can be used to approximate the value of an improper integral, provided the integral exists. However, some changes of variable generally must be made to achieve theoretical order of convergence, if required. Due to the existence of damping terms, the dependence of characteristic roots m_k $\left(k = 1, 2, 3, 4\right)$ on p is very complicated and hence the inversion of integral transform is quite difficult task analytically. But this can be conveniently managed through numerical evaluations of the inversion integrals (Sharma and Kumar [14]; Thakur et al. [11]). In order to obtain the solution of the instant problem in the physical domain, we have to invert the integral transforms in equations (27) and (28) numerically.

NUMERICAL RESULTS AND DISCUSSION:

The piezothermoelastic material PZT-5A has been chosen for the purpose of numerical calculations, physical data for which is given by Thakur et al. [11]. For computation purpose the intensity of the mechanical load $\left(\sigma_0^*\right)$ has been considered as unity. The non-dimensional temperature change and stresses has been computed by taking values of thermal relaxation times as $t_0 = 0.5$ and $t_1 = 0.3$ in the context of LS, CT and GL models of thermoelasticity. The computations are carried out for two instants of times $t = 0.1$ and $t = 0.25$ for thermally insulated half-space at $z = 1.0$. The computer simulated quantities have been plotted in Figs. 1 to 3.

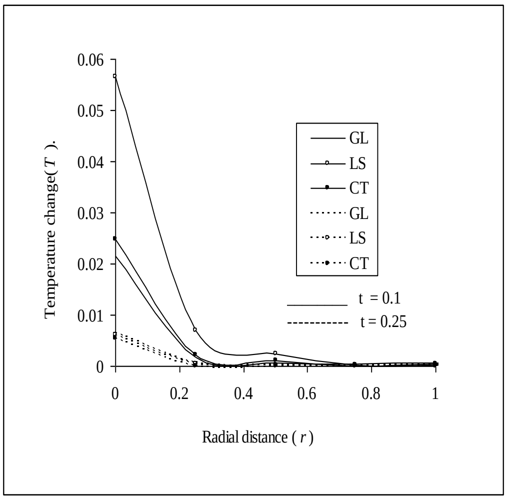

Fig. 1: Variation of non-dimensional temperature change with radial distance.

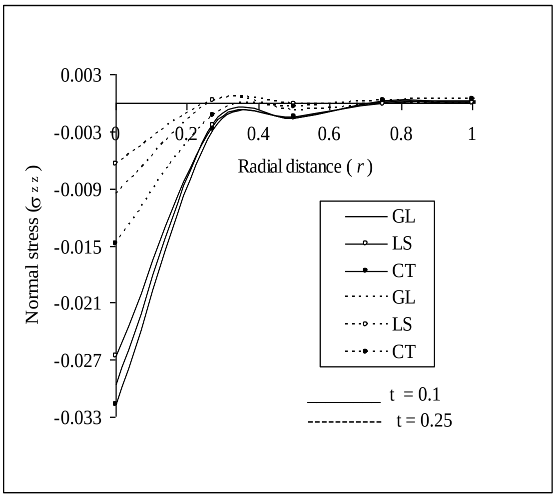

Fig. 2: Variation of non-dimensional normal stress with radial distance.

Fig. 1 presents the variation of absolute temperature change (T) in the context of GL, LS and CT models of thermoelasticity due to continuous point mechanical load. The magnitude of temperature change (T) decreases with radial distance and observes oscillating behavior to vanish at a certain value of radial distance $(r \geq 1.0)$. The oscillating behavior of the temperature change is attributed to compression and expansion of the solid.

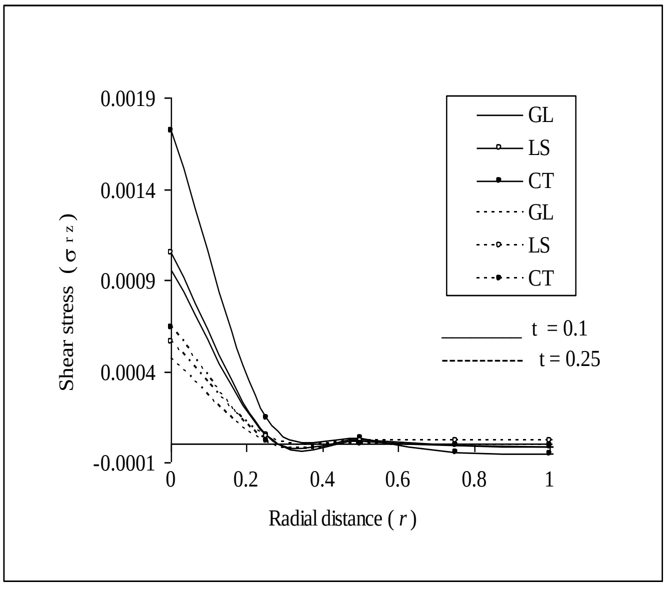

Fig. 3: Variation of non-dimensional shear stress with radial distance

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From Fig. 2 it is noticed that the magnitude of normal stress (σ_{zz}) is quite large in the domain $0 \le r \le 0.2$, decreases afterward and finally dies out in an oscillating manner at certain value of radial distance $(r \ge 1.0)$ due to compression and expansion of the solid. Fig. 3 reveals that the shear stress (σ_{rz}) follows oscillatory behavior with varying amplitude due to continuous load. The shear stress has maximum magnitude near the vicinity of the load which decreases and ultimately dies out in an oscillating fashion with increasing radial distance.

CONCLUDING REMARKS:

The magnitudes of all the considered field functions have been observed to be quite large near the vicinity of point source which follow decreasing trends as we move away from the source and are noticed to vanish after certain value of radial distance, which shows the existence of wave-fronts. There exists no wave-front of temperature and stress distributions in case of coupled (CT) thermoelastic model due to diffusive nature of heat conduction equation. The shear stress development is comparatively small as compared to normal stress and temperature change. This means major portion of the energy is carried by the normal stress and thermal waves and only meager amount propagates in the form of shear stress waves as expected.

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APPENDIX

The quantities F, a_k and A_k $(k=1,2,3)$ used in equation (16) are obtained as:

$$F=\frac{\left(1-\varepsilon\tau^*\varepsilon_\eta p_1^2\right)\left(c_1+\varepsilon\bar{\beta}^2\tau^*\right)+\varepsilon_\eta\left(1-p_1\varepsilon\bar{\beta}\tau^*\right)^2}{\left(c_1+\varepsilon_\eta\right)},$$

$$a_1=\frac{l_3+\bar{\varepsilon}c_1c_2q^2+\varepsilon_\eta\left\{\left(1+c_1e_1^2+2c_2e_2-2c_3e_1\right)q^2+p^2\right\}}{l_1},$$

$$a_2=\frac{\bar{\varepsilon}l_3q^2+l_2l_4+\varepsilon_\eta\left\{\left(c_2e_1^2+c_2e_2^2-2c_3e_1e_2\right)q^4+\left(2e_2l_2+e_1^2p^2\right)q^2\right\}}{l_1},$$

$$a_3=\frac{q^2l_2\left\{\bar{\varepsilon}l_4+\varepsilon_\eta e_2^2q^2\right\}}{l_1},$$

$$A_1=\frac{\left\{E_1L_3+\bar{\bar{\varepsilon}}C_1C_2q^2\right\}+\varepsilon_\eta\left\{\left(2EC_2\bar{e}_2-2C_3E\bar{e}_1+\bar{e}_1^2C_1+E^2\right)q^2+E^2\bar{p}^2\right\}}{L_1}$$

$$A_2=\frac{\bar{\bar{\varepsilon}}q^2L_3+E_1L_2L_4+\varepsilon_\eta\left\{\left(\bar{e}_1^2C_2+\bar{e}_2^2C_2-2C_3\bar{e}_1\bar{e}_2\right)q^4+\left(2E\bar{e}_2L_2+\bar{e}_1^2\bar{p}^2\right)q^2\right\}}{L_1},$$

$$A_3=\frac{q^2L_2\left\{\varepsilon_\eta\bar{e}_2^2q^2+\bar{\bar{\varepsilon}}L_4\right\}}{L_1}$$

Where

$$\varepsilon_\eta=\frac{1}{\eta_3},\ l_1=c_2\left(c_1+\varepsilon_\eta\right),\ l_2=q^2+p^2,\ l_3=Pq^2+J\,p^2,\ l_4=c_2\,q^2+p^2,\ L_1=C_2\left(C_1E_1+\varepsilon_\eta E^2\right),$$

$$L_2=q^2+\bar{p}^2,\ L_3=P'q^2+J'\bar{p}^2,\ L_4=C_2q^2+\bar{p}^2,\ F=\left(1-\varepsilon\tau^*\right)^3,$$

$$C_1=\frac{c_1-\tau^*\bar{\beta}^2\varepsilon}{1-\varepsilon\tau^*},\ C_2=\frac{c_2}{1-\varepsilon\tau^*},\ C_3=\frac{c_3-\tau^*\bar{\beta}\varepsilon}{1-\varepsilon\tau^*},\ \bar{e}_1=\frac{e_1+\varepsilon p_1\tau^*}{1-\varepsilon\tau^*},\ \bar{e}_2=\frac{e_2}{1-\varepsilon\tau^*},$$

$$E=\frac{1+\varepsilon\bar{\beta}\,p_1\tau^*}{1-\varepsilon\tau^*},\ E_1=\frac{1+\varepsilon p_1^2\varepsilon_\eta\tau^*}{1-\varepsilon\tau^*},\ \bar{\bar{\varepsilon}}=\frac{\bar{\varepsilon}}{1-\varepsilon\tau^*},\ \bar{p}^2=\frac{p^2}{1-\varepsilon\tau^*},\ P=c_1+c_2^2-c_3^2,$$

$$J=c_1+c_2,\ P'=\frac{\left(1+\varepsilon\tau^*\right)\left(c_1+\varepsilon\bar{\beta}^2\tau^*\right)+c_2^2-\left(c_3+\varepsilon\bar{\beta}\tau^*\right)^2}{\left(1+\varepsilon\tau^*\right)^2},\ J'=\frac{\left(J+\varepsilon\bar{\beta}^2\tau^*\right)}{\left(1+\varepsilon\tau^*\right)},$$

$$\tau^*=\begin{cases}1, & \text{for LS and CT theory}\\ \dfrac{\tau_1}{\tau_0}, & \text{for GL theory}\end{cases}$$

The quantities λ_k $(k=1\,\text{to}\,13)$ used in equation (21) are defined as:

$$\lambda_1=\eta_3\bar{\beta}-p_1,\ \lambda_2=\bar{\beta}+p_1c_1,\ \lambda_3=1+c_1\eta_3,\ \lambda_4=p_1e_2-\bar{\varepsilon}\bar{\beta}\eta_3,\ \lambda_5=p_1c_2+\bar{\beta}e_2,$$

$$\lambda_6=\eta_3\left(c_1\bar{\varepsilon}+c_2\right),\ \lambda_7=p_1e_1-\eta_3,\ \lambda_8=\bar{\beta}\eta_3\left(1+\bar{\varepsilon}c_2\right)-p_1\left(1+c_2e_2-c_3e_1\right)-\left(e_1+\eta_3c_3\right)+e_1^2\bar{\beta},$$

$$\lambda_9=\left(e_1e_2+\bar{\varepsilon}c_3\eta_3\right),\ \lambda_{10}=\left(c_1e_1+\bar{\beta}c_2e_2-\bar{\beta}c_3e_1-c_3\right),\ \lambda_{11}=e_1c_2-c_3e_2,$$

$$\lambda_{12}=\bar{\varepsilon}\eta_3c_1c_2+2c_2e_2-2c_3e_1+c_1e_1^2,\ \lambda_{13}=c_2e_2-2e_1c_3+2.$$

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ON THE BOUNDS FOR OSCILLATIONS IN DOUBLE DIFFUSIVE CONVECTION WITH CROSS-DIFFUSIONS EFFECTS IN POROUS MEDIUM: DARCY MODEL

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ABSTRACT: In the present paper, the stability analysis of double-diffusive convection problems (Veronis and Stern Types) with cross-diffusions effects (Soret and Dufour effects) have been carried out mathematically in porous medium using Darcy model. The eigen values equations governing these problems have been casted into mathematically tractable forms for stability analysis using some linear transformations. The stability of the oscillatory modes and consequently the bounds for the complex growth rate for arbitrary neutral or unstable oscillatory perturbations are derived which are valid for each combinations of rigid (slip free) and dynamically free (stress-free) boundaries. Various consequences of the derived results are also worked out.

KEYWORDS: Double-Diffusive Convection; oscillatory motions; porous medium, Darcy model, complex growth rate; eigenvalues problem; Soret effect; Dufour effect;

INTRODUCTION:

A broader range of dynamical behaviour is observed in the convective motions that may occur in a gravitational field containing two components (for example, temperature and solute) of different diffusivities that affect the density of the fluid and the phenomenon is known as double-diffusive convection. These phenomena of double-diffusive convection occur when the temperature and concentration gradients are of comparable magnitude and operate on different scales and lead to large scale convection. These kinds of double-diffusion processes are found in astrophysics (big Helium-stars), the earth core, metal alloy, refilling of gas reservoirs, etc. Double-diffusive convection is also of importance in various other fields of practical importance such as high quality crystal production oceanography, production of pure medication, solidification of molten alloys, limnology and engineering.

The double diffusive process was first recognized by Arons and Stommel [1] through his '*thought experiment*' with ocean flow/circulation in mind. Two fundamental configurations have been studied in the context of thermohaline instability problem, the first one by Stern[2],wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and the second one by Veronis [3], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing. Stern found that the steady motion is the preferred mode of onset of instability whereas Veronis observed that oscillatory mode of instability is the preferred mode of convective instability. Since then numerous authors have investigated the double diffusive convection problems under varying assumptions of hydrodynamics both numerically and analytically. For a broader view of the subject of

double-diffusive convection one may refer to Turner [4], Brandt and Fernando [5], Schmitt[6] andNield [7].
The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects[8,9]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, whereas the analogous effect that arises from a concentration gradient which produces a heat flux is called the Dufour effect. When considering the isothermal coupled diffusion between a solvent and two solutes, we need to include these two cross diffusion effects as each property gradient has a significant influence on the flux of the other property. The stability of Dufour-Soret driven double-diffusion convection in a horizontal layer of a fluid subjected to thermal and solutal gradients has been investigated theoretically by means of a linear stability analysis by many authors including Groot and Mazur [8] and Fitts [9]and McDougall [10].

Convective instability in porous media has been studied with great interest for more than half century. Convection given by buoyancy has found increased applications in underground coal gasification, solar energy extraction and many more areas as discussed by Nield and Bejan [11], Ingham [12] in industrial application, harmful particles can be filtered from a fluid stream by passing it through.

When we consider flow in porous medium, we have to take into consideration some additional complexities, which are principally due to the interactions between the fluids and the porous material. When a fluid permeates a porous medium, we cannot follow analytically the actual path of an individual fluid particle, because of the fluid-rock boundary conditions, which must be considered. Thus in aporous medium one generally considered. The fluid motion in termsof volume or ensemble average of the motion of individual fluid elements over regions of space. This is usually done by famous Darcy's law and as a result of this the viscous term in the equation of fluid motion will be replaced by the resistance term $-\frac{\mu}{K_1}\vec{q}$ where, μ is the dynamic viscosity of fluid,k_1 is the permeability of the medium and $\vec{q}$ is the seepage velocity of the fluid. A macroscopic equation which describes incompressible creeping flow of a Newtonian fluid of viscosity μ through a macroscopically homogenous and isotropic porous medium of permeability is well known Darcy's equation [13]

$$-\frac{\mu}{K_1}\vec{q} = \nabla p$$

where, p is interstitially averaged pressure within the porous medium and $\vec{q}$ is the filter velocity (or Darcian velocity).

The stability of flow of a fluid through porous medium was studied by Lapwood [14] and Wooding [15] and have studied double diffusive convection in porous medium by considering the Darcy flow model which is relevant to densely packed, low permeability porous medium.

The problem of double diffusive convection in porous medium has been extensibly investigated and growing volume of work in this area is well documented by Ingham and Pop[12].In this Paper we have studied double–diffusive convection problem in porous mediumby considering the Darcy flow model which is relevant to densely packed, low permeability porous medium. The aim of the present analysis is to investigate the effect of cross-diffusions terms (Dufour and Soret effects) on the onset of Double Diffusive Convective instability.

However, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. Hence, non-Darcy effects on double diffusive convection in porous media have received a great deal of attention in recent years.

PHYSICAL CONFIGURATION AND EIGENVALUE PROBLEM:
Consider a horizontal layer of viscous fluid saturating a porous medium between two parallel boundaries at$z = 0$ and $z = 1$,which are heated and salted from below in the force field of gravity. The temperature and concentration difference between the bounding planes are ΔT and ΔS, respectively. We assume the Oberback-Bosssinesq approximation is valid and flow in porous medium is governed by Darcy's law. The extra effect we consider here is that of the coupled fluxes of the two properties due to irreversible thermodynamic effects. The governing equations for the Double Diffusive convection problem with cross-diffusions effects in a horizontal fluid saturating a porous medium under Boussinesq approximation are given by

$$\frac{\partial u_i}{\partial t} = 0 \tag{1}$$

$$\frac{\rho_0}{\epsilon}\left[\frac{\partial u_i}{\partial t} + \frac{u_i}{\epsilon}\frac{\partial u}{\partial x_j}\right] = -\frac{\partial p}{\partial x_i} + \rho g - \frac{\mu}{\kappa}u_i \tag{2}$$

$$E'\frac{\partial T}{\partial t} + u_i\frac{\partial T}{\partial x_j} = \kappa\frac{\partial^2 T}{\partial {x_j}^2} + S_1\frac{\partial^2 S}{\partial {x_j}^2} \tag{3}$$

$$E''\frac{\partial S}{\partial t} + u_i\frac{\partial S}{\partial x_j} = \eta_s\frac{\partial^2 S}{\partial {x_j}^2} + D_1\frac{\partial^2 T}{\partial {x_j}^2} \tag{4}$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(S - S_0)] \tag{5}$$

Here, $u_i = (u, v, w)$ is the velocity,T is the temperature, $S$is the solute concentration, p is the pressure,ρ is the density,$T_{0,}S_0$ and ρ_0 are referencetemperature, concentration and density respectively, $g$is the accerlation due to gravity,μ is the fluid viscosity, ε *is* the porosity, κ is the effective thermal diffusivity and η_s is the solute diffusivity. The suffix '0' denotes the values of the various parameters at some suitably reference temperature T_0 and concentration C_0. Further,

$E' = \epsilon + (1 - \epsilon)\frac{\rho_s c_s}{\rho_0 c_f}$is a constant related to heat and E'' is a constant analogous to concentration κ_1 is the permeability of the porous medium, where $\rho_{S\,,}C_S$ *and* ρ_0, C_f stand for density and heat capacity of the solid (porous matrix) material and fluid, respectively.

Following the usual steps of linear stability theory, the non dimensional linearized perturbation equations and boundary conditions governing The aim of the present analysis is to investigate the effect of cross-diffusions terms (Dufour and Soret effects) on the onset of Double Diffusive Convective instability

The following non-dimensional linearized perturbation equations

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)(D^2-a^2)w=-R_T a^2\theta+R_S a^2\varphi \qquad (6)$$

$$[D^2-a^2-E'p]\theta+D_T[D^2-a^2]\varphi \qquad (7)$$

$$\left[D^2-a^2-\frac{E''p}{\tau}\right]\varphi+S_T[D^2-a^2]\theta=-\frac{w}{\tau} \qquad (8)$$

where

$a^*=kd\,,\theta^*=\frac{\hat{\theta}\kappa}{\beta_1 d^2}\,,n=\frac{p^*\kappa}{d^2}\varphi^*=\frac{\hat{\varphi}\kappa}{\beta_2 d^2}\,,w^*=\hat{w},R_T=\frac{g\alpha\beta_1 d^4}{\vartheta\kappa}\,,R_S=\frac{g\alpha'\beta_2 d^4}{\nu\kappa},S_T=\frac{S_f\beta_1}{\tau\kappa\beta_2},D_T=\frac{D_f\beta_2}{\kappa\beta_1}$

together with the boundary conditions

Case1:Both boundaries are dynamically free

$$w=\theta=\varphi=0=D^2\text{w at z = 0 and z = 1} \qquad (9)$$

Case2: Both boundaries are rigid

$$w=\theta=\varphi=0=Dw\text{ at z = 0 and z = 1} \qquad (10)$$

Case3: Lower boundary rigid and upper boundary free

$$w=\theta=\varphi=0=Dw\ at\ z=0\ and\ w=\theta=\varphi=0=D^2w\ at\ z=1 \qquad (11)$$

or

$$w=0=\theta=\varphi=Dw\ at\ z=0\ and\ w=0=\theta=\varphi=D^2w\ at\ z=1 \qquad (12)$$

we use the linear transformations to transform the above eigenvalue problem to a mathematically tractable form.

Let us define the linear transformation given by;

$$w'=\left(S_TE''+BE'\right)w,\theta'=E\theta+F\varphi,\varphi'=S_T\theta+B\varphi \qquad (13)$$

where, E=$\left(\frac{S_TE''+BE'}{E'D_T+AE''}\right)A\,,F=\left(\frac{S_TE''+BE'}{AE''+E'D_T}\right)D_T,B=-\frac{AE'}{\tau E''}$

and A is any positive root of equation

$$A^2E''+A\left(\tau E'-E''\right)-\tau S_TD_TE'=0$$

The system of equations (6)-(8) together with boundary conditions (9)-(12) upon using the linear transformation defined by (13) assume the following forms;

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)(D^2-a^2)w=-\hat{R}_T a^2\theta+\hat{R}_S a^2\varphi \qquad (14)$$

$$\left[\hat{K}_1(D^2-a^2)-E'p\right]\theta=-w \qquad (15)$$

$$\left[\hat{K}_2(D^2-a^2)-\frac{E''p}{\tau}\right]\varphi=-\frac{w}{\tau} \qquad (16)$$

and boundary conditions remains the same as described above by(9)-(12).

Here, $\hat{K}_1=1+\frac{\tau S_TD_TE'}{AE''}\ and\ \hat{K}_2=1+\frac{S_TD_TE''}{\tau E'B}$ are non-negative constant, since $0\le\frac{S_TD_T}{A}\le 1$ as $S_T\ and\ D_T>0$are Soret and Dufour numbers and A is a constant as defined earlier.

Also,$\hat{R}_T=\frac{(D_TE'+AE'')(R_TB+R_SS_T)}{AB-S_TD_T}\,,\hat{R}_S=\frac{(S_TE''+BE')(R_TD_T+R_SA)}{AB-S_TD_T}$ is the modified thermal Rayleigh number and modified concentration Rayleigh number respectively.

Further, the dashes have been dropped in writing the perturbed quantities in the above equations for the sake of convenience.

Remarks A.. We note that system of equations (14)- (16) yields the following non-dimensional linearized perturbation equations governing;

(i) Dufour Driven Double-Diffusive Convection**(DDDDC)** problem with porous media, whenS_T=0.We have

$$\hat{R}_T = R_T\left(E'' + \frac{D_T E'}{A}\right), \hat{R}_S = E'\left(R_S + \frac{R_T D_T}{A}\right), \hat{K}_1 = 1\ \hat{K}_2 = 1$$

And equations (14)-(16) reduce to

$$\left(\frac{p}{\sigma\varepsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R_T\left(E'' + \frac{D_T E'}{A}\right)a^2\theta + E'\left(R_S + \frac{R_T D_T}{A}\right)a^2\varphi \quad (17)$$

$$[(D^2 - a^2) - E'p]\theta = -w \quad (18)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \quad (19)$$

(ii) Soret Driven Double-Diffusive Convection (**SDDDC)** problem with porous media, when $D_T = 0$. We have

$$\hat{R}_T = E''\left(R_T + \frac{R_S S_T}{B}\right), \hat{R}_S = \frac{R_S}{B}\left(S_T E'' + BE'\right), \hat{K}_1 = 1, \hat{K}_2 = 1$$

And equations (14)- (16) reduce to

$$\left(\frac{p}{\sigma\epsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -E''\left(R_T + \frac{R_S S_T}{B}\right)a^2\theta + \frac{R_S}{B}\left(S_T E'' + BE'\right)a^2\varphi \quad (20)$$

$$[(D^2 - a^2) - E'p]\theta = -w \quad (21)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \quad (22)$$

(iii) Double-Diffusive Convection (**DDC)** problem with porous media, When $S_T = 0, D_T = 0$.We have $\hat{R}_T = E''R_T$, $\hat{R}_S = R_S E'$,

equations (14)- (16) reduces to

$$\left(\frac{p}{\sigma\epsilon} + \frac{1}{P_1}\right)(D^2 - a^2)w = -R_T E'' a^2\theta + R_S E' a^2\varphi \quad (23)$$

$$[(D^2 - a^2) - E'p]\theta = -w \quad (24)$$

$$\left[(D^2 - a^2) - \frac{E''p}{\tau}\right]\varphi = -\frac{w}{\tau} \quad (25)$$

MATHEMATICAL ANALYSIS:

On the Stability of Oscillatory modes:

We shall now investigate the stability of the oscillatory modes for Double-Diffusive convection problem with cross-diffusions for Darcy model in porous medium.

Theorem1. If $(p, w, \theta, \varphi), p = p_r + ip_i, p_i \neq 0$, $\hat{R}_T > 0, \hat{R}_S > 0$ is a non-trivial solution of equations (14)-(16)together with one of the boundary conditions(9)-(12) and $\hat{R}_T \leq \frac{27\pi^4 P_1\tau\hat{K}_2\hat{K}_1 + 16\pi^2\sigma K_1 \in E''}{4\epsilon E''\sigma P_1}$,then$p_r < 0$.

Proof: Multiplying both side of equation (14) by w^* and then integrating the resulting equation over the range of $z\epsilon[0,1]$, we have

$$\left(\frac{p}{\sigma\varepsilon} + \frac{1}{P_1}\right)\int_0^1 w^*(D^2 - a^2)w\,dz = -\hat{R}_T a^2\int_0^1 \theta w^* dz + \hat{R}_S a^2\int_0^1 \varphi w^* dz \quad (26)$$

Taking the complex conjugate of both side of equations (15) and (16) and then using the resulting equations in equation (26), we get

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1 w^*(D^2-a^2)w dz = \hat{R}_T a^2 \int_0^1 \theta\left[\hat{K}_1(D^2-a^2)-E'p^*\right]\theta^* dz - \hat{R}_S a^2 \tau \int_0^1 \varphi\left[\hat{K}_2(D^2-a^2)-\frac{E''p^*}{\tau}\right]\varphi^* dz \quad (27)$$

Now, integrating the various terms of equation (27) by parts an appropriate number of times and making use of either of boundary conditions (9)-(12), we get

$$\left(\frac{p}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1 [|Dw|^2+a^2|w|^2]\,dz - \hat{R}_T a^2 \int_0^1 \left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)+E'p^*|\theta|^2\right]dz + \hat{R}_S a^2 \tau \int_0^1 \left[\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)+E'p^*|\varphi|^2\right]dz = 0 \quad (28)$$

Equating the real and imaginary parts of both side of equation (28)and cancelling$p_i \neq 0$ throughout from the obtained imaginary part of the equation, wehave

$$\left(\frac{p_r}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1 [|Dw|^2+a^2|w|^2]\,dz - \hat{R}_T a^2 \int_0^1 \left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)+E'p_r|\theta|^2\right]dz + \hat{R}_S a^2 \int_0^1 \left[\tau\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)+E''p_r|\varphi|^2\right]dz = 0 \quad (29)$$

and

$$\left[\frac{1}{\varepsilon\sigma}\int_0^1 (|Dw|^2+a^2|w|^2)dz + \hat{R}_T a^2 E' \int_0^1 |\theta|^2 dz - \hat{R}_S a^2 E'' \int_0^1 |\varphi|^2\,dz\right] = 0 \quad (30)$$

If permissible,$p_r \geq 0$.Multiplying equation (30) by $p_r$and adding the resulting equation to equation (27), we have

$$\left(\frac{2p_r}{\sigma\varepsilon}+\frac{1}{P_1}\right)\int_0^1 [|Dw|^2+a^2|w|^2] - \hat{R}_T a^2 \int_0^1 \left[\hat{K}_1(|D\theta|^2+a^2|\theta|^2)\right]dz + \hat{R}_S a^2 \int_0^1 \left[\tau\hat{K}_2(|D\varphi|^2+a^2|\varphi|^2)\right]dz = 0 \quad (31)$$

Equation also (30) implies that

$$\frac{1}{\sigma\varepsilon}\int_0^1 (|Dw|^2+a^2|w|^2)\,dz < \hat{R}_S a^2 E'' \int_0^1 |\varphi|^2\,dz \quad (32)$$

Since,w, θ, φ vanish at $z = 0$ and $z = 1$,therefore by the use of Rayleigh Ritz inequality, we have

$$\int_0^1 |Dw|^2\,dz \geq \pi^2 \int_0^1 |w|^2 dz \quad (33)$$

$$\int_0^1 |D\theta|^2\,dz \geq \pi^2 \int_0^1 |\theta|^2\,dz \quad (34)$$

$$\int_0^1 |D\varphi|^2\,dz \geq \pi^2 \int_0^1 |\varphi|^2\,dz \quad (35)$$

Combining inequalities (33) and (34), we get

$$\frac{(\pi^2+a^2)}{\varepsilon\sigma}\int_0^1 |w|^2 dz < \hat{R}_S a^2 E'' \int_0^1 |\varphi|^2\,dz \quad (36)$$

Also upon using inequality (35), we can have

$$\int_0^1 [|D\varphi|^2+a^2|\varphi|^2]\,dz \geq (\pi^2+a^2)\int_0^1 |\varphi|^2\,dz \quad (37)$$

Also, upon using inequality (36) and (37) we get

$$\hat{R}_S a^2 E'' \int_0^1 [|D\varphi|^2+a^2|\varphi|^2]\,dz \geq \frac{(\pi^2+a^2)^2}{\varepsilon\sigma}\int_0^1 |w|^2\,dz \quad (38)$$

Further, multiplying equation (15) by its complex conjugate and then integrating the resulting equation over the range of z, we get

$$\int_0^1 \left[\hat{K}_1(D^2-a^2)-E'p\right]\theta dz \int_0^1 \left[\hat{K}_1(D^2-a^2)-E'p^*\right]\theta^* dz = \int_0^1 ww^* dz \quad (39)$$

$\int_0^1 \left|\widehat{K}_1{}^2(D^2-a^2)\theta\right|^2 dz + 2p_r\,\widehat{K}_1E'\int_0^1[|D\theta|^2+a^2|\theta|^2]\,dz + (E')^2|p|^2\int_0^1|\theta|^2\,dz =$
$\int_0^1|w|^2\ dz$ (40)

Since $p_r \geq 0$, therefore inequality (40) yields

$\int_0^1 \left|\widehat{K}_1{}^2(D^2-a^2)\theta\right|^2 dz \leq \int_0^1|w|^2dz$ (41)

Now, in view of inequality (34), we can have

$$\int_0^1 |(D^2-a^2)\theta|^2 = \int_0^1[|D^2\theta|^2+2a^2|D\theta|^2+a^4|\theta|^2]\,dz \geq (\pi^2+a^2)^2\int_0^1|\theta|^2\,dz \tag{42}$$

Using inequality (42) in inequality (41),we have

$\widehat{K}_1{}^2(\pi^2+a^2)^2\int_0^1|\theta|^2\,dz \leq \int_0^1|w|^2\,dz$ (43)

Also, we know that

$\int_0^1|w|^2dz = \int_0^1\{|w|^2\}^{\frac{1}{2}}\,dz\int_0^1\{|w|^2\}^{\frac{1}{2}}\,dz$ (44)

which upon using inequalities(41) and (43) yields

$\int_0^1|w|^2dz \geq (\pi^2+a^2)\widehat{K}_1{}^2\int_0^1\{|(D^2-a^2)\theta|^2\}^{\frac{1}{2}}\,dz\int_0^1[|\theta|^2]^{\frac{1}{2}}\,dz$ (45)

$$\geq \widehat{K}_1{}^2(\pi^2+a^2)\int_0^1|-\theta^*(D^2-a^2)\theta dz|$$

$\geq\ \widehat{K}_1{}^2(\pi^2+a^2)\int_0^1[|D\theta|^2+a^2|\theta|^2]\,dz$ (46)

Since, $p_r \geq 0$ and for given $\frac{p_r}{\varepsilon}$ positive,equation (31) can be written as

$\frac{1}{P_1}\int_0^1[|Dw|^2+a^2|w|^2]\,dz + \widehat{R}_S a^2\tau\widehat{K}_2\int_0^1[|D\varphi|^2+a^2|\varphi|^2] < \widehat{R}_T\,a^2\widehat{K}_1\int_0^1[|D\theta|^2+a^2|\theta|^2]\mathrm{d}z$ (47)

Using inequalities (33), (38), (46) in inequality (47),

$\left[\frac{\widehat{K}_1(\pi^2+a^2)^2}{a^2P_1} + \frac{(\pi^2+a^2)^3\tau\widehat{K}_2\widehat{K}_1}{a^2\varepsilon E''\sigma} - \widehat{R}_T\right]\int_0^1|w|^2dz < 0$ (48)

Since, minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 is $4\pi^2$, and $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$, therefore inequality (48) yields that

$\left[\frac{K_1 4\pi^2}{P_1} + \frac{27\pi^4\tau\widehat{K}_2\widehat{K}_1}{4\varepsilon E''\sigma} - \widehat{R}_T\right]\int_0^1|w|^2\,dz < 0$ (49)

which can be written as

$$\left[\frac{16\pi^2K_1\varepsilon E'' + 27\pi^4P_1\tau\widehat{K}_2\widehat{K}_1}{4\varepsilon E''\widehat{R}_TP_1}\right] < 1$$

or, we have

$\widehat{R}_T \geq \frac{(16\pi^2\widehat{K}_1\varepsilon E''\sigma + 27\pi^4P_1\tau\widehat{K}_2\widehat{K}_1)}{4\varepsilon E''P_1\sigma}$ (50)

This is a contradiction to the hypothesis of the theorem.
Hence, we must have $p_r < 0$.
This completes the proof.

Using Remark A (i-iii), we have the following corollaries;

Corollary 1. Under the hypothesis of Theorem1, for DDDDC problem with porous medium,if $R_TE'' + \frac{R_TD_TE'}{A} \leq \frac{[27\pi^4\tau P_1 + 16\pi^2\epsilon E''\sigma]}{4P_1\epsilon E''\sigma}$, *then* $p_r < 0$.

Corollary 2. Under the hypothesis of Theorem 1, for SDDDC with porous medium $R_T E^{''} + \frac{E^{''} R_S S_T}{B} \leq \frac{[27\pi^4 \tau P_1 + 16\pi^2 \epsilon E^{''} \sigma]}{4P_1 \epsilon E^{''} \sigma}, p_i \neq 0,$ then $p_r < 0$.

Corollary 3. Under the hypothesis of Theorem4, For DDC with porous medium, $\hat{R}_T = R_T E^{''}, \hat{R}_S = R_S E^{'}, p_i \neq 0, R_T E^{''} \leq \frac{[27\pi^4 \tau P_1 + 16\pi^2 \varepsilon E^{''} \sigma]}{4P_1 \varepsilon E^{''} \sigma}, then\ p_r < 0.$

It is remarkable to note that when we consider the compliment of the above sufficient condition as stated in Theorem above, for the stability of the oscillatory motions i.ewhen $\hat{R}_T > \frac{27\pi^4 P_1 \tau \hat{K}_2 \hat{K}_1 + 16\pi^2 \hat{K}_1 \in E^{''} \sigma}{4\epsilon E^{''} P_1 \sigma}$, the oscillatory modes of growing amplitudemay exists. Hence, it becomes important to prescribe the bounds for the growth rate of these motions. In the following Theorem, we have derived such bounds which arrest the complex growth rate of the arbitrary neutral or unstable $p_r \geq 0$ oscillatory motions $(p_i \neq 0)$.

In the following theorem, we have derived bounds which arrest the complex growth of the arbitrary neutral or unstable $p_r \geq 0$ oscillatory motions $p_i \neq 0$.

Bound for the complex Growth Rate:

Theorem 2. If (p, w, θ, φ) , $p = p_r + ip_i$, $p_r \geq 0$, $p_i \neq 0$, $\hat{R}_T > 0, \hat{R}_S > 0$, is a non-trivial solution ofequations(14)-(16) together with one of the boundary conditions(9)-(12)and for $H > 1$, then $|p| < \frac{\hat{K}_1(\pi^2+a^2)\sqrt{H^2-1}}{E^{'}}$, where

$H = \frac{4\hat{R}_T P_1 \varepsilon \sigma E^{''}}{[27\pi^4 \tau \hat{K}_2 \hat{K}_1 + 16\pi^2 P_1 \varepsilon E^{''} \hat{K}_1]}$.

Proof : Since $p_r \geq 0$, therefore from equation (42), we can have

$$\int_0^1 \left|\hat{K}_1^{\,2}(D^2 - a^2)\theta\right|^2 dz + (E^{'})^2 |p|^2 \int_0^1 |\theta|^2 dz < \int_0^1 |w|^2\, dz \tag{51}$$

Using inequality (44) in inequality (51), we get

$$\hat{K}_1^{\,2}(\pi^2 + a^2)^2 \left[1 + \frac{(E^{'})^2 |p|^2}{\hat{K}_1^{\,2}(\pi^2+a^2)^2}\right] \int_0^1 |\theta|^2\, dz < \int_0^1 |w|^2 dz \tag{52}$$

Since

$$\int_0^1 [|D\theta|^2 + a^2|\theta|^2] dz = \left|-\int_0^1 \theta^*(D^2 - a^2)\,\theta\right| dz \leq \int_0^1 |\theta|\,|(D^2 - a^2)\theta| dz$$

$\leq \int_0^1 \{|\theta|^2\}^{\frac{1}{2}} dz \int_0^1 \{|(D^2 - a^2)\theta|^2\}^{\frac{1}{2}}\, dz$ (Using Schwartz inequality)

$$\leq \frac{1}{\hat{K}_1^{\,2}(\pi^2+a^2)} \left[1 + \frac{(E^{'})^2 \sigma^2 |p|^2}{\hat{K}_1(\pi^2+a^2)}\right]^{\frac{-1}{2}} \int_0^1 |w|^2 dz \tag{53}$$

Making use of inequalities (39), (43) and(53) in equation (47) we get

$$\left(\frac{1}{P_1}\right) \int_0^1 [|Dw|^2 + a^2|w|^2]\, dz + \hat{R}_S a^2 \tau \hat{K}_2 \int_0^1 [|D\varphi|^2 + a^2|\varphi|^2]\, dz < \hat{R}_T a^2 \hat{K}_1 \int_0^1 [|D\theta|^2 + a^2|\theta|^2]\, dz \tag{54}$$

which on simplification yields

$$\left[\left(\frac{1}{P_1}\right) \frac{\hat{K}_1(\pi^2+a^2)^2}{a^2} + \frac{\tau \hat{K}_2 (\pi^2+a^2)^3 \hat{K}_1}{\varepsilon E^{''} \sigma a^2}\right] \left[1 + \frac{(E^{'})^2 |p|^2}{\hat{K}_1(\pi^2+a^2)}\right]^{\frac{1}{2}} < \hat{R}_T \tag{55}$$

Since, minimum values of $\frac{(\pi^2+a^2)^2}{a^2}$ and $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $4\pi^2$ and $\frac{27\pi^4}{4}$, inequality (55) yields that

$$\left[\frac{4\pi^2}{P_1}+\frac{\tau\widehat{K}_2 27\pi^4\widehat{K}_1}{\varepsilon E'' 4\sigma}\right]\left[1+\frac{(E')^2|p|^2}{\widehat{K}_1(\pi^2+a^2)}\right]^{\frac{1}{2}}<\widehat{R}_T$$

which yields

$$|p|<\frac{\widehat{K}_1(\pi^2+a^2)}{E'}\sqrt{H^2-1} \qquad (56)$$

This completes the proof of the theorem.

Theorem above from point of view of hydrodynamic stability theory may be stated as:

The complex growth rate $p=p_r+ip_i$ of an arbitrary oscillatory perturbation of growing amplitude $(p_r\geq 0)$ lies inside a semi-circle in the right-half of the $p_r p_i$ −plane whose centre is at the origin and whose radius is $\frac{\widehat{K}_1(\pi^2+a^2)}{E'}\sqrt{H^2-1}$.

Using Remarks A. (i-iii),we have the following corollaries

Corollary 4. For **DDDDC** problem with porous medium, $p_i\neq 0$, the complex growth rate is given by, $|p|<\frac{(\pi^2+a^2)}{E'}\sqrt{H'^2-1}$, where, $H'=\frac{4R_T P_1\epsilon E''\sigma\left(E''+\frac{D_T E'}{A}\right)}{[27\pi^4\tau P_1+16\pi^2\epsilon E''\sigma]}$

Corollary 5. For **SDDDC** with porous medium, $p_i\neq 0$, the complex growth rate is given by, $|p|<\frac{(\pi^2+a^2)}{E'}\sqrt{H''^2-1}$, where, $H''=\frac{4P_1\epsilon E''\sigma\left(R_T E''+\frac{E'' R_S S_T}{B}\right)}{[27\pi^4 P_1(\tau)+16\pi^2\epsilon E''\sigma]}$.

Corollary 6. For **DDC** with porous medium, $p_i\neq 0$, the complex growth rate is given by, $|p|<\frac{(\pi^2+a^2)}{E'}\sqrt{H'''^2-1}$. where, $H'''=\frac{4P_1\epsilon E'' R_T E'}{[27\pi^4 P_1(\tau)+16\pi^2\epsilon E'']}$.

RESULTS AND CONCLUSIONS:

We studied the Double-Diffusive Convection problem saturating porous medium (Darcy Model) in order to study the effects of cross diffusion terms. For general nature of boundary conditions, a sufficient condition for the stability of oscillatory modes is obtained. Moreover, when compliment of this condition is supposed to be true, the bounds for growth rate for arbitrary perturbations of growing amplitude are obtained and various consequences of these results for DDDDC, SDDDC, DDC are obtained in the presence of porous medium. However, the presence of porous media prepones the onset of convection.

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WAVES PROPAGATION IN HOMOGENEOUS ISOTROPIC PLATE IN CONTEXT OF TWO TEMPERATURE GENERALIZED THERMOELASTICITY

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ABSTRACT:
This paper concerns with investigation of propagation of thermoelastic waves in homogeneous isotropic plate in context of two temperature generalized theory of thermoelasticity. The surfaces of plate are subjected to stress free, rigid insulated and isothermal boundary conditions. Secular equations in closed form and isolated mathematical conditions for symmetric and skew symmetric wave mode propagation are derived. The results for uncoupled, coupled, Lord-Shulman and Green-Lindsay theories of thermoelasticity have been obtained as particular cases from the derived secular equations. The effect of two temperatures on the propagation of thermoelastic waves is presented graphically and the results are compared theoretically as well as numerically with the one temperature theory as well as existing previous literature.

Keywords: Generalized Thermoelasticity, Two Temperature, Rayleigh-Lamb waves, Secular Equations

1. INTRODUCTION:

The classical uncoupled theory of thermoelasticity, the heat conduction equation is parabolic in nature and hence it predicts infinite speed of propagation for thermal signals and the elastic deformation is independent of temperature. Biot [1] introduced the coupled theory of thermoelasticity in 1956. Generalized Theories of Thermoelasticity predict finite speed for the propagation of thermal signals. The first generalized theory of thermoelasticity has been given by introducing one relaxation time in Fourier’s heat conduction law [2]. Another generalized theory of thermoelasticity is proposed by improving the classical energy equation and the stress-strain temperature relations [3]. Other models of generalized thermoelasticity are given in [4-6]. Model proposed in [5] is non-linear and applicable in low temperatures such as laser pulse heating. The theory of heat conduction, which depends on two distinct temperatures viz the conductive temperature and the thermodynamic temperature has been described in [7] and [8]. The difference between two temperatures is proportional to heat supply for in case of no heat supply and time independent situations, the two temperatures are equal. The wave propagation in the two temperature thermoelasticity theory [9]. Harmonic plane waves in two temperature theory have been investigated in [10]. Two temperature generalized thermoelasticity theory has been formulated by Youssef [11]. The effect of thermal relaxation time on plane wave propagation under two temperature theory has also been investigated [12].

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In this article, Youssef's model of two temperature generalized thermoelasticity has been used to investigate the wave propagation.

2. FORMLATION OF PROBLEM:

Let us consider an infinite homogeneous, isotropic thermoelastic plate of thickness 2d at uniform temperature T_0. The origin of the coordinate system is taken on the middle surface of the plate such that z-axis is in thickness direction.

The surfaces z = ±d are assumed to be Stress free insulated or isothermal. We have considered the x-z plane as the plane of incident and assume that the solutions are explicitly independent of y coordinate, but they depend implicitly so that the transverse component of displacement does not vanish.

The field equations in the context of generalized thermoelasticity with two temperatures in absence of body forces and heat sources are:
Equations of motions

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z}+\mu\frac{\partial^2 u}{\partial z^2}-\gamma(1+\nu\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\mu\frac{\partial^2 w}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial z\partial x}+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\gamma(1+\nu\frac{\partial}{\partial t})\frac{\partial\theta}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2})=\rho\ddot{v} \tag{3}$$

Generalized heat conduction equation is

$$\begin{aligned}&K(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2})=\rho C_E(\frac{\partial\theta}{\partial t}+\tau_0\frac{\partial^2\theta}{\partial t^2})+\\&\gamma T_0\left(\frac{\partial}{\partial t}+n_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)\end{aligned} \tag{4}$$

where ϕ and θ are given by

$$\phi-T=a\phi_{,ii} \quad \& \quad \theta=\left|T-T_0\right| \tag{5}$$

Constitutive relations for non-vanishing stresses are

$$\begin{aligned}&\sigma_{zz}=(\lambda+2\mu)\frac{\partial w}{\partial z}+\lambda\frac{\partial u}{\partial x}-\gamma(\theta+\nu\frac{\partial\theta}{\partial t})\\&\sigma_{yz}=2\mu\frac{\partial v}{\partial z},\quad \sigma_{xz}=2\mu(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})\end{aligned} \tag{6}$$

$$\sigma_{zz}=\frac{\partial w}{\partial z}+(1-2\delta^2)\frac{\partial u}{\partial x}-(\theta+\gamma\frac{\partial\theta}{\partial t})=0$$

where n_0 is constant; λ, μ are Lame's parameters, K is the thermal conductivity; ρ and C_E are the density and specific heat at constant strain respectively, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the linear thermal expansion. Taking $\tau_0 = \nu = a = 0$, one can obtain results for coupled thermoelasticity, In case of Lord & Shulman theory $n_0 = 1, \tau_0 > 0, \nu = 0, a = 0$ and Green & Lindsay theory $n_0 = 0, \tau_0 > 0, \nu > 0, a = 0$.

Introducing non dimensional quantities

$$x' = \frac{x\omega^*}{c_1},\ z' = \frac{z\omega^*}{c_1},\ u' = \frac{u\omega^* c_1 \rho}{\gamma T_0}, w' = \frac{w\omega^* c_1 \rho}{\gamma T_0}$$

$$t' = \omega^* t,\ \nu' = \omega^* \nu,\ \tau_0' = \omega^* \tau_0,\ \gamma = (3\lambda + 2\mu)\alpha_t$$

$$a' = \frac{a\omega^{*2}}{c_1^2},,\ \delta^2 = \frac{\mu}{(\lambda + 2\mu)}, \epsilon = \frac{T_0 \gamma^2}{\rho C_E (\lambda + 2\mu)}, \theta' = \frac{\theta}{T_0},\ \phi' = \frac{\phi}{T_0} \tag{7}$$

where

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ \omega^* = \frac{C_E(\lambda + 2\mu)}{K} \tag{8}$$

Equations (1), (2) and (4) are obtained as

$$\frac{\partial^2 u}{\partial x^2} + (1 - \delta^2)\frac{\partial^2 w}{\partial x \partial z} + \delta^2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial}{\partial x}(\theta + \nu \frac{\partial \theta}{\partial t}) = \frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\frac{\partial^2 w}{\partial z^2} + (1 - \delta^2)\frac{\partial^2 u}{\partial x \partial z} + \delta^2 \frac{\partial^2 w}{\partial x^2} - \frac{\partial}{\partial z}(\theta + \nu \frac{\partial \theta}{\partial t}) = \frac{\partial^2 w}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = (\frac{\partial \theta}{\partial t} + \tau_0 \frac{\partial^2 \theta}{\partial t^2}) + \in \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \tag{11}$$

Further, the mechanical boundary conditions at z = ±d in non-dimensional form are given by

$$\sigma_{zz} = \frac{\partial w}{\partial z} + (1 - 2\delta^2)\frac{\partial u}{\partial x} - (\theta + \gamma \frac{\partial \theta}{\partial t}) = 0 \tag{12}$$

$$\sigma_{xz} = \delta^2 (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) = 0,\ \sigma_{yz} = \frac{\partial v}{\partial z} = 0\ .$$

and the thermal conditions are given as $\frac{\partial \phi}{\partial z} + h\phi = 0$ (13)

where h denotes surface heat transfer coefficient. The limiting case $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ leads isothermal boundaries.

3. SOLUTION OF THE PROBLEM:

In order to solve equations (9) to (11), introducing potential functions Φ and Ψ such that

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial z},\ w = \frac{\partial \Phi}{\partial z} - \frac{\partial \Psi}{\partial x} \tag{14}$$

and consider the solutions of the form

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$$\left(\Phi,\Psi,\phi\right)=\left[f(z),g(z),h(z)\right]\exp[i\xi(x-ct)] \tag{15}$$

where $c=\omega/\xi$ is the phase velocity, ω is the frequency and ξ is the wave number. Inserting (14) in equations (9) to (11), applying the solution defined by (15) in resulting equations and then solving the resulting system of differential equations, the expressions for u, v, w and ϕ are obtained as

$$\begin{aligned} u(x,z,t)=&[i\xi(C_3\cos m_1z+C_4\sin m_1z+C_5\cos m_2z+C_6\sin m_2z)\\ &-\beta C_7\sin\beta z+\beta C_8\cos\beta z]\exp[i\xi(x-ct)] \end{aligned} \tag{16}$$

$$\begin{aligned} w(x,z,t)=&[-m_1C_3\sin m_1z+m_1C_4\cos m_1z-m_2C_5\sin m_2z+\\ &m_2C_6\cos m_2z-i\xi(C_7\cos\beta z+C_8\sin\beta z)]\exp[i\xi(x-ct)] \end{aligned} \tag{17}$$

$$\phi(x,z,t)=\begin{bmatrix} \frac{1}{\alpha_1}(C_3\cos m_1z+C_4\sin m_1z)+\\ \frac{1}{\alpha_2}(C_5\cos m_2z+C_6\sin m_2z) \end{bmatrix}\exp[i\xi(x-ct)] \tag{18}$$

$$\begin{aligned} \theta(x,z,t)=&[\frac{\{1+a(\xi^2+m_1^2)\}}{\alpha_1}(C_3\cos m_1z+C_4\sin m_1z)+\\ &\frac{\{1+a(\xi^2+m_2^2)\}}{\alpha_2}(C_5\cos m_2z+C_6\sin m_2z)]\exp[i\xi(x-ct)] \end{aligned} \tag{19}$$

where

$$\alpha_1=\frac{-i\omega t_2\{1+a(\xi^2+m_1^2)\}}{\alpha^2-m_1^2},\ \alpha_2=\frac{-i\omega t_2\{1+a(\xi^2+m_2^2)\}}{\alpha^2-m_2^2} \qquad m_1^2=[\frac{1}{2}(A+\sqrt{A^2-4B})],\ m_2^2=[\frac{1}{2}(A-\sqrt{A^2-4B})]$$

$$A=\frac{2\xi^2-\omega^2\left(1+t_0-i\in\omega t_2t_3+a\left(\omega^2t_1+2i\xi^2\omega t_2t_3-2\xi^2t_1\right)\right)}{-1+a\omega^2(t_1-i\in\omega t_2t_3)},$$

$$B=\frac{-\xi^4-\omega^4t_0+\omega^2\xi^2\left(1+t_0-i\in\omega t_2t_3+a\left(-\omega^2t_1-i\in\xi^2\omega t_2t_3+\xi^2t_1\right)\right)}{-1+a\omega^2(t_1-i\in\omega t_2t_3)} \qquad t_0=\tau_0+i\omega^{-1},t_1=1+i\omega^{-1},t_2=\nu+i\omega^{-1},t_3=n_0\tau_0+i\omega^{-1}$$

$$\alpha^2=\xi^2\left(c^2-1\right),\ \beta^2=\xi^2\left(\frac{c^2}{\delta^2}-1\right).$$

4. SECULAR EQUATIONS:

Using the above expressions (16) to (19) and applying boundary conditions (12) and (13), a homogeneous system of linear equations in terms of unknowns C_i is obtained. The obtained system of equations will have non trivial solution iff the determinant of coefficient matrix vanishes which gives the secular equations for stress free thermally insulated and isothermal boundaries, respectively are obtained as

$$\begin{aligned} &\left(\frac{\tan m_1d}{\tan m_3d}\right)^{\pm1}-\frac{m_1(\alpha^2-m_1^2)(1+a(\xi^2+m_2^2))}{m_2(\alpha^2-m_2^2)(1+a(\xi^2+m_1^2))}\left(\frac{\tan m_2d}{\tan m_3d}\right)^{\pm1}\\ &\quad=\frac{4\xi^2\beta m_1(m_2^2-m_1^2)(1+a(\xi^2+\alpha^2))}{(\xi^2-\beta^2)^2(\alpha^2-m_2^2)(1+a(\xi^2+m_1^2)} \end{aligned} \tag{20}$$

$$\left(\frac{\tan m_1 d}{\tan m_3 d}\right)^{\pm 1} - \frac{m_2(\alpha^2 - m_1^2)(1 + a(\xi^2 + m_2^2))}{m_1(\alpha^2 - m_2^2)(1 + a(\xi^2 + m_1^2))}\left(\frac{\tan m_2 d}{\tan m_3 d}\right)^{\pm 1}$$
$$= \frac{(\xi^2 - \beta^2)^2(m_2^2 - m_1^2)(1 + a(\xi^2 + \alpha^2))}{4\xi^2\beta m_1(\alpha^2 - m_2^2)(1 + a(\xi^2 + m_1^2)}$$

In above secular equations, the superscripts +1 and −1 correspond to skew symmetric and symmetric wave modes respectively.

5. NUMERICAL R ESULT AND DISCUSSIONS

In order to describe theoretical results presented in above equations, numerical results are presented. Here, we have chosen copper material for evaluation of numerical results and physical data for which is as given below

$$\lambda = 7.76 \times 10^{10}\,\mathrm{Kg\,m^{-1}s^{-2}},\ \mu = 3.86 \times 10^{10}\,\mathrm{Kg\,m^{-1}s^{-2}}$$
$$\varepsilon = 0.0168,\ \rho = 8954\,\mathrm{Kg\,m^{-3}},\ C_E = 383.1\,\mathrm{J\,Kg^{-1}K^{-1}}$$
$$k = 386\,\mathrm{W\,m^{-1}K^{-1}},\ \alpha_t = 1.78 \times 10^{-5}\,\mathrm{K^{-1}},\ T_0 = 293\mathrm{K}$$
$$\tau_0 = 6.131 \times 10^{-13}\mathrm{s},\ \nu = 8.765 \times 10^{-13}\mathrm{s},\ d = 1.0$$

The phase velocities of symmetric and skew symmetric modes of wave propagation have been computed for various values of wave number from the dispersion relation (20) for stress free insulated boundary conditions for Lord and Shulman theory of thermoelasticity.

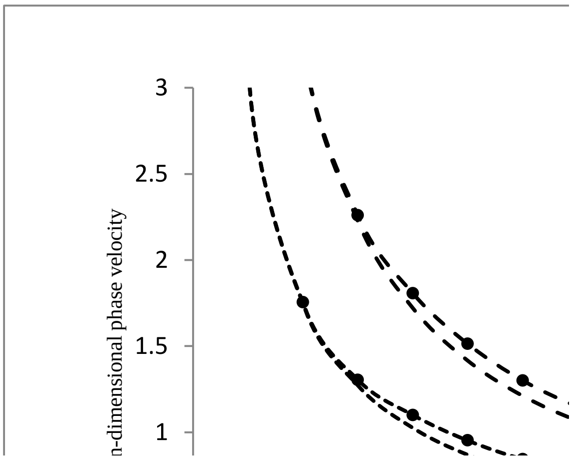

Figure 1: Variation of phase velocity with wave number

Figure 1 presents the variation of non-dimensional phase velocity with wave number for n =0, 1, 2 in case of skew symmetric mode. Curves with dark circles as markers correspond to two temperature theory in both figures. The phase velocity in all the three wave modes has higher magnitude in two temperature theory as compared to corresponding one temperature theory of thermoelasticity. Similar behavior has been observed for other theories as well. Figure 2 presents the variation of phase velocity with wave number for symmetric modes of vibrations. In symmetric modes of vibration phase velocity in case of two temperature thermoelasticity is having smaller magnitude.

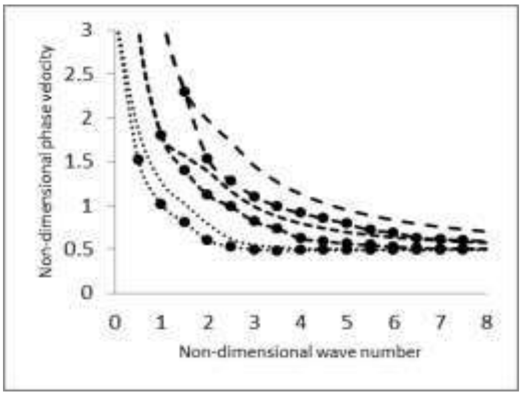

Figure 2: Variation of phase velocity with wave number

5. CONCLUSIONS:

The trends of variation of the phase velocity in two and one temperature thermoelasticity are similar however in skew symmetric modes magnitude is higher but in symmetric modes it is reversed.

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GRAVITATIONAL INSTABILITY IN VISCOELASTIC MEDIUM WITH DISSIPATIVE EFFECTS

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ABSTRACT:
In this paper, the gravitational instability of a self-gravitating viscoelastic medium has been analyzed in the presence of dissipative effects, using standard Jeans mechanism. A dispersion relation is derived for viscoelastic medium from the perturbation equations of the problem, using normal mode analysiswhen the bulk viscosity affects the first order dynamics. The instability conditions under both strongly (kinetic) and weakly (hydrodynamic) coupling limitsare obtained from the dispersion relation. It is observed for strongly coupled viscoelastic medium, the instability criterion gets modified due to the presence of dissipative effect which is represented by the Stokes-Kirchoff's factor and the critical wave number decreases, hence have stabilizing effect on the onset of instability. However, for weakly coupledfluid, the Jeans Criterion governs the onset of instability.

KEY WORDS: Gravitational instability, Jeans criteria, Dissipation energy. Normal mode analysis, viscoelastic medium, wave propagation, Coupling limits.

1. INTRODUCTION:

Generally accepted theoretical framework for the formation of the solar systems and the other planetary systems, in the domain of astrophysics and geophysics, is that of the Gravitational Instability(GI). GI is the key idea in explaining the way in which structures evolve in the universe. The star formation begins with small density fluctuations in an initially nearly uniform medium that are amplified by gravity in process called gravitational instability.During last century, the problem of self-gravitational instability has been a broad area of research in astrophysics, plasma physics and many other crucial phenomena of the interstellar medium. GI is a theory that recognizes the gravity as the only force capable of creating structures by accumulating material in space. Whenever the internal pressure of a gas is too weak to balance the self-gravitational force of a mass density perturbation, a collapse occurs; such mechanism was first studied by Jeans [1, 2] in terms of the wavelengths of a fluctuation. The dynamical mechanism responsible behind aggregation of matter and formation of various astrophysical structures, such as comets, asteroids, etc., in astro-comic environments are widely described and studied by gravitational or Jeans instability. The Jeans instability was later rigorously investigated by anumber of other authors including; Eddington [3], Mestel and Spitzer [4] and Chandrasekhar [5]. Dhiman and Dadwal [6,7] havealso studied the gravitational instability of a stratified homogeneous medium under varying assumptions of hydrodynamic and hydromagnetic, however Chandrasekhar [5] has given a detailed account of the Jeans instability under the influence of rotation or/and magnetic field and has derived the Jeans criteria for the various problems.For more details on gravitational instability, one may refer to Jeans [2].

TheJeans Instability in various kinds of plasma environment, *e.g.* gaseous plasma, fluid plasmaand dusty plasma have been investigated by many authors. In astrophysical domain, there are so many objects including interstellar medium which are composed of viscoelastic fluidsand possess both viscous and elastic properties. Elastic effects in fluid play an important role similar to thermal pressure, which lower the growth rate of gravitational instability. Janaki et al. [8] have discussed the Jeans instability of viscoelastic medium and found that the threshold for the onset of instability appears at higher wavelength in viscoelastic medium than in the case of ordinary gaseous medium. Prajapati and Chhajalni[9] have also investigated the linear self-gravitational instability of finitely conducting, magnetized viscoelastic fluid using the modified generalized hydrodynamic model.Rosenberg and Shukla [10] used the Generalized Hydrodynamic Model (GH) to describe the behavior of viscoelastic fluid and reported that the GH model describes the effects of strong correlations through the introduction of viscoelastic coefficients. Further,Kaw and Sen[11] suggested that the viscoelastic properties of the medium are characterized by the relaxation time τwhich provides a characteristic timescale to distinguish two classes of low frequency modes; one when the frequency $\sigma \ll \frac{1}{\tau}$ known as *hydrodynamic limit*(*weakly couplinglimit*) and the other frequency $\sigma \gg \frac{1}{\tau}$ known as *kinetic limit* (*strongly couplinglimit*) where $\sigma$is the wave frequency and τ is the viscoelastic relaxation time. Recently, Dhiman and Sharma [12] have investigated the effect of rotation on the growth rate of magneto gravitational instability of a viscoelastic medium under both of these limits and investigated the growth rate of the instability for both longitudinal and transverse modes of wave propagation.

Janaki [8] discussed that a central idea in the study of instability by including various factors is to find the ways of arresting the gravitational collapse.Heat in the interstellar medium play important role in the gravitational collapse which means dissipation energy effects of viscoelastic medium on GI. The viscous coefficients are not constant and we have to express their dependence on the state parameters of the fluid. Since we are interested to treat isotropic and homogeneous perturbative cosmological models. In fact, in such models there is no displacement of matter with respect to each other and this kind of viscosity represents the energy dissipation due to this effect(*cf.*Carlevaro and Montani [13]).

The present paper is motivated by the above discussions and aimed to study the effect of dissipative energy on the onset of gravitational instability of the viscoelastic fluid. For this, following [13], we shall consider the bulk viscosity expressed (in the first order)in terms of the thermo dynamical parameters of the fluid without neglecting the shear viscosity effect.We have followed the normal mode analysis to analyze linear perturbation of density in the governing equations and have obtained and a general dispersion relation. The stability criteria for both strongly and weakly coupled fluids under kinetic and hydrodynamical limitsare obtained from the dispersion relation in terms of wave numbers of the self-gravitating system,using the Hurwitz Criterion given by Guillemin[14].

2. Mathematical Formulation of the Problem:

We have considered the GH model to treat the viscoelastic properties ofan infinite homogeneous,self-gravitatingviscoelastic fluid. Under these assumptions, the generalized basic equations of continuity, motion and Poisson equation governing the physical problem (*cf*. Janaki et al. [8] and Dhiman and Sharma [12])are given by:

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\boldsymbol{u})=0 \tag{1}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial\boldsymbol{u}}{\partial t}+(\boldsymbol{u}.\nabla)\boldsymbol{u}\right)-\rho\nabla\phi+{c_s}^2\nabla\rho\right]=\mu\nabla^2\boldsymbol{u}+\left(\zeta+\frac{\mu}{3}\right)\nabla(\nabla.\boldsymbol{u}) \tag{2}$$

$$\nabla^2\phi=-4\pi G(\rho-\rho_0) \tag{3}$$

In the above equations;$\boldsymbol{u}$and $\boldsymbol{r}$ represent respectively the velocity and position vectors, τ,ρ,ϕ,μ,G andc_s respectively denote the relaxation time, density of fluid, gravitational potential, coefficient of viscosity, the universal gravitational constant and the speed of sound in isothermal medium. Further, the term $\left(\zeta+\frac{\mu}{3}\right)$ is the coefficient of bulk viscosity and $\left(1+\tau\frac{\partial}{\partial t}\right)$denotes the Frenkel's term or viscoelastic operator which accounts for the relaxation effects(*cf*. Frenkel [15]).

Following Carlevaro and Montani [13], the fundamental hypothesis of Jeans modelfor a static and uniform solution for the zeroth-order dynamics are;

$$u_0=0,\rho_0=\text{const. },\phi_0=const. \tag{4}$$

Mathematically, we can say that when the density and pressure of the medium $\rho_0\,,p_0$ are constant, and the mean velocity v_0 is zero, it follows from equation (2) that $\nabla\phi_0=0$. On the other hand, Poisson equation (3) requires that $\nabla^2\phi_0=-4\pi G\rho_0$. This assumption contradicts the gravitational equation andthese two requirements are inconsistent unless $\rho_0\ =0$. This is known as *Jeans Swindle*. Physically, there are no pressure gradients in a homogeneous medium to balance gravitational attraction (*cf.* Larson [16]). Thus, in order to avoid the*Jeans Swindle*, we have taken Poisson equation as;

$$\nabla^2\phi=-4\pi G(\rho-\rho_0)$$

where the density $-\rho_0$is a repulsion term and may be regarded as a Newtonian analogue of Einstein's cosmological constant (*cf.*Kiessling [17]).

3.Linearized Perturbation Equations and Dispersion Relation:

To investigate the instability of the self-gravitating system governed by basic equations (1)-(3), let the initial stationary state solution (4) be slightly perturbed by giving infinitesimal small perturbations $\delta\rho(\boldsymbol{r},t),\delta\phi,\delta\boldsymbol{v}(\boldsymbol{r},t)$ in the density ρ_0, gravitational potential ϕ_0, and velocity$\boldsymbol{u}$, respectively. In treating bulk-viscosity perturbations, we use the expansions

$\zeta=\zeta_0+\delta\zeta$, where $\zeta_0=\zeta(\rho_0)=z{\rho_0}^s=constant$ and

$$\delta\zeta=\delta\rho\left(\frac{\partial\zeta}{\partial\rho}\right)+\cdots=zs{\rho_0}^{s-1}\delta\rho+\cdots$$

Here,$\zeta=ze^s$ defines the intensity of *dissipative effects,* with $s=constant$ and z is a parameterwhich defines the intensity of viscous effects.

Thus, the perturbations are now represented by;

$$\rho=\rho_0+\delta\rho\,,\phi=\phi_0+\ \delta\phi\,,\boldsymbol{u}=0+\ \delta\boldsymbol{v},\zeta=\zeta_0+\delta\zeta \tag{5}$$

Using these perturbed quantities in equations (1)-(3) and then linearizing the resulting equations, we get the following linearized perturbation equations of continuity, motion and Poisson equation respectively,for the present problem for the case of viscoelastic medium with dissipative effects;

$$\frac{\partial \delta\rho}{\partial t} + \nabla.(\rho_0 \delta \boldsymbol{v}) = 0 \tag{6}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho_0\left(\frac{\partial \delta \boldsymbol{v}}{\partial t}\right) - \rho_0 \nabla\, \delta\phi + {c_s}^2 \nabla \delta\rho\right] = \mu \nabla^2 \delta\boldsymbol{v} + \left(\zeta_0 + \frac{\mu}{3}\right)\nabla(\nabla.\delta\boldsymbol{v}) \tag{7}$$

$$\nabla^2\, \delta\phi = -4\pi G\, \delta\rho \tag{8}$$

Using equation (6) in (7), we get

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho_0\left(\frac{\partial \delta \boldsymbol{v}}{\partial t}\right) - \rho_0 \nabla\, \delta\phi + {c_s}^2 \nabla \delta\rho\right] = \mu \nabla^2 \delta\boldsymbol{v} - \frac{\left(\zeta_0 + \frac{\mu}{3}\right)}{\rho_0}\nabla\left(\frac{\partial \delta\boldsymbol{\rho}}{\partial t}\right) \tag{9}$$

Taking the divergence of both sides of the above equation and using equation (8), we get

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[-\left(\frac{\partial^2 \delta\rho}{\partial t^2}\right) + 4\pi G \rho_0 \delta\rho + {c_s}^2 \nabla^2 \delta\rho\right] = -\frac{\mu}{\rho_0}\nabla^2\left(\frac{\partial \delta\rho}{\partial t}\right) - \frac{\left(\zeta_0 + \frac{\mu}{3}\right)}{\rho_0}\nabla^2\left(\frac{\partial \delta\boldsymbol{\rho}}{\partial t}\right) \tag{10}$$

To study the properties of $\delta\rho$, we now consider a solution of the form;$\delta\rho \sim$ exp $[-i(\omega t - \boldsymbol{k}.\boldsymbol{r})]$. Here,$\omega$ is the frequency and$\boldsymbol{k}$is the wave vector of the mode under consideration.

Using the above dependence of $\delta\rho$ in the above equation, we have

$$(1 - i\omega\tau)\left[\omega^2 + {\omega_j}^2 - k^2 {c_s}^2\right] = -i\omega \frac{k^2}{\rho_0}\left(\zeta_0 + \frac{4\mu}{3}\right) \tag{11}$$

Equation (11) can be written as

$$(1 + \sigma\tau)\left[-\sigma^2 + {\omega_j}^2 - k^2 {c_s}^2\right] - \sigma \frac{k^2}{\rho_0}\left(\zeta_0 + \frac{4\mu}{3}\right) = 0 \tag{12}$$

where,$\sigma = -i\omega$ is frequency of harmonic disturbance.

Following Dhiman and Sharma [12], the above equation (12)under *strongly coupling limit* (*kinetic limit)* $\tau\sigma \gg 1$, yields the following dispersion relation

$$\sigma^2 + \frac{k^2}{\tau\rho_0}\left(\zeta_0 + \frac{4\mu}{3}\right) + k^2 {c_s}^2 - {\omega_j}^2 = 0 \tag{13}$$

where, ${\omega_j}^2 = 4\pi G \rho_0$ is the *Jean's frequency*.

Equation (13) can be written in the following form

$$\sigma^2 + k^2\left({c_s}^2 + \frac{{D_v}^2}{\tau}\right) - {\omega_j}^2 = 0 \tag{14}$$

This is the required *dispersion relation*.

Here, ${D_v}^2 = \frac{\left(\zeta_0 + \frac{4\mu}{3}\right)}{\rho_0}$ is the square of *modified* Stokes-Kirchoff's factor (*cf.*García-Colín and Sandoval-Villalbazo [18]) representing the dissipative effects.

It is to note that the dispersion relation (14) is modified version of the relation obtained by Janaki et al. [8]. Now, invoking the Hurwitz criterion as discussed by Guillemin [14] for the sign of roots, which implies that if the constant term of equation (14) is negative, we have the condition for the onset of gravitationalinstability given as;

$$k^2\left({c_s}^2 + \frac{{D_v}^2}{\tau}\right) - {\omega_j}^2 < 0 \tag{15}$$

The above condition of instability is the modified form of instability criterion given byJanaki et al. [8] for the case of viscoelastic medium and now represent the effect of dissipative energy in the form of Stokes-Kirchoff's factor. Thus, the bulk and shear viscosityof the viscoelastic medium with dissipative effects modifies this criterion of Jeans instability.

The expression of critical Jeans wave number is given by;

$$k < k_j \; where \;\; k_j = \frac{\omega_j}{\sqrt{{c_s}^2 + \frac{{D_v}^2}{\tau}}}$$

It is obvious that as the viscosity with dissipative effects in the viscoelastic fluid increases the corresponding critical Jeans wave number decreases.
Further under *weakly coupling limit* (*hydrodynamic limit*)when $\tau\sigma \ll 1$, the dispersion relation (12) yields the following dispersion relation

$$\sigma^2 + k^2 \frac{{D_v}^2}{\tau}\sigma + k^2 {c_s}^2 - {\omega_j}^2 = 0 \tag{16}$$

Again, by using Guillemin [14] criterion,we have the following condition for instability given by;

$$k^2 {c_s}^2 - {\omega_j}^2 < 0 \tag{17}$$

which is the same Jeans Criterion as obtained by Chandrasekhar [5] for gaseous medium.

4. RESULTS AND DISCUSSION:

In the present analysis, the effect of bulk and shear viscosity on the onset of gravitational instability have been studied by considering the bulk viscosity as the first-order dynamics via a power-law function of the energy density. Here, we have derived a general dispersion relation form the linearized perturbation equations using normal mode analysis, under both strongly and weakly coupling limits for viscoelastic medium. The instability conditions are derived for both of these cases. It is observed that under the strongly coupling limit, the instability criterion obtained by Janaki et al. [8] gets modified due to the presence of dissipative effect of viscosity and is represented by the Stokes-Kirchoff's factor. Hence, for the viscoelastic medium the dissipative effect of bulk viscosity decreases the critical wave number and hence postpones the onset of instability. Further, in case of weakly coupling limit, theJeans Criterion as obtained by Chandrasekhar governs the onset of instability.

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ON THE WEIRDNESS OF SOME NUMBERS IN MATHEMATICS

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ABSTRACT:
An endless mysterious numbers are known to exist having various kind of strangeness. Kaprekar and Indian mathematician revealed an interesting property of the number 6174, which was subsequently named the Kaprekar constant. This number is truly a strange numbers. At first glance, it might not seem so obvious, but rearranging the digits of every four digit number where the digits are not all the same, also if one or more of the digits is zero to make it a four digit number, they must be embedded in the left side of minimum number to obtain the largest and smallest numbers from the digits can make. On using *Kaprekar's operation* that is subtracting the smallest number from the largest to obtain a new number, and carry on repeating the operation for each new number, reaches 6174 under Kaprekar's process, and in at most seven steps. Similarly using *Kaprekar's operation* for 3-digit numbers then most sequences (i.e., other than repeated digits such as 111) will terminate in the value 495 in at most 6 iterations. In this article Computer Algebra System's new algorithm is being used to demonstrate this operation for any number of digits for unlimited number of iterations. Various properties of the results obtained are discussed and compared with the results available in the literature. Results obtained are also exhibited graphically.

1. INTRODUCTION:

Mathematics, mainly theory of numbers, is no more complicated than any other subject. If one's have the enthusiasm, endurance, and time to work on and do the effort, then number theory is worth studying and we will find that this branch of mathematics can be amusing and entertaining. Science of numbers is for eternity at the heart of mathematics and has a natural attraction from the beginning of to the studentship. Mathematics is the conception and is part of the character of the human intelligence; it arises from the individual mind as it comes into contact with the world. Every individual demonstrate mathematical inclination and tends to guess, desires to identity, quantify, difference, similarity, and summarize, to make classify and generalize. A branch of pure mathematics.

Mathematics is not all regarding numbers–higher mathematics such as, real analysis, abstract algebra, complex analysis and topology, etc. is scarcely build upon numbers, but on definitions and concepts. However, the numbers are the most important component of mathematics since the beginning of its history.

Even after nearly seven decades Kaprekar constant continues to fascinate mathematicians, mathematics scholar, teachers and recreational mathematics. Kaprekar [1-3] revealed an exciting and amazing process, known as Kaprekar's operation. This process when functional to any positive four digit number not all the same i.e. avoiding numbers with four identical digits like 9999, converges to the same number 6174 in at most seven iterations. To understand the process, choose an four digits integer N (all digits of N are not all equal), and arranges its digits in descending and ascending order, subtract them, repeat the above procedure certain number of times (at the most 7 times). Thus the Kaprekar process for any four digit number is remarkable for two reasons. Firstly process converges to the same number 6174 and in no more than 7 iterations. Yutaka Nishiyama [4] described that at fleeting look, the number 6174 is might not seem so obvious, but it is really a mysterious number. Naranan [12] has also discussed this magical number. Patil and Shah [5] have derived the Kaprekar numbers and its analog equations. A variation on the two-digit kaprekar routine is studied by Anne Ludington Young [8]. Various authors have studied problems and intriguing questions linked with the above process. (See [6]-[11].)

In this paper Computer Algebra System's innovative algorithm is being used to demonstrate this operation for a number of any numbers of digits which can be checked for unlimited iterations. Various properties of the results obtained for a number consisting of 2-digitd, 3-digits ….. *n*-digits are discussed and compared with the results available in the literature. Results obtained are also exhibited graphically that how the results obtain in the process converges to constants (2-digitd, and 4-digits) and behaviors of all two digits number.

2. The Kaprekar Process:

Consider any number where all the digits are different, firstly reshuffle the digits to make the largest (arranging the digits in a descending order) and smallest (arranging the digits in a ascending order) numbers. Also if you consider say 5, and want to apply Kaprekar operation for three 3-digits or 4-digits number, then arrange the number as 005 or 0005 respectively. Secondly subtract the smaller number from the larger one For the resulting number, repeat the procedure, and keep repeating.

2.1 Four digit number:

Try N = 2019 (the current year)
9210-0129 = 9081
9810-0189 = 9621
9621-1269 = 8352
8532-2358 = 6174
7641-1467 = 6174

Interestingly it took four iterations to converge 6174 and to repeats it i.e after reaching at 6174 it creates an infinite loop on the constant 6174.
Let us try N= 2510
5210 − 0125 = 5085
8550 − 0558 = 7992
9972 − 2799 = 7173
7731 − 1377 = 6354
6543 − 3456 = 3087
8730 − 0378 = 8532
8532 − 2358 = 6174
7641 − 1467 = 6174

it took seven iterations (maximum) to converge 6174 and to repeats it.

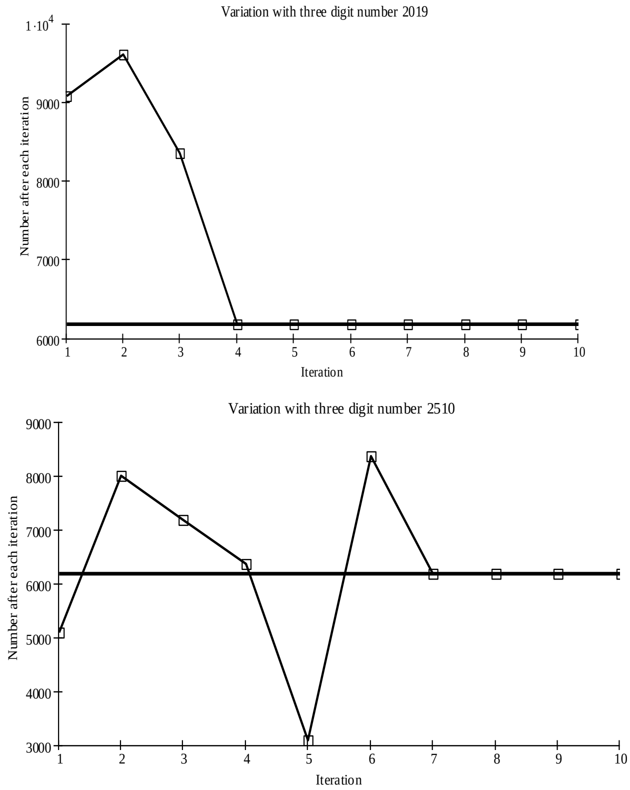

Figure 1. Variation of numbers 2019 (a) and 2510 (b) with iteration using Kaprekar's operation for 4-digits numbers.

2.2 Frequency and Number of iterations of 4-digits numbers converges to constant:
Many researchers have worked on the Kaprekar's operation by writing algorithm in various software and computer algebra systems to work out for all the 4-digits. Yutaka Nishiyama [2012] have checked all 4-digits 8991 numbers and found that every four digit number where the digits aren't all equal converges to 6174 in at most seven steps under Kaprekar's process and consequently concluded that if you do not reach 6174 after using Kaprekar's operation seven times, then categorically there is a mistake in the computation and try it again.

Number of iterations	0	1	2	3	4	5	6	7
Frequency	1	356	5192	2124	1124	1379	1508	1980

2.3 Three digit number:
On considering any three digit numbers not all the digits are same, the identical phenomenon occurs. In this case the process converges to the same number 495 and in no more than 6 iterations.
For example applying Kaprekar's operation to the three digit number 753 gives the following:

$$753 \rightarrow 396 \rightarrow 594 \rightarrow 495$$

The number 495 is the unique kernel for the operation on three digit numbers, and all three digit numbers reach 495 using the operation. Why don't you check it yourself?

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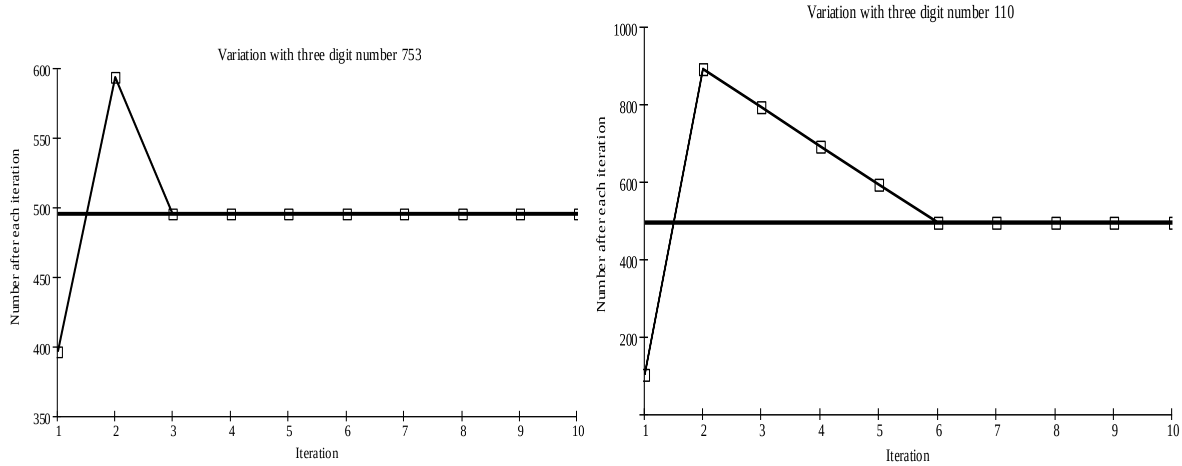

Figure 2. Variation of numbers 753 (a) and 110 (b) with iteration using Kaprekar's operation for 3-digits numbers.

2.2 Two digit number:

In case of four and three digit numbers reach a unique kernel 6174 and 495 respectively. On attempting the process for a two digit number, unlike for three and four digit numbers, there is no unique kernel for two digit numbers. Outcome is given in the following table:

2.3 Kaprekar operation on two digits number:

For all numbers 01-98 except where the digits are same (that is not 11, 22,,...,99).

Table:1 For all numbers 01-98 except where the digits are same (that is not 11, 22,,...,99) Kaprekar's operation for 2-digits numbers.

Pattern outcome	**Number (01-98) except where the digits are same**									
{9,81,63,27,45}	**01**	**12** **10**	**23** **21**	**34** **32**	**45** **43**	**56** **54**	**67** **65**	**78** **76**	**89** **87**	**98**
{**18**,63,27,45,9,81}	**02**	**13**	**20, 24**	**31,35**	**42,46**	**53.57**	**64, 68**	**75, 79**	**86**	**97**
{27,45,9,81,63}	**03**	**14**	**25**	**30,36**	**41,47**	**52,58**	**63, 69**	**74**	**85**	**96**
{**36**,7,45,9,81,63 }	**04**	**15**	**26**	**---,37**	**40,48**	**51,59**	**62**	**73**	**84**	**95**
{45,9,81,63,27}	**05**	**16**	**27**	**--, 38**	**---,49**	**50,---**	**61**	**72**	**83**	**94**
{**54**,9,81,63,27,45}	**06**	**17**	**28**	**--,39**	**Ä**	**Ä**	**60**	**71**	**82**	**93**
{63,27,45,9,81}	**07**	**18**	**29**	**Ä**	**Ä**	**Ä**	**Ä**	**70**	**81**	**92**
{**72**,27,45,9,81,63}	**08**	**19**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**80**	**91**
{81,63,27,45,9}	**09**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**Ä**	**90**

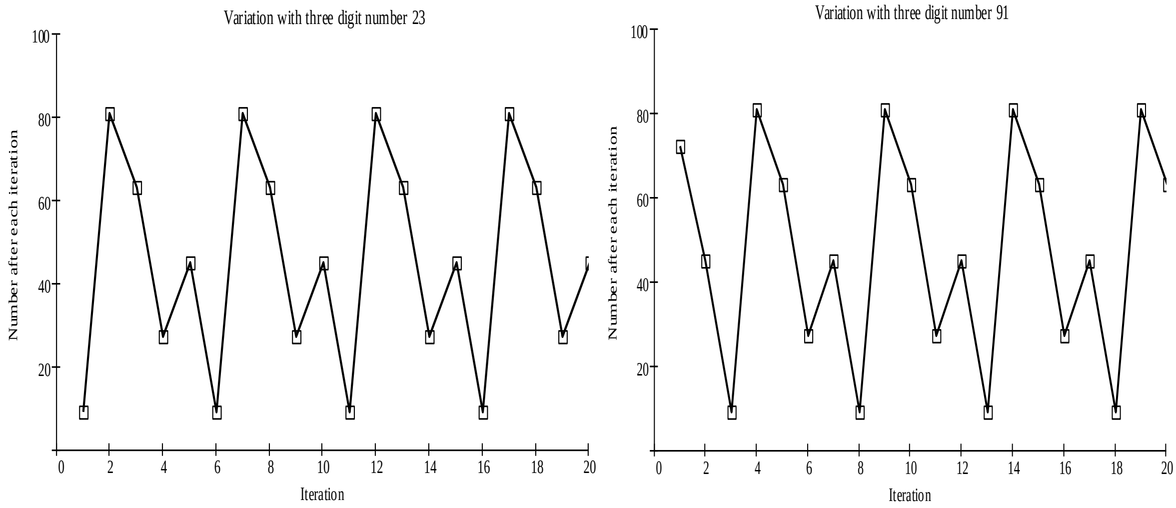

Figure 3. Variation of numbers 23 (a) and 91 (b) with iteration using Kaprekar's operation for 2-digits numbers.

2.4 Five digits, six and for any digit number...

Thus there are unique kernel for 3-digit and 4-digit numbers. Algorithm in the computer algebra system take no time to check what happens for six or more digits, and output becomes extremely monotonous, and found that there is no kernel for 2-digits, 5-digits, 7-digit numbers where as there are two kernels for 6-digits, 8-digits, 9-digit ……and 10-digit numbers having three kernels, which is in agreement with algorithm by

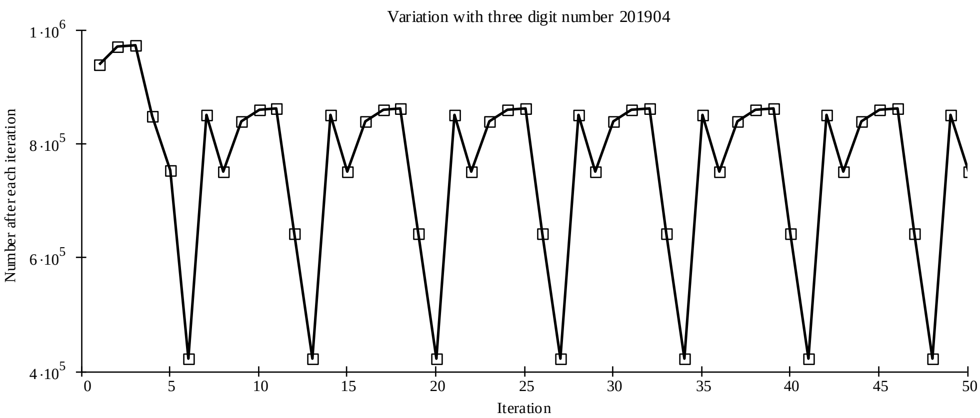

Figure 4. Variation of number 201904 with iteration using Kaprekar's operation for 6-digits numbers.

The computer algebra system (Mathcad) implementation of the method developed here is shown below:

ORIGIN := 1

KLVerma(N1, n) :=
```
P ← N1
dgt ← length(P)
sum ← 0
Diff ← 1
for j ∈ 1..n
    L ← sort(P)
    NumA ← Σ_{j=1}^{dgt} L_j · 10^(dgt−j)
    M ← reverse(sort(P))
    NumD ← Σ_{j=1}^{dgt} M_j · 10^(dgt−j)
    NNum ← NumD − NumA
    for i ∈ 1..dgt
        d_i ← mod(trunc(NNum / 10^(i−1)), 10)
        i ← i + 1
    for i ∈ 1..dgt
        for p ∈ 1..1
            M_{i,p} ← d_i
            M
    P ← M
    Sum_j ← Σ_{k=1}^{dgt} d_k
    Diff_j ← Σ_{i=1}^{dgt} d_{dgt−i+1} · 10^(dgt−i)
    j ← j + 1
((Sum_j, Diff_j, j))
```

3. RESULTS AND CONCLUSIONS:

From the algorithm written in computer algebra system for the Kaprekar's operation and the graphical representation in figures 1-4, of the results obtained it appears that Kaprekar's operation every number converges to a unique kernel only for three and four digit numbers.

Whereas for two digits numbers under the Kaprekar's operation it is observed that Number and on reversing the digit of a number has the same representation. Numbers which are congruent to ±2, ±4, ±6, ±8 show extra two digit number in beginning before settling for a repeating sequence. First and last two digit numbers and numbers preceding and succeeding (11, 22,,...,99) also have same representation. Sequence of each number which is repeating in any pattern has sum 9. i.e repeating two digit numbers with digit sum 9 are only {9, 27, 36, 45, 63, 81} whereas 18, 36, 54 and72 are not repeating. Interestingly non-repeating two digit numbers 18, 36, 54 and72 of sequences (congruent to ±2, ±4, ±6, ±8) has property that 20-02 =18, 40-04=36, 60-06-54 and 80-08=72. Any two digit number, if the sum is not 9 then it is not in any repeating sequence. Number of two digits which has representation I to IX has terms in AP are 18,16.,14,…,2. In j^{th} Pattern each number is $\pm j$ (modulo 11), j =1,2,…9 . Every column of the table has 9 numbers. Variation of number 6-digit number 201904 upto 50 iteration using Kaprekar's exhibited in Figure 4. Various progresses in science and mathematics have been driven by 'mistakes'. Mathematicians, all over the world are still working that some great theory of mathematics might lie behind this process also.

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A SHORT STUDY OF PRIME NUMBERS

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ABSTRACT:
In this paper, a review of some general properties of prime numbers has been given. Distance between two neighboring prime numbers and simple distance between two prime numbers have been defined and some results about the distance between two prime numbers have been presented. All prime numbers have one of the digits 1, 3, 7, 9 at unit place ; a short explanation of this concept has been considered, Fermat's theorems play an important role in the study of prime numbers, thus some results have been derived using a Fermat's theorem and application of these results has been given. Idea of Mersin's numbers has also been considered. A study of sum of consecutive prime numbers has also been done.

KEYWORDS: prime, composite, neighboring, consecutive, odd, even, divisible.

INTRODUCTION:

Here in this paper, we shall study some simple properties of prime numbers. The natural number system has been the main topic of human interest for the beginning of the civilization. The natural number system can be considered as the seed of mathematics. Once begin to play with natural number system, you will across various interesting results of natural numbers. Once dive deeply in the natural number system, you will dive in this system again and again because innumerable interesting results attract you greatly.

Among the natural numbers, there exists special type of numbers called prime numbers. A prime number is that natural which is divisible by one and itself. 1 is not considered a prime number, 2 is the only even prime number and all other primes are odd. Here one thing is note worthy that the existence of even numbers is due to the even prime 2 because if there were not 2, there won't have been even natural numbers. We can easily decide that natural numbers which are divisible by 2, 3, 5, and 11, because all even numbers are divisible by 2, all those numbers, the sum of whose digits is divisible by 3 , are divisible by 3, all those natural numbers ending with 5 or 0 are divisible by 5, and all those natural numbers, the difference of the sums of alternating digits of that number, is 0, are divisible by 11. We can call them separable prime numbers. There exists innumerable conjectures about prime numbers, but there exists no short cut formula for determining that a particular number is a prime number, there exist only long and time consuming algorithms for deciding a particular natural number to be a prime number. Prime numbers are infinite in numbers and their distribution does not follow any rule. Now we study some interesting results regarding prime numbers.

SOME GENERAL PROPERTIES OF PRIME NUMBERS:

First and last even prime number is 2, and all other prime numbers are odd. So we can write every odd prime number as $2n + 1$. Further every prime greater than or equal to 5 can be expressed as $6n \pm 1$, this can be proved as:
Consider three consecutive naturals $2n, 2n + 1, 2n + 2$. If any one of them is prime then that

will be $2n+1$. Any one of three consecutive naturals is divisible by 3. If $2n+1$ is prime then it will not be divisible by 3. Thus if $2n$ is divisible by 3, then $n = 3k\ , k \in N$. Therefore $2n+1 = 6k+1\ \forall\ k \in N$

If $2n+2$ is divisible by 3, then $n+1 = 3k\ , k \in N\ \Rightarrow n = 3k-1\ , k \in N$.

Therefore $2n+1 = 2(3k-1)+1 = 6k-2+1 = 6k-1\ \ \forall\ k \in N$.

So we find that every prime number ≥5, can be expressed in the form $6n \pm 1\ \forall\ n \in N$.

Therefore if p≥ 5 be a prime number then $p = 6n \pm 1\ \forall\ n \in N$.

Further each prime number ≥ 5 is sum of two consecutive natural numbers such that one of them is divisible by 3. This can be easily verified as $6k+1 = 3k+3k+1\ \forall\ k \in N$ and $6k-1 = 3k-1+3k\ \forall\ k \in N$.

Now consider $(6n \pm 1)^2 = 36n^2 \pm 12n + 1$.

$$= 12n(3n \pm 1) + 1. \tag{1}$$

If n is even let $n = 2k, k \in N,$ then $3n \pm 1 = 6k \pm 1$. (2)

Using the values given in (2) in (1), we have

$$(6n \pm 1)^2 = 24k(6k \pm 1) + 1 \tag{3}$$

If n is odd let $n = 2k-1, k \in N,$ then $3n \pm 1 = 3(2k-1) \pm 1$

$$= 6k-2\ or\ 6k-4$$

$$2(3k-1)or2(3k-2)$$

∴ if $n = 2k-1$, then $3n \pm 1 = 2(3k-1or2)$ (4)

Using the values given in (4) in (1), we have

$$(6n \pm 1)^2 = 24(2k-1)(3k-1or2) + 1 \tag{5}$$

From (3) and (5), we conclude that if $p \geq 5$ be a prime number then p^2-1 is always divisible by 24.

One interesting property of the natural numbers is that all those naturals which end with 0 or 2 or4 or 6 or 8 , that are always divisible by 2 and all those naturals which end with 0 or 5 , that are always divisible by 5. We can find associated natural number by placing 0 or 2 or 4 or 6 or 8 at the end of a prime numbers and dividing the resulting number again and again by 2 until the obtained number has not factor 2. Similarly we can find associated natural number by placing 0 or 5 at the end of a prime numbers and dividing the resulting number again and again by 5 until the obtained number has not factor 5.

Consider a prime number 5, place 0 at the end of 5, we get 50, divide 50 by 2, we get 25, therefore 25 is the associated natural of prime 5 under the division by 2. Similarly, divide 50 by 5, we get 10, divide 10 by 5, we get 2, therefore 2 is the associated natural of prime 5 under the division by 5.

If we get a prime number on subtracting a prime number from a even number then both the primes are called complementary primes of the used even number. For example 11 and 13 are complementary primes of 24. There may be more than one pair of complementary primes. For example, 5, 19 and 7, 17 are other complementary primes of 24.

DISTANCE BETWEEN TWO NEIGHBORING PRIME NUMBERS:

Two prime numbers are called neighboring prime numbers if there exists no other prime number between them. For example, 2 and 3, 3 and 5, 5 and 7, 7 and 11 etc are neighboring

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prime numbers.
Distance between two neighboring prime numbers, is defined as number of composite numbers between them. Distance between neighboring prime numbers 2 and 3 is 0, distance between neighboring prime numbers 3 and 5 is 1, distance between neighboring prime numbers 5 and 7 is 1, distance between neighboring prime numbers 113 and 127 is 13 and so on.

It can be easily seen that if p_1 and p_2 (p_1<p_2) are two neighboring prime numbers then the distance between them is $p_2 - p_1 - 1$ or $|p_1 - p_2| - 1$, let $d_p(p_1, p_2) = |p_1 - p_2| - 1$.
2 and 3 are the only neighboring prime numbers such that distance between them is 0, minimum distance between any two odd neighboring prime numbers, is 1, but it is not yet known that what is maximum distance between any two odd neighboring prime numbers.

Maximum distance between any two neighboring prime numbers up to 100, is 7, and maximum distance between any two neighboring prime numbers between 100 and 200, is 13 and it is between two neighboring prime numbers 113 and 127.

It can be proved that the maximum distance between any two neighboring odd primes, is finite. Suppose, if possible, the distance $d = d_p(p_1, p_2) = p_2 - p_1 - 1$, between any two neighboring odd primes p_1 and p_2 (p_1<p_2) , is infinite, this implies $p_2 - p_1 - 1 = d = \infty$
$\Rightarrow p_2 = \infty$ $\Rightarrow$ there exists no prime number after p_1
$\Rightarrow$ there are finite number of prime numbers which is a contradiction as there are infinite number of prime numbers. Therefore our supposition that the distance between any two neighboring prime numbers is infinite, is wrong and hence the distance between any two neighboring prime numbers is finite.

DISTANCE BETWEEN ANY TWO PRIME NUMBERS:
Distance between any two prime numbers is defined as the number of natural numbers occurring between them whether primes or composites. If $p \; and \; q (p < q)$ are two prime numbers, then distance between them is denoted by $d_p(p, q)$ and defined as $d_p(p, q) = q - p - 1 = |p - q| - 1$. Distance between prime numbers 7 and 29 is $d_p(7, 29) = 29 - 7 - 1 = |7 - 29| - 1 = 21$.

$d_p(3, 5) = 1, d_p(5, 7) = 1$, thus 3, 5, 7 are equidistant prime numbers and 5 is the central prime of 3 and 5. The numbers 5, 11, 17, 23, 29 are equidistant prime numbers as $d_p(5, 11) = d_p(11, 17) = d_p(17, 23) = d_p(23, 29) = 6$ here 17 is the central prime. There are infinite equidistant triples of prime numbers such as 3, 5, 7; 3, 7, 11; 3, 11, 19; 3, 13, 23; 3, 23, 43 etc. are equidistant triples of primes with 3 as first prime. We can call the ordered set of equidistant prime numbers as a chain of equidistant prime numbers and number of prime numbers in this chain, is called the length of the chain.

SIMPLE DISTANCE BETWEEN ANY TWO PRIMES:
Simple distance between any two prime number is the difference of the larger and smaller primes i.e. if $p > q$ are two primes then simple distance between them is $d(p, q) = p - q$. Equidistant prime numbers remain equidistant under any of the two definitions of distance between any two primes.

Now a question arises that what will be the maximum length of the equidistant chain of prime numbers, it may be 5 because if we assume that first prime of the chain is $6n-1$ and d is the common difference then equidistant chain of prime numbers will be of the form $6n-1,\ 6n-1+d, 6n-1+2d, 6n-1+3d, 6n-1+4d, 6n-1+5d$, then one of these prime will be divisible by 3 or 5. Thus the maximum length of equidistant chain of primes may be 5.

DIGITS AT THE UNIT PLACE OF THE PRIME NUMBERS:

1, 3, 7, 9 are the digits which are at unit place in prime numbers, 5 has been excluded from odd digits. 13, 17, 37, 79 are prime numbers when their order is reversed; we get primes 31, 71, 73, 97. Consider all the permutation of 1, 3, 7; which are 137, 173, 317, 371, 713, 731; then 137, 173, 317 are primes but 371, 713, 731 are not primes. Consider now the permutations of 1, 3, 9 ; which are 139, 193, 319, 391, 913, 931; then 139, 193, 319 are primes but 391, 913, 931 are not primes. . Consider now the permutations of 1, 7, 9; which are 179, 197, 719, 791, 917, 971; then 179, 197, 719, 971 are primes but 791, 917 are not primes. Consider now the permutations of 3, 7, 9; which are 379, 397, 739, 793, 937, 973; then 379, 397, 739, 937 are primes but 793, 973 are not primes. Similarly we can check the all the permutations of 1, 3, 7, 9 for prime numbers.

RESULTS RELATED TO FERMAT' S THEOREM:

There are many results related to prime numbers due to Fermat 's theorems (Hardy and Wright [1]) one of them is the following theorem stated as: "if p is a prime number and n is any positive integer, then $n^p \equiv n(mod\ p)$". Some more results relating to prime numbers can be derived from this theorem.

Theorem-1: If $m, n\ \in N$such that $m+n=p\ (prime), m^p+n \equiv 0(mod\ p)$.

Proof: By the Fermat's theorem as stated above, we have

$$m^p \equiv m(mod\ p)$$

$\Rightarrow m^p - m = kp\ ,\ k \in N$

$\Rightarrow m^p - m + m + n = kp + m + n\ ,\ k \in N$

$\Rightarrow m^p + n = kp + p\ ,\ k \in N$

$\Rightarrow m^p + n = (k+1)p\ ,\ k \in N$

$\Rightarrow m^p + n \equiv 0(mod\ p)$.

Theorem-2: If $m, n\ \in N$such that $m+n=p\ (prime), n^p+m \equiv 0(mod\ p)$.

Proof: By the Fermat's theorem as stated above, we have

$$n^p \equiv n(mod\ p)$$

$\Rightarrow n^p - n = kp\ ,\ k \in N$

$\Rightarrow n^p - n + n + m = kp + n + m\ ,\ k \in N$

$\Rightarrow n^p + m = kp + p\ ,\ k \in N$

$\Rightarrow n^p + m = (k+1)p\ ,\ k \in N$

$\Rightarrow n^p + m \equiv 0(mod\ p)$.

Above stated Fermat's theorem can be stated in another way as ""if p is a prime number and n is any positive integer, then $n^{p-1} \equiv 1(mod\ p)$". From this, we can derive the following result.

Theorem-3: if $k, l,\ m, n \in N\ and\ p$ be a prime such that $m^k + n \equiv 0(mod\ p)$, then

$$m^{k+l(p-1)} + n \equiv 0(mod\ p)$$

Proof: Given $m^k + n \equiv 0(mod\ p)$
$\Rightarrow m^k + n = rp\ ,\ r \in N$
$\Rightarrow m^k = rp - n\ ,\ r \in N$ (6)
Also $m^{p-1} \equiv 1(mod\ p)$
$\Rightarrow\ m^{l(p-1)} \equiv 1(mod\ p)$
$\Rightarrow\ m^{l(p-1)} = 1 + sp, s \in N$ (7)
Therefore, by (6) and (7), we get
$m^k\ m^{l(p-1)} = (rp - n\ )(1 + sp)$
$\Rightarrow m^{k+l(p-1)} = rp + rsp^2 - n - nsp$
$\Rightarrow m^{k+l(p-1)} + n = rp + rsp^2 - nsp$
$\Rightarrow m^{k+l(p-1)} + n = p(r + rsp - ns)$
$\Rightarrow m^{k+l(p-1)} + n \equiv 0(mod\ p)$ (8)
If we take $k = l = 1\ in$ (8), we get $m^p + n \equiv 0(mod\ p)$ which is the result of theorem-1
Application of these results is given below as:
$5 + 2 = 7(a\ prime) \Rightarrow 5^7 + 2\ and\ 2^7 + 5$ are divisible by 7, this can be easily verified.
By Fermat's theorem, $10^{p-1} \equiv 1(mod\ p)$
$\Rightarrow$ a prime number p divides a number in which any one of the digit out of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; is repeated $p - 1$ times.
e.g. consider the number 111111 in which digit I is repeated $7 - 1 = 6$ times, thus 111111 is divisible by 7 and $\frac{111111}{7} = 15873$ or $15873 \times 7 = 111111$
Similarly $\frac{777777777777}{13} = 59829059829$
$\Rightarrow 59829059829 \times 13 = 777777777777$
$65359477124183 \times 17 = 1111111111111111$
It should be noted that if $\frac{p-1}{2} + 1 = \frac{p+1}{2}$ is again a prime then the prime number p divides the number in which any one of the digit out of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; is repeated $\frac{p-1}{2}$ times.
e.g. $\frac{13-1}{2} + 1 = \frac{13+1}{2} = 7$ and 7 is prime therefore 13 divides 111111, 222222,….,999999 each containing 6 digits. Since $\frac{111111}{13} = 8547$
Another example of this type is $p = 37$ such that $\frac{37-1}{2} + 1 = \frac{37+1}{2} = 19$ and 19 is prime therefore 37 divides 111111111111111111, 222222222222222222,…., 999999999999999999 each containing 18 digits. Since $\frac{111111111111111111}{37} = 3003003003003003$. In fact 37 divides all numbers of type 111, 111111, 111111111, ….
Now we express the natural numbers as a geometrical expression which has fixed repetition i.e. number of the form 237237237237, 825382538253 etc.
Now

$$1 + 10^k + 10^{2k} + \cdots + 10^{(n-1)k} = \frac{10^{nk}-1}{10^k-1}, p \neq 2, 5.$$

Therefore $1010101 = 1 + 10^2 + 10^4 + 10^6 = \frac{10^8-1}{10^2-1}$
Using Fermat's theorem, we can derive the following result:
$10^{k(p-1)} \equiv 1(mod\ p)$
$\Rightarrow 10^{k(p-1)} - 1 \equiv 0(mod\ p)$

$\Rightarrow (10^k - 1)\left(\frac{10^{k(p-1)}-1}{10^k-1}\right) \equiv 0(mod\ p)$
$\Rightarrow p$ divides $\frac{10^{k(p-1)}-1}{10^k-1}$ (9)
From (9), we find that $\frac{10^{2(7-1)}-1}{10^2-1} = 10101010101$ is divisible by 7 and $\frac{10101010101}{7} = 1443001443$. Similarly, that $\frac{10^{2(13-1)}-1}{10^2-1} = 101010101010101010101$ is divisible by 13 and $\frac{101010101010101010101}{13} = 7770007770007770007777$.
Similarly 13 divides 1001001001001001 as $\frac{1001001001001001}{13} = 77000077000077$
If p divides $\frac{10^{k(p-1)}-1}{10^k-1}$, then p divides $n\left(\frac{10^{k(p-1)}-1}{10^k-1}\right)$, $n \in N$.
Therefore 171717171717, 191919191919, 232323232323, 179179179179179179, 1753175317531753175317531753 etc. are divisible by 7 and 13.
Prime numbers of the form $2^{2^n} + 1, n \in N$ are called Fermat's numbers and prime numbers of the form $2^n - 1$ are called Mersin's numbers (Hardy and Wright [1]). Thus $2^{2^1} + 1$ =5, $2^{2^2} + 1$ =17, $2^{2^3} + 1$ =257 etc. are Fermat's numbers. Similarly $2^2 - 1$ =3, $2^3 - 1$ =7, $2^5 - 1 = 31$, $2^7 - 1 = 127$ etc. are Mersin's numbers.
Now consider the numbers of the form 11111, 1111111, 11111111111, ….etc. where 1 is repeated as a prime number of times i.e. 5, 7, 11, 13,… times.
Then 11111=41$\times$ 271, here 1 is repeated 5 times and $\frac{41-1}{8} = 5$, some relation exists among 11111, 41 and 5.
Similarly 1111111=239$\times$ 4649
From this it seems that 11111111111=5 digits prime number $\times$ 6 digits prime number
And 1111111111111=6 digits prime $\times$ 7 digits number prime
There is possibility that 5 digits prime factor of 11111111111 will start from 4 and end with 1, 6 digits prime factor of 11111111111 will start from 2 and end with 1, 6 digits prime factor of 1111111111111 will start from 2 and end with 9, 7 digits prime factor of 1111111111111 will start from 4 and end with 9.

Also 101010101, 1001001001001,... are divisible by 41 and 271. Similarly 1010101010101, 1001001001001001001,… are divisible by 239 and 4649. We notice that $41 - 1 = 40$, $271 - 1 = 270$ both are divisible by 5, similarly $239 - 1 = 238$, $4649 - 1 = 4648$ both are divisible by 7.

SUM OF CONSECUTIVE PRIME NUMBERS:
Let us check the sum of prime numbers from the first prime number 2 to succeeding consecutive primes as follows:

Let S_p, denotes sum of all prime numbers from 2 to p so that
$S_2 = 2, S_3 = 2 + 3 = 5, S_5 = 2 + 3 + 5 = 10, S_7 = 2 + 3 + 5 + 7 = 17,$ $S_{11} = 2 + 3 + 5 + 7 + 11 = 28, S_{13} = 2 + 3 + 5 + 7 + 11 + 13 = 41$, $S_{17} = 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58$, $S_{19} = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 58$, etc. we prepare table to write these sums conveniently as follows

Prime numbers	**Cumulative sum**	**Prime numbers**	**Cumulative sum**	**Prime numbers**	**Cumulative sum**
2	2=S_2	59	440= S_{59}	137	1988= S_{137}
3	5= S_3	61	501= S_{61}	139	2127= S_{139}
5	10= S_5	67	568= S_{67}	149	2276= S_{149}
7	17= S_7	71	639= S_{71}	151	2427= S_{151}
11	28= S_{11}	73	712= S_{73}	157	2584= S_{157}
13	41= S_{13}	79	791= S_{79}	163	2747= S_{163}
17	58= S_{17}	83	874= S_{83}	167	2914= S_{167}
19	77= S_{19}	89	963= S_{89}	173	3087= S_{173}
23	100= S_{23}	97	1060= S_{97}	179	3266= S_{179}
29	129= S_{29}	101	1161= S_{101}	181	3447= S_{181}
31	160= S_{31}	103	1264= S_{103}	191	3638= S_{191}
37	197= S_{37}	107	1371= S_{107}	197	3835= S_{197}
41	238= S_{41}	109	1480= S_{109}	199	4034= S_{199}
43	281= S_{43}	113	1593= S_{113}	211	4245= S_{211}
47	328= S_{47}	127	1720= S_{127}	223	4468= S_{223}
53	381= S_{53}	131	1851= S_{131}	227	4695= S_{227}

From this table of sum of all primes from 2 to p(prime), we find some interesting facts e.g. 10, 100 are sum of consecutive primes; the primes 5, 17, 41, 197, 281 are sum of consecutive primes, but it is interesting that there is no prime after 281, which is the sum of the consecutive primes up to S_{227} or perhaps further, beginning from 2.

CONCLUSION:

There are innumerable interesting facts, conjectures about natural numbers and prime numbers which can be explored. In this paper, there are unsolved problems like maximum distance between any two neighboring prime numbers, maximum length of the chain of a prime numbers with same common difference (distance). One can try to solve these problems. More results on prime numbers can be derived by using the property that if p divides $\frac{10^{k(p-1)}-1}{10^k-1}$, then p divides $n\left(\frac{10^{k(p-1)}-1}{10^k-1}\right)$, $k, n \in N$. Two more problems are that there may be no prime of the form 111, 1111, 11111, 111111, 1111111,… and 111, 11111, 1111111, 11111111111,.. have only two prime factors.

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CALIBERATING MICROPOLARITY ON TRANSFERENCE OF SH-WAVES IN AN ELASTIC LAYER OVERLYING SIZE-DEPENDENT SUBSTRATE

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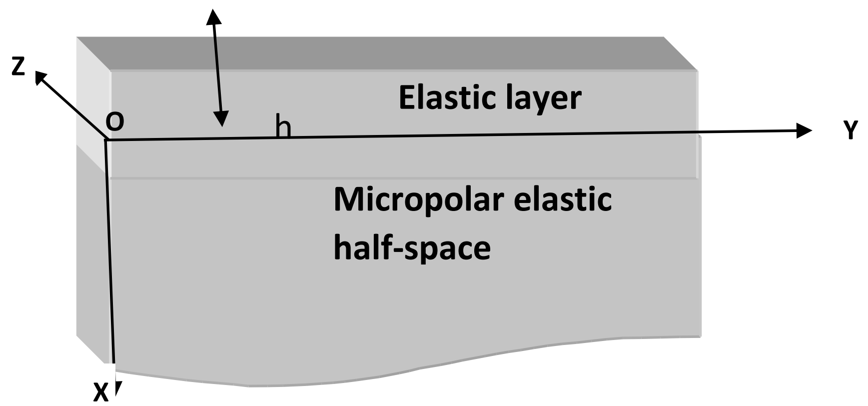
Research aim:
Considering micro structure of modern sensing devices the aim of this present study is to analyse the propagation behavior of horizontally polarized shear waves in an elastic layer overlying semi-infinite micropolar elastic substrate. Here micropolar theory is preferred instead of classical model due to its capability to expound size effects on micro scale. The closed form expression of dispersion relation for the existence of horizontally polarized shear waves is obtained analytically. Possible particular case has been derived and validated with the existing result of the classical Love wave equation. To exemplify the competence of the problem numerical computations are executed by considering aluminium-epoxy as a micropolar elastic material. The substantial effects of micropolarity and thickness of elastic layer have been examined and depicted graphically on the phase velocity profiles of the wave for the considered structure.

Brief literature survey:
Surface wave propagation in a composite structure has numerous applications in geoscience, engineering, ocean acoustics, NDE techniques, as well as in designing of surface acoustic wave (SAW) devices. Numerous researchers have investigated the propagation behavior of SH-waves in layered structure comprising of elastic material (Liu et al., 2001, Sahu et al., 2014). Detailed investigations have been done on the propagation behavior of SH-waves in different materials based on classical model. But due to inadequacy of classical model to explain the behaviour of materials viz. aluminium epoxy, cellular solids, platelet composites, polymers and many other having complex microstructures, micropolar theory is preferred as it has ability to explain size effects on microscale by auditing the additional degree of freedom. Certain discrepancies were occurred and unexplained by the classical theory. In order to remove these shortcomings Voigt (1887) introduced the concept of couple stresses in elasticity by introducing additional couple vector to explicate particle interactions in a body. Eringen and Suhubi (1964) introduced the theory of linear and nonlinear micropolar elastic continua and Eringen (1966) generalized the classical theory of elasticity by considering three rotational degrees of freedom in addition to three classical displacement degrees of freedom. Due to the practical applicability in various fields of science and technology such as acoustics, seismology, aerospace and marine structures an extensive study is available on the wave propagation phenomenon in micropolar solids.Propagation of Love-type surface waves in homogeneous micropolar elastic media were studied by Midya (2004). Tomar (2005) investigated wave propagation in a micropolar elastic plate with voids. Kaur et al. (2017) examined shear wave propagation in vertically homogeneous viscoelastic layer over a micropolar elastic half-space. Kundu et al. (2017) investigated Love wave propagation in heterogeneous micropolar media.

Problem formulation:
This paper studies the propagation characteristics of horizontally polarized shear waves in a structure comprised of elastic layer perfectly bonded over semi-infinite micropolar elastic substrate. Both materials are isotropic in nature. An elastic layer of thickness h overlying semi-infinite micropolar elastic substrate is considered. The Cartesian coordinate system is considered such that shear wave is propagating in the y-direction and x-axis is positive in vertically downward direction as shown in fig. 1. The elastic material is polarized along z-direction perpendicular to x-y plane. The displacement components will be independent of z-coordinate. Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ are the mechanical displacement components for an elastic layer and the micropolar substrate respectively.



The governing equation for the elastic layer in the absence of body forces is given by:

$$\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} = \frac{1}{\alpha_1^2}\frac{\partial^2 u_3}{\partial t^2} \tag{1}$$

The governing equations and constitutive relations for micropolar elastic half-space (Eringen, 1966) in the vector form in the absence of body forces are

$$(\lambda + \mu)\nabla(\nabla.\vec{v}) + (\mu + \kappa)\nabla^2\vec{v} + \kappa\left(\nabla \times \vec{\phi}\right) = \rho\frac{\partial^2\vec{v}}{\partial t^2}, \tag{4}$$

$$(\alpha + \beta + \gamma)\nabla\left(\nabla.\vec{\phi}\right) - \gamma\nabla \times \left(\nabla \times \vec{\phi}\right) + \kappa(\nabla \times \vec{v}) - 2\kappa\vec{\phi} = ǰ\rho\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{5}$$

$$\sigma_{ij} = \lambda v_{k,k}\delta_{ij} + \mu(v_{i,j} + v_{j,i}) + \kappa(v_{j,i} - \epsilon_{ijk}\phi_k), \tag{6}$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{7}$$

where i, j, k=1, 2, 3.α, β, γ, κ are micropolar material parameters. λ and μ are Lame's constants. ρ is the mass density, ǰ is micro inertia.$\sigma_{ij}$and m_{ij} are the stress tensor and couple stress tensor. δ_{ij} is kronecker delta. ϵ_{ijk} is alternating tensor.$\vec{\phi} = \left(\vec{\phi}_1, \vec{\phi}_2, \vec{\phi}_3\right)$ is the microrotation vector.

The admissible boundary conditions will be applied and dispersion relation will be obtained by taking the determinant of the coefficients of unknown variables equal to zero, for the propagation of SH-waves in an elastic layered structure. In the absence of micropolar constants the classical Love wave equation (Love, 1911) is obtained from the dispersion relation for the sake of validation.

Solution methodology:
The solutions are obtained analytically by using general differential equations with corresponding boundary conditions.

Significant conclusions:
- Existence of shear waves in the composite structure consisting of an elastic layer bonded perfectly to the micropolar elastic material and micropolarity has significant effect on the propagation of SH-waves. As micropolarity constant $\left(\frac{\kappa}{\mu}\right)$increases phase velocity of the wave increases.
- Thickness of elastic layer also has eminent effect on the phase velocity profile of the SH-wave. As the thickness of elastic layer is increasing, the phase velocity of the wave also increases.

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ON THE REGION OF COMPLEX GROWTH RATES IN TRIPLY DIFFUSIVE CONVECTION IN POROUS MEDIUM

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ABSTRACT:
In the present paper, the problem of triply diffusive convection in porous medium is studied by using Darcy-Brinkman model. Upper bounds for the complex growth rates of an arbitrary oscillatory disturbance of growing amplitude, neutral or unstable, are obtained which are important especially when both the boundaries are not free so that exact solutions in the closed form are not obtainable. It is further proved that the result obtained herein is uniformly valid for any combination of rigid and free boundaries.

KEYWORDS: Triply diffusive convection; Porous Medium; Darcy-Brinkman model; Rayleigh number; Concentration Rayleigh number.

INTRODUCTION:
The convective fluid motion under the simultaneous action of a uniform vertical temperature gradient and a gravitationally opposite uniform vertical concentration gradient (known as double diffusive convection) has been an area of great research activity for the last many decades. Double diffusive convection is now well known. For a broad view of the subject one may refer to Stern (1960), Veronis (1965), Nield (1967), Baines and Gill (1969), Turner (1974),Huppert and Turner (1981),Banerjee et al. (1981, 1993a,b), Gupta et al. (1982, 1983, 2001), Brandt and Fernado (1996), Basu and Layek (2013), Radko (2013), Kellner and Tilgner (2014), Yang et al. (2015,2016), Chen et al. (2016,2017), Safi and Benissaad (2017) etc.

The presence of more than two components in a fluid having different diffusive properties, can lead to convective instabilities, often well before a fluid system would become statically unstable. It is now well established that (Griffiths (1979a), Moroz (1989), Pearlstein et al. (1989), Lopez et al. (1990), Terrones (1993), Prakash et al. (2014a,b,c,d,e, 2015a,b,c,d, 2016a), Patil et al. (2018a,b),etc.) the small concentration of a third component with a smaller mass diffusivity can have a significant effect upon the nature of instability; and 'diffusive convection' (oscillatory modes) and direct 'salt finger' modes (steady convection) may simultaneously exist under a wide range of conditions, even if the over-all density distribution is gravitationally stable.

The subject of systems having more than two components in porous medium has also attracted many researchers recently due to its importance in the study of geothermally heated lakes, earth core, solidification of molten alloys, underground water flow, natural phenomena such as contaminant transport, warming of stratosphere and magmas and their laboratory models and sea water etc. For the broad view of the subject one may be referred to Vafai (2005), Straughan et al. (2008),Rionero (2013), Abdullah et al. (2018),Zhao et al. (2014), Ghalambaz et al. (2016), Prakash et al. (2016b,c, 2017), Roy (2017), Hewitt (2017), Raghunatha et al. (2018)etc.

Since instability in triply diffusive configurations in porous and non-porous media may occur in the form of oscillatory motions, the problem of deriving the upper limits for the linear growth rate of an arbitrary neutral or unstable oscillatory disturbance of growing amplitude in triply diffusive convection has its own importance in fluid dynamics, especially when at least one of the boundaries is rigid so that exact solutions in the closed form are not derivable as was possible for the cases treated by Griffiths (1979a), Poulikakos (1985) and Rudraiah and Vortmeyer (1982).

In the present paper we have studied triply diffusive convection (with heat and one concentration component as destabilizing agent and the second concentration component as stabilizing agent) in a sparsely distributed porous medium by using Darcy-Brinkman model. Darcy flow model is relevant only to densely packed, low permeability porous medium. Also, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. The Brinkman (1947) extension of the Darcy's law has overcome these difficulties by adding a viscous like term to the equations.

Mathematical Formulation and Analysis:
Consider a viscous finitely heat conducting Boussinesq fluid layer, saturating a sparsely distributed porous medium, of infinite horizontal extension statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1(< T_0)$ and uniform concentrations S_{10},S_{20} and$S_{11}(< S_{10})$,$S_{21}(> S_{20})$(see Fig.1.). It is assumed that the saturating fluid is incompressible and the porous medium is a constant porosity medium. It is further assumed that the cross-diffusion effects of the stratifying agencies can be neglected. The Darcy-Brinkman model has been used to investigate the triply diffusive convection in porous medium.

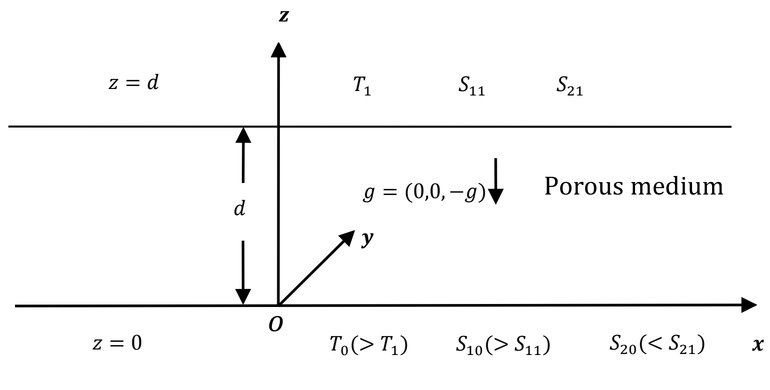

Fig.1. Physical configuration.

The basic hydrodynamics equations that govern the motion of a triply diffusive fluid layer in porous medium in their non-dimensional form with $R > 0, R_1 > 0,\ R_2 < 0$ are given(Vafai (2005), Prakash et al. (2017)):

$$\Lambda(D^2 - a^2)^2 w - (p + D_a^{-1})(D^2 - a^2)w = \ Ra^2\theta - R_1 a^2\phi_1 + |R_2|a^2\phi_2, \quad (1)$$

$$(D^2 - a^2 - E\sigma p)\theta = -w, \quad (2)$$

$$\left(D^2 - a^2 - \frac{E_1\sigma\, p}{\tau_1}\right)\phi_1 = \ -\frac{w}{\tau_1} \quad (3)$$

and

$$\left(D^2 - a^2 - \frac{E_2 \sigma\, p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}. \tag{4}$$

Equations (1) – (4) are to be solved by using the following boundary conditions

$w = \theta = \phi_1 = \phi_2 = Dw = 0$ at $z = 0$ and $z = 1$(both the boundaries are rigid) (5)

Or $w = \theta = \phi_1 = \phi_2 = D^2 w = 0$ at $z = 0$ and $z = 1$(both the boundaries are free) (6)

or $\left.\begin{array}{l} w = \theta = \phi_1 = \phi_2 = Dw = 0 \text{ at } z = 0 \text{ (lower boundary is rigid)} \\ \text{and } w = \theta = \phi_1 = \phi_2 = D^2 w = 0 \text{ at } z = 1 \text{ (upper boundary is free)} \end{array}\right\}$ (7)

or $\left.\begin{array}{l} w = \theta = \phi_1 = \phi_2 = D^2 w = 0 \text{ at } z = 0 \text{ (lower boundary is free)} \\ \text{and } w = \theta = \phi_1 = \phi_2 = Dw = 0 \text{ at } z = 1 \text{ (upper boundary is rigid)} \end{array}\right\},$ (8)

where z is the real independent such that $0 \le z \le 1$, D is the differentiation w.r.t. z, a^2 is square of the wave number, $\sigma > 0$ the Prandtl number, $\tau_1 > 0$ and $\tau_2 > 0$ are the Lewis numbers for two concentration components with mass diffusivity κ_1, κ_2 respectively, $D_a > 0$ is the Darcy number, $R > 0$ is the Rayleigh number, $R_1 > 0$ and $R_2 < 0$ are the two concentration Rayleigh numbers, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are the real constants, w is the vertical velocity, θ is the temperature, ϕ_1 and ϕ_2 are the respective concentrations of the two components, $E_1 > 0$ and $E_2 > 0$ are constants.

It may further be noted that equations (1) – (8) describe an eigenvalue problem for p and govern triply diffusive convection in a porous medium for any combination of dynamically free and rigid boundaries.

MATHEMATICAL ANALYSIS:

Now we derive upper bounds for the complex growth rate in triply diffusive convection in porous medium. We prove the following theorem.

Theorem. If $R > 0,\ R_1 > 0, R_2 < 0,\ \tau_1 > \tau_2,\ E_1 > 0,\ E_2 > 0, \sigma > 0, p_r \ge 0, p_i \ne 0$, then a necessary condition for the existence of non-trivial solution $(w,\ \theta,\ \phi_1,\ \phi_2,\ p)$ of equations (1) – (4) together with either of the boundary conditions (5) – (8) is that

$$|p|^2 < \frac{R_1}{E_1 \sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{E_1^2 \sigma^2}.$$

Proof: Multiplying equation (1) by w^* (the superscript $*$ hence forth denotes the complex conjugation) on both sides and integrating over vertical range of z, we obtain

$$\Lambda \int_0^1 w^* (D^2 - a^2)^2 w\, dz - (p + D_a^{-1}) \int_0^1 w^* (D^2 - a^2) w\, dz = Ra^2 \int_0^1 w^* \theta\, dz - R_1 a^2 \int_0^1 w^* \phi_1\, dz + |R_2| a^2 \int_0^1 w^* \phi_2\, dz. \tag{9}$$

Making use of equations (2) – (4), and the fact, that $w(0) = 0 = w(1)$, we can write

$$Ra^2 \int_0^1 w^* \theta\, dz = -Ra^2 \int_0^1 \theta\, (D^2 - a^2 - E\, \sigma\, p^*)\theta^* dz, \tag{10}$$

$$R_1 a^2 \int_0^1 w^* \phi_1 dz = -R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{E_1 \sigma\, p^*}{\tau_1}\right)\phi_1^* dz \tag{11}$$

and

$$|R_2| a^2 \int_0^1 w^* \phi_2 dz = -|R_2| a^2 \tau_2 \int_0^1 \phi_2 \left(D^2 - a^2 - \frac{E_2 \sigma\, p^*}{\tau_2}\right)\phi_2^* dz. \tag{12}$$

Combining equations (9) – (12), we obtain

$$\Lambda \int_0^1 w^* (D^2 - a^2)^2 w\, dz - (p + D_a^{-1}) \int_0^1 w^* (D^2 - a^2) w\, dz = -Ra^2 \int_0^1 \theta\, (D^2 - a^2 - E\, \sigma\, p^*)\theta^* dz + R_1 a^2 \tau_1 \int_0^1 \phi_1 \left(D^2 - a^2 - \frac{E_1 \sigma\, p^*}{\tau_1}\right)\phi_1^* dz - |R_2| a^2 \tau_2 \int_0^1 \phi_2 \Big(D^2 - a^2 -$$

$\left.\frac{E_2\sigma\, p^*}{\tau_2}\right)\phi_2^* dz.$ (13)

Integrating various terms of equation (13), by parts, for an appropriate number of times and making use of the boundary conditions (5) – (8) and the equality

$\int_0^1 \psi^* D^{2n}\psi dz = (-1)^n \int_0^1 |D^2\psi|^2 dz,$ (14)

$\psi = w(n = 1,2)$ or $\psi = \theta,\ \phi_1, \phi_2\ (n = 1)$,

we may rewrite equation (13) in the form

$\Lambda \int_0^1 (|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2)\, dz + (p + D_a^{-1}) \int_0^1 (|Dw|^2 + a^2|w|^2)\, dz =$
$Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + E\sigma p^*|\theta|^2)\, dz - R_1 a^2 \tau_1 \int_0^1 \Big(|D\phi_1|^2 + a^2|\phi_1|^2 +$
$\frac{E_1\sigma\, p^*}{\tau_1}|\phi_1|^2\Big)\, dz + |R_2| a^2 \tau_2 \int_0^1 \Big(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{E_2\sigma\, p^*}{\tau_2}|\phi_2|^2\Big)\, dz.$ (15)

Equating imaginary parts of both sides of equation (15), and cancelling $p_i(\neq 0)$ throughout, we have

$\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz = -\ Ra^2 E\sigma \int_0^1 |\theta|^2 dz + R_1 a^2 E_1 \sigma \int_0^1 |\phi_1|^2 dz - |R_2| a^2 E_2 \sigma \int_0^1 |\phi_2|^2 dz.$ (16)

Since $w,\ \theta,\ \phi_1, \phi_2$ satisfy $w(0) = 0 = w(1),\ \theta(0) = 0 = \theta(1),\ \phi_1(0) = 0 = \phi_1(1)$ and $\phi_2(0) = 0 = \phi_2(1)$, by Rayleigh-Ritz Inequality, we have

$\int_0^1 |Dw|^2\, dz \ \geq\ \pi^2 \int_0^1 |w|^2 dz,$ (17)

$\int_0^1 |D\theta|^2\, dz \ \geq\ \pi^2 \int_0^1 |\theta|^2 dz,$ (18)

$\int_0^1 |D\phi_1|^2\, dz \ \geq\ \pi^2 \int_0^1 |\phi_1|^2 dz$ (19)

and

$\int_0^1 |D\phi_2|^2\, dz \ \geq\ \pi^2 \int_0^1 |\phi_2|^2 dz.$ (20)

Using inequality (17), we have

$\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz \ \geq (\pi^2 + a^2) \int_0^1 |w|^2 dz.$ (21)

Combining equations (16) and (21), we have

$R_1 E_1 \sigma a^2 \int_0^1 |\phi_1|^2 dz \ \geq (\pi^2 + a^2) \int_0^1 |w|^2 dz.$ (22)

Now, multiplying equation (3) by its complex conjugate and integrating the resulting equation for a suitable number of times and using the boundary conditions on ϕ_1, namely, $\phi_1(0) = 0 = \phi_1(1)$, we obtain

$\int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2)\ dz + \frac{2E_1\sigma p_r}{\tau_1} \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz +$
$\frac{E_1^2\sigma^2|p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz = \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz.$ (23)

Now, we have (Banerjee et al (1993b))

$\int_0^1 |D^2\phi_1|^2\, dz \ \ \geq\ \pi^4 \int_0^1 |\phi_1|^2\, dz.$ (24)

Using (19) and (24), we have

$\int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2)\ dz \ \geq (\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz.$ (25)

Substituting (25) in (23), we have

$(\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz + \frac{2E_1\sigma p_r}{\tau_1} \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz + \frac{E_1^2\sigma^2|p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz \leq \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz.$ (26)

Since $p_r \geq 0$, it follows from equation (26) that

$$(\pi^2 + a^2)^2 \int_0^1 |\phi_1|^2 dz + \frac{E_1^2\sigma^2|p|^2}{\tau_1^2}\int_0^1 |\phi_1|^2 dz \le \frac{1}{\tau_1^2}\int_0^1 |w|^2 dz \tag{27}$$

which gives

$$R_1E_1\sigma a^2 \int_0^1 |\phi_1|^2 dz \le \frac{R_1}{E_1\sigma}\frac{a^2}{|p|^2}\int_0^1 |w|^2 dz - \frac{(\pi^2+a^2)^2}{|p|^2}\frac{\tau_1^2}{E_1^2\sigma^2}.R_1E_1\sigma a^2\int_0^1 |\phi_1|^2 dz. \tag{28}$$

Using (22) in (28), we have

$$R_1E_1\sigma a^2 \int_0^1 |\phi_1|^2 dz \le \frac{R_1}{E_1\sigma}\frac{a^2}{|p|^2}\int_0^1 |w|^2 dz - \frac{(\pi^2+a^2)^3}{|p|^2}\frac{\tau_1^2}{E_1^2\sigma^2}\int_0^1 |w|^2 dz. \tag{29}$$

Utilizing (29) in (16), we obtain

$$\int_0^1 |Dw|^2 dz + Ra^2E\sigma\int_0^1 |\theta|^2 dz + |R_2|a^2E_2\sigma\int_0^1 |\phi_2|^2 dz + a^2\left\{1 - \frac{R_1}{E_1\sigma}\frac{1}{|p|^2} + \frac{(\pi^2+a^2)^3}{a^2}\frac{\tau_1^2}{|p|^2E_1^2\sigma^2}\right\}\int_0^1 |w|^2 dz \le 0. \tag{30}$$

Since minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$ for $a^2 = \frac{\pi^2}{2}$, inequality (30) yields

$$\int_0^1 |Dw|^2 dz + Ra^2E\sigma\int_0^1 |\theta|^2 dz + |R_2|a^2E_2\sigma\int_0^1 |\phi_2|^2 dz + a^2\left\{1 - \frac{R_1}{E_1\sigma}\frac{1}{|p|^2} + \frac{27\pi^4}{4}.\frac{\tau_1^2}{|p|^2E_1^2\sigma^2}\right\}\int_0^1 |w|^2 dz \le 0. \tag{31}$$

Inequality (31) clearly implies that

$$|p|^2 < \frac{R_1}{E_1\sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{E_1^2\sigma^2}. \tag{32}$$

This proves the theorem.

The above theorem states, from the physical point of view that the complex growth rate (p_r, p_i) of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triply diffusive fluid layer saturating a porous medium using Darcy-Brinkman model with one of the components as heat with diffusivity κ must lie inside a semicircle in the right half of the (p_r, p_i) – plane whose centre is origin and radius equals $\sqrt{\frac{R_1}{E_1\sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{\sigma^2E_1^2}}$, where $R_1(> 0)$ and $R_2(< 0)$ are the Rayleigh numbers for the two concentration components with diffusivities κ_1 and κ_2 , σ is the Prandtl number, E_1 and E_2 are constants. This result is uniformly valid for any combinations of rigid or free boundaries.

A general plot showing the region of complex growth rate is given below:

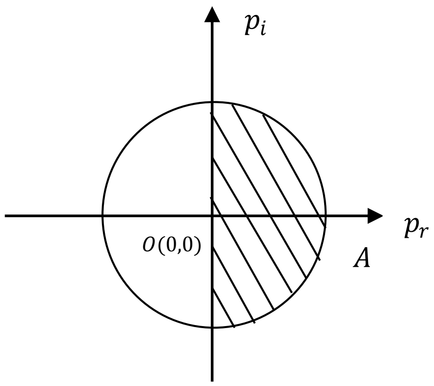

Fig.2. Shaded region shows that the region of complex growth rate

$$(OA)^2 < \frac{R_1}{E_1\sigma} - \frac{27\pi^4}{4}\frac{\tau_1^2}{\sigma^2 E_1^2}.$$

CONCLUSIONS:

The linear stability theory is used to derive the upper bounds for the complex growth rate in triply diffusive convection of a fluid layer in porous medium heated from below. The Darcy-Brinkman model has been used which is more compatible for the flow through high porosity medium. These bounds are important especially when both the boundaries are not free so the exact solutions in the closed form are not obtainable. Further, since the result derived herein involve only the non-dimensional quantities and are independent of the wave number are, thus, of uniform applicability.

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THE MATHEMATICS OF CRYPTOGRAPHY AND NETWORK SECURITY

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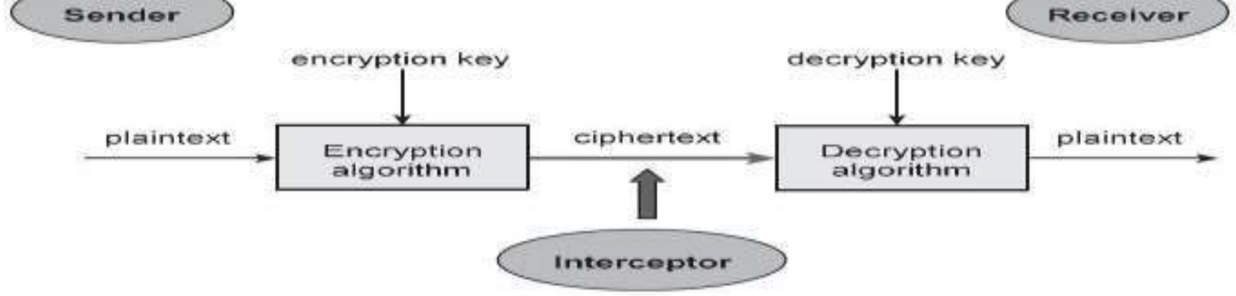
INTRODUCTION:

In the current orientation of the world, the technologies have advanced so much that most of the individuals prefer using the internet as the primary mean to relegate data from one end to another across the world. There are many possible ways to broadcast data using the internet: via e-mails, chats, etc. The data transition is made very cinch, fast and accurate using the internet. However, one of the main challenge with sending data over the internet is the "security threat" it poses i.e. the personal or privy data can be bagged or hacked in many ways. Therefore, it becomes very important to take data security into consideration, as it is one of the most necessary factors that need attention during the process of data transferring [1].

Plain or normal text, which is send over the network, is firstly get transformed into cipher text so that only the sender and the recipient can use the information.
Cipher Text, it is a data that has been encrypted. Cipher text is unreadable until it has been converted into Plain text with key.
Encryption, the process of encoding plain text messages into cipher text messages.
Decryption, transformation process of cipher text again into plain text.

Decryption is just opposite to encryption. In computer-to-computer communications, the computer at sender's end usually transforms a plain text messages into cipher text messages by performing encryption. Then this message is sent to the receiver over the network. The receiver's computer takes the encrypted message and performs the decryption process to obtain plain text.

The process of encryption and decryption is known as **Cryptography**. In general, cryptography is the art and science of achieving security by encoding message to make them non-readable. It can be used to hide the meaning of information in any form. It can also be applied to software, graphics or voice.



Cryptography Process

Cryptography is the art of secret coding. The basic service provided by cryptography is the ability to send the information between participants in a way that prevents others reading it.

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The main purpose of the cryptography is used not only to provide confidentiality, but also to provide solutions for other problems like: data integrity, authentication, non-repudiation [2].

Network Security:

System and Network Technology is a key technology for a wide variety of applications. It is a critical requirement in current situation networks, there is a significant lack of security methods that can be easily implemented. There exists a "communication gap" between the developers of security technology and developers of networks. Network design is a developed process that is depends on the Open Systems Interface (OSI) model. The OSI model has several advantages when designing network security. It offers modularity, ease-of-use, flexibility, and standardization of protocols. The protocols of different layers can be easily combined to create stacks, which allow modular development. In contrast to secure network, design is not a well- developed process. There is not a methodology to manage the complexity of security requirements. When considering about network security, it should be emphasized that the complete network is secure. It does not only concern with the security in the computers at each end of the communication chain. When transferring from one node to another node data the communication channel should not be vulnerable to attack. A hacker will target the communication channel, get the data, and decrypt it and re‐insert a duplicate message. Though securing the network is just as important as securing the computers and encrypting the message. While developing a secure network, the following needs to be considered [3].

Network Security Services:

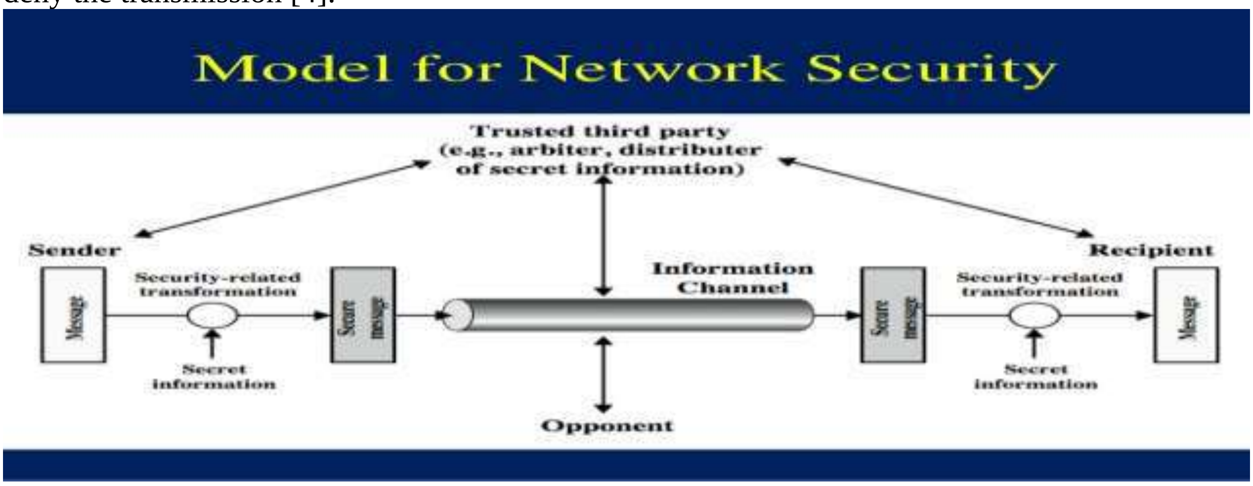
The classification of security services are as follows:

Confidentiality: Ensures that the information in a computer system and transmitted information are accessible only for reading by authorized parties. e.g. printing, displaying and other forms of disclosure.

Authentication: Ensures that the origin of a message or electronic document is correctly identified, with an assurance that the identity is not false.

Data Integrity: Ensures that only authorized parties are able to modify computer system assets and transmitted information. Modification includes writing, changing status, deleting, creating and delaying or replaying of transmitted messages.

Non-repudiation: Requires that neither the sender nor the receiver of a message be able to deny the transmission [4].



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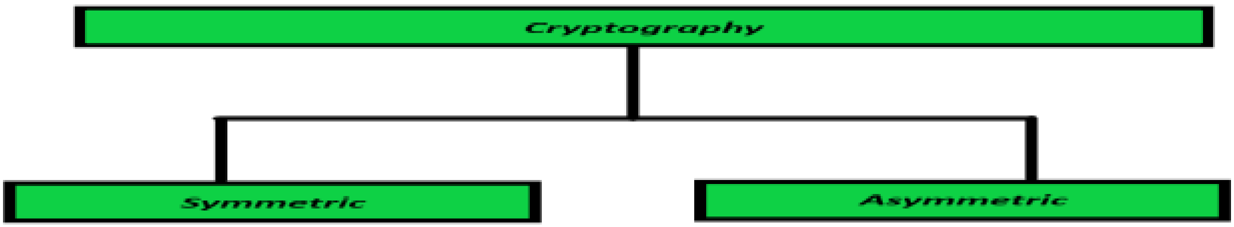
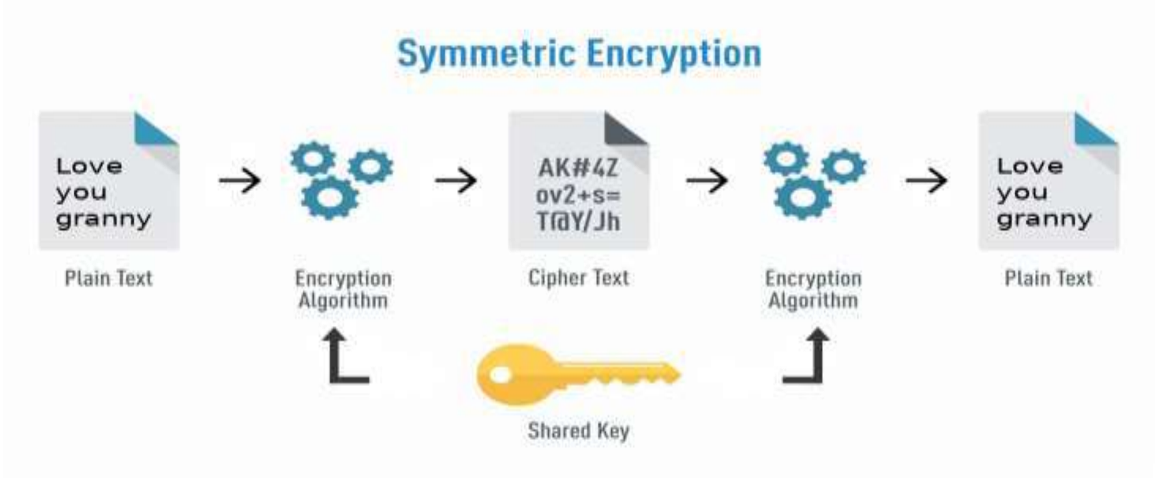
Cryptography is the methods that allow information to be sent in a secure from in such a way that the only receiver able to retrieve this information. Presently continuous researches on the new cryptographic algorithms are going on. However, it is a very difficult to find out the specific algorithm, because we have already known that they must consider many factors like security, the features of algorithm, the time complexity and space complexity [2].

Techniques/ Methods/ Types of Cryptography:
There are two main categories of cryptography depending on the type of security keys used to encrypt/decrypt the data. These two categories are asymmetric and Symmetric encryption techniques i.e. ***Symmetric and Asymmetric Encryptions*:**



1) Symmetric Encryption:
It is also called as single key cryptography. It uses a single key. In this encryption process the receiver and the sender has to agree upon a single secret (shared) key. Given a message (called plaintext) and the key, encryption produces unintelligible data, which is about the same length as the plaintext was. Decryption is the reverse of encryption, and uses the same key as encryption [5].



ADVANTAGES AND DISADVANTAGES OF SYMMETRIC CRYPTOSYSTEM
ADVANTAGES:

- A symmetric cryptosystem is faster.
- In Symmetric Cryptosystems, encrypted data can be transferred on the link even if there is a possibility that the data will be intercepted. Since there is no key transmitted with the data, the chances of data being decrypted are null.
- A symmetric cryptosystem uses password authentication to prove the receiver’s identity.
- A system only, which possesses the secret key, can decrypt a message.

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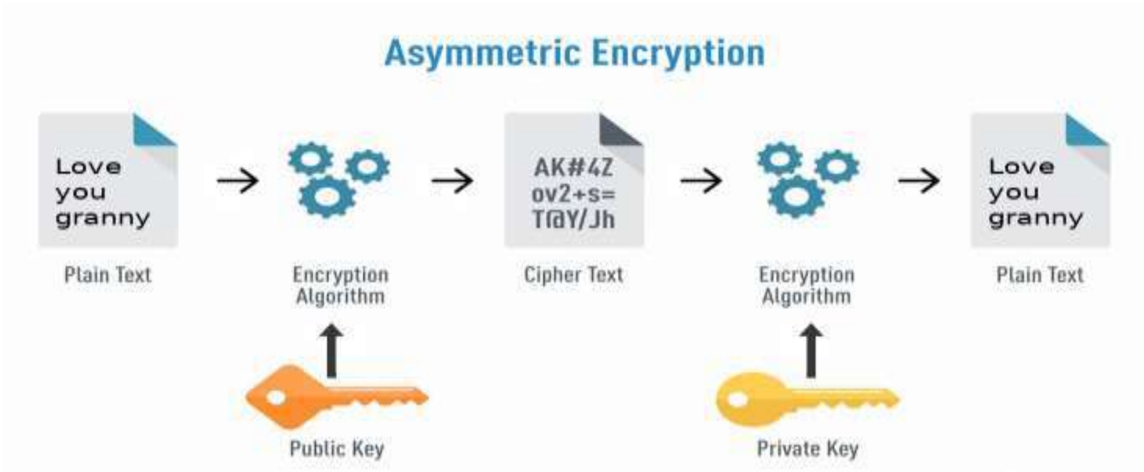
DISADVANTAGES:

- Symmetric cryptosystems have a problem of key transportation. The secret key is to be transmitted to the receiving system before the actual message is to be transmitted. Every means of electronic communication is insecure as it is impossible to guarantee that no one will be able to tap communication channels. Therefore, the only secure way of exchanging keys would be exchanging them personally.
- Cannot provide digital signatures that cannot be repudiated [6].

2) Asymmetric Encryption:
It is also called as public key cryptography. It uses two keys: public key, which is known to the public, used for encryption and private key, which is known only to the user of that key, used for decryption. The public and the private keys are related to each other by any mathematical means. In other words, data encrypted by one public key can be encrypted only by its corresponding private key. Encryption and decryption procedure [5].



ADVANTAGES AND DISADVANTAGES OF ASYMMETRIC CRYPTOSYSTEM
ADVANTAGES

- In asymmetric or public key, cryptography there is no need for exchanging keys, thus eliminating the key distribution problem.
- The primary advantage of public-key cryptography is increased security: the private keys do not ever need to be transmitted or revealed to anyone.
- Can provide digital signatures that can be repudiated.

DISADVANTAGES
A disadvantage of using public-key cryptography for encryption is speed: there are popular secret-key encryption methods which are significantly faster than any currently available public-key encryption method [6].

MAJOR TYPES OF ATTACKS
Many attacks are possible over any ongoing communication within a network.
Some major types of attacks are explained below: -
(a) Security Threats: - Security threats are attacks where the system of the user is hampered in some manner that leads to loss of confidential data. This includes activities like service denying, attacking with viruses, malwares, spywares and Trojan horses. In addition, activities

like intruding database or accessing Internet without permission.

(b) Data capturing and cryptanalysis: - This attack is performed while data is travelling in communication channels. The confidential data is captured or stolen from the channels and cryptanalysis is performed on it to extract the original data.

(c) Unauthorized Installing of Applications: - Installing unauthorized or uncertified applications within the system leads to virus intrusion and security breaching. To avoid it only certified applications must be allowed and unwanted applications such as audios, videos, games or other Internet applications must be avoided.

(d) Unauthorized Access: - Intrusion of any unauthorized person within the network resources or in data records leads to loss of confidential information. Hence proper authentication techniques for user's identity must be used and only resources must be monitored and checked from time-to-time.

(e) Virus Infection: - When network or resources are attacked with viruses, malware, Trojan horses or spywares leads to loss or manipulation of confidential data. It may sometimes destroy different resources and components of the network by effecting their source codes or hardware.

CYBER SECURITY TECHNIQUES:

To overcome or undo the attacks on networks different technologies are used these days. Some of the major techniques are given below:-

(a) **Authentication:** - All data and documents received must be authenticated if they are sent by trusted sender or not. They must also be checked for unwanted breaching or alterations within data.

(b) **Antivirus: -** Antivirus software must be installed and updated on regular time intervals. Also network and systems checks must be conducted regularly.

(c) **Firewalls:** - This software keeps track of inward and outward traffic of any system. It also inform user about unpermitted access and usage.

(d) **Access Control:** - Each user must have their particulars like username and passwords so that only intended users may log in.

(e) **Cryptography:** - It is the technique of encoding plain text into cipher text before transmitting it over channel for avoiding stealing of confidential data [7].

CONCLUSION AND FUTURE WORK:

With the explosive growth in the Internet, network and data security have become an inevitable concern for any organization whose internal private network is connected to the Internet. The security for the data has become highly important. User's data privacy is a central question over cloud. With more mathematical tools, cryptographic schemes are getting more versatile and often involve multiple keys for a single application.

In this concern, Cryptography plays a vital role in computer science area because the amount of work done is only kept secret. There are various techniques and algorithm studied and different types of research have been done. This paper further studied that symmetric key cryptography are faster than asymmetric systems. However, asymmetric key cryptography are more scalable and provide more authentication and non- repudiation easily.

In the future, work can be done on key distribution and management as well as optimal cryptography algorithm for data security over clouds.

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A TECHNIQUE TO SOLVE SIMULTANEOUS LINEAR EQUATIONS.

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ABSTRACT:
Linear equations play an important role in Science and Technology, Engineering and various social problems.Here in this paper, simple method has been developed to solve the problems occurring in the simultaneous linear equations having two or three unknowns. Some problems based on simultaneous linear equations have been solved also.

KEYWORDS :- Linear equations, simultaneous equations, co-efficients.

INTRODUCTION :-
There are so many methods to solve simultaneous linear equations having two or three unknowns like Graphical Method, Substitution Method, Elimination Method, Cross multiplication Methods, Crammer Rule, Matrix Method, Gauss Jacob's Method, Gauss Jordan Method, Gauss elimination method, Gauss Seidal Method etc. In this paper, we introduce a technique to solve simultaneous linear equations having two or three unknowns which is the modification of crammer's rule.

1. **Method to solve the simultaneous linear equations having two unknowns (Variables)**.

Consider two linear equations having two variables such that
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$

Method :
Write constants which is as follows :

$$\begin{matrix} a_1 & b_1 & b_1 & c_1 & c_1 & a_1 \\ b_2 & a_2 & c_2 & b_2 & a_2 & c_2 \end{matrix}$$

(1) $\Delta = a_1b_2 - b_1a_2$
(2) $\Delta_1 = b_1c_2 - c_1b_2$
(3) $\Delta_2 = c_1a_2 - a_1c_2$

Now to find the value of $x \ and \ y$
$\therefore \ x = -\frac{\Delta_1}{\Delta}, \quad y = -\frac{\Delta_2}{\Delta}$

***Some problems based on two un-knowns:**
Example 1. Solve the following system of equations : $2x + 3y = 9$, $3x - 2y = 7$.
Solution : Given system of equations are $2x + 3y = 9$, $3x - 2y = 7$
Here $a_1 = 2 \quad b_1 = 3 \quad c_1 = 9$ and $a_2 = 3 \quad b_2 = -2 \quad c_2 = 7$

Method : $\begin{matrix} a_1 & b_1 & b_1 & c_1 & c_1 & a_1 \\ b_2 & a_2 & c_2 & b_2 & a_2 & c_2 \end{matrix}$

i.e. $\begin{matrix} 2 & 3 & 3 & 9 & 9 & 2 \\ -2 & 3 & 7 & -2 & 3 & 7 \end{matrix}$

$\Delta = a_1b_2 - b_1a_2 = 2\times -2 - 3 \times 3 = -4 - 9 = -13$

$\Delta_1 = b_1c_2 - c_1b_2 = 3\times 7 - 9 \times -2 = 21 + 18 = 39$
$\Delta_2 = c_1a_2 - a_1c_2 = 9\times 3 - 2 \times 7 = 27 - 14 = 13$
Now to find the value of x *and* y
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{39}{-13} = 3$
$y = -\frac{\Delta_2}{\Delta} = -\frac{13}{-13} = 1$
Hence the solution.

Example 2. Solve the following system of equations : $x - 2y = 4, -3x + 5y = -7$.
Solution : Given system of equations are $x - 2y = 4, -3x + 5y = -7$
Here $a_1 = 1 \quad b_1 = -2 \quad c_1 = 4$ and $a_2 = -3 \quad b_2 = 5 \quad c_2 = -7$

Method :	a_1	b_1	b_1	c_1	c_1	a_1
	b_2	a_2	c_2	b_2	a_2	c_2
i.e.	1	-2	-2	4	4	1
	5	-3	-7	5	-3	-7

$\Delta = a_1b_2 - b_1a_2 = 1\times 5 - (-2) \times (-3) = 5 - 6 = -1$
$\Delta_1 = b_1c_2 - c_1b_2 = -2\times -7 - 4 \times 5 = 14 - 20 = -6$
$\Delta_2 = c_1a_2 - a_1c_2 = 4\times -3 - 1 \times -7 = -12 + 7 = -5$
Now to find the value of x *and* y
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{-6}{-1} = -6$
$y = -\frac{\Delta_2}{\Delta} = -\frac{-5}{-1} = -5$
Hence the solution.

Example 3. Solve the following system of equations : $3x - 2y = 4, 4x - 3y = 5$.
Solution : Given system of equations are $3x - 2y = 4$, $4x - 3y = 5$
Here $a_1 = 3 \quad b_1 = -2 \quad c_1 = 4$ and $a_2 = 4 \quad b_2 = -3 \quad c_2 = 5$

Method :	a_1	b_1	b_1	c_1	c_1	a_1
	b_2	a_2	c_2	b_2	a_2	c_2
i.e.	3	-2	-2	4	4	3
	-3	4	5	-3	4	5

$\Delta = a_1b_2 - b_1a_2 = 3\times -3 - (-2) \times 4 = -9 + 8 = -1$
$\Delta_1 = b_1c_2 - c_1b_2 = -2\times 5 - 4 \times -3 = -10 + 12 = 2$
$\Delta_2 = c_1a_2 - a_1c_2 = 4\times 4 - 3 \times 5 = 16 - 15 = 1$
Now to find the value of x *and* y
$\therefore x = -\frac{\Delta_1}{\Delta} = -\frac{2}{-1} = 2$
$y = -\frac{\Delta_2}{\Delta} = -\frac{1}{-1} = 1$
Hence the solution.

2. **Method to solve the simultaneous linear equations having three unknowns (Variables)**.

Consider three linear equations having three variables such that

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Method :

(1) First of all Write constants i.e. $a\,,b\,,c$ which is as follows :

$$\begin{array}{cccccc} a_1 & a_1 & b_1 & b_1 & c_1 & c_1 \\ b_2 & b_3 & c_2 & c_3 & a_2 & a_3 \\ c_3 & c_2 & a_3 & a_2 & b_3 & b_2 \end{array}$$

(1) $\Delta = (a_1b_2c_3 - a_1b_3c_2) + (b_1c_2a_3 - b_1c_3a_2) + (c_1a_2b_3 - c_1a_3b_2)$.

(II) Write constants i.e. $b\,,c\,,d$ which is as follows :

$$\begin{array}{cccccc} b_1 & b_1 & c_1 & c_1 & d_1 & d_1 \\ c_2 & c_3 & d_2 & d_3 & b_2 & b_3 \\ d_3 & d_2 & b_3 & b_2 & c_3 & c_2 \end{array}$$

$\Delta_1 = (b_1c_2d_3 - b_1c_3d_2) + (c_1d_2b_3 - c_1d_3b_2) + (d_1b_2c_3 - d_1b_3c_2)$.

(III) Write constants i.e. $c\,,d\,,a$ which is as follows :

$$\begin{array}{cccccc} c_1 & c_1 & d_1 & d_1 & a_1 & a_1 \\ d_2 & d_3 & a_2 & a_3 & c_2 & c_3 \\ a_3 & a_2 & c_3 & c_2 & d_3 & d_2 \end{array}$$

$\Delta_2 = (c_1d_2a_3 - c_1d_3a_2) + (d_1a_2c_3 - d_1a_3c_2) + (a_1c_2d_3 - a_1c_3d_2)$.

(IV) Write constants i.e. $d\,,a\,,b$ which is as follows :

$$\begin{array}{cccccc} d_1 & d_1 & a_1 & a_1 & b_1 & b_1 \\ a_2 & a_3 & b_2 & b_3 & d_2 & d_3 \\ b_3 & b_2 & d_3 & d_2 & a_3 & a_2 \end{array}$$

$\Delta_3 = (d_1a_2b_3 - d_1a_3b_2) + (a_1b_2d_3 - a_1b_3d_2) + (b_1d_2a_3 - b_1d_3a_2)$.

Now to find the value of $x\,,y\ and\ z$

$\therefore x = \frac{\Delta_1}{\Delta}\,, y = -\frac{\Delta_2}{\Delta}\ , \quad z = \frac{\Delta_3}{\Delta}$.

*** Some problems based on three un-knowns:**

Example 1. Solve the following system of equations : $3x + 2y - z = 4, -x - y + 3z = 6, 5x - 3y + z = 2$.

Solution :- Given system of equations are $3x + 2y - z = 4, -x - y + 3z = 6, 5x - 3y + z = 2$

Here $a_1 = 3, b_1 = 2, c_1 = -1, d_1 = 4$, $a_2 = -1, b_2 = -1, c_2 = 3, d_2 = 6$, $a_3 = 5, b_3 = -3, c_3 = 1, d_3 = 2$

Method

(I)

$$\begin{array}{cccccc} a_1 & a_1 & b_1 & b_1 & c_1 & c_1 \\ b_2 & b_3 & c_2 & c_3 & a_2 & a_3 \\ c_3 & c_2 & a_3 & a_2 & b_3 & b_2 \end{array}$$

i.e.

$$\begin{array}{cccccc} 3 & 3 & 2 & 2 & -1 & -1 \end{array}$$

$-1 \quad -3 \quad 3 \quad 1 \quad -1 \quad 5$
$1 \quad 3 \quad 5 \quad -1 \quad -3 \quad -1$

$\Delta = (3 \times -1 \times 1 - 3 \times -3 \times 3) + (2 \times 3 \times 5 - 2 \times 1 \times -1) + \{-1 \times -1 \times -3 - (-1) \times 5 \times -1\}$
$= (-3 + 27) + (30 + 2) + (-3 - 5)$
= 24+32-8
= 56-8 = 48
Δ=48

(II) $b_1 \quad b_1 \quad c_1 \quad c_1 \quad d_1 \quad d_1$
$c_2 \quad c_3 \quad d_2 \quad d_3 \quad b_2 \quad b_3$
$d_3 \quad d_2 \quad b_3 \quad b_2 \quad c_3 \quad c_2$
i.e.
$2 \quad 2 \quad -1 \quad -1 \quad 4 \quad 4$
$3 \quad 1 \quad 6 \quad 2 \quad -1 \quad -3$
$2 \quad 6 \quad -3 \quad -1 \quad 1 \quad 3$

$\Delta_1 = (2 \times 3 \times 2 - 2 \times 1 \times 6) + \{-1 \times 6 \times -3 - (-1) \times 2 \times -1\} + (4 \times -1 \times 1 - 4 \times -3 \times 3)$
$= (12 - 12) + (18 - 2) + (-4 + 36)$
$= 0 + 16 + 32 = 48$
$\Delta_1 = 48$

(III) $c_1 \quad c_1 \quad d_1 \quad d_1 \quad a_1 \quad a_1$
$d_2 \quad d_3 \quad a_2 \quad a_3 \quad c_2 \quad c_3$
$a_3 \quad a_2 \quad c_3 \quad c_2 \quad d_3 \quad d_2$

i.e.

$-1 \quad -1 \quad 4 \quad 4 \quad 3 \quad 3$
$6 \quad 2 \quad -1 \quad 5 \quad 3 \quad 1$
$5 \quad -1 \quad 1 \quad 3 \quad 2 \quad 6$

$\Delta_2 = \{-1 \times 6 \times 5 - (-1) \times 2 \times -1\} + (4 \times -1 \times 1 - 4 \times 5 \times 3) + (3 \times 3 \times 2 - 3 \times 1 \times 6)$
$= (-30 - 2) + (-4 - 60) + (18 - 18)$
= -32-64+0 = -96+0 = -96
$\Delta_2 = -96$

(IV) $d_1 \quad d_1 \quad a_1 \quad a_1 \quad b_1 \quad b_1$
$a_2 \quad a_3 \quad b_2 \quad b_3 \quad d_2 \quad d_3$
$b_3 \quad b_2 \quad d_3 \quad d_2 \quad a_3 \quad a_2$
i.e.
$4 \quad 4 \quad 3 \quad 3 \quad 2 \quad 2$
$-1 \quad 5 \quad -1 \quad -3 \quad 6 \quad 2$
$-3 \quad -1 \quad 2 \quad 6 \quad 5 \quad -1$

$\Delta_3 = (4 \times -1 \times -3 - 4 \times 5 \times -1) + (3 \times -1 \times 2 - 3 \times -3 \times 6) + (2 \times 6 \times 5 - 2 \times 2 \times -1)$
$\quad = (12 + 20) + (-6 + 54) + (60 + 4)$
$\quad = 32 + 48 + 64 = 144$
$\Delta_3 = 144$

Now to find the value of $x\ , y\ and\ z$
$\therefore x = \frac{\Delta_1}{\Delta} = \frac{48}{48} = 1$
$y = -\frac{\Delta_2}{\Delta} = -\frac{-96}{48} = 2$
$z = \frac{\Delta_3}{\Delta} = \frac{144}{48} = 3$
$\therefore x = 1, y = 2, z = 3$
Hence the solution.

Example 2. Solve the following system of equations : $6x - 9y - 20z = -4, 4x - 15y + 10z = -1, 2x - 3y - 5z = -1.$
Solution :- Given system of equations are $6x - 9y - 20z = -4, 4x - 15y + 10z = -1, 2x - 3y - 5z = -1$
Here $a_1 = 6, b_1 = -9, c_1 = -20, d_1 = -4$, $a_2 = 4, b_2 = -15, c_2 = 10, d_2 = -1$, $a_3 = 2, b_3 = -3, c_3 = -5, d_3 = -1$
Method
(I)

a_1	a_1	b_1	b_1	c_1	c_1
b_2	b_3	c_2	c_3	a_2	a_3
c_3	c_2	a_3	a_2	b_3	b_2

i.e.

6	6	-9	-9	-20	-20
-15	-3	10	-5	4	2
-5	10	2	4	-3	-15

$\Delta = (6 \times -15 \times -5 - 6 \times -3 \times 10) + (-9 \times 10 \times 2 - (-9) \times -5 \times 4) + \{-20 \times 4 \times -3 - (-20) \times 2 \times -15\}$
$\quad = (450 + 180) + (-180 - 180) + (240 - 600)$
$\quad = 630 - 360 - 360$
$\quad = 630 - 720$
$\Delta = -90$

(II)

b_1	b_1	c_1	c_1	d_1	d_1
c_2	c_3	d_2	d_3	b_2	b_3
d_3	d_2	b_3	b_2	c_3	c_2

i.e.

-9	-9	-20	-20	-4	-4
10	-5	-1	-1	-15	-3
-1	-1	-3	-15	-5	10

$\Delta_1 = (-9 \times 10 \times -1 - (-9) \times -5 \times -1) + \{-20 \times -1 \times -3 - (-20) \times -1 \times -15\} + (-4 \times \quad -15 \times -5 - (-4) \times -3 \times 10)$

$= (90 + 45) + (-60 + 300) + (-300 - 120)$

$= 135 + 240 - 420 = 375 - 420$

$\Delta_1 = -45$

(III)

$$\begin{array}{cccccc} c_1 & c_1 & d_1 & d_1 & a_1 & a_1 \\ d_2 & d_3 & a_2 & a_3 & c_2 & c_3 \\ a_3 & a_2 & c_3 & c_2 & d_3 & d_2 \end{array}$$

i.e.

$$\begin{array}{cccccc} -20 & -20 & -4 & -4 & 6 & 6 \\ -1 & -1 & 4 & 2 & 10 & -5 \\ 2 & 4 & -5 & 10 & -1 & -1 \end{array}$$

$\Delta_2 = \{-20 \times -1 \times 2 - (-20) \times -1 \times 4\} + (-4 \times 4 \times -5 - (-4) \times 2 \times 10) + (6 \times 10 \times -1 - 6 \times \quad -5 \times -1)$

$= (40 - 80) + (80 + 80) + (-60 - 30)$

$= -40 + 160 - 90 = -130 + 160 = 30$

$\Delta_2 = 30$

(IV)

$$\begin{array}{cccccc} d_1 & d_1 & a_1 & a_1 & b_1 & b_1 \\ a_2 & a_3 & b_2 & b_3 & d_2 & d_3 \\ b_3 & b_2 & d_3 & d_2 & a_3 & a_2 \end{array}$$

i.e.

$$\begin{array}{cccccc} -4 & -4 & 6 & 6 & -9 & -9 \\ 4 & 2 & -15 & -3 & -1 & -1 \\ -3 & -15 & -1 & -1 & 2 & 4 \end{array}$$

$\Delta_3 = (-4 \times 4 \times -3 - (-4) \times 2 \times -15) + (6 \times -15 \times -1 - 6 \times -3 \times -1) + (-9 \times -1 \times 2 - \quad (-9) \times -1 \times 4)$

$= (48 - 120) + (90 - 18) + (18 - 36)$

$= -72 + 72 - 18 = -18$

$\Delta_3 = -18$

Now to find the value of $x, y \text{ and } z$

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-45}{-90} = \frac{1}{2}$

$y = -\frac{\Delta_2}{\Delta} = -\frac{30}{-90} = \frac{1}{3}$

$z = \frac{\Delta_3}{\Delta} = \frac{-18}{-90} = \frac{1}{5}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Hence the solution.

Example 3. Solve the following system of equations : $3x - 4y + 5z = 25, 2x + 3y + z = 4, x - y + 2z = 9$.

Solution :- Given system of equations are $3x - 4y + 5z = 25, 2x + 3y + z = 4, x - y + 2z = 9$

Here $a_1 = 3, b_1 = -4, c_1 = 5, d_1 = 25$, $a_2 = 2, b_2 = 3, c_2 = 1$, $d_2 = 4$, $a_3 = 1, b_3 =$

$-1, c_3 = 2, d_3 = 9$

Method

(I)

a_1	a_1	b_1	b_1	c_1	c_1
b_2	b_3	c_2	c_3	a_2	a_3
c_3	c_2	a_3	a_2	b_3	b_2

i.e.

3	3	-4	-4	5	5
3	-1	1	2	2	1
2	1	1	2	-1	3

$\Delta = (3 \times 3 \times 2 - 3 \times -1 \times 1) + (-4 \times 1 \times 1 - (-4) \times 2 \times 2) + (5 \times 2 \times -1 - 5 \times 1 \times 3 \})$

$= (18 + 3) + (-4 + 16) + (-10 - 15)$

$= 21 + 12 - 25$

$= 33 - 25$

$\Delta = 8$

(II)

b_1	b_1	c_1	c_1	d_1	d_1
c_2	c_3	d_2	d_3	b_2	b_3
d_3	d_2	b_3	b_2	c_3	c_2

i.e.

-4	-4	5	5	25	25
1	2	4	9	3	-1
9	4	-1	3	2	1

$\Delta_1 = (-4 \times 1 \times 9 - (-4) \times 2 \times 4) + (5 \times 4 \times -1 - 5 \times 9 \times 3) + (25 \times 3 \times 2 - 25 \times -1 \times 1)$

$= (-36 + 32) + (-20 - 135) + (150 + 25)$

$= -4 - 155 + 175 = -159 + 175$

$\Delta_1 = 16$

(III)

c_1	c_1	d_1	d_1	a_1	a_1
d_2	d_3	a_2	a_3	c_2	c_3
a_3	a_2	c_3	c_2	d_3	d_2

i.e.

5	5	25	25	3	3
4	9	2	1	1	2
1	2	2	1	9	4

$\Delta_2 = (5 \times 4 \times 1 - 5 \times 9 \times 2) + (25 \times 2 \times 2 - 25 \times 1 \times 1) + (3 \times 1 \times 9 - 3 \times 2 \times 4)$

$= (20 - 90) + (100 - 25) + (27 - 24)$

$= -70 + 75 + 3 = -70 + 78 = 8$

$\Delta_2 = 8$

(IV)

d_1	d_1	a_1	a_1	b_1	b_1
a_2	a_3	b_2	b_3	d_2	d_3

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i.e.

$$\begin{array}{cccccc} b_3 & b_2 & d_3 & d_2 & a_3 & a_2 \\ 25 & 25 & 3 & 3 & -4 & -4 \\ 2 & 1 & 3 & -1 & 4 & 9 \\ -1 & 3 & 9 & 4 & 1 & 2 \end{array}$$

$\Delta_3 = (25 \times 2 \times -1 - 25 \times 1 \times 3) + (3 \times 3 \times 9 - 3 \times -1 \times 4) + (-4 \times 4 \times 1 - (-4) \times 9 \times 2)$

$= (-50 - 75) + (81 + 12) + (-16 + 72)$

$= -125 + 93 + 56 = -125 + 149$

$\Delta_3 = 24$

Now to find the value of x , y and z

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{16}{8} = 2$

$y = -\frac{\Delta_2}{\Delta} = -\frac{8}{8} = -1$

$z = \frac{\Delta_3}{\Delta} = \frac{24}{8} = 3$

$\therefore x = 2, y = -1, z = 3$

Hence the solution.

CONCLUSION:

This method is another technique which is helpful to find the solution of the simultaneous linear equations of two or three un-knowns or variables as compared to other methods. This is also the simplest approach to find the solution of the simultaneous linear equations of two or three un-knowns or variables.

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A CHARACTERIZATION THERMOSOLUTAL CONVECTION IN COUPLE-STRESS FLUID IN A POROUS MEDIUM IN THE PRESENCE OF A ROTATION AND MAGNETIC FIELD

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ABSTRACT:
The thermosolutal instability of a couple-stress fluid heated from below in a porous medium acted upon by uniform vertical magnetic field and rotation is investigated. Following the linearized stability theory and normal mode analysis, the paper mathematically established the conditions for characterizing the oscillatory motions which may be neutral or unstable for rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, and a constant vertical adverse temperature gradient, are necessarily non-oscillatory in the regime

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\le 1 ,$$

where the R_s is the thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, ε is the porosity, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter. The result is important since it holds for all wave numbers and the exact solutions of the problem investigated are not obtainable in closed form when both the boundaries are perfectly conducting and rigid. A similar characterization theorem is also proved for Stern type of configuration.

KEY WORDS: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Rayleigh number; Chandrasekhar Number; Taylor number.
MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05.

1. INTRODUCTION:

A detailed account of the theoretical and experimental study of the onset of thermal instability in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1] and the Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion, except in the external force term. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [2]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can name the food processing industry, the chemical processing industry, solidification, and the centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [3]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat Tabor and Matz [4] and some Antarctic lakes Shirtcliffe [5]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust Lister [6]. Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice - versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context Mc Donnel [7]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov [8] and Linden [9] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [10] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [11] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge.

The theory of couple-stress fluid has been formulated by Stokes [12]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze - film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded – bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [12], couple-stresses appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [13] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [14] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [15] have considered a couple - stress fluid with suspended particles heated from below. In another study, Sunil et al. [16] have considered a couple- stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [17] have considered the thermal instability of a layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Pellow and Southwell [18] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [19] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [20] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [21]. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [22] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in porous medium in the presence of uniform rotation and uniform magnetic field has been considered in the present paper. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, and a constant vertical adverse temperature gradient, are necessarily non-oscillatory in the regime

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1 ,$$

where the R_s is the thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, ε is the porosity, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter. The result is important since it holds for all wave numbers and the exact solutions of the problem investigated are not obtainable in closed form when both the boundaries are perfectly conducting and rigid. A similar characterization theorem is also proved for Stern [23] type of configuration.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta^{/}\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

Let p, ρ, T, C, α, $\alpha^{/}$, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid (Chandrasekhar [1]; Joseph [2]; Stokes [12]) are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T , \quad (3)$$

$$E^{/}\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)C=\kappa^{/}\nabla^{2}C, \tag{4}$$

$$\rho = \rho_0 [1 - \alpha (T-T_0) + \alpha^{/} (C-C_0)], \tag{5}$$

Where the suffix zero refers to values at the reference level z = 0 and in writing equation (1), use has been made of Boussinesq approximation. Here E = $\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^{/}$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o, C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The magnetic permeability μ_e, the kinematic viscosityν, couple-stress viscosity $\mu^{/}$, the thermal diffusivity κ and the solute diffusivity $\kappa^{/}$ are all assumed to be constants.

The Maxwell's equations yield

$$\in\frac{d\vec{H}}{dt}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\in\eta\nabla^{2}\vec{H}, \tag{6}$$

and $\nabla\cdot\vec{H}=0,$ (7)

where $\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\in^{-1}\vec{q}\cdot\nabla$ stands for the convective derivative.

The steady state solution is

$\vec{q}(u,v,w)=(0,0,0)$, T = T_0-βz, C = C_0 - $\beta^{/}$z,

$$\rho = \rho_0 (1+ \alpha\beta z - \alpha^{/}\beta^{/}z). \tag{8}$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, δρ, θ, γ , $\vec{h}(h_x,h_y,h_z)$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,0)$ and velocity $\vec{q}(0,0,0)$. The change in densityδρ, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = - \rho_0 (\alpha\theta - \alpha^{/} \gamma). \tag{9}$$

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\vec{g}(\alpha\theta-\alpha^{/}\gamma)-\frac{1}{k_1}\left(\nu-\frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{10}$$

$$\nabla.\vec{q} = 0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^{2}\theta, \tag{12}$$

$$E^{/}\frac{\partial\gamma}{\partial t}=\beta^{/}w+\kappa^{/}\nabla^{2}\gamma, \tag{13}$$

$$\in\frac{\partial\vec{h}}{\partial t}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\in\eta\nabla^{2}\vec{h}, \tag{14}$$

and $\nabla.\vec{h} = 0.$ (15)

3. NORMAL MODES ANALYSIS:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$[w,\theta,h_z,\gamma,\varsigma,\xi]=[W(z),\Theta(z),K(z),\Gamma(z),Z(z),X(z)]$ exp(ik$_x$x + ik$_y$y + nt), (16)

where k_x, k_y are the wave numbers along the x- and y- directions respectively, k= ($\sqrt{k_x^2+k_y^2}$) is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\varsigma=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively. $W(z),K(z),\Theta(z),Z(z)$ and $X(z)$ are the functions of z only.

Using (16), equations (10)-(15), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left(D^2-a^2\right)\left[\frac{F}{P_l}\left(D^2-a^2\right)-\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\right]W=Ra^2\Theta-R_sa^2\Gamma+T_ADZ-Q\left(D^2-a^2\right)DK, \quad (17)$$

$$\left[\frac{F}{P_l}\left(D^2-a^2\right)-\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\right]Z=-DW-QDX, \quad (18)$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta=-W, \quad (19)$$

$$\left(D^2-a^2-E'p_3\sigma\right)\Gamma=-W, \quad (20)$$

$$\left(D^2-a^2-p_2\sigma\right)K=-DW, \quad (21)$$

and

$$\left(D^2-a^2-p_2\sigma\right)X=-DZ, \quad (22)$$

Where we have introduced new coordinates (x',y',z') = (x/d, y/d, z/d) in new units of length d and $D=d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a=kd,\sigma=\frac{nd^2}{\nu}$, $p_1=\frac{\nu}{\kappa}$, is the thermal Prandtl number;, $p_3=\frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $p_2=\frac{\nu}{\eta}$ is the magnetic Prandtl number; $P_l=\frac{k_1}{d^2}$ is the dimensionless medium permeability, $F=\frac{\mu'/(\rho_0d^2)}{\nu}$, is the dimensionless couple-stress parameter; $R=\frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $R_s=\frac{g\alpha'\beta'd^4}{\kappa'\nu}$ is the thermosolutal Rayleigh number; $Q=\frac{\mu_eH^2d^2}{4\pi\rho_0\nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A=\frac{4\Omega^2d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W=W_\oplus$, $\Theta=\frac{\beta d^2}{\kappa}\Theta_\oplus$, $\Gamma=\frac{\beta'd^2}{\kappa'}\Gamma_\oplus$, $Z=\frac{2\Omega d}{\nu\varepsilon}Z_\oplus$, $K=\frac{Hd}{\varepsilon\eta}K_\oplus$, $X=\left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_\oplus$ and $D_\oplus=dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, are perfectly conducting and maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (17)--(22), must possess a solution are

W = 0 =DW =Z=DX =K= $\Theta = \Gamma$, on both the horizontal boundaries, (23)

Equations (17)-(22), along with boundary conditions (23), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, K and Z satisfy $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \quad \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz, \quad \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz,$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \text{ and } \int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz, \tag{24}$$

Further, for $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, Banerjee et al. [25] have shown that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz, \quad \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz. \tag{25}$$

4. MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^4} \int_0^1 |DW|^2 dz \quad .$$

Proof: Multiplying equation (19) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Θ namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2\right\} dz + E\sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of} \left\{\int_0^1 \Theta^* W dz\right\},$$

$$\leq \left|\int_0^1 \Theta^* W dz\right| \leq \int_0^1 \left|\Theta^* W\right| dz \leq \int_0^1 \left|\Theta^*\right| |W| dz,$$

$$\leq \int_0^1 |\Theta| |W| dz \leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{26}$$

(Utilizing Cauchy-Schwartz-inequality),

So that the fact that $\sigma_r \geq 0$, we obtain from the above that

$$a^2 \int_0^1 |\Theta|^2 dz \leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{a^2}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{27}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (26) on utilizing (27) and (24), gives

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz, \tag{28}$$

This completes the proof of lemma 1.

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz \ .$$

Proof: Multiplying equation (20) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{|D\Gamma|^2 + a^2|\Gamma|^2\right\}dz + E^{/}\sigma_r p_3\int_0^1 |\Gamma|^2 dz = \text{Real part of}\left\{\int_0^1 \Gamma^* W dz\right\},$$

$$\leq \left|\int_0^1 \Gamma^* W dz\right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*||W| dz,$$

$$\leq \int_0^1 |\Gamma||W| dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{29}$$

(Utilizing Cauchy-Schwartz-inequality),

So that, since $\sigma_r \geq 0$, we obtain from the above that

And thus, we get

$$a^2\int_0^1 |\Gamma|^2 dz \leq \left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{\int_0^1 |\Gamma|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{a^2}\left\{\int_0^1 |W|^2 dz\right\}^{\frac{1}{2}}, \tag{30}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (29) on utilizing (30) and (24), gives

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^4}\int_0^1 |DW|^2 dz, \tag{31}$$

This completes the proof of lemma 2.

Lemma 3: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\}dz \leq \frac{\pi^2}{(2\pi^2 - 1)}\int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (21) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0) = 0 = K(1)$, it follows that

$$\int_0^1 \left\{ |DK|^2 + a^2|K|^2 \right\} dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of} \left\{ \int_0^1 K^* DW dz \right\} \leq \left| \int_0^1 K^* DW dz \right|$$

$$\leq \int_0^1 |K^* DW| dz,$$

$$\leq \int_0^1 |K^*||DW| dz \leq \frac{1}{2}\int_0^1 \left(|K|^2 + |DW|^2 \right) dz, \tag{32}$$

This gives that

$$\int_0^1 |DK|^2 dz \leq \frac{1}{2}\int_0^1 \left(|K|^2 + |DW|^2 \right) dz, \tag{33}$$

Inequality (33) on utilizing (24), gives

$$\int_0^1 |K|^2 dz \leq \frac{1}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz, \tag{34}$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (32) on utilizing (34), give

$$\int_0^1 \left(|DK|^2 + a^2|K|^2 \right) dz \leq \frac{\pi^2}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz, \tag{35}$$

This completes the proof of lemma 3.

Lemma 4: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \leq \frac{P_l}{(2\pi^2 F - P_l)} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (18) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (22) and appropriate boundary conditions (23), it follows that

$$\frac{F}{P_l}\int_0^1 \left\{ |DZ|^2 + a^2|Z|^2 \right\} dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 |Z|^2 dz + Q\int_0^1 \left\{ |DX|^2 + a^2|X|^2 \right\} dz + Qp_2\sigma_r \int_0^1 |X|^2 dz$$

$$= \text{Real part of } \left\{ \int_0^1 DW^* Z dz \right\} \leq \left| \int_0^1 DW^* Z dz \right|,$$

$$\leq \int_0^1 |DW^* Z| dz \leq \int_0^1 |DW^*||Z| dz,$$

$$= \int_0^1 |DW||Z| dz \leq \frac{1}{2}\int_0^1 \left(|Z|^2 + |DW|^2 \right) dz, \tag{36}$$

This gives that

$$\frac{F}{P_l}\int_0^1 |DZ|^2 dz \le \frac{1}{2}\int_0^1 \left(|Z|^2 + |DW|^2\right) dz, \tag{37}$$

inequality (36) on utilizing (37), gives

$$\int_0^1 |Z|^2 dz \le \frac{P_l}{(2\pi^2 F - P_l)}\int_0^1 |DW|^2 dz, \tag{38}$$

This completes the proof of lemma 4**.**
Now we prove the following theorems:

Theorem 1: If $R\rangle\, 0$, $R_s\rangle 0$ $F\rangle\, 0$, $Q\rangle 0$, $T_A\rangle\, 0$, $P_l\rangle 0$, $p_1\rangle 0$, $p_2\rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution $\left(W,\Theta,\Gamma,K,Z,X\right)$ of equations (17) – (22), together with boundary conditions (23) is that

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Q p_2 \pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)}\rangle 1 \ .$$

Proof: Multiplying equation (17) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*\left(D^2 - a^2\right) W dz$$

$$= Ra^2\int_0^1 W^*\Theta dz - R_s a^2\int_0^1 W^*\Gamma dz + T_A\int_0^1 W^* DZ dz - Q\int W^* D(D^2 - a^2) K dz\ , \tag{39}$$

Taking complex conjugate on both sides of equation (19), we get

$$\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* = -W^*, \tag{40}$$

Therefore, using (40), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz, \tag{41}$$

Taking complex conjugate on both sides of equation (20), we get

$$\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* = -W^*, \tag{42}$$

Therefore, using (42), we get

$$\int_0^1 W^*\Gamma dz = -\int_0^1 \Gamma\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* dz, \tag{43}$$

Also taking complex conjugate on both sides of equation (18), we get

$$\frac{F}{P_l}\left(D^2 - a^2\right)Z^* - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)Z^* + QDX^* = -DW^*, \tag{44}$$

Therefore, using (44), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2 - a^2\right) Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 Z^* Z dz + Q\int_0^1 ZDX^* dz,$$

Integrating by parts the third term on left hand side and using equation (22), and appropriate boundary condition (23), we get

$$\int_0^1 W^* DZdz = \frac{F}{P_l}\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 Z^* Zdz + Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz\,, \qquad (45)$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^* = -DW^*\,, \qquad (46)$$

Therefore, equation (46), using appropriate boundary condition (23), we get

$$\int_0^1 W^* D\left(D^2-a^2\right)Kdz = -\int_0^1 DW^*\left(D^2-a^2\right)Kdz = \int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz\,, \qquad (47)$$

Substituting (41), (43), (45) and (47), in the right hand side of equation (39), we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2 Wdz - \left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 W^*(D^2-a^2)Wdz = -Ra^2\int_0^1 \Theta\left(D^2-a^2-Ep_1\sigma^*\right)\Theta^* dz$$

$$+R_s a^2\int_0^1 \Gamma(D^2-a^2-E' p_3\sigma^*)\Gamma^* dz + \frac{T_A F}{P_l}\int_0^1 Z(D^2-a^2)Z^* dz - T_A\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 ZZ^* dz$$

$$+T_A Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz - Q\int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz\,, \qquad (48)$$

Integrating the terms on both sides of equation (48) for an appropriate number of times and making use of the appropriate boundary conditions (23), we get

$$\frac{F}{P_l}\int_0^1\left\{\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right\}dz + \left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2+Ep_1\sigma^*|\Theta|^2\right)dz - R_s a^2\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2+E' p_3\sigma^*|\Gamma|^2\right)dz$$

$$-\frac{T_A F}{P_l}\int_0^1\left\{|DZ|^2+a^2|Z|^2\right\}dz - T_A\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1|Z|^2 dz - T_A Q\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$$

$$-T_A Qp_2\sigma\int_0^1|X|^2 dz - Q\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1\left(|DK|^2+a^2|K|^2\right)dz\,, \qquad (49)$$

now equating imaginary parts on both sides of equation (49), and cancelling $\sigma_i(\neq 0)$ throughout from imaginary part, we get

$$\frac{1}{\varepsilon}\int_0^1\left\{|DW|^2+a^2|W|^2\right\}dz = -Ra^2Ep_1\int_0^1|\Theta|^2 dz + R_s a^2 E' p_3\int_0^1|\Gamma|^2 dz$$

$$+\frac{T_A}{\varepsilon}\int_0^1|Z|^2 dz + Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz - T_A Qp_2\int_0^1|X|^2 dz\,, \qquad (50)$$

Now R $\rangle$ 0, $Q\rangle 0$ $\varepsilon\rangle 0$ $p_2\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (31), (35) and (38), the equation (50) gives,

$$\frac{1}{\varepsilon}\left[1-\left\{\frac{P_l T_A}{(2\pi^2 F-P_l)}+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\varepsilon\left(\frac{R_s E' p_3}{\pi^4}\right)\right\}\right]\int_0^1|DW|^2 dz + I_1\langle 0\,, \qquad (51)$$

Where $I_1 = \dfrac{a^2}{\varepsilon}\int_0^1|W|^2 dz + Ra^2Ep_1\int_0^1|\Theta|^2 dz + T_A Qp_2\int_0^1|X|^2 dz$, is positive definite.

and therefore , we must have

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\rangle 1 . \tag{52}$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then } \varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\rangle 1 . \tag{53}$$

And this completes the proof of the theorem. Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 1, can be put in the form as follow:-

Corollary 1: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and rotation in a porous medium heated from below is that, $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, where R_s is the Thermosolutal Rayliegh number, T_A is the Taylor number, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium heated from below, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, for rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 2: If $(W,\Theta,\Gamma,K,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (17) – (22), with $R\rangle 0$ and,

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1 ,$$

Then $\sigma_i=0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \le 1.$

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Veronis type, we can state the above theorem as follow:-

Corollary 3: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Veronis type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium heated from below is that the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1$, for rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in couple-stress fluid heated from below, i. e. when $T_A = 0 = R_s =$ Q. (Sunil et al $[16]$).

(ii). Magneto-rotatory-thermal convection in couple-stress fluid heated from below (R_s =0), if

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \le 1 ,$$

(iii) Magneto-thermosolutal convection of Veronis (1965) type in couple-stress fluid heated from below
(T_A = 0), if

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) \le 1 .$$

(iv) Rotatory-thermosolutal convection of Veronis (1965) type in couple-stress fluid heated from below (Q= 0), if

$$\varepsilon\left(\frac{R_s E^{/} p_3}{\pi^4}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1$$

A similar theorem can be proved for thermosolutal convection in couple-stress fluid configuration of Stern type in a porous medium as follow:

Theorem 2: If $R \langle 0, R_s \langle 0$, F$\rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, Z, \sigma)$ of equations (17) – (22), together with boundary conditions (23) is that

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\rangle 1.$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (17) – (22) and proceeding exactly as in Theorem 1 and utilizing the inequality (28), we get the desired result.
Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 2, can be put in the form as follow:-

Corollary 4: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium is that, $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1$, where R is the Thermal Rayliegh number, the Taylor number T_A, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number, p_1 is the thermal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the couple-stress parameter, for rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermal Rayliegh number R, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F, satisfy the inequality $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1$, for rigid boundaries at the top and bottom of the fluid
The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 5: If $(W,\Theta,\Gamma,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r\ge 0$ is a solution of equations (17) – (22), with R$\rangle$ 0 and,

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1 \quad ,$$

Then $\sigma_i=0$.
In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r=0\Rightarrow\sigma_i=0$ if $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Stern's type, we can state the above theorem as follow:-

Corollary 6: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal couple-stress fluid configuration of Stern type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium is that the Thermal Rayliegh number R, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the couple-stress parameter F must satisfy the inequality $\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\rangle 1$, for rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in couple-stress fluid i. e. when $T_A = 0 =$R=Q.

(ii). Magneto-rotatory thermal convection couple-stress fluid (R=0), if

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1,$$

(iii). Magneto-thermal convection of Stren (1960) type in couple-stress fluid (T_A = 0), if

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)\le 1 \ .$$

(iv). Rotatory-thermal convection of Stren (1960) type in couple-stress fluid (Q= 0), if

$$\varepsilon\left(\frac{|R|Ep_1}{\pi^4}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1$$

5. CONCLUSIONS:

Theorem 1 mathematically established that the onset of instability in a thermosolutal couple-stress fluid configuration of Veronis (1965) type in the presence of uniform vertical magnetic field and vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the couple-stress parameter F satisfy the inequality $\varepsilon\left(\frac{R_sE^{/}p_3}{\pi^4}\right)+\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1$, for rigid boundaries at the top and bottom of the fluid

The essential content of the theorem 1, from the point of view of linear stability theory is that for the thermosolutal configuration of Veronis (1965) type of couple-stress fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for rigid boundaries at the top and bottom of the fluid, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\varepsilon\left(\dfrac{R_s E^{/} p_3}{\pi^4}\right)+\varepsilon\left(\dfrac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\dfrac{P_l T_A}{(2\pi^2 F-P_l)}\leq 1$, and in particular PES is valid.

The similar conclusions can be drawn for the thermosolutal configuration of Stern (1960) type of couple-stress fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for rigid boundaries at the top and bottom of the fluid from Theorem 2.

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APPLICATION OF NUMERICAL ANALYSIS IN VARIOUS ENGINEERING FIELDS

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ABSTRACT:
Mathematics is an important part of the most branches of engineering fields. In this paper, several examples of application of Numerical Analysis in chemical engineering, mechanical engineering, electrical engineering and computer engineering. Numerical methods in chemical engineering deals with a broad range of problems related to quantum mechanical calculation of atoms or molecules. In mechanical engineering, numerical method deals with problems relating to linearization, finding root of the function, solving system of equation and optimization. Illumination problems in electrical engineering and in computer engineering to solve problem by using MATLAB and Mathematica.

KEYWORDS: Numerical methods, illumination, linearization, quantum mechanical.

INTRODUCTION:
Numerical analysis provide a way to solve problems quickly and easily.Whether the goal is integration or solution of complex differential equation. In engineering, application of numerical analysis in quantum mechanics of atoms and molecules, in first order irreversible series reaction, in illumination problems, in linearization problem and optimization.

Applications of Numerical analysis in chemical engineering:-

1. Quantum mechanics of atoms and molecules, Monte Carlo and Dynamics

Energy of an n-electron can be portioned into energy of one electron moving in the average field of (HF-SCF) electrons and nuclei. This is called Hartree-Fock Model. This approach allows that two electrons with different spins can found at the same spatial point. In this approach, the total Hamiltonian of the system is portioned into two pieces, zeroth order H_0 which is the Hartree-Fock Hamiltonian and a perturbation V. The exact energy is then expressed as an infinite sum of contribution of increasing complexity. The energy E_R of a molecular system is obtained as a solution of the electronic part of Schrodinger for a fixed configuration R of a nuclei

$$H(r, R)\ \psi(r) = E_R\psi_R(r)$$

The n- electron wave function $\psi_R(r)$ describes the motion of the electron in the nuclei. Due to electron-electron interaction term in Hamiltonian, this equation cannot be solved without approximations. The Hamiltonian function approximation assumes that n-particles wave function $\Psi(r)$ can be written as antisymmtrized product of n- electron function $\Psi i\ (r_i)$

$$\begin{vmatrix} \Psi_1 (r_1) \ \psi_2 (r_1) \ \ldots\ldots \Psi n (r_1) \\ \Psi (r_{1,}r_{2}\ ,r_{3\ldots}r_{n)}\ \psi_1(r_2) \quad \psi_2(r_2) \ \ldots\ldots\ldots \quad \Psi n\ (r_2) \\ \vdots \qquad \vdots \qquad \vdots \\ \Psi_1 (r_n)\psi_2 (r_n)\ldots\ldots\ \Psi n\ (r_n) \end{vmatrix}$$

Equation (1) is called Slater determinant. The set of orbitals that yield the lowest energy of a molecular system in the sense of vibrational principle is given by the following set of H.F. integro differential equations

$F(r_i)\,\Psi_i(r_i) = E_i\Psi_i(r_i)$

The orbitals $\Psi_i(r_i)$are called molecular orbitals and $F(r_i)$ is the Fock operator which comprises the differential operator of the kinetic energy and the electron-electron interaction term. The expansion of molecularorbitals $\Psi i\ (r_i)$ into a finite series of the basic function $\chi_i(r_i)$

$$\Psi_i(r_i) = \chi_i(r_i)\, C_i = (\chi_1(r_i), \chi_2(r_2), \chi_3(r_3) \ldots\ldots \chi_m(r_i)) \begin{pmatrix} C_{1i} \\ C_{2i} \\ \vdots \\ C_{mi} \end{pmatrix}$$

(2)First order irreversible series reactions:
Numerical Analysis applied in diffusion/reaction problems. Consider the first order irreversible series reactions

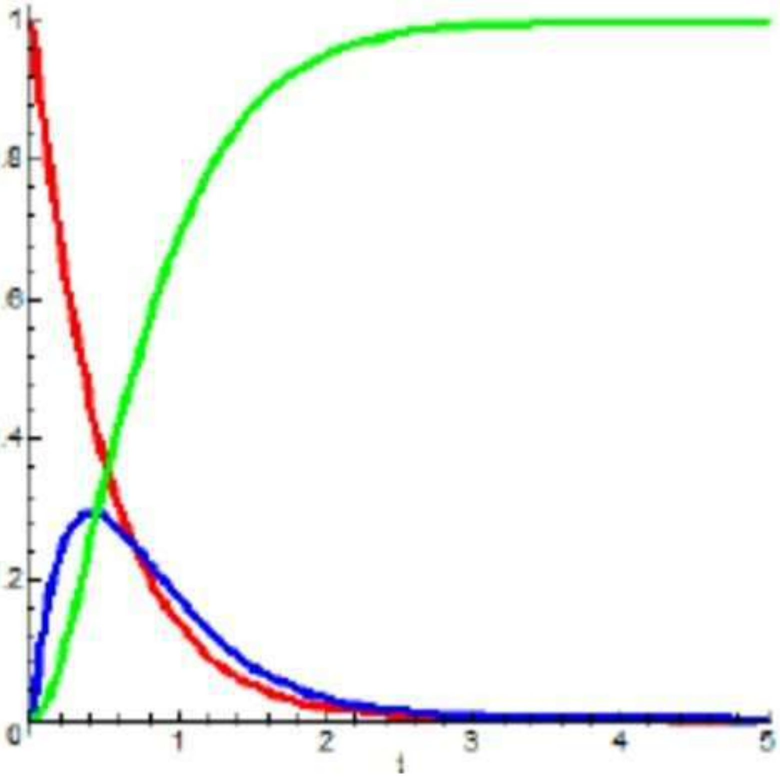
$$A \xrightarrow{K_1} B \xrightarrow{K_2} C$$

governing equation for the scheme are given as

$$\frac{dC_A}{dt} = -K_1 C_A \qquad \text{(i)}$$

$$\frac{dC_B}{dt} = K_1 C_A - K_2 C_B \qquad \text{(ii)}$$

Where $K_1$and K_2 are rate constants and the initial conditions are $C_A(0) = 1$ mol /L, $C_B(0) = 0, C_C(0) = 0$. The concentration of species C(t) at any time is given by the material balance equation, $C_C = 1 - C_A - C_B$



Using Maple, the solution of equation (i) and (ii) given by

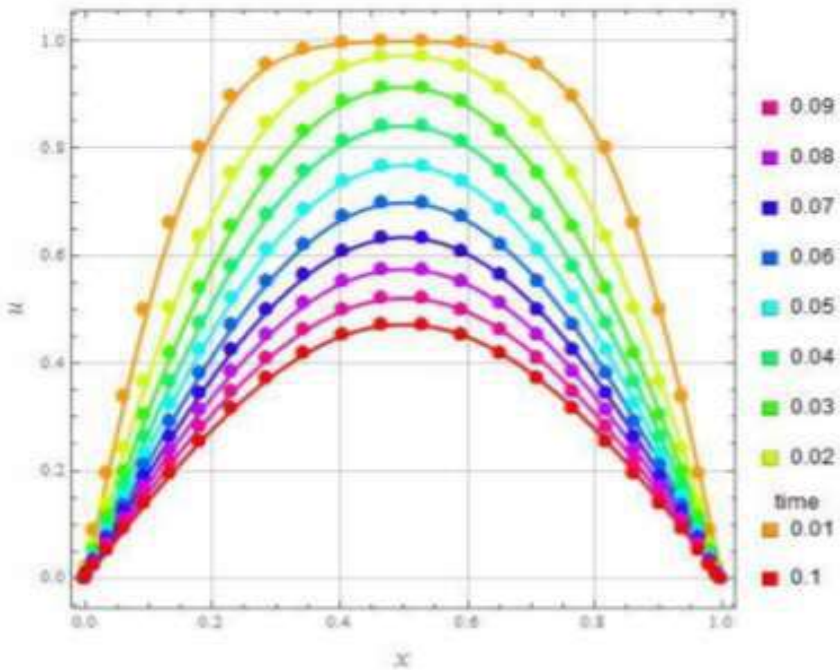
$C_A = e^{-k_1 t}$
$C_B = k\,(e^{-k_1 t} - e^{-k_2 t}) / k_2 - k_1$
As t approaches infinity, both C_A and C_B will approaches zero and C_C approaches 1

(3) Transient One Dimensional Heat Conduction:
Consider one dimensional heat equation given by

$$\frac{1}{\alpha}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{with } t \geq 0 \text{ and } 0 \leq x \leq 1$$

This equation represents heat conduction in a rod. The boundary conditions are such that the temperature u is equal to 0 at both ends of the rod.
u(x=0, t) = 0 and u(x=1, t) =0, fort ≥ 0

One can take the thermal diffusivity α equal to 1 cm^2/s the condition is given by
u(x, 0) =1 for0 ≤ x ≤ 1
The temperature can be found ND Solve (solid colored curve or Chebyshev Collocation dots) In various values of time, t ranging from 0.01 to 0.1 with a span of 0.01. One can set the number of interior pointsused by the Chebyshev Collocation dots method.



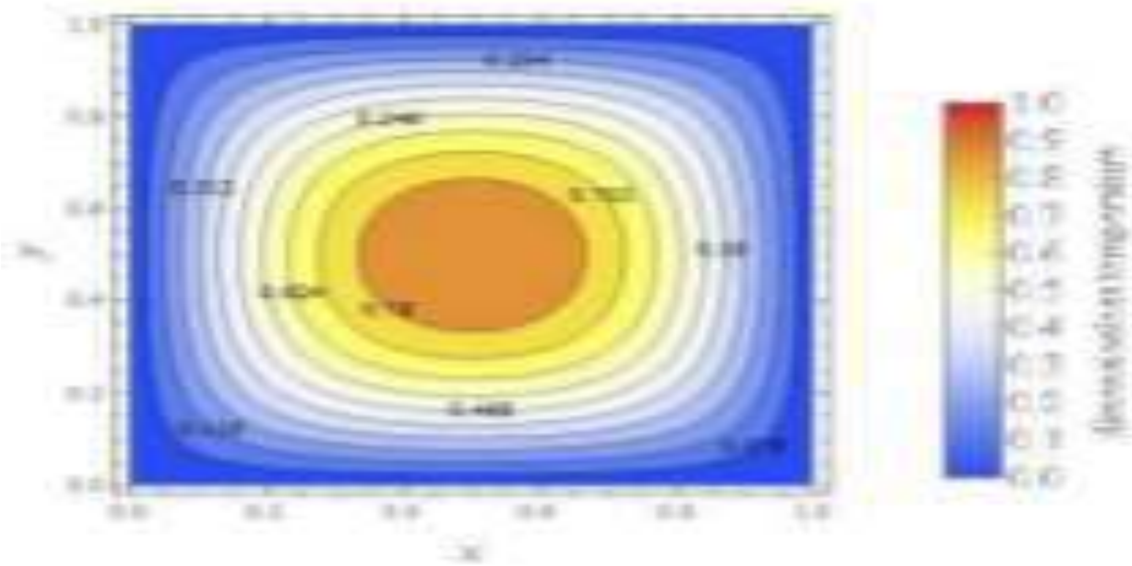
Temperature profile for t ranging from 0.01 to 0.1 with a span of 0.01

(4) Transient Two Dimensional Heat Conduction
Consider two dimensional heat conduction given by

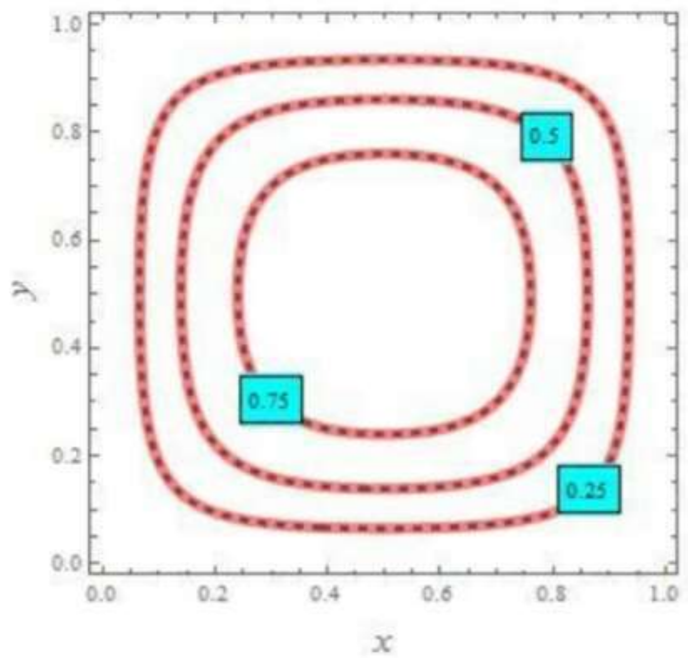
$$\frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \text{with } t \geq 0 \text{ and } 0 \leq x, y \leq 1$$

which represents heat conduction in a two dimensional domain. The boundary conditions are s.t. temperature u is equal to 0 on all the edges of the domain
u(x=0,y, t) = 0 and u(x=1,t) =0, for t ≥ 0 and
u(x=0, y, t) = 0 and u(x, y =1,t) =0, for t ≥ 0

One can take the thermal diffusivity α equal to 1 cm^2/s. The condition is given by and u(x, y, 0) =0, for $0 \le x, y \le 1$
The dimensional temperature can be found by using either ND Solve (solid colored curve or Chebyshev Collocation dots)



Contour plot of the solution obtained using Chebyshev Collocation dot



Three contours of the dimensionless temperature, (i.e. 0.25, 0.5, and 0.75) at t =0.02

Application in Mechanical Engineering:

1. Linearization System: Taylor's series expansion provides a convenient way to approximate a nonlinear equation or function with a linear equation.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots\ldots$$

Linearization System in Swinging Pendulum:
Sum of forces in tangential direction:

$$\sum F_t = W_t = mg\sin\theta = ma_t = m\frac{d^2\theta}{dt^2}L = m\ddot{\theta}$$

$$\ddot{\theta} - g\sin\theta/L = 0$$

Linearize $\sin\theta$

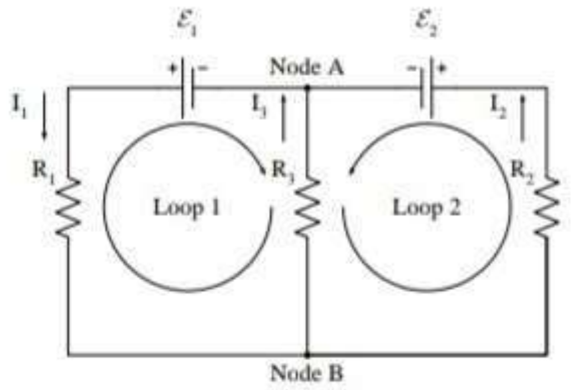
$$\sin\theta \approx \sin(0) + \cos(0)(\theta - 0) = \theta$$

Linear System in circuit analysis:
Kirchhoff's Laws: - The sum of all voltages changes around any closed loop is zero.

$\sum_{i=1}^{ne} \Delta V_i = 0$

The sum of all currents at any node is zero

$\sum_{i=1}^{nb} \Delta I_i = 0$



Application of these two laws to an electrical circuit facilitates the formulation of a system of n-linear equation when n-unknown quantities exist. Given that $R_1=2\Omega$, $R_2=4\Omega$, $R_3=1\Omega$, $\varepsilon_1=6v$, $\varepsilon_2=9v$ and using equation from loop-1, loop-2 and node A we find

$$6 - 2I_1 - I_3 = 0$$
$$9 + 4I_2 - I_3 = 0$$
$$-I_1 + I_2 + I_3 = 0$$

$$\begin{pmatrix} -2 & 0 & -1 \\ 0 & 4 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -9 \\ 0 \end{pmatrix}$$

$$AX = B$$

2. Nonlinear Systems Example: Turbine Blade Analysis

● Turbine blades are components of gas turbine engines (used for aircraft and electricity generation)
● Subject to high temperature, high inertial forces and high drag forces
● Structural and thermal analyses must be performed simultaneously (coupled non-linear equations).

Methods apply to arbitrary non-linear equations (black-box function)

$$T(x) = f_1(L)$$
$$L = f_2(T(x))$$

3. Optimization:
To find the input variable to a function such that the function is minimized (or maximized),

possibly subject to constraints.

Negative Null Form: min f(x)

subject to $g(x) \le 0$

$h(x) = 0$

4. Engineering Design:

1. Maximize performance criteria subject to failure constraints: Minimize bicycle frame weight subject to structural failure constraints by varying frame shape and thickness
2. Minimize cost subject to performance and failure constraint. Minimize vehicle cost subject to acceleration, top speed, handling and comfort and safety constraints by varying vehicle design variables.

Application in Electrical Engineering:

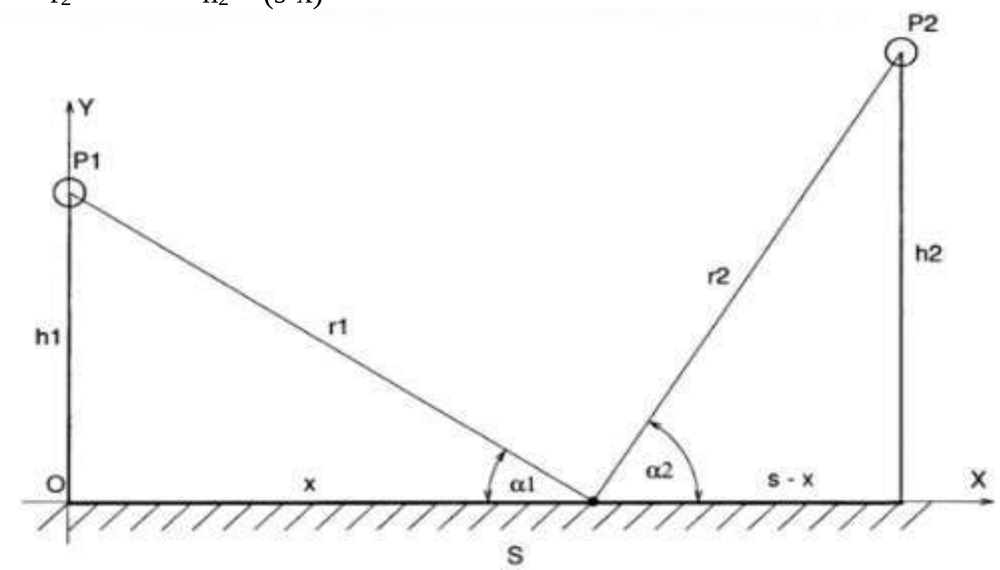
Illumination Problems: a courtyard is illuminated by two lights, where P_i is the illumination power and h_i is the height of lamp. The coordinates of the lamp are $(0,h_1)$ and (s,h_2),where s is the horizontal distance between two light sources. Let X= (x,0) be a point on the courtyard somewhere between two lights. We have to find a point X which will get minimum illumination rom two lamps.

$$r_1^2 = h_1^2 + x^2, \qquad r_2^2 = h_2^2 + (s-x)^2$$

The light intensities from the two lamps at X are given by

$$l_1(x) = \frac{P_1}{r_1^2} = \frac{P_1}{h_1^2 + x^2}$$

$$l_2(x) == \frac{P_2}{r_2^2} = \frac{P_1}{h_2^2 + (s-x)^2}$$



The illumination $ll_1(x)$ at point x from each lamp is given by

$ll_i(x) = l_i(x)$

$$ll_i(x) = \frac{P_1 h_1}{(h_1^2 + x^2)½}$$

$$ll_i(x) = \frac{P_2 h_2}{(h_1^2 + (s-x)^2)½}$$

writing $P_1 = p$, $P_2 = q$, $h_1 = a$, $h_2 = b$.
The total illumination is given
'Remove the radicals by squaring the expression

$$\frac{P^2a^2x^2}{(a^2+x^2)^5} = \frac{q^2a^2(s-x)^2}{(b^2+(s-x)^2)^5}$$

Expanding this will produce a degree-12 polynomial. For an illustration, take p=1 kW, q= 2 kW, a =4 m, b= 5m and s = 10m. Substituting the values to the above equation produces after expanded and arranging the result. $F(x) = 21x^{12} - 100x^{12} - \ldots\ldots\ldots\ldots..\ 52428000 + 2621440000 = 0$

Numerical solution produces three roots between $0 \le x \le 10$.
The value that makes C(x) minimum can be found by testing the first derivative of F(x) with x.
If F'(x)>0, C(x) will be minimum.

Application in Computer Engineering:-
MATLAB is a highly level language and interactive environment that enables you to perform computationally intensive tasks faster with traditional programming languages such as C,C^{++}. MATLAB (for matrix laboratory) is a numerical computing environment. MATLAB supports matrix generation operation.

Matrix Generation:
Entering a vector: A vector is a special case of a matrix. The elements of vectors in MATLAB are enclosed by square brackets and are separated by spaces or columns. For example to enter a row vector v type

```
>>v   = [1 4 7 10 13]
v    =
          1   4   7    10   13
```

Columns vectors are created in a similar way, however, semicolon (;) must separates the components of a column vector,

```
> w   = [1; 4;7;10; 13]
              1
             4
             7
 10
 13
```

On the other hand, a row vector is converted to a column vector using the transpose operator. Thetranspose operation is denoted by an apostrophe or a single quote (').

```
>>w =v'
        w   =
             1
              4
              7
             10
   13
```

Thus v (1) is the first element of vector, v (2) is the second element and so forth. To access blocks of elements, we use MATLAB's colon notation (:). For example, to access the first

three elements of v,
>>v (1:3)

ans =
1 4 7
All elements from the third through the last elements,
>> v (3, end)
ans =
7 10 13
Where end signifies the last element in the last element in the vector. If v is a vector, writing
>> v (:)
produces a column vector,whereas writing
>>v(1: end)
produces a row vector.

Entering a matrix:
A matrix is an array of numbers.To type a matrix into MATLAB you must

- Begin with a square bracket, [
- Separate elements in a row with spaces or commas (,)
- Use a semicolon (;) to separate rows
- End the matrix with another square bracket,].

For example, to enter a matrix A, such as

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Type,
>> A = [1 2 3; 4 5 6; 7 8 9]
MATLAB then displays the 3× 3 matrix as follows

A =
1 2 3
4 5 6
7 8 9

Matrix indexing:
The element of row i and column j of the matrix A isdenoted by A (i, j). Thus, A (i, j) in MATLAB refersto the element A_{ij} of the matrix A. The first index is the row number and the second index is the column number .For example, A(1,3) is an element of first row and third column .Here, A(1, 3) =3 .

Correcting any entry is easy through indexing .Here we substitute A(3,3) =9 by A(3,3) =0. The result is
>>A (3, 3) =0

A =

1	2	3
4	5	6
7	8	0

Single elements of a matrix are accessed as A (i, j), where $i \geq 1$ and $j \geq 1$ .Zero or negative subscripts are not supported in MATLAB.

MATHEMATICA:

It is a modern technical computing system spanning most area of technical computing including geometry, data science and visualization.Mathematica is a program for symbolic computation.

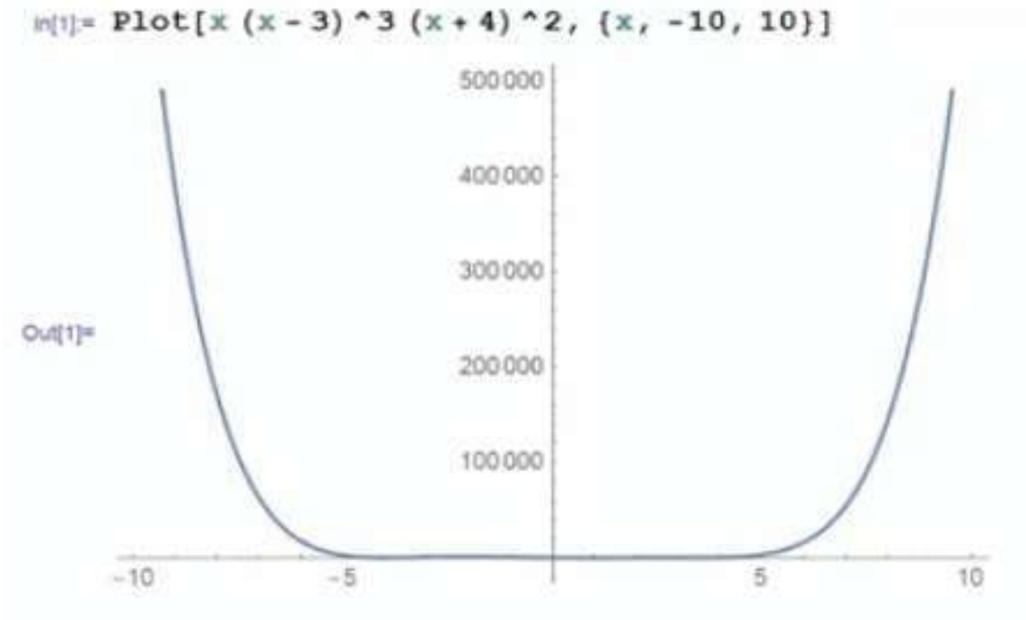
It does symbolic manipulation. Solve [$ax^2 + bx + c = 0$, x >0

{ {$x \to -b - \sqrt{b^2 - 4ac} / 2a$} , {$x \to -b + \sqrt{b^2 - 4ac} / 2a$ } }

It solves math problems Dsolve[{

$x''[t] + 3x'[t] + 2x[t] = 0$,$x[0] = 1$, $x'[0] = 0$}, x [t], t]

Plotting graph: Plot [x^ 2, {(x, -3, 3)],plot range –{[-3, 3), (-4, 9)] }



CONCLUSION:

In this paper, the applications of Numerical Analysis in engineering fields have been presented. The problems are from real life .It is expected that the problems presented in this paper can motivate engineering students to understand mathematics better. Mathematics should be enjoyable as it has helped engineering evolved.

Engineering + Math = Everything.

Engineering –Math = Nothing.

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ROLE OF MATHEMATICS IN MANAGEMENT

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ABSTRACT:
Research paper aims to describe the scope of mathematics in management; authors described the role of mathematics earlier and modern context in management. The role of Management science is also discussed. The authors also attempted to explore the importance of mathematics in various fields of management and effective decision making by means of using mathematics techniques, models and theories. Secondary data is used to get insight and discuss the role of mathematics in Management. Mathematics plays a major role in business management because it helps maximize profit by using techniques such as analyzing production costs, determining ideal pricing, discerning sales patterns and projecting future sales. Having strong skills in mathematics means an individual can analyze all of a company's finances and make changes to save the company money and time, and ultimately make a higher profit. Thus we concluded Mathematics is an important part of managing business

KEYWORDS: Mathematics, Management, Techniques and Decision Making

INTRODUCTION:
Management is the process of reaching organisational goals by working with and through people and other organisational resources. Mathematics is used in most aspect of daily life and Management. Many of the top jobs such as business consultants, computer consultants, airline pilot’s company directors require a solid understanding of basic mathematics for effective performance. It also play important role in business like business mathematics by commercial enterprises to record and manage business operations.

Until the mid 1950's, the role of mathematics (and statistics to a lesser extent) in Business and Management programs was minimal: some elementary business mathematics which included a smattering of the mathematics of finance using interest tables and some algebra, as well as some cookbook statistics. Beginning roughly at that time, with the infusion of funds from various sources, business schools entered the scientific age. Numerous mathematical techniques -- some new, some not so new -- were assembled to help solve various problems of management.

Mathematics typically used in management includes elementary arithmetic’s such as fractions Decimal and percentages, elementary algebra, statistics and probability. Business management can be made more effective by use of more advanced mathematics such as calculus and linear programming

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OBJECTIVES OF STUDY:

- To describe the role of Mathematics in Management
- To provide a overview on use of mathematics in Business and management programme and processes

Managerial Decision models-A Decision making Process:

The role of mathematics in Business decisions has very important now days in the process of managerial decision models. To turn to the specific aspects of the quantitative decision making process, it is possible to recognize three distinct phases in every decision situation. First is carefully defined the problem, second is a conceptual model to be generated and third is the selection of the appropriate quantitative model they may lead to a solution. Lastly a specific algorithm is selected. Algorithms are the orderly delineated sequences of mathematical operations that lead to a solution. The algorithms generate the decision which is subsequently implemented managerial action program. The entire process is shown :

Defined problem ——> Conceptual model —> Quantitative Model —-> Algorithms—-> Decision —- Action programs.

Some basic question that reveals the use of math in daily life are-

You need to use math to calculate compound interest rates (to see how much your savings can grow).

You also need to use math to understand the monthly percentages, which are added to your credit cards or bank loans,

or you could end up paying Rs10,000 in 5 year's time for borrowing Rs2,000 today! This is a good reason to understand mathematics

Another meaning of Business mathematics sometimes called commercial math, consumer math is group of practical subjects used in management and everyday life in schools, these subjects are often taught to students who are not planning a university education, in the United States they are typically offered in high schools and in schools that grants associate degree.

A Business Organisation Manager should have quite knowledge of elementary arithmetic, including fraction decimals and percentages. Elementary algebra is often important as well as in the context of solving practical business problems, the practical applications typically includes checking accounts , price discounts mark-up and mark down , payroll calculation , simple and compound interest , consumer and business credit and mortgages.

Role of Applied Mathematics in Management:

Management is mainly dependent on applied mathematics a branch of mathematics, such as statistics is used to reach effective decision. The various statistical techniques such as calculation of positional averages and calculated averages are of meanwhile importance in business management. Some other techniques are discussed below

Regression and Correlation are statistical techniques used to make prediction about Demand, production, investment, consumption, prices, profit and sales in the business. it also used to estimate birth rate , death rate and tax rate etc. Through correlation we can predict about the future eg. If there is heavy rainfall, than we can expect increase in sale of umbrellas thus enable us to study the relationship between two variables.

Index Numbers are very helpful in framing suitable policies by government and business to study prices and cost of living index for future planning and progress of country.

Index numbers of import and export prices are used to measure change in the trade of a country so economic and business policies are guided by indexes. Index numbers are also important to comparing living standard of people and forecasting future economic trends in the demand of commodity, volume of production etc. These are also widely used in adjustment of wages earned over a time and suggest a possible increase in wages which the worker accept, and useful in deflating the national income in inflation situations and help the government in adjusting its policies in such situation.

Wholesale price index number gives us the change in the value of money. Cost of living index number measure real income of people so that dearness allowances may be adjusted .Index number of stock prices measure economic changes in purchasing power of money over a stock.

Probability is widely used in business decision. The theory of probability has its origin in betting and games of chance but now it is used to take many managerial decisions. Probability theory is the backbone insurance companies because life tables are based of these theories. It is also widely used in time and motion studies, marketing decisions, input- output analysis, business forecasting, trade cycles, and analysis of population.

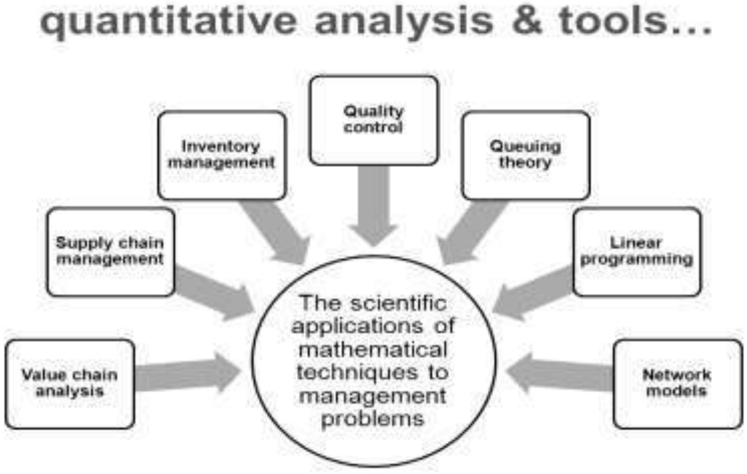
In the current time customer attitude are changing, their expectations from the business organisations are also changing. In such demanding situations when business manager feel the pressure from all side, the solution to their problem can be extracted through business research. Business research refers to systematic collection and analysis of data with purpose of finding answers to problem faced by management. Business research can be carried out to with the objective to explore, to describe, to diagnose phenomenon. Quantitative research one of the branch of business research involves the measurement of phenomenon in quantitative terms which mainly dependent on quantitative data and application of parametric and non- parametric test to test the significance of hypothesis and to solve the business problem.

Management Science:

A particular discipline called Management Science sprang up. It incorporated various techniques -- mathematical programming, linear algebra, network methods, queuing theory, stochastic processes, statistics, recursive relations, and computer simulation to solve various management problems. Drawing upon the above techniques, management science has as its philosophy the solving of a problem. As mathematics began to be established in business schools, applications of mathematics to various management problems became prevalent. Among them were the applications of quadratic programming to financial portfolio analysis and to the planning of production, inventories, and work force as well as the application of linear programming to advertising media selection. Queuing theory was used to analyze service facilities, such as restaurants and banks.

Other applications include the transportation method of linear programming, the economic lot size formula trading off inventory and order/set-up costs, CPM-PERT networks in project management, the use of learning curves, exponentially-weighted moving averages as a means of forecasting, and gravity models for site selection.

Models that are used most in practice include linear programming and computer simulation. These models have been applied in industry for approximately twenty years. Other models are also in use, but not nearly as wide-spread a use as linear programming and simulation. Business schools differ in the extent to which they employ quantitative techniques. Some business school programs painstakingly avoid any reference to concepts of calculus and awkwardly work around instances where such concepts would be useful. For example, they might illustrate a simple maximization by enumerating all possibilities or by graphing the appropriate function. My own background (undergraduate Medical, Post graduate in management, and M.Phil in Mangement, Persuing PhD. in management) and experiences make me a staunch supporter of the mathematical end of the spectrum to solve the business Problem and Business management



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I would now like to describe our BBA program at the HPU Shimla and describe what I see to be its strengths and weaknesses. In our undergraduate program, students take a two-semester sequence in Statistics for business decision and Quantitative techniques and linear algebra offered by the BBA department. They also take a two semester sequence in statistics and computing which covers BASIC programming, probability and statistics, and the use of the computer for statistical analysis. These courses are followed by a course titled Production and Operations Management, which includes a treatment of the production activities of a firm and an introduction to the philosophy and methodology of Management Science. In this course students use packaged computer programs for solving problems; they also write computer programs in the solution of certain other problems. Subsequent undergraduate courses build on the above foundation. For example, economics makes use of the calculus (both differential and integral), certain functional areas (such as marketing and finance) use some of the models used in the above courses, and several courses use the computer skills.

USE OF MATHEMATICS IN BUSINESS MANAGEMENT:

Business and mathematics go hand in hand this is because business deals with money and money encompasses everything in itself. There is a need for everyone to manage money as some point or the other to take decisions which requires everyone to know mathematics. Business mathematics is used by commercial enterprises to record and manage business operations. Commercial organizations use mathematics in accounting,, marketing, sales forecasting, and financial analysis. It helps you know the financial formulas, fractions; measurements involved in interest calculation, hire rates, salary calculation, tax calculation etc. which help complete business tasks efficiently. Business mathematics also includes statistics and provides solution to business problems.

Business is always surrounded with challenges which need to be dealt with in a proper fashion so that they do not arise in future. These problems that occur on a daily basis can be effectively solved with the help of mathematical models. Hence mathematics not only helps to calculate but also analyze business problems and work upon them. Learning and using business mathematics enables a person to think out of the box, sharpens one’s thinking and helps in precisely formulating and structuring relationships.

In order to known a business it requires skill more than the developing a product or providing a service. If a business has to survive ad succeed it needs to look after the finances and make necessary arrangements for it to prosper as well. Understanding business mathematics is important to maintain profitable operations and accurate keeping of records. It is required right from the start for pricing products/services till the end when we need to check if the budget was met. Let’s look at situations where business mathematics is required:

PRODUCTION COSTS CALCULATION:

Before one formally starts production and establishes its business it is very important to estimate the costs that would be incurred in relation to the manufacturing such as the cost of raw materials, machinery, rent, administrative expenses etc. In addition to these basic expenses there are other associated costs such as marketing, warehousing, interest and repayment of loans etc. Once all he expenses relating to production have been included I would be easy to estimate the profit from it to sustain and remain competitive in the market. Accurately determining the cost associated with each item will make the base for the business strong.

PRICE DETERMINATION:

When you have successfully determined the costs, the next task is to price the products correctly so that it generates right amount of cash flows for future requirements of the business. Charging the correct selling price would ensure that the product remains profitable.

PROFIT MEASUREMENT:

These require determining the net profit by subtracting the operating costs from the total amount of sales/revenue during a period of time. What also needs to deducted are the tax, depreciation, discount expenses. This helps to find out if the products are being charged enough to continue the business operations and expand.

FINANCIAL ANALYSIS:

You need to project the revenue and expenses of a business if we need to analyze the financial health of a business. We need to do sensitivity analysis of how an increase or decrease in sales figure or pricing could affect the business. It helps in determining how each employee contributes to the business and how I would affect. Using business mathematics helps in making these interpretations ad take the business to a higher level.

Learn marketing concepts and apply in real world. Plan and implement promotional campaigns. Become a successful marketing manager with our practical training.

Since all corporations require managing their money and business many mathematicians find employment in these fields. Mathematicians follow a logical thinking; follow a problem-solving approach to business.

CONCLUSION:

Mathematics plays a major role in business management because it helps maximize profit by using techniques such as analyzing production costs, determining ideal pricing, discerning sales patterns and projecting future sales. Having strong skills in mathematics means an individual can analyze all of a company's finances and make changes to save the company money and time, and ultimately make a higher profit.

The study of mathematics is essentially just studying number patterns, and in business, this means knowing how to manipulate numbers and make meaning out of large data sets. All companies need some sort of mathematician to look at the company's expenses, sales and cash flow. If a company has good documentation of where their money is coming from and going to, using mathematics, an individual is able to see inefficiencies in the company's operations and make important changes. Thus we conclude that Mathematics is an important part of managing business.

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EFFECT OF ASCORBIC ACID ON THE SIZE OF CUS NANOPARTICLES

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ABSTRACT:
Among other metal sulfide nanoparticles such as CdS, ZnS, PbS, HgS and SnS, CuS has attracted the attention of researchers due to its unique properties like high solar absorbance, antibacterial activity, etc. CuSnanoparticles are less toxic, environmentally benign and can be easily synthesized using simple chemical methods. In this study we have reported sonochemical synthesis and X-ray diffraction, UV-vis and Photoluminescence spectroscopic characterization of copper sulfide nanoparticles.

INTRODUCTION:
Copper sulfides (Cu_xS) have received greatword-wide attention due to their opto-electronic properties [1]. Cu_xS materials (where x >1) are p-type semiconductors and when x = 1 it exhibits metal-like behavior. Depending upon the values of x ($1 \leq x \leq 2$), the properties of Cu_xS can be tuned towards applications in different fields, such as solar cells, Li-ion rechargeable batteries, catalysts, sensors, etc. [2]. Covellitephase of copper sulfide is one of the most studied coppersulfides due to its absorption band gap in the nearinfrared region. In this study we are reporting the sonochemical synthesis of CuS nanoparticles using ethanol as reaction medium. Synthesized samples have been characterized using X-ray diffraction (XRD), UV-vis and Photoluminescence (PL) spectroscopic methods. The results obtained reveals the hexagonal structure of CuS nanoparticles.

MATERIALS & METHODS:
Copper chloride ($CuCl_2$) and sodium sulfide (Na_2S) were used as copper and sulfur precursors respectively without further purification. Calculated quantities of $CuCl_2$(0.1M) and Na_2S (0.4M) were dissolved in ethanol in to separate beakers. Copper chloride beaker was kept under sonication and sodium sulfide solution was added drop-wise to it and solution was continuously sonicated for one hour. In another reaction ascorbic acid (0.1M) was mixed with copper chloride solution before addition of sodium sulfide. The precipitates thus obtained in both reactions were filtered and dried under vacuum oven for 16 hours. In this way we have synthesized two samples of CuS labeled as CuS1 and CuS2. CuS1 was synthesized in the presence of ascorbic acid and CuS2 was synthesized in the absence of ascorbic acid under similar conditions.Bothsamples have been characterized using XRD, UV-vis and Photoluminescence spectroscopic techniques.

RESULTS AND DISCUSSION:
X-ray diffraction Study:
Both samples have been characterized withXRD method to determine the crystal structure. XRD pattern reveals the presence of (100), (101), (102), (103), (006), (110), (108) and (116) crystal planes in these nanoparticle samplesas shown in Figure 1. These crystal planes are indexed to hexagonal structure of CuS nanoparticles. The crystallite size was calculated using Debye Scherrer formula

$$D = \frac{c\,\lambda}{\beta\, cos\theta} \qquad (1)$$

Where c = 0.94is the correction factor, β is the full width at half maximum(FWHM) of the highest peak in radians, θ is the XRD peak position and λ = 1.540593 Å.

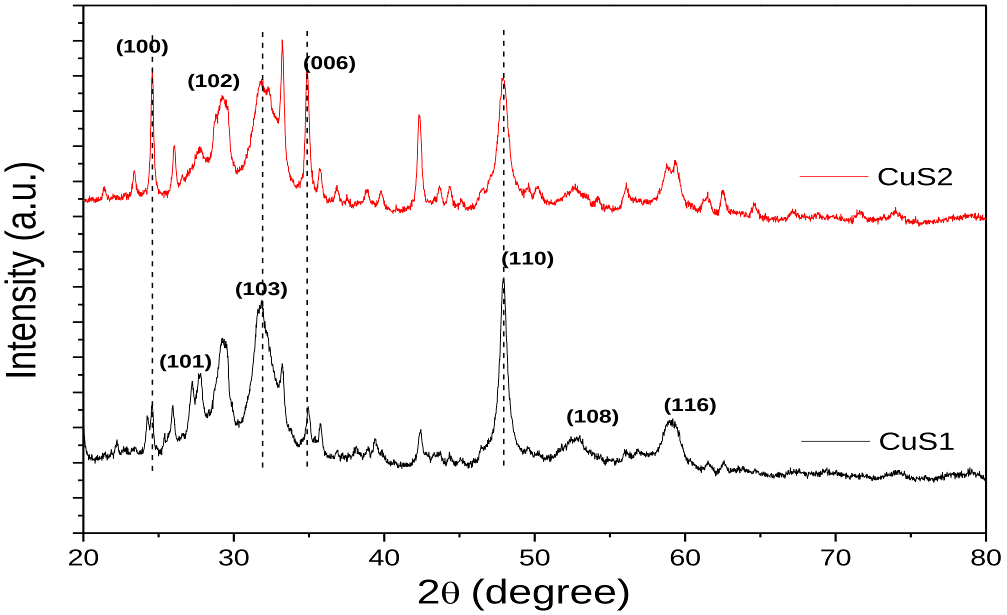

Figure 1.XRD pattern of CuS1 and CuS2 samples.

The average crystallite size for CuS1 and CuS2 are found to be ~12nm and~18nm respectively. It was observed that CuS1 has smaller crystallite size than CuS2 which indicates better nanoparticle formation in the presence of ascorbic acid.

UV-Vis and Photoluminescence spectroscopy:

The pattern of UV-vis spectrum of both samples has been shown in Figure2 (a). The bandgap of samples has been estimated using Tauc's plot and shown in inset of Figure2 (a). The bandgap value for both nanoparticle samples was found to be ~3.8eV.

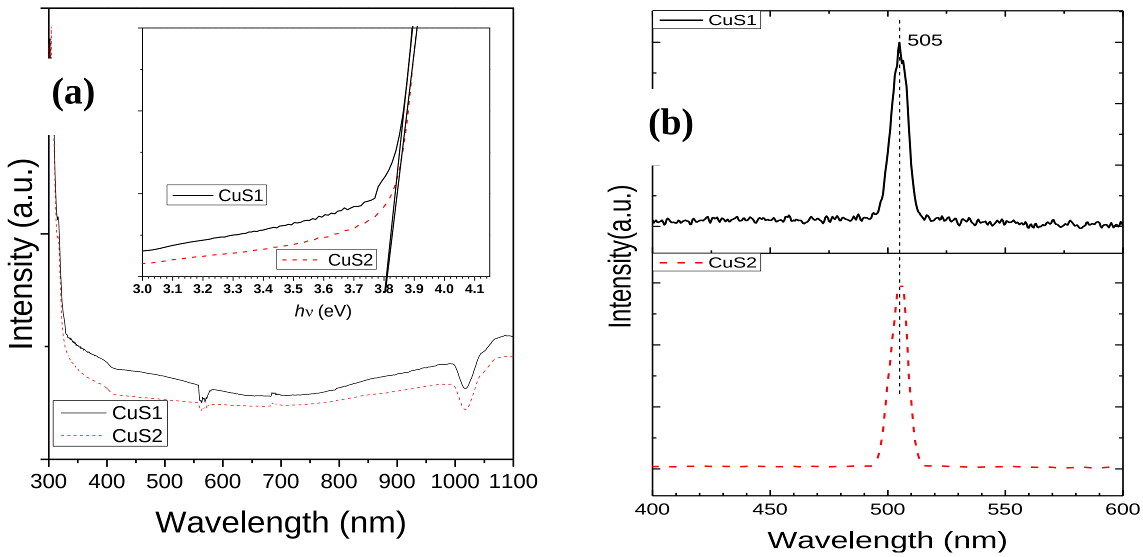

Figure 2.(a) UV-vis spectrum of CuS1 and CuS2. Inset shows bandgap estimation using Tauc's plot.(b)Photoluminescence spectrum of CuS1 and CuS2.

PL spectrum of CuS1 and CuS2 indicates peak position at 505nm as shown in Figure 2 (b).

CONCLUSIONS:

In this work we have reported the sonochemical synthesis of CuS nanoparticles with and without ascorbic acid in ethanol as solvent. XRD characterization reveals the crystalline nature and hexagonal structure of CuS nanoparticles synthesized by two different approaches. Smaller nanoparticle size was obtained for CuS prepared in the presence of ascorbic acid. Bandgap estimation for both samples gives approximately same value 3.8eV.

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ESTABLISHING THE PRINCIPLE OF EXCHANGE OF STABILITIES OF VERONIS AND STERN TYPES PROBLEMS WITH VARIABLE GRAVITY USING POSITIVE OPERATOR

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ABSTRACT:
Weinberger used a method of a Positive Operator, a generalization of a positive matrix, to establish the Principle Of Exchange of Stabilities (PES). Wherein, the resolvent of the linearized stability operator is analyzed which is in the form of a composition of certain integral operators In the present paper, Thermohaline Problem of Veronis Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer, $\frac{\Re}{\Re_s} \geq 1$ and Lewis number $\tau < 1$. Simultaneously thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.
KEY WORDS: Veronis and Stern Types Problems, variable gravity, Principle of Exchange of Stabilities, Lewis number.

1. INTRODUCTION

In classical thermal instability problems, it has been assumed that the driving density differences are produced by the spatial variation of single diffusing property i.e. heat. Recently, it has been shown that a new phenomenon occurs when the simultaneous presence of two or more components with different diffusivities is considered. This problem has been probed when we think about ocean where both heat and salt (or some other dissolved substances) are important. This problem has been termed as *thermosolutal convection (or thermohaline convection)*. In these problems the solute is commonly, but not necessarily, a salt. Related effects have now been observed in other contexts, and the name '*double–diffusive convection*', has been used to cover this wide range of phenomena. Much of the theoretical work in this field has been developed directly from the linear stability calculations for a simple fluid heated from below. Two fundamental configurations have been studied in the context of the thermohaline instability problem, one by Stern [1960], wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing, and another by Veronis [1965], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing. The main results derived by Stern and Veronis for their respective configurations are that both allow the occurrence of a steady motion or an oscillatory motion of growing amplitude, provided the destabilizing concentration gradient or the temperature gradient is sufficiently large. Stern [1960] was the first to consider the case of linear opposing gradients of two properties between horizontal boundaries at fixed concentrations, and since then many others, including Gershuni and Zhukhovitskii [1963]. The problem of thermosolutal convection in a layer of fluid heated from below and subjected to a stable solute gradient has been studied by Veronis [1965]. The minimum requirements for the occurrence of thermosolutal convection are the following:

i) The fluid must contain two or more components having different molecular diffusivities. It is the differential diffusion that produces the density differences required to derive the motion.

ii) The components must make opposing contributions to the vertical density gradient.

Further, he found that the analogous non–dimensional parameters accounting for uniform salinity gradient are given by $S = \frac{g\alpha'\beta' d^4}{\upsilon\kappa'_T}$ and Schmidt number $q = \frac{\upsilon}{\kappa'_T}$, where α', β', κ'_T denote the coefficient of analogous solvent expansion, uniform solute gradient and solute diffusivity, respectively. The main large–scale engineering applications of double–diffusive concepts are to solar ponds, shallow artificial lakes that are density stratified. Linear calculations have also been made for a variety of boundary conditions by Nield [1967] and for an unbounded fluid by Walin [1964]. A study of the onset of convection in a layer of sugar solution, with a stabilizing concentration gradient, when the layer is heated from below, has been made by Shirtcliff [1967]. He found that the first stage of the development of convection layers similar to those described by Turner and Stommel [1964]. Nield [1967] has studied the problem of thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above. When the solute gradient is stabilizing, Sani [1965] has found that finite amplitude subcritical instability (convection at a thermal Rayleigh number less than that given by the linear theory) is possible. A direct analogue of heat/salt diffusive convection has been used to explain the properties of large stars with helium–rich core, which is heated from below and thus convecting. Spiegel [1972] has shown that variations in the helium/hydrogen ratio can produce a density gradient that limits the helium transport by double–diffusive convection, though, whether this may be in layers is still unclear. Another example of double–diffusive convection is when metals solidify, since as metals solidify, undesirable inhomogeneities on the microscopic scale can be produced by several mechanisms, among which is double–diffusive convection. It was shown by Turner [1973, 1974] that the form of the resulting motions depends on whether the deriving energy comes from the component having the higher or lower diffusivity. When one layer of fluid is placed above another (denser) layer having different diffusive properties, two basic types of convective instabilities arise, in the 'diffusive' and 'finger' configurations. In both the cases, the double–diffusive fluxes can be much larger than the vertical transport in a single–component fluid because of the coupling between diffusive and convective processes.The salinity gradient is not constant with depth and this has prompted theoretical studies (Walton [1982]) of the breakdown, which is found to occur preferentially (in agreement with observations) in a thin layer where the salinity gradient is a minimum. A recent comprehensive review of thermosolutal convection in porous media has been conducted by Nield and Bejan[1999]. For a broad and latest review of the subject one may be referred to Turner and Brandt and Fernando [1996] and Gupta *et al.*[2001, 2002]. Dhiman, *et al.*[20102] have also dealt with the problem On the Stationary Convection of Thermohaline Problems of Veronis and Stern Types.

In the present paper, Thermohaline Problem of Veronis Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number $\tau < 1$.But thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.

2. MATERIALS AND METHODS:

Mathematical Formulation of the Physical Problem.

Basic Hydrodynamical Equations Governing The Physical Configuration

The basic hydrodynamic equations that govern the above physical configurations under Boussinesq approximation for the present problem are given by (c. f. Veronis [1965]);

Equation of Continuity

$$\nabla.\vec{v} = 0 \tag{1}$$

Equations of Motion

$$\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right] = -\frac{1}{\rho_0}\nabla p + \left(1 + \frac{\delta\rho}{\rho_0} + \frac{\delta\rho'}{\rho_0}\right)\vec{X} + \upsilon\nabla^2\vec{v} \tag{2}$$

Equation of Heat Conduction

$$\frac{\partial T}{\partial t} + (\vec{v}.\nabla)T = K_T\nabla^2 T. \tag{3}$$

Equation of Mass Diffusion

$$\frac{\partial S}{\partial t} + (\vec{v}.\nabla)T = K_S\nabla^2 S \tag{4}$$

Equation of State $\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(S - S_0)\right]$ (5)

In the above equations; $\vec{v}$ is the velocity vector; p is the pressure; $\vec{X} = -g(z)\hat{\lambda}$ is the external force field (gravity); T is the temperature; S is the concentration; $\upsilon = \frac{\mu}{\rho_0}$ is the coefficient of kinematic viscosity; $K_T = \frac{K}{\rho_0 c_v}$ is the thermal diffusivity; K_S is the salt diffusivity and

$$\delta\rho = -\rho_0\alpha(T - T_0) \tag{6}$$

$$\delta\rho' = \rho_0\alpha'(S - S_0) \tag{7}$$,

are the variation in density due to temperature and concentration variations.

Following the usual steps of the linearized stability theory, it is easily seen that the non dimensional linearized perturbation equations governing the physical problem described by equations (1)-(4) can be put into the following forms, upon ascribing the dependence of the perturbations of the form $exp\left[i(k_x x + k_y y) + \sigma t\right]$,

$(\boldsymbol{\sigma = \sigma_r + i\sigma_i})$ (c.f. Chandrasekhar [1961] and Siddheshwar and Krishna [2001]);

$$\left(D^2 - k^2\right)\left(D^2 - k^2 - \frac{\sigma}{Pr}\right)w = g(z)R_T k^2\theta - g(z)R_s k^2\phi \tag{8}$$

$$\left(D^2 - k^2 - \sigma\right)\theta = -R_T w \tag{9}$$

$$\left(D^2 - k^2 - \frac{\sigma}{\tau}\right)\phi = -\frac{R_s}{\tau} w \tag{10}$$

together with following dynamically free and thermally and electrically perfectly conducting boundary conditions

$$w = 0 = \theta = D^2 w \quad \text{at} \quad z = 0 \ and \ z = 1 \tag{11}$$

where, $\Re = R^2{}_T = \dfrac{g_0\alpha\beta d^4}{K_T\upsilon}$ is the thermal Rayleigh number, $Pr = \dfrac{\upsilon}{\kappa}$ is the Prandtl $\Re_s = R^2{}_s = \dfrac{g_0\alpha'\beta' d^4}{K_T\upsilon}$ is the salinity Rayleigh number, $\tau = \dfrac{K_S}{K_T}$ is the Lewis number.

3. THE METHOD OF POSITIVE OPERATOR:

We seek conditions under which solutions of equations (8)-(10) together with the boundary conditions (11) grow. The idea of the method of the solution is based on the notion of a 'positive operator', a generalization of a positive matrix, that is, one with all its entries positive. Such matrices have the property that they possess a single greatest positive eigenvalue, identical to the spectral radius. The natural generalization of a matrix operator is an integral operator with non-negative kernel. To apply the method, the resolvent of the linearized stability operator is analyzed. This resolvent is in the form of certain integral operators. When the Green's function Kernels for these operators are all nonnegative, the resulting operator is termed positive. The abstract theory is based on the Krein –Rutman theorem, which states that;

"If a linear, compact operator A, leaving invariant a cone $\hbar$, has a point of the spectrum different from zero, then it has a positive eigen value λ, not less in modulus than every other eigen value, and this number corresponds at least one eigen vector $\phi \in \hbar$ of the operator A, and at least one eigen vector $\varphi \in \hbar^*$ of the operator A^*". For the present problem the cone consists of the set of nonnegative functions.

To apply the method of positive operator, formulate the above equations (8) - (10) together with boundary conditions (11) in terms of certain operators as;

$$\tilde{M}\left(\tilde{M} + \frac{\sigma}{Pr}\right)w = g(z)R_T k^2\theta - g(z)R_s k^2\phi \tag{12}$$

$$\left(\tilde{M} + \sigma\right)\theta = -R_T w \tag{13}$$

$$\left(\tilde{M} + \frac{\sigma}{\tau}\right)\phi = -\frac{R_s}{\tau} w \tag{14}$$

The domains are contained in B, where

B $= L^2(0,1) = \left\{ \phi \mid \int_0^1 |\phi|^2 \, dz < \infty \right\}$,

with scalar product $\langle \phi, \varphi \rangle = \int_0^1 \phi(z) \, \overline{\varphi(z)} dz$, $\phi, \varphi \in$ B ; and norm $\|\phi\| = \langle \phi, \phi \rangle^{1/2}$.

We know that $\boldsymbol{L}^2(\mathbf{0}, \mathbf{1})$ is a Hilbert space, so, the domain of M is dom M = $\{\phi \in \mathrm{B} / \ \mathrm{D}\phi, \mathrm{m}\phi \in \mathrm{B}, \ \ \phi(0) = \phi(1) = 0\}$.

We can formulate the homogeneous problem corresponding to equations (8)-(10) by eliminating θ from (12) -(14) as;

$$w = k^2 \tilde{M}^{-1} \left(\tilde{M} + \frac{\sigma}{\Pr} \right)^{-1} g(z) \left(\Re(\tilde{M} + \sigma)^{-1} - \frac{\Re_S}{\tau} (\tilde{M} + \frac{\sigma}{\tau})^{-1} \right) w \tag{15}$$

$$w = K(\sigma) w \tag{16}$$

where

$$K(\sigma) = k^2 \tilde{M}^{-1} \left(\tilde{M} + \frac{\sigma}{\Pr} \right)^{-1} g(z) \left(\Re(\tilde{M} + \sigma)^{-1} - \frac{\Re_S}{\tau} (\tilde{M} + \frac{\sigma}{\tau})^{-1} \right) w \tag{17}$$

is the linearized stability operator. Further $\Re$ and $\Re_s$ respectively are the thermal and concentration Rayleigh numbers. In the present problem the linearized stability operator $K(\sigma)$ consists of three different operators, namely $\tilde{M}^{-1}, \left(\tilde{M} + \frac{\sigma}{\Pr} \right)^{-1}$ and $\left(\Re(\tilde{M} + \sigma)^{-1} - \frac{\Re_S}{\tau} (\tilde{M} + \frac{\sigma}{\tau})^{-1} \right)$, however the operator $\left(\Re(\tilde{M} + \sigma)^{-1} - \frac{\Re_S}{\tau} (\tilde{M} + \frac{\sigma}{\tau})^{-1} \right)$ which is the difference of two operators $\Re(\tilde{M} + \sigma)^{-1}$ and $\frac{\Re s}{\tau} (M + \frac{\sigma}{\tau})^{-1}$ needs special attention in regards to the positivity of the operator.

Defining, $T\left(\frac{\sigma}{\Pr} \right)$ exists for $\sigma \in T_{k\sqrt{\Pr}} = \{ \sigma \in C \mid \mathrm{Re}(\sigma) > -k^2 \Pr, \mathrm{Im}(\sigma) = 0 \}$

and $\left\| T\left(\frac{\sigma}{\Pr} \right) \right\|^{-1} > |\sigma + k^2 \Pr|$ for $\mathrm{Re}(\sigma) > -k^2 \Pr$. and define $T\left(\frac{\sigma}{\tau} \right)$ by $T\left(\frac{\sigma}{\tau} \right) f = \int_0^1 g\left(z, \xi; \frac{\sigma}{\tau} \right) f(\xi) d\xi$, where g $\left(z, \xi, \frac{\sigma}{\tau} \right)$ is Green's function kernel for the operator $\left(\tilde{M} + \frac{\sigma}{\tau} \right)^{-1}$

Let, $L(\sigma) = \left(\Re(\tilde{M}+\sigma)^{-1} - \frac{\Re_S}{\tau}(\tilde{M}+\frac{\sigma}{\tau})^{-1}\right)$. The operator $L(\sigma)$ exists for $\sigma \in L_k = \{\sigma \in C \mid \operatorname{Re}(\sigma) > \max.\{-k^2(1,\tau)\}, \operatorname{Im}(\sigma) = 0\}$ and $\|L(\sigma)\|^{-1} > |\sigma + k^2|$ for $\{\operatorname{Re}(\sigma) > \max\{-k^2(1,\tau)\}, \operatorname{Im}(\sigma) = 0\}$. $L(\sigma) = [\Re T(\sigma) - \frac{\Re_s}{\tau} T(\frac{\sigma}{\tau})]$ is an integral operator with Green's function kernel $g'(z,\xi,\sigma) = \frac{\Re \cosh[r(1-|z-\xi|)] - \Re\cosh[r(-1+z+\xi)]}{2r\sinh r} - \frac{\frac{\Re_s}{\tau}\cosh[r'(1-|z-\xi|)] - \frac{\Re_s}{\tau}\cosh[r'(-1+z+\xi)]}{2r'\sinh r'}$

for $\sigma > \max.\{-k^2(1,\tau)\}$.

$K(\sigma)$ defined in (17), which is a composition of certain integral operators, is termed as linearized stability operator. K (σ) depends analytically on σ in a certain right half of the complex plane. It is clear from the composition of K (σ) that it contain an implicit function of σ .

We shall examine the resolvent of the K (σ) defined as $[I - K(\sigma)]^{-1}$

$$[I - K(\sigma)]^{-1} = \{I - [I - K(\sigma_0)]^{-1}[K(\sigma) - K(\sigma_0)]\}^{-1}[I - K(\sigma_0)]^{-1} \tag{18}$$

If for all σ_0 greater than some a,

Remark (1) $[I - K(\sigma_0)]^{-1}$ is positive,

(2) $K(\sigma)$ has a power series about σ_0 in $(\sigma_0 - \sigma)$ with positive coefficients; i.e., $\left(-\frac{d}{d\sigma}\right)^n K(\sigma_o)$ is positive for all n, then the right side of (18) has an expansion in $(\sigma_0 - \sigma)$ with positive coefficients. Hence, we may apply the methods of Weinberger [1969] and Rabinowitz , to show that there exists a real eigenvalue σ_1 such that the spectrum of $K(\sigma)$ lies in the set $\{\sigma : \operatorname{Re}(\sigma) \le \sigma_1\}$. This is result is equivalent to PES, which was stated earlier as "the first unstable eigenvalue of the linearized system has imaginary part equal to zero."

4. RESULTS AND DISCUSSION:
THE PRINCIPLE OF EXCHANGE OF STABILITIES (PES)

It is clear that $K(\sigma)$ is a product of certain operators. Condition (1) can be easily verified by following the analysis of Herron [2001,2002] for the present operator $K(\sigma)$, i.e. $K(\sigma)$ is a linear, compact integral operator, and has a power series about σ_0 in $(\sigma_0 - \sigma)$ with positive coefficients. Thus, $K(\sigma)$ is a positive operator leaving invariant a cone (set of non negative functions). Moreover, for σ real and sufficiently large, the norms of the operators $T(0)$ and $T(\Pr\sigma)$ become arbitrarily small. So, $\|K(\sigma)\| < 1$. Hence, $[I - K(\sigma)]^{-1}$ has a convergent Neumann series, which implies that $[I-K(\sigma)]^{-1}$ is a positive operator. This is the content of condition (P1).

To verify condition (2), we note that. Green's function kernel g$\left(z,\xi,\frac{\sigma}{\Pr}\right)$ is the Laplace transform of the Green's function PrG$(z,\xi;\Pr t)$ for the initial-boundary value problem

$$\left(-\frac{\partial^2}{\partial z^2}+k^2+\frac{1}{\Pr}\frac{\partial}{\partial t}\right)G=\delta(z-\xi,t), \qquad (19)$$

where, $\delta(z-\xi,t)$ is Dirac –delta function in two-dimension,
with boundary conditions $G(0,\xi;\Pr t)=G(1,\xi;\Pr t)=G(z,\xi;0)=0$,
then $G(z,\xi;\Pr t)\geq 0$.
and Green's function kernel $g'(z,\xi;\sigma)$ is the Laplace transform of the Green's function $G'(z,\xi,t)$ defined by

$$G'(z,\xi,t)=\Re G(z,\xi;t)-\Re_s G(z,\xi;\tau t), \qquad (20)$$

If $\Re\geq\Re_s$ with $\tau<1$,
then $G'(z,\xi,t)\geq 0$
With boundary conditions $G(0,\xi;t)=G(1,\xi;t)=G(z,\xi;0)=0$, (21)
Using the similar result proved in Herron [2000,2001] by direct calculation of the inverse Laplace transform, we have $K(\sigma)$ is a positive operator for all real $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and $\Re\geq\Re_s$ with $\tau<1$ together with g(z) positive in the flow domain.

Theorem. The PES holds for (12)- (14) when g(z) is nonnegative through out the fluid domain and $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and $\Re\geq\Re_s$ with $\tau<1$
Proof: As $[I-K(\sigma)]$ is a nonnegative compact integral operator for $\sigma_0>\max.\{-k^2(1,\Pr,\tau)\}$ and for $\Re\geq\Re_s$ with $\tau<1$, which satisfied all the conditions of the Krein-Rutman theorem and hence it has a positive eigen value σ_1, which is an upper bound for the absolute values of all the eigenvalues, and the corresponding eigen function $\phi(\sigma)$ is nonnegative, which is essentially the contents of condition (2) stated in Remark 1.

We observe that
$[I-K(\sigma)][\phi(\sigma)]=(1-\sigma_1)\phi\geq 0$,
Thus, if $[I-K'''(\sigma)]$ is nonnegative, then $\sigma_1\leq 1$. The methods of Weinberger and Rabinowitz [1969] apply thereby showing that there exits a real eigenvalue $\sigma_1\leq 1$ such that the spectrum of $K(\sigma)$ lies in the set $\{\sigma\ |\mathrm{Re}(\sigma)\leq\sigma_1\}$.

This is equivalent to the PES.

5. CONCLUSIONS:

In this paper we have investigated the Thermohaline Convection Problem with variable gravity of Veronis Type.

It is established that if g(z) is positive; throughout the flow domain, then

PES is valid for Veronis' Thermohaline Convection if $\Re \geq \Re_s$ or $\lambda = \frac{\Re}{\Re_s} \geq 1$ and $\tau < 1$;
But thermohaline Problem of Stern Type heated from below with variable gravity is analyzed and it is established by the method of positive operator of Weinberger and uses the positivity properties of Green's function that principle of exchange of stabilities is valid for this general problem, when g(z) is non-negative throughout the fluid layer , $\frac{\Re}{\Re_s} \geq 1$ and Lewis number greater than one.

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HYDROMAGNETIC UNSTEADY FLOW PAST AN INFINITE VERTICAL ACCELERATED PLATE WITH INDUCED MAGNETIC FIELD IN THE PRESENCE OF HEAT SOURCE AND VARIABLE TEMPERATURE

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ABSTRACT:
A theoretical analysis of an unsteady viscous incompressible electrically conducting fluid in an infinite vertical accelerated porous plate is presented. It is assumed that the plate is electrically non-conducting and the applied magnetic field is of uniform strength $(H_0)$and perpendicular to the plate. The plate temperature is raised with time $(t' > 0)$.The coupled non-linear partial differential equation are solved by using Laplace transform technique.The solution are expressed in terms of exponential andcomplimentary error function and the effect of flow parameters on velocity, temperature and induced magnetic field are presented through graphs.

KEY WORDS: Accelerated plate, Suction, Induced magnetic field, Heat source.

INTRODUCTION:

Free and forced convection flow of an electrically conducting fluid past a porous vertical surface under the influence of magnetic field occur in many industrial and technical applications which include plasma studies, the boundary layer control in aerodynamics , petroleum industries, MHD power generator, cooling of nuclear reactors and crystal growth. Alom[1] analyzed the Steady heat and mass transfer by mixed convection flow from induced magnetic field, constant heat and mass fluxes. Ahmed[2] presented the study of heat and mass transfer on free convective three-dimensional unsteady flow over a porous vertical plate. The analytic study of induced magnetic field with radiating fluid over a porous vertical plate was presented by Ahmed[3]. Ahmed and Chung[4] presented mixed convective three-dimensional heat and mass transfer flow with transversely periodic suction velocity. Ahmed *et al.*[5] analyzed the model of MHD mixed convective radiating fluid with viscous dissipative heat. Ahmed *et al*.[6] analyzed the MHD mixed convection and mass transfer from an infinite vertical porous plate with chemical reaction in the presence of heat source. Ahmed *et al*.[7] have presented a mathematical model of megnetohydrodynamic transient free and forced convective flow with induced magnetic field effects. Ahmed *et al*.[8] studied the effects of chemical reaction and radiation on an unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer. Beg *et al.*[9]obtained local non-similarity numerical solutions for the velocity, temperature and induced magnetic field distribution in forced convection hydromagnetic boundary layer, over an extensive range of magnetic Prandtl numbers and Hartmann numbers. Ghosh *et al.*[10] considered an exact solution for the hydromagnetic natural convection boundary layer flow past an infinite vertical flat plate under the influence of transverse magnetic field with magnetic induction effects.Hussain *et al.*[11] studied the effect of radiation on free convection from a porous vertical plate. Prakash *et al.*[12-13] investigated the Dufour effects on unsteady hydro magnetic radiative and diffusion-thermo on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with variable temperature and mass diffusion.Raptis *et al.*[15-18] have studied MHD flow past a steadily moving infinite vertical porous plate with constant heat flux in the presence of radiation. MHD Couette flow and heat transfer in a rotating system have studied by Seth *et al.*[19] Seshaiah *et al.*[20] investigated the effects of induced magnetic field on free convective flow of radiative, dissipative fluid past a porous plate with temperature gradient heat source. Singha[21] analyzed the problem of MHD free convective flow of an electrically conducting fluid between the two heated parallel plates in the presence of an induced magnetic field. Singh and Singh[22] presented MHD effects on heat and mass transfer in flow of a viscous with induced magnetic field. MHD effects on flow of viscous fluid with induced magnetic field studied by Singh and Singh[23].

The objective of present study is to investigate the effect of induced magnetic field, heat source and variable temperature inhydromagnetic flow past a porous vertical accelerated plate.

FORMULATION OF PROBLEM:
Consider an unsteady, free convective flow of an incompressible, electrically conducting viscous fluid past an accelerated infinite vertical plate with variable temperature. We introduce a co-ordinate system with origin at the accelerated plate which is subjected to constant suction velocityV_0, the x'axis is taken along the plate in the upward vertical direction, y' axis is taken along normal to the plate directed in to the fluid region.

Let$(u', v', 0)$ be the fluid velocity and$\left(h'_x, h'_y, 0\right)$ be component of magnetic induction vector at a point(x', y', z') in the fluid. The $x'$is taken along the plate in the upward direction,y' is normal to the plate in to the fluid region. Since the plate is infinite in length inx' direction, therefore all the physical quantities except the pressure are assumed to be independent ofx'. Let$(u', v', 0)$be the fluid velocity at the$(x', y', z')$when time$(t' > 0)$. Initially$(i.e when t' \leq 0)$the plate is at rest relative to the fluid velocity $(u' = 0)$and thefluid at the plate's surface has the same temperature as at theedge of the boundary layer i.e. T'_∞ respectively. At time $(t' > 0)$the plate is accelerated with a velocity $u' = at'$ in its own planeand the temperature at the plate is raised linearly with respect to time

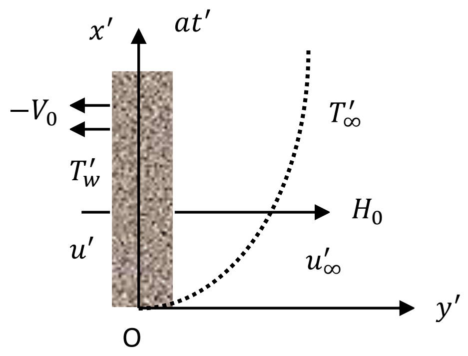

Fig.1-Geometrical configuration of the problem.

Under Boussinesq approximation the equations governing the flow are, Conservation of mass

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Gauss Law of magnetism

$$\frac{\partial h_y'}{\partial y'} = 0 \text{ which holds for } h_y' = H_0 = \text{constant} \tag{2}$$

Conservation of momentum

$$\rho\frac{\partial u'}{\partial t'} + \rho v'\frac{\partial u'}{\partial y'} = -\frac{\partial p}{\partial x'} - \rho g + \mu\frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0\frac{\partial h'}{\partial y'} \tag{3}$$

Conservation of energy

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{S'}{\rho c_p}(T' - T_\infty') \tag{4}$$

Conservation of magnetic induction

$$\frac{\partial h_x'}{\partial t'} + v'\frac{\partial h_x'}{\partial y'} = \frac{1}{\sigma\mu_e}\frac{\partial^2 h_x'}{\partial y'^2} + H_0\frac{\partial u'}{\partial y'} \tag{5}$$

Since there is no large velocity gradient here, the viscous term in equation (3) vanishes for small $\mu$and hence for the outer flow, beside there is no induced magnetic field along x'-direction gradient, so we have

$$-\frac{\partial p}{\partial x'} = \rho_\infty g \text{...(6)}$$

By eliminating the pressure term from equations (3) and (6), we obtain

$$\rho\frac{\partial u'}{\partial t'} + \rho v'\frac{\partial u'}{\partial y'} = (\rho_\infty - \rho)g + \mu\frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0\frac{\partial h'}{\partial y'}. \tag{7}$$

By using Boussinesq approximation we have,

$$\rho_\infty - \rho = \rho_\infty\beta(T' - T_\infty') \tag{8}$$

On using (8) in the equation (7) and noting that is approximately equal to 1, the momentum equation reduces to

$$\rho\frac{\partial u'}{\partial t'} + \rho v'\frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty') + \mu\frac{\partial^2 u'}{\partial y'^2} + \mu_e H_0\frac{\partial h'}{\partial y'} \tag{9}$$

The flow is governed by the following initial and boundary conditions:

$$t' \le 0 \quad u' = 0, T' \to T_\infty', = 0, h_x' = 0 \quad \text{for all } y\text{...(10)}$$

$$t' > 0\begin{Bmatrix} u' = at', T' = T_\infty' + (T_w' - T_\infty')At', h_x' = 0, at y' = 0 \\ u' \to 0, T' \to T_\infty', \to 0,\ h_x' \to 0 \quad at y' \to \infty \end{Bmatrix} \tag{11}$$

whereu' and v'- denotes the velocity component in the boundary layer in direction x'-axis and y'-axis respectively; T'- the temperature inside the boundary layer;T_w'- temperature at the plate;T_∞'- the temperature of the free stream; β-thermal expansion; ρ- density of fluid, $A = \left(\frac{a^2}{v}\right)^{\frac{1}{3}}$;$a$- acceleration of the plate;t'- time;h_x'- induced magnetic field along x'direction;H_0-external applied magnetic field; g- acceleration due to gravity; μ_e-magnetic permeability; μ-kinematic viscosity,α- magnetic diffusivity.

From equation of continuity (1), it is clear that suction velocity normal to the plate is constant. Hence from the equation of the continuity we obtained:

$$v' = -V_0 \tag{12}$$

Governing equations in non-dimensional form are:

$$\frac{\partial u}{\partial t} - w_0'\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + M\frac{\partial h}{\partial y} + G_r\theta. \tag{13}$$

$$P_r\frac{\partial\theta}{\partial t} - w_0 P_r'\frac{\partial u}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + P_r Q_0\theta\text{...} \tag{14}$$

$$\frac{\partial h}{\partial t} - w_0{}' \frac{\partial h}{\partial y} = \frac{1}{P_{rm}} \frac{\partial^2 h}{\partial y^2} + M \frac{\partial u}{\partial y} \ldots \tag{15}$$

where the non-dimensional quantities used above are:

$$\left.\begin{array}{l} u = \frac{u'}{(\nu a)^{\frac{1}{3}}}, t = t'\left(\frac{a^2}{\nu}\right)^{\frac{1}{3}}, y = y'\left(\frac{a}{\nu^2}\right)^{\frac{1}{3}}, h = \frac{h_x'}{(\nu a)^{\frac{1}{3}}}\left(\frac{\mu_e}{\rho}\right)^{\frac{1}{2}} \\ w_0 = \frac{V_0}{(\nu a)^{\frac{1}{3}}}, M = \frac{\mu_e}{\rho}\frac{H_0}{(\nu a)^{\frac{1}{3}}}, G_r = \frac{g\beta(T' - T_\infty')}{a}, P_r = \frac{\kappa}{\mu c_p} \\ P_{rm} = \sigma\mu_e\nu, Q_0 = \frac{s'}{\rho c_p (a^2/\nu)^{\frac{1}{3}}}, \alpha = (\sigma\mu_e)^{-1} \end{array}\right\} \ldots \tag{16}$$

The initial and boundary condition in dimensionless form are as follows:

$$t \le 0, u = 0, \theta = 0, h = 0 \quad \ldots(17)$$

$$t > 0 \left\{\begin{array}{l} u = t, \theta = t, h = 0 \; at y = 0 \\ u = 0, \theta = 0. h = 0 \;\; at y \to \infty \end{array}\right\} \ldots(18)$$

Method of solution

The dimensionless governing equations (13) to (15) subjected to boundary conditions (17) and (18) are solved by using Laplace transform technique and transform to following set of equations:

$$\frac{d^2\bar{u}}{dy^2} + w_0 \frac{\partial \bar{u}}{\partial y} - S\bar{u} + M\frac{d\bar{h}}{dy} = -G_r\bar{\theta} \ldots \tag{19}$$

$$\frac{d^2\bar{\theta}}{dy^2} + w_0 P_r \frac{d\bar{\theta}}{dy} + P_r(Q_0 - S)\bar{\theta} = 0 \tag{20}$$

$$\frac{1}{P_{rm}}\frac{d^2\bar{h}}{dy^2} + w_0 \frac{\partial \bar{h}}{\partial y} - S\bar{h} + M\frac{d\bar{u}}{dy} = 0 \tag{21}$$

Corresponding boundary conditions:

$$\left.\begin{array}{l} \bar{u} = \frac{1}{S^2}, \bar{\theta} = \frac{1}{S^2}, \bar{h} = 0 \quad at y = 0 \\ \bar{u} = 0, \bar{\theta} = 0, \bar{h} = 0 \quad at y \to \infty \end{array}\right\} \tag{22}$$

Following Pande *et. al.*[14] we assume magnetic Prandtl number$P_{rm} = 1$, i.e. $\nu = \alpha$ which is a plausible assumption in most of the hydromagnetic problems, the solution of the equations (19) to (21) under the boundary condition $(22)$are given by following expressions.

$$\bar{\theta} = \frac{e^{-\left(\frac{w_0 P_r + \sqrt{w_0{}^2 P_r{}^2 - 4P_r(Q_0 - S)}}{2}\right)y}}{S^2} \ldots(23)$$

$$\bar{u} = \frac{1}{2}\left[\begin{array}{c} \frac{e^{-A_2 y}}{S^2} + \frac{G_r}{(P_r - 1)}\left\{\frac{M_1 e^{-A_2 y}}{S} + \frac{M_2 e^{-A_2 y}}{S^2} + \frac{M_3 e^{-A_2 y}}{S - \alpha_1} + \frac{M_4 e^{-A_2 y}}{S - \beta_1}\right\} \\ + \frac{e^{-A_3 y}}{S^2} + \frac{G_r}{(P_r - 1)}\left\{\frac{M_5 e^{-A_3 y}}{S} + \frac{M_6 e^{-A_3 y}}{S^2} + \frac{M_7 e^{-A_3 y}}{S - \alpha_2} + \frac{M_8 e^{-A_3 y}}{S - \beta_2}\right\} \\ - \frac{G_r}{(P_r - 1)}\left\{\begin{array}{c} (M_1 + M_5)\frac{e^{-A_1 y}}{S} + (M_2 + M_6)\frac{e^{-A_1 y}}{S^2} \\ + \frac{M_3 e^{-A_1 y}}{S - \alpha_1} + \frac{M_4 e^{-A_1 y}}{S - \beta_1} + \frac{M_7 e^{-A_1 y}}{S - \alpha_2} + \frac{M_8 e^{-A_1 y}}{S - \beta_2} \end{array}\right\} \end{array}\right] \ldots \tag{24}$$

$$\bar{h} = \frac{1}{2}\left[\begin{array}{c} \frac{-e^{-A_2y}}{S^2} - \frac{G_r}{(P_r-1)}\left\{\frac{M_1e^{-A_2y}}{S} + \frac{M_2e^{-A_2y}}{S^2} + \frac{M_3e^{-A_2y}}{S-\alpha_1} + \frac{M_4e^{-A_2y}}{S-\beta_1}\right\} \\ + \frac{e^{-A_3y}}{S^2} + \frac{G_r}{(P_r-1)}\left\{\frac{M_5e^{-A_3y}}{S} + \frac{M_6e^{-A_3y}}{S^2} + \frac{M_7e^{-A_3y}}{S-\alpha_2} + \frac{M_8e^{-A_3y}}{S-\beta_2}\right\} \\ + \frac{G_r}{(P_r-1)}\left\{\begin{array}{c} (M_1+M_5)\frac{e^{-A_1y}}{S} + (M_2+M_6)\frac{e^{-A_1y}}{S^2} \\ + \frac{M_3e^{-A_1y}}{S-\alpha_1} + \frac{M_4e^{-A_1y}}{S-\beta_1} - \frac{M_7e^{-A_1y}}{S-\alpha_2} - \frac{M_8e^{-A_1y}}{S-\beta_2} \end{array}\right\} \end{array}\right] \ldots \quad (25)$$

Taking inverse Laplace transforms of the equations(23)to(25),we obtained the following expression for the velocity, temperature and induced magnetic field:

$$u = \left[\begin{array}{c} -Z_4(\eta_1e^{X_1}erfc\eta_1 - \eta_2e^{X_2}erfc\eta_2) + Z_5(e^{X_1}erfc\eta_1 + e^{X_2}erfc\eta_2) \\ +Z_6(e^{X_3}erfc\eta_3 + e^{X_4}erfc\eta_4) + Z_7(e^{X_5}erfc\eta_5 + e^{X_6}erfc\eta_6) \\ -Z_8(\eta_7e^{X_7}erfc\eta_7 - \eta_8e^{X_8}erfc\eta_8) + Z_9(e^{X_7}erfc\eta_7 + e^{X_8}erfc\eta_8) \\ +Z_{10}(e^{X_9}erfc\eta_9 + e^{X_{10}}erfc\eta_{10}) + Z_{11}(e^{X_{11}}erfc\eta_{11} + e^{X_{12}}erfc\eta_{12}) + \\ Z_{12}(\eta_{13}e^{X_{13}}erfc\eta_{13} - \eta_{14}e^{X_{14}}erfc\eta_{14}) - Z_{13}(e^{X_{13}}erfc\eta_{13} + e^{X_{14}}erfc\eta_{14}) \\ -Z_6(e^{X_{15}}erfc\eta_{15} + e^{X_{16}}erfc\eta_{16}) - Z_7(e^{X_{17}}erfc\eta_{17} + e^{X_{18}}erfc\eta_{18}) \\ -Z_{10}(e^{X_{19}}erfc\eta_{19} + e^{X_{20}}erfc\eta_{20}) - Z_{11}(e^{X_{21}}erfc\eta_{21} + e^{X_{22}}erfc\eta_{22}) \end{array}\right] \ldots .(26)$$

$$\theta = 2Z_3[-\eta_{13}e^{X_{13}}erfc\eta_{13} - \eta_{14}e^{X_{14}}erfc\eta_{14}] \quad (27)$$

$$h = \left[\begin{array}{c} Z_4(\eta_1e^{X_1}erfc\eta_1 - \eta_2e^{X_2}erfc\eta_2) - Z_5(e^{X_1}erfc\eta_1 + e^{X_2}erfc\eta_2) \\ -Z_6(e^{X_3}erfc\eta_3 + e^{X_4}erfc\eta_4) - Z_7(e^{X_5}erfc\eta_5 + e^{X_6}erfc\eta_6) \\ -Z_8(\eta_7e^{X_7}erfc\eta_7 - \eta_8e^{X_8}erfc\eta_8) + Z_9(e^{X_7}erfc\eta_7 + e^{X_8}erfc\eta_8) \\ +Z_{10}(e^{X_9}erfc\eta_9 + e^{X_{10}}erfc\eta_{10}) + Z_{11}(e^{X_{11}}erfc\eta_{11} + e^{X_{12}}erfc\eta_{12}) - \\ Z_{14}(\eta_{13}e^{X_{13}}erfc\eta_{13} - \eta_{14}e^{X_{14}}erfc\eta_{14}) + Z_{15}(e^{X_{13}}erfc\eta_{13} + e^{X_{14}}erfc\eta_{14}) \\ +Z_6(e^{X_{15}}erfc\eta_{15} + e^{X_{16}}erfc\eta_{16}) + Z_7(e^{X_{17}}erfc\eta_{17} + e^{X_{18}}erfc\eta_{18}) \\ -Z_{10}(e^{X_{19}}erfc\eta_{19} + e^{X_{20}}erfc\eta_{20}) - Z_{11}(e^{X_{21}}erfc\eta_{21} + e^{X_{22}}erfc\eta_{22}) \end{array}\right] \quad (28)$$

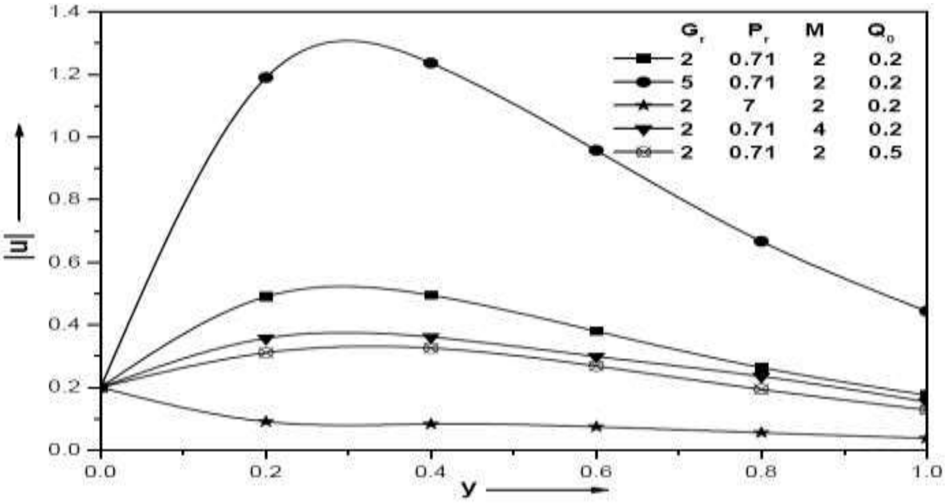

Fig.2- Variation of amplitude of velocity with $t = 0.2$..

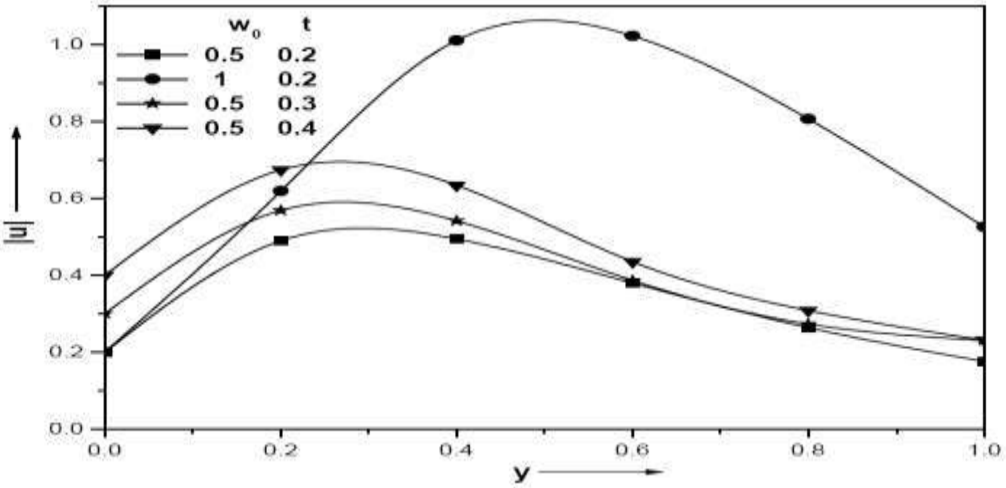

Fig.3- Variation of amplitude of velocity at $G_r = 2, P_r = 0.71, M = 2, Q_0 = 0.2$.

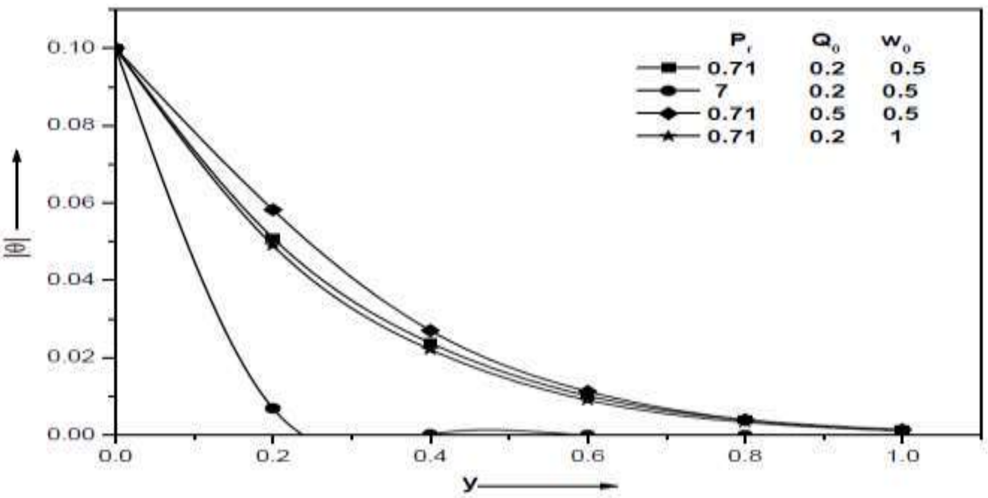

Fig.4- Variation of amplitude of temperature at $t = 0.2$.

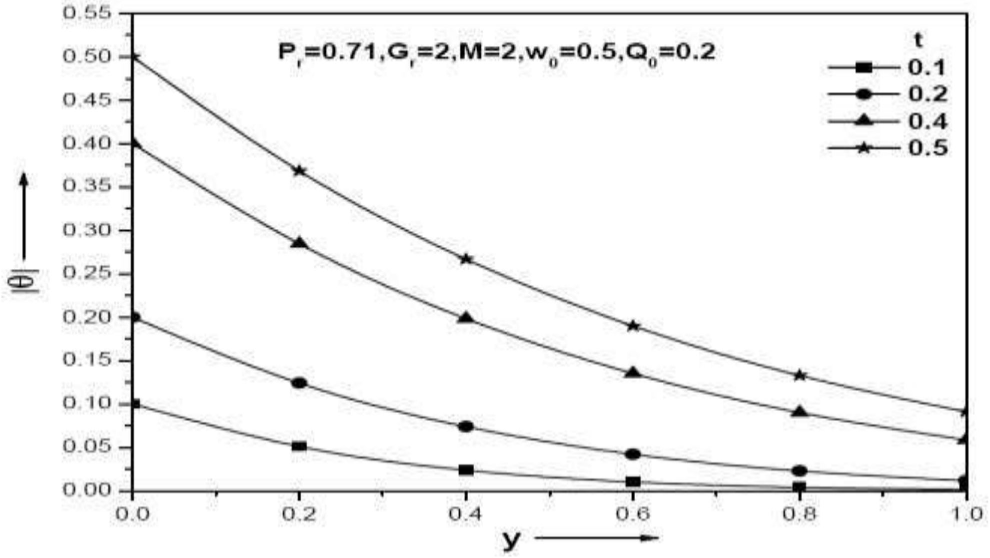

Fig.5- Variation of amplitude of temperature.

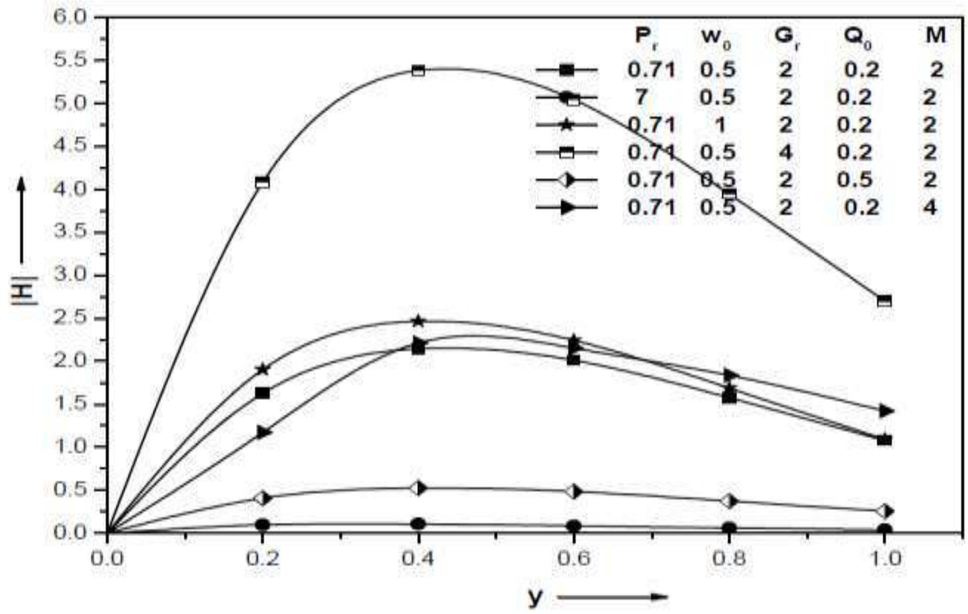

Fig.6- Variation of amplitude of induced magnetic field at $t = 0.2$.

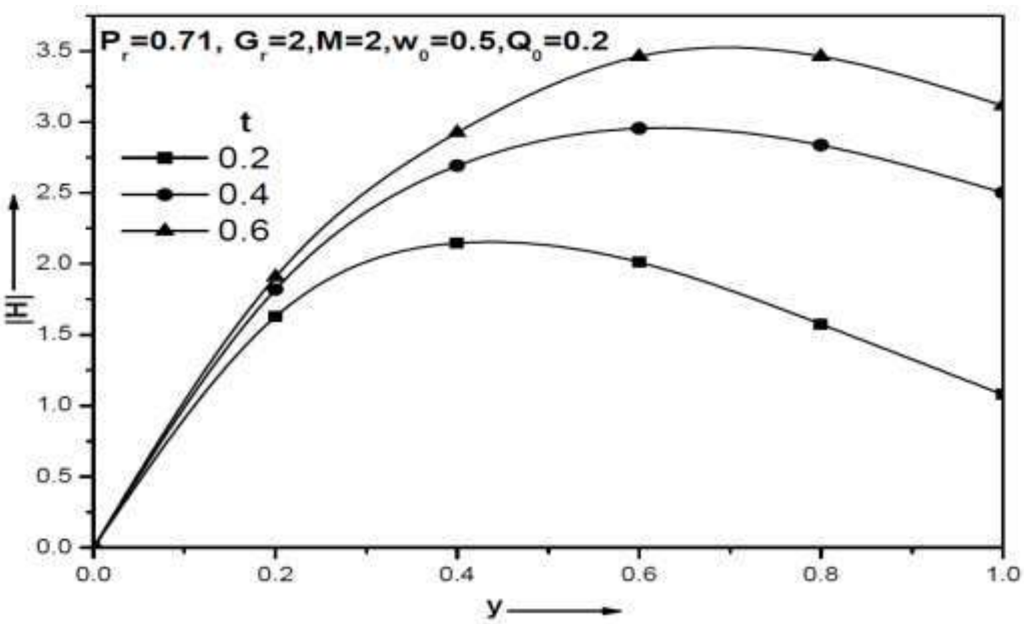

Fig.7- Variation of amplitude of induced magnetic field.

RESULTS AND DISCUSSION:

In order to have a physical view of the problem, we computed the numerical calculations for non-dimensional amplitude of velocity profile, amplitude of temperature profile and amplitude of induced magnetic field at the plate for different values of physical parameters involved and these value have been demonstrated in graphs. Our investigations are restricted to Prandtl number 0.71 which corresponds to air. The values of other parameters namely Grashoff number, Hartmann number, suction parameter, heat source parameter and timeare chosen arbitrarily. Our results are in good agreement with the result of Ahmed S[3] in the absence of mass transfer and viscous dissipation.

From fig.2 observed that with the increase of Grashoff number (G_r)amplitude of velocity increases and with increase of Prandtl number(P_r), Hartmann number and heat source parameter amplitude of velocity decreases. Fig.3. Clearly show that increase in the value of suction parameter and time cause rise in amplitude of velocity.From fig.4&5 it is noticed that fluid temperature rises due to the increasing value of heat source parameter, time and it decreases with the increase of Prandtl number $(P_r)$and suction parameter(w_0). From fig. 6 & 7 it is observed that amplitude of induced magnetic field rises with the increase in suction parameter, Grashoff number, time and it fall with the rise in Prandtl number, heat source parameter(Q).

CONCLUSIONS:

The main conclusion of this study is:

(i) An increase in Hartmann number andheat source parameter leads to decelerate the flow velocity.

(ii) It is observedthat with the increase in suction parameter the induced magnetic field and the velocity of the fluid increase.

(iii) Fluid temperature increases with the increase in heat source parameter and decrease with Prandtl number.

(iv) Fluid temperature and its velocity increases with the time.

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REMOTE SENSING AND THEIR HISTORICAL DEVELOPMENT

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ABSTRACT:
The science of remote sensing has emerged as one of the most fascinating subjects over the past three decades. Earth observation from space through various remote sensing instruments has provided a vantage means of monitoring land surface dynamics, natural resources management, and the overall state of the environment itself.
Remote sensing is defined, for our purposes, as the measurement of object properties on the earth's surface using data acquired from aircraft and satellites. Remote sensing systems, particularly those deployed on satellites, provide a repetitive and consistent view of the earth that is invaluable to monitoring the earth system and the effect of human activities on the earth.

KEYWORDS: Fascinating, Monitoring, Deployed, Vantage.

INTRODUCTION:
A huge quantity of Earth observation and geospatial data is produced daily by numerous satellites launched by several worldwide space agencies. The processing of remote sensing data requires several steps. The extraction of prominent information from remote sensing data requires a coordinated use of many applications and algorithms. Sharing computational resources among different scientists represents the sole way to approach the problem in order to achieve good performance.

In this paper I describe advantages and disadvantages of remote sensing and their problems andhistorical development. The science of remote sensing has emerged as one of the most fascinating subjects over the past three decades. Earth observation from space through various remote sensing instruments has provided a vantage means of monitoring land surface dynamics, natural resources management, and the overall state of the environment itself.

WHAT IS REMOTE SENSING:
Remote sensing is the science of obtaining information about objects or areas from a distance, typically from aircraft or satellites. Remote sensors can be either passive or active. Passive sensors respond to external stimuli. They record natural energy that is reflected or emitted from the Earth's surface.

Remote means something which is far away and sensing means getting information or getting data.It is the process of detecting and monitoring the physical characteristics of an area by measuring its reflected and emitted radiation at a distance from the targeted area. Special cameras collect remotely sensed imagesof the Earth, which help researchers "sense" things about the Earth. It is obtaining information about an area or phenomenon through a device that does not touch the area or phenomenon under study. Your eyes are a good example of remote sensing instrument. Many devices are onboard satellites that monitor the Earth from space.

Some Examples Are

- Cameras on satellites and airplanes take images of large areas on the Earth's surface, allowing us to see much more than we can stand on the ground.
- Sonar systems on ships can be used to create images of the ocean floor without needing to travel to the bottom of the ocean.
- Cameras on satellites can be used to make images of temperature changes in the oceans

Some specific uses of remote sensing

- Large forest fires can be mapped from space, allowing rangers to see a much larger area than from the ground.
- Tracking clouds to help predict the weather or watch erupting volcanos, and help watch for dust storms.
- Tracking the growth of a city and changes in farmland or forests over several years or even decades.
- Mapping the ocean bottom - Discovery and mapping of the rugged topography of the ocean floor (e.g., huge mountain ranges, deep canyons, and the “magnetic striping” on the ocean floor)

How it work: It has seven steps to work the remote sensing.

Energy source: The first requirement for remote sensing is to have an energy source which illuminate or provide electromagnetic energy to the target of interest.

Radiation and atmosphere: As the energy travels from its source to the target, it will come in contact with or interact with the atmosphere it passes through; this interaction may take place a second as the energy travels from the target to the sensor.

Interaction with target: once the energy makes it way to the target through the atmosphere, it interacts with the target depending upon the properties of both the target and radiation.

Recording of energy by the sensor: after the energy has been scattered by, emitted from the target, we require a sensor to collect and record the electromagnetic radiation.

Transmission, reception and processing: the energy required by the sensors has to be transmitted, often in electronic form, to receive and processing section where the data are produced into an image.

Interpretation and analysis: The proceed image is interpreted in two types visually and digitally. To exact information about the target which was illuminated.
Applications: The final element of remote sensing process is achieved when we apply the information- we have been able to extract from the imagery about the target in order to better understand and reveal some new information.

SENSORS: Remote sensing instruments are of two primary types - active sensors and passive sensors:

Active sensors: They provide their own source of energy to illuminate the objects they observe. LIDAR, RADAR etc.
Passive sensors, they detect natural energy (radiation) that is emitted or reflected by the object or scene being observed. ASTER, Quickbard, Ikonos, Landsat etc.

ADVANTAGES AND DISADVANTAGES:
ADVANTAGES:

- Large area coverage.
- Some remote sensors operate in all seasons, at night and in bad weather.
- Remote sensing allows repetitive coverage which comes in handy when collecting data on dynamic themes such as water, agricultural fields and so on.
- Remote sensors "see" over a broader portion of the spectrum than the human eye.
- Remotely sensed data can easily be processed and analyzed fast using a computer and the data utilized for various purposes.

DISADVANTAGES:

- Remote sensing is a fairly expensive method of analysis especially when measuring or analyzing smaller areas.
- Remote sensing requires a special kind of training to analyze the images. It is therefore expensive in thetechnology since extra training must be accorded to the users of the technology.
- It is expensive to analyze repetitive photographs if there is need to analyze different aspects of the photography features.
- It is humans who select what sensor needs to be used to collect the data, specify the resolution of the data and calibration of the sensor, select the platform that will carry the sensor and determine when the data will be collected. Because of this, it is easier to introduce human error in this kind of analysis.

QUESTION: WHY USE SATELLITES TO STUDY THE EARTH?

ANSWER: Consistent, routine, global measurements

- Overview of information on the hemispheric, regional, national and local scales – the "big picture"
- Provide information in areas where there are no grounds – based measurements
- Advance warning of impending environmental events and disasters.
- Visual appeal: a picture is worth thousand words.

HISTORICAL DEVELOPMENT:
The history of remote sensing began with the invention of photograph. The term photograph is derived from two words 'phos' means light and 'graphy' means writing.

- In year 1038 AD AL Hazen an Arabian Mathematician explain the principle of camera obscura to observe solar eclipse.
- After 1666 Newton experimenting with a prism and found that when a white light passes through a prism then it splits into seven different colours i.e. (VIBGYOR)
- In 1800 Sir William Herschel measures the temperature of light splits with a prism into spectrum of visible colours. He had discovered thermal infrared electromagnetic radiation.

- In 1858 GFT Nadar takes the first aerial photograph from a captative balloon from an altitude of 1200 feet in Paris.
- In 1889 Arthur Batut takes the first photo by using a kite in France.
- In 1903 BP Crop uses pigeon to transmit message and take arial photograph.
- In 1957 SPUTNIK 1 launched by Russia (world first artificial satellite launched 4 October 1957).
- In 1958 EXPLORER 1 launched by USA.
- In 1970 DONG FANG HONG 1 launched by China.
- In 1975 ARYABHATTA, in 1979 BHASKAR 1, in 1980 ROHINI 1 launched by India.
- In 1982 INSAR 1A launched by India.
- In 1998 IRS (INDIAN REMOTE SENSING)
- In 2001 GSAT 1
- In 2004 EDUSAT
- In 2005 CHARTOSAT
- In 2008 CHANDRAYAAN
- In 2013 MANGALYAN launched by India (5^{th} Nov. 2013 - 24 sept. 2014)
- And recently in 2019 CHANDRAYAAN 2 launched by ISRO in India that was the biggest achievement for India
- The Chandrayaan 2 has achieved 95% of itsmission objectives, the lander’s unsuccessful bud to touch-down on the lunar surface notwithstanding. And this all thanks to the chandrayaan-2 orbiter. Everything is not lost. The payloads will conduct

CONCLUSION:

Remote sensing is the gathering of information concerning the earth’s surface that does not involve contact with the surface or object under study. The techniques include aerial photography, multi-spectral, and infrared imagery, and radar. With the help of remote sensing, we can able to get accurate information about the earth’s surface including its components like forests, landscapes, water resources, oceans, etc. This information helps the researchers to their research activities about the earth’s components concerning its sustainable management and conservation and so on.

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A CROSS SECTIONAL HOSPITAL BASED SURVEY IN NORTH INDIA ON THE KNOWLEDGE, ATTITUDE ANDPRACTICE TOWARDS CERVICAL CANCER AMONG WOMEN ATTENDING THE OUTPATIENT DEPARTMENT OF OBSTETRICS AND GYNECOLOGY

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ABSTRACT:
Cervical cancer is the second most common cancer in women worldwide after breast cancer. India contributes to one-fourth of the global burden. Cervical cancer related deaths among women in India are often due to late diagnosis of disease. Knowledge about disease and early screening is the most effective method of cervical cancer prevention. Lack of awareness, negative attitude and poor screening are the major causes to increase the incidence of disease.
AIM: to assess the knowledge, attitude and practice among the general population towards cervical cancer screening and prevention.
MATERIAL AND METHOD: Hospital based cross sectional study. A total of 200 consecutive subjects attending the Gynecology OPD in a tertiary care hospital were enrolled after informed consent.
STATISTICAL ANALYSIS: descriptive statistics were used to represent the socio-demographic characteristics and levels of knowledge, attitude and practice levels.
RESULTS: most of the subjects enrolled in the study (78%) had heard about cervical cancer and majority of them had heard from friends and family (41%) , media (12%). Only 54% women knew about symptoms , while only 33% knew the risk factors. 60% of the women knew about screening methods. More than half (52%) had a positive attitude towards screening but more than two thirds (78%) are not practicing screening methods.
CONCLUSION: Although women are having good knowledge towards cervical cancer screening and prevention, still there are many who need to be educated regarding the same. Amongst the ones who have knowledge, there is a gap to transform it into practice. There is a need for more educational programs for different strata of society for larger reforms.

INTRODUCTION:

Cervical cancer is the second most common cancer in the women around the world[1]. It is second only to breast cancer. India contributes to one fourth of the global burden. According to National Institute of Cancer Prevention and Research, one woman dies every 8 minutes in India because of Cervical Cancer[2]. Cervical cancer causes 10% of all cancer related deaths in India. The stage at diagnosis is the main factor for the outcome and 5-year survival rates. The average 5 year survival after diagnosis is 48%. In India, most of the cases are diagnosed at a much advanced stage, and hence have a poor survival rate. The main reason for delayed diagnosis is lack of awareness amongst women regarding the screening and prevention of cervical cancer. Screening for cervical cancer is essential as the women are mostly asymptomatic until the disease has far progressed. The screening methods not only make

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earlier diagnosis of cancer of cervix but also help in catching the premalignant lesions of the cervix. The most common symptoms are inter-menstrual bleeding (bleeding in between periods), persistent back pain, pelvic pain, foul smelling discharge, bleeding after intercourse, urinary urgency, weight loss etc. Cervical cancer is caused by Human Papilloma Virus (HPV) infection, particularly HPV 16 and 18. These HPV 16 and 18 cause 75% of the cases globally[6]. Other risk factors include multiple partners, early age of marriage, multiple child births and HIV infection.

Although there have been many advances like Pap smear test and HPV DNA analysis in the screening for cervical cancer, there have been multiple hurdles for the same. The major problems include lack of awareness and misconceptions regarding female cancers and gynecological diseases and socio-economic limitations. According to latest guidelines issued by the American College of Obstetricians and Gynecologists, women between age 21-29 years should have a Pap test done every 3 years, and those between 30-65 years should have co-testing (Pap test and HPV DNA testing) every 5 years[3].The prevention and control of disease depends upon awareness, knowledge, preventive measures and screening procedures[4]. There is lack of information regarding knowledge, attitude and practice towards cervical cancer and its screening and hence this study was conducted.

AIM : to assess the knowledge, the practice and the overall attitude of women regarding cervical cancer and screening in women attending the department of Obstetrics and Gynecology at a tertiary care hospital and to assess the relationship of these with socio-demographic characters of the population.

MATERIAL AND METHODS:

It is a hospital based cross-sectional study conducted over a period of 3 months in the outpatient department of Obstetrics and Gynecology in a tertiary care hospital between July 2019 to September 2019. 200 parous women were recruited in the study who were over 18 years of age and were willing to participate in the study. All women who participated in the study were assessed using a preformed questionnaire about cervical cancer, screening and prevention.

The questionnaire comprised of 4 parts: socio-demographic characters, knowledge, practice and attitude. The demographic details included age, residence (rural/urban), and socio-economic stage using modified Kuppuswamy's scale[5]. Knowledge regarding the disease was identified using a questionnaire which had ten questions and each question was awarded 1 mark for appropriate knowledge and nil for either misconceptions or no knowledge.

S.No.	Knowledge about	1 mark	Zero marks
1.	Awareness of Ca cervix	Yes	No
2.	Symptoms	Any 2	≤1
3.	Risk factors	Any 1	0
4.	Prevention method	Any 1	0
5.	Screening methods	Yes	No
6.	Description about screening methods	yes	No
7.	Eligibility for screening	yes	no
8.	Frequency of screening	yes	No
9.	Vaccination	yes	No
10.	Treatment available	yes	No

Score of (A) 8 – 10 was considered good knowledge
(B) 5 – 7 = moderate knowledge
(C) <5 = Poor knowledge

Attitude was assessed using 8 statements related to cervical cancer, its screening and management. The responses were assessed using Likert scale. In Likert scale the responses ranged from strongly agree (5), agree (4), neither agree nor disagree (3), disagree (2) and strongly disagree (1).

S. no.	**Statement**	**5**	**4**	**3**	**2**	**1**
1.	Cervical cancer is highly prevalent in India					
2.	Any adult woman including you can acquire this					
3.	Cervical cancer does not spread via contact					
4.	Symptoms like intermenstrual bleeding and foul discharge pv need consultation by a doctor					
5.	Screening helps prevent it					
6.	Screening is not harmful					
7.	Screening is not expensive and is					
8.	HPV vaccination can prevent cancer					

Maximum score expected is 40 from all statements and minimum is 8. A score of ≥20 is considered a positive attitude and <20 as a negative attitude.

Practice was assessed by response towards screening in last 3 years. If patient had undergone screening in last 3 years, it was considered as regular practice, more than 3 years since last screen were considered irregular and the ones who were never screened as no practice.

STATISTICAL ANALYSIS:

Epi- info software was used to analyze collected data. Frequencies and proportions were used to represent the socio-demographic profile , knowledge, attitude and practice for carcinoma of the cervix ad its screening procedures in the study population.

RESULTS:

Among the 200 women who answered the questionnaire, most were between 35 to 45 years of age (66%). The mean age of women was 38.4 years. Majority of the women were multi-parous, i.e. with more than 1 live birth. The study population comprised of 74% women belonging to the rural area. Amongst the subjects 13% women had received no formal education, 34% were having matriculate certificate and the rest were graduate. 82% of the study population were home-makers, and 26% belonged to upper lower class, 42% into the lower middle class and 32% to the upper middle class according the modified kuppuswamy scale.

78% had heard about the cervical cancer and majority of them 41% ha heard from friends and family members, 12% from media and rest 25% from medical personnel. Regarding knowledge about signs and symptoms of the cancer of the cervix, only 54% of the women knew about 2 or more symptoms. 40% knew about foul smelling discharge from vagina and irregular spotting. Only 33% of the women had an idea about the risk factors contributing to cervical cancer. Out of these 200 women 60% of them knew about screening methods mostly Pap smear only. On assessing the attitude towards screening and prevention of the same, 52% had a positive attitude towards it, only 22% were practicing the screening procedure. The 22% women who had been previously screened , they were screened once only and had no information regarding the protocol for screening.

DISCUSSION:

The present study explored the knowledge, attitude and practice among women attending the obstetrics and gynecology department in a tertiary care hospital in northern India, with a resource limited setting. Here the responders were mostly from the lower socio-economic strata, and many of them had no formal education.

The study found out that more than two-thirds of the population had heard about cervical cancer, which was similar to the studies conducted by Chande HM et al[6]. and Abdullahi et al[7]. the results of this study are in contrast with other studies conducted in southern India by G.Narayana et al[8] and in other under-developed countries by Anorlu[9], Yifru and Asheber[10] in which more than one fourth of the population had no information regarding cervical cancer.

This study found out that more than half of the population knew about the signs and symptoms of the disease, which are consistent with the findings of Mukama et al[11]. and with G.narayana[8] et al. Still there is lack of awareness regarding the risk factors for the same.

More than two thirds of the women showed a positive attitude towards cervical cancer and more than half of the about its screening, which is again consistent with findings of G.Narayana[8] et al. Despite positive attitude among women, there is a huge difference between attitude and practice. Less than one fourth of the women had been screened in the last 3 years and only 6% had been screened more than once. None of the women knew about the screening protocol and the need for repeated screening.

Younger women i.e. between the ages 30-35 years had a positive attitude and good knowledge about signs, symptoms and screening methods. Similar findings were in the study conducted by Ogunbode and Ayinde[12].

STRENGTHSAND LIMITATIONS:

There is need to conduct such studies at greater levels with larger sample size as these data are useful to design educational programs regarding cervical cancer. The najor limitation was a smaller sample size and a shorter time frame.

CONCLUSION:

The study concludes that though women have fairly good knowledge and attitude towards cervical cancer and its screening, there is a need to bridge the gap between knowledge and practice. There also is a need to educate the masses about preventive HPV vaccination.

CONFLICTS OF INTEREST :

none.

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A REVIEW OF DEEP VEIN THROMBOSIS TREATMENT

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ABSTRACT:
Using the pretest probability score calculated from the Wells Clinical Prediction Rule, patients are stratified into three risk groups – high, moderate or low. The results from duplex ultrasound are incorporated as well. If the patient is in the high or moderate risk and the duplex ultrasound study is positive, treat for DVT. If the duplex study is negative and the patient is in low risk, DVT has been ruled out. If the patient is in high risk but the ultrasound study was negative, the patient still has a significant probability of DVT. However, if the patient is in low risk but the ultrasound study is positive, then before treating for DVT it is better to go for second confirmatory study such as a Venogram.

KEYWORDS: Deep Vein Thrombosis, Venogram.

Introduction of Deep Vein Thrombosis (DVT):
DVT usually occurs in a deep leg vein, a larger vein that runs through the muscles of the calf and the thigh.

It can cause pain and swelling in the leg and may lead to complications such as pulmonary embolism. This is a serious condition that occurs when a piece of blood clot breaks off into the bloodstream and blocks one of the blood vessels in the lungs (see below).

DVT and pulmonary embolism together are known as venous thromboembolism (VTE).

Deep vein thrombosis (DVT) is a blood clot that develops within a deep vein in the body, usually in the leg

We aim to critically review the current state of knowledge on this subject, with a view to updating clinicians on the prophylaxis and treatment of DVT.

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Symptoms of DVT:

In some cases, there may be no symptoms of DVT. If symptoms do occur they can include:

- pain, swelling and tenderness in one of your legs (usually your calf)
- a heavy ache in the affected area
- warm skin in the area of the clot
- red skin, particularly at the back of your leg below the knee

DVT usually (although not always) affects one leg. The pain may be worse when you bend your foot upward towards your knee.

Objectives of DVT treatment:

The primary **objectives** for the **treatment** of **deep venous thrombosis** (**DVT**) are to prevent pulmonary embolism (PE), reduce morbidity, and prevent or minimize the risk of developing the postthrombotic syndrome (PTS).

If left untreated, about one in 10 people with a DVT will develop a pulmonary embolism. A pulmonary embolism is a very serious condition which causes:

- breathlessness which may come on gradually or suddenly
- chest pain which may become worse when you breathe in
- sudden collapse

Both DVT and pulmonary embolism require urgent investigation and treatment.

Seek immediate medical attention if you have pain, swelling and tenderness in your leg and you develop breathlessness and chest pain.

As well as age, there are also a number of other risk factors, including:

- having a history of DVT or pulmonary embolism
- having a family history of blood clots
- being inactive for long periods such as after an operation or during a long journey
- blood vessel damage a damaged blood vessel wall can result in the formation of a blood clot
- having certain conditions or treatments that cause your blood to clot more easily than normal such as cancer (including chemotherapy and radiotherapy treatment), heart and lung disease, thrombophilia and hughes syndrome
- being pregnant your blood also clots more easily during pregnancy
- being overweight or obese

Treatment Options:

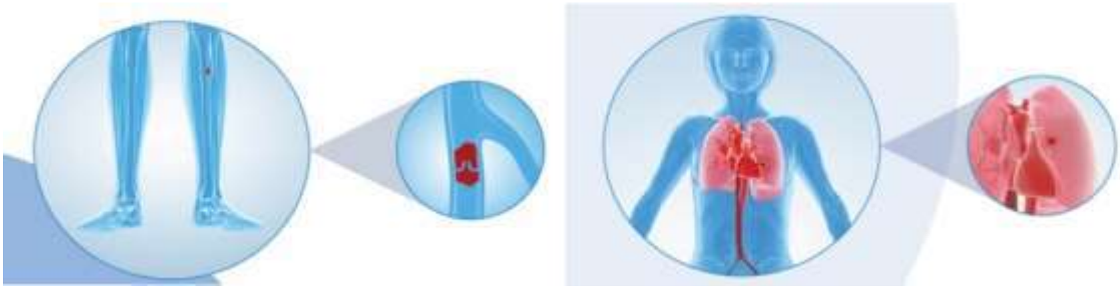
- Anticoagulation
- Thrombolytic Therapy for DVT
- Surgery for DVT
- Filters for DVT
- Compression Stockings

Anticoagulation:

Treatment for DVT usually involves taking anticoagulant medicines, which reduce the blood's ability to clot and stop existing clots getting bigger.

Heparin and warfarin are two types of anticoagulant often used to treat DVT. Heparin is usually prescribed first because it works immediately to prevent further clotting. After initial treatment, you may also need to take warfarin to prevent another blood clot forming.

A number of anticoagulants, known as directly acting oral anticoagulants (DOACs), may also be used to treat conditions such as DVT. These medications include rivaroxaban and apixaban, and they've been shown to be as effective as heparin and warfarin with less serious side effects.

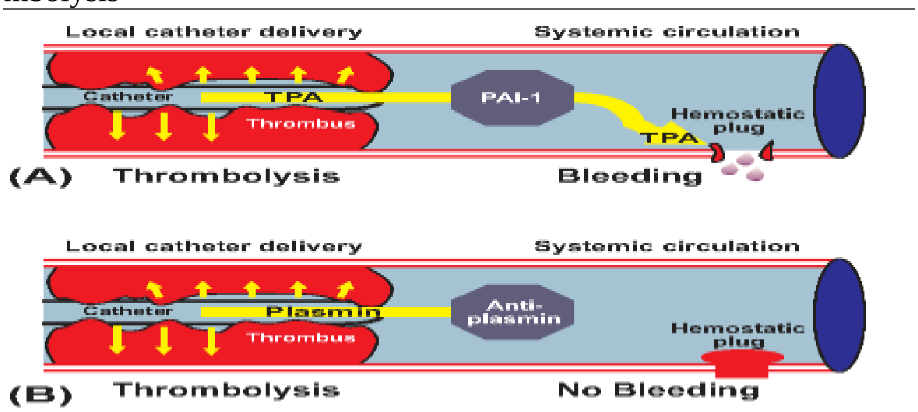


Anticoagulant Treatment for DVT

Thrombolytic Therapy for DVT:

A mechanical thrombectomy device can remove venous clots, but it is recommended only as an option when the following conditions apply: "Iliofemoral DVT, symptoms for less the seven days (criterion used in the single randomized trial), good functional status, life expectancy of greater than equal to one year, and both resources and expertise are available. Thrombolytic Therapy does not prevent

- Clot propagation
- Rethrombosis, or
- Subsequent embolisation
- Heparin therapy and oral anticoagulant therapy always must follow a course of thrombolysis



Thrombolytic Therapy for DVT:

Surgery for DVT:

Open surgical thrombectomy has a very limited role in the management of acute DVT. The limitations to the procedure are obvious, and the results are ambiguous at best. For this reason, open surgical thrombectomy for DVT is reserved as a last resort for those patients with threatened limb loss secondary to extensive DVT.

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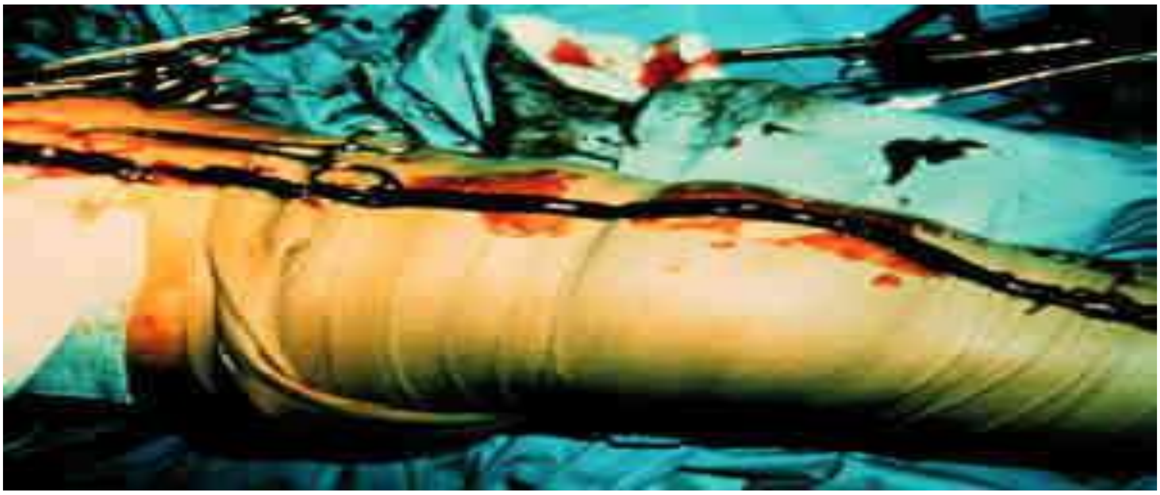
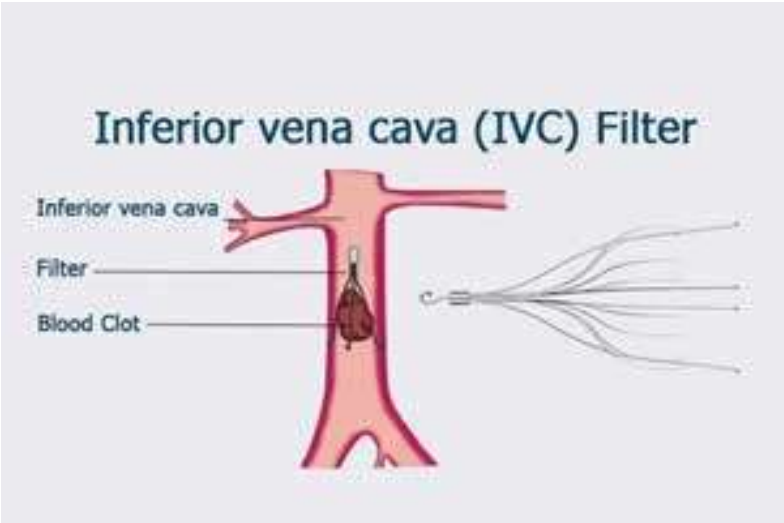


This patient underwent a thrombectomy. The thrombus has been laid over the approximate location in the leg veins where it developed

Filters for DVT:

An IVC **filter** is a small metal device that can stop blood clots in your veins from moving. It's used for conditions in which there's a chance that a blood clot could enter your lungs, such as **deep vein thrombosis** (**DVT**). It's placed in your body's main vein, called the inferior vena cava (IVC)



COMPRESSION STOCKINGS:

A compression stocking is a device that uses pressure to help prevent and ease symptoms of various thrombotic conditions such as deep vein thrombosis (DVT) and Post Thrombotic Symdrome (PTS). Compression stockings are specially made, snug-fitting, stretchy stockings that gently squeeze the leg.

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Complications:

- Acute pulmonary embolism
- Postthrombotic Syndrom
- Blood clot in the kidney, called renal vein thrombosis
- Blood clot in the heart, leading to heart attack
- Blood clot in the brain, leading to stroke
- Chronic venous insufficiency

CONCLUSION:

Using the pretest probability score calculated from the Wells Clinical Prediction Rule, patients are stratified into three risk groups – high, moderate or low. The results from duplex ultrasound are incorporated as well. If the patient is in the high or moderate risk and the duplex ultrasound study is positive, treat for DVT. If the duplex study is negative and the patient is in low risk, DVT has been ruled out. If the patient is in high risk but the ultrasound study was negative, the patient still has a significant probability of DVT. However, if the patient is in low risk but the ultrasound study is positive, then before treating for DVT it is better to go for second confirmatory study such as a Venogram.

An ultrasound scan can be used to detect clots in your veins. A special type of ultrasound called a Doppler ultrasound can also be used to find out how fast the blood is flowing through a blood vessel. This helps doctors identify when blood flow is slowed or blocked, which could be caused by a blood clot. A venogram may be used if the results of a D-dimer test and ultrasound scan can't confirm a diagnosis of DVT. During a venogram, a liquid called a contrast dye is injected into a vein in your foot. The dye travels upthe leg and can be detected by X-ray , which will highlight a gap in the blood vessel where a clot is stopping the flow of blood.

TREATING DVT

Treatment for DVT usually involves taking anticoagulant medicines, which reduce the blood's ability to clot and stop existing clots getting bigger.

Heparin and warfarin are two types of anticoagulant often used to treat DVT. Heparin is usually prescribed first because it works immediately to prevent further clotting. After initial treatment, you may also need to take warfarin to prevent another blood clot forming.

A number of anticoagulants, known as directly acting oral anticoagulants (DOACs), may also be used to treat conditions such as DVT. These medications include rivaroxaban and apixaban, and they've been shown to be as effective as heparin and warfarin with less serious side effects.

You'll also be prescribed compression stockings to wear every day, which will improve your symptoms and help prevent complications.
These may include:

- not smoking
- eating a healthy, balanced diet
- taking regular exercise
- maintaining a healthy weight or losing weight if you're obese

There's no evidence to suggest that taking aspirin reduces your risk of developing DVT.
See your GP before embarking on long-distance travel if you're at risk of getting a DVT, or if you've had a DVT in the past.

When taking a long-distance journey (six hours or more) by plane, train or car, you should take steps to avoid getting DVT, such as drinking plenty of water, performing simple leg exercises and taking regular, short walking breaks.

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AN OVERVIEW OF THE APPLICATIONS OF MAGNETIC NANOPARTICLES IN THE BIOMEDICAL FIELD

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ABSTRACT:
In this paper, the biomedical applications of magnetic nanoparticles are summarized. As a result of the special physical properties, the magnetic nanoparticles offer some attractive possibilities in biomedicine. They have controllable sizes ranging from a few nanometers up to tens of nanometers, which places them at dimensions that are smaller than or comparable to those of a cell (10–100µm), a virus (20–450nm), a protein (5–50nm) or a gene (2nm wide and 10–100nm long). This means that they can 'get close' to a biological entity of interest. Firstly, a short introduction of magnetic nanoparticles is presented. Secondly, the applications of magnetic nanoparticles in biomedical field are summarized. In medicine, the application of magnetic nanoparticles in magnetic separation technique, as therapeutic agent in hyperthermia, a targeted drug delivery carrier as well as contrast agents in magnetic resonance imaging (MRI) are explained. Lastly, a perspective of the magnetic nanoparticles in biomedicine in future is also described.

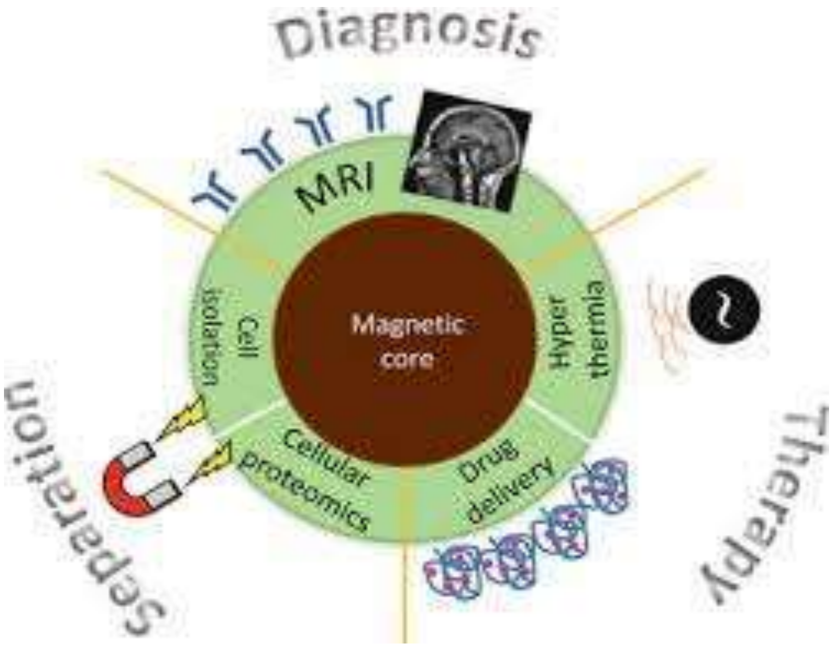
KEYWORDS: Magnetic nanoparticles, magnetic separation techniques, magnetic hyperthermia, targeted drug delivery, MRI contrast agent.

1. INTRODUCTION:

Magnetic nanoparticles consist of magnetic elements such as iron, cobalt, nickel, manganese, gadolinium, and their alloys, oxide compounds, cation complexes along with polymers etc., which shows ferromagnetism, paramagnetism or even superparamagnetism. The physical and chemical properties of magnetic nanoparticles largely rely on the chemical components, crystal structures, sizes and shapes, sometimes the source of magnetic nanoparticles synthesized. Besides the four basic effects on general nanomaterials (the quantum size effect, the surface effect, the small size effect and the macroscopic quantum tunnelling effect), the magnetic nanoparticles also possess some special magnetic properties such as superparamagnetism, high coercivity, low Curie temperature and high magnetic susceptibility.

The magnetic nanoparticles are widely used in catalysis, mineralogy (such as the selection of ores), informatics (such as data storage), environmental science (such as pollutants concentration), as well as biomedicine due to their attractive properties in physics and chemistry. With the rapid development of nanotechnology, magnetic nanoparticles are particularly attractive for biology and medicine fields. The small size is able to benefit the interaction between magnetic nanoparticles and bioentities and enhances the ability of biomolecules to cover the nanoparticle surface. The magnetic nanoparticles abide by magnetic Coulomb's law and are easily controlled by an external magnetic field. This long-distance interaction combined with the penetrativity of the magnetic field is helpful for developing applications in transportation and immobilization of magnetic nanoparticles-labelledbioentities in human tissues (such as delivering antitumor drugs or shuttling the isotope atoms to some targeted areas).The magnetic nanoparticles are able to respond the cyclic variation of the external magnetic field, obtain the energy from the excitation field and transport the heat to the targeted area, for instance different tumors. As a matter of fact, magnetic nanoparticles are able to be used as a heat enhancement agent for chemotherapy and radiotherapy, because the heated tissues can destroy effectively the malignant cancer cells.

Due to the good magnetic guidance, biocompatibility, biodegradability, and functional groups, the magnetic nanoparticles can easily be conjugated with many functional molecules or units, such as enzyme, antibody, cell, DNA and RNA. Moreover, the surface of magnetic nanoparticles is easily modifiable based on different needs. The usually used modification agents are poly (ethylene glycol) (PEG), glucosan, polyvinyl pyrrolidone (PVP), aliphatic acid, polyvinyl alcohol (PVA), peptides, gelatin, chitosan, methylsilanes and lipids, such as liposomes, etc. These modification agents improve the biocompatibility of magnetic nanoparticles, decrease aggregation, prevent protein absorption, prolong their time in blood circulation system, reduce their toxicity, and enhance their targeting. As a result, the magnetic nanoparticles are widely used in biology and medicine.



2. APPLICATIONS OF MAGNETIC NANOPARTICLES IN BIOMEDICAL FIELD
2.1. MAGNETIC BIOSEPARATIONTECHNIQUE:
Magnetic bioseparation technique is a separation technique, in which bioentities attached to the surface of nanomaterials with magnetic susceptibility are extracted from their bio-environment using an external magnetic force. At present, the usually used magnetic nanomaterials for bioseparation purpose are the superparamagnetic nanoparticles, because they are easily magnetized under an external magnetic field. After the removal of external magnetic field, the bioentities captured by the superpapramagnetic nanoparticles are immediately diffused into the solutions once again. There are two steps related to the magnetic bioseparation technique: (i) The bioentities are conjugated and labeled with the (superpara-) magnetic nanomaterials; (ii) The labeled bioentities are separated from the solutions in a magnetic separator. The magnetic bioseparation technique has been widely used in the separation and purification of bioentities of different types, for example cells, bacteria, proteins and nucleic acids.

2.2. MAGNETIC HYPERTHERMIA

Magnetic hyperthermia is a method of cancer treatment based on magnetic nanoparticles producing heat under an alternating magnetic field. Under an alternating magnetic field of appropriate amplitude and frequency, the magnetic nanoparticles which are put inside or around a tumor area will raise the tumor temperature. If the temperature is over 45˚, it could kill the tumor cells by necrosis. Or if the temperature is reached around 42˚, it could improve the efficiency of chemotherapy. The concept of magnetic hyperthermia using small particles was proposed in 1957 by Gilchrist and others, but most of the studies were inexact thermometry and poor AC magnetic field parameters in animal models. In 1993 Jordan and his colleagues found that iron oxide nanoparticles exhibit an extraordinary specific absorption rate (SAR [W/g]). This was the renaissance of a cancer treatment method using hyperthermia techniques and made a big step since then. The size and the component of nanoparticles have a great influence on their hyperthermia properties. The usually used magnetic nanoparticles for hyperthermia are composed of iron oxide nanoparticles, though some metallic nanoparticles of cobalt, iron or FeCo compared to iron oxide increase their magnetization and the maximum SAR values. Hardly were these metallic nanoparticels used because of their toxicity concerned.

2.3. TARGETED DRUG DELIVERY:

The idea of using magnetic particles as a carrier tool for drug delivery was conceived in the late 1970s by Widder, Senyei and their colleagues. The basic premise of this idea is that therapeutic agents such as targeted chemotherapy or therapeutic nucleic acid molecules are attached to, or encapsulated within a magnetic particle core. And this magnetic particle core with a layer of polymer or metal coating enables to be functionalized easily. After being functionalized, the magnetic particle-therapeutic agent conjugate is injected or administrated orally into the blood-stream system. Under the guidance of a strongly external magnetic field, the magnetic particle-therapeutic agent composites are guided to the area of the targeted sites and the drugs are delivered.

2.4. MRI CONTRAST ENHANCEMENTS:

Magnetic resonance imaging (MRI) is a non-invasive way greatly depending on the relaxation properties of proton nuclei in water and lipids showing images of the inside of an object. MRI is widely used in imaging to show pathological or other physiological changes of living tissues in the body. In comparison with CT, MRI has some advantages: (i) to employ nonionizing radio frequency (RF) signals to obtain its images and is best matched to noncalcified tis-sues in the body, (ii) to be able to detect various features in tissues via varying scanning parameters, (iii) to create cross-sectional images in any images besides oblique planes, (iv) to be superior to detect and identify tumors, (v) to be best suited for multiple times examination successively within a short period of time, (vi) to provide multiple contrastmechanisms for example: T1 weighted, T2 weighted, and T2* weighted MR images.

In MRI, T1 relaxation also called spin-lattice or longitudinal relaxation that is a time constant of nuclear spins returning to equilibrium. When nuclei are from the high-energy state to the low energy what is related to loss of energy to the surrounding nuclei. T1 relaxation is characterized by the longitudinal return of the net magnetization to its ground state of maximum length along with a direction of the main magnetic field. T1 is usually around 1s for tissue. T2 relaxation also named spin-spin or trans-verse relaxation that is a time constant of signal decay. T2 relaxation happens during spins in the high and low energy state exchanging energy but not releasing energy to the surrounding lattice. The magnetic moments interact with each other making a decrease in the transverse magnetization or decay after nuclei release their excess energy. T2 is usually less than 100 ms for general tissue. T2* is a time that occurs for the transverse magnetization to decay to 37% of its original magnitude. It is produced under an inhomogeneous magnetic field and happens in all magnets. It is characterized by inhomogeneous B_0 and loss of transverse magnetization at a rate greater than T2.

3. FUTURE PERSPECTIVES:

We have summarized applications of magnetic nanoparticles relied on their magnetic properties in biology and medicine of five aspects: magnetic separation, magnetic hyperthermia treatment, targeted drug delivery, MRI enhancement media and magnetofection technique.

(1)In magnetic separation field, these magnetic nanoparticles are being widely used in cells, bacteria, protein, and nucleic acids etc. based on their interactions with the above-mentioned biomoieties integrated with rapidly developingbiosensor techniques. This technique has entered many laboratories for bioanalytics, cell biology, molecular biology, biomedical analysis, and bioassay. There are some related commercial parts or devices for sale, for example magnetic separation beads, and magnetic-based micro total analysis system (μTAS), etc. In future, the magnetic separation technique should enable to integrate with the intelligent technology to carry out separation, purification, detection and analysis all in one line and the separation devices should also be minimized.

(2)As for magnetic hyperthermia treatment area, although there have been over 17 years after Jordan and their colleagues made a break-through for hyperthermia treatment of tumors based on magnetic nanoparticles, magnetic hyperthermia treatment for tumors in humans is only within a Germany hospital in Berlin, despite having been proven to be effective in different animal models from many laboratories and medical schools around the world. As a result, the magnetic hyperthermia treatment techniques will enter more and more hospitals to benefit for the patients in the near future after the face front problems such as the treatment temperature control and the magnetic nanomaterials with high SAR developed, etc., being overcome.

(3)About targeted drug delivery field based on magnetic nanoparticles, in such systems after the targeted molecules for crossing the cell membrane or traveling mechanosensitive ion channels in the membrane such as peptides, antibodies, and nucleic acids being attached on the surface of magnetic nanoparticles to form a multifunctional nanocomposite, the nanocomposite is able to shuttle the therapeutic drugs into the tumor nidi, release the drugs and make a therapy. It is a future and final tendency that the delivery drugs targeted, the release of the drugs controllable and the therapy at the tumor nidi are concentrated on a multifunctional nanocomposite or nanodevice based on magnetic nanoparticles.

(4) Although there are a lot of exciting and interesting applications of magnetic nanoparticles in biology and medicine, considerable problems remainto be resolved before they benefit more for the health of human beings, for instance (i) the precisely control of the size of magnetic nanoparticles; (ii) the solubility in aqueous solutions that is a bottleneck for their

applications both *in vitro* and *in vivo*; (iii) the biocompatibility, which is a barrier of the magnetic nanoparticles to be used in biomedicine *in vivo*; (iv) the stability, which is a key factor whether the magnetic nanoparticles enable to produce their functions; (v) the toxicity that is a gate whether the magnetic nanoparticles enable to be used in clinic later. As a result, the magnetic nanoparticles would attract much more interest and could lead to new opportunities in both biology and medicine after the above-mentioned challenges are resolved finally.

(5) To magnofection technique, it was introduced about two decades, compared with other transfection techniques based on cell biology, this technique provides a simple way to make a lot of cells at once with very high transfection rate. Up to now, magnetofection technique has been applied to all types of natural and artificial nucleic acids, nonviral transfection, viruses and been tested on many cell lines including different stem cells and primary cells. Improving the transfection rate is still the goal of magnetofection based on magnetic nanoparticles in the near future.

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FIBER OPTICS COMMUNICATION

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ABSTRACT:
Fiber optics is the major building block in the telecommunication infrastructure. Fiber optic systems are important telecommunication infrastructure for world-wide broadband networks. Wide bandwidth signal transmission with low delay is a key requirement in present day applications. Optical fibers provide enormous and unsurpassed transmission bandwidth with negligible latency, and are now the transmission medium of choice for long distance and high data rate transmission in telecommunication networks. This paper gives an overview of fiber optic communication systems including their key technologies, and also discuss their technological trend towards the next generation.

INTRODUCTION :
The major driving force behind the widespread use of fiber optics communication is the high and rapidly increasing consumer and commercial demand for more telecommunication capacity (larger than both wireless connections and copper cable). Advances in technology have enabled more data to be conveyed through a single optical fiber over long distances. The transmission capacity in optical communication networks are significantly improved using wavelength division multiplexing.

A desirable feature for future optical networks is the ability to process information entirely in the optical domain for the purpose of amplification, multiplexing, de-multiplexing, switching, filtering and correlation, since optical signal processing is more efficient than electrical signal processing. Several new classes of optical communication networks are presently emerging. For example, Code Division Multiple Access networks using optical signal processing techniques have recently being introduced.

Despite the associated benefits of utilizing optical fiber for communication (such as its high reliability over long distances, low attenuation, low interference, high security, very high information capacity, longer life span and ease of maintenance), research is still on going to further improve on the present fiber optics communication system, and also to solve some of the challenges facing it. Future optical communication systems are envisioned to be more robust than the present system.

Basic Principles of Fiber Optic Communication:
Fiber optic communication is a communication technology that uses light pulses to transfer information from one point to another through an optical fiber. The information transmitted is essentially digital information generated by telephone systems, cable television companies and computer systems. An optical fiber is a dielectric cylindrical waveguide made from low-loss materials, usually silicon dioxide. The core of the waveguide has a refractive index a little higher than that of the outer medium (cladding), so that light pulses is guided along the axis of the fiber by total internal reflection. Fiber optic communication systems consists of an optical transmitter to convert an electrical signal to an optical signal for transmission through the optical fiber, a cable containing several bundles of optical fibers, optical amplifiers to boost the power of the optical signal, and an optical receiver to reconvert the received optical

signal back to the original transmitted electrical signal.

Optical fibers fall into two major categories, namely : step index optical fiber, which include single mode optical fiber and multimode optical fiber, and graded index optical fiber. Single mode step index optical fiber. Single mode step index optical fiber has a core diameter less than 10 micrometers and only allows one light path. Multimode step index optical fiber has a core diameter greater than or equal to 50 micrometers and allows several light paths, this leads to modal dispersion.

Evolution of Fiber Optics Communication:
Optical fiber was first developed in 1970 by Corning Glass Work. At the same time, GaAs semiconductor lasers were also developed for transmitting light through the fiber optic cables. The first generation fiber optic system was developed in 1975, it used GaAs semiconductor lasers, operated at a wavelength of 0.8□m, and bit rate of 45 Megabits/second with 10Km repeater spacing.

In the early 1980's, the second generation of fiber optic communication was developed, it used InGaAsP semiconductor lasers and operated at a wavelength of 1.3 □m. By 1987, these fiber optic systems were operating at bit rates of up to 1.7 Gigabits/second on single mode fiber with 50 Km repeater spacing. The third generation of fiber optic communication operating at a wavelength of 1.55 □m was developed in 1990. These systems were operating at a bit rate of up to 2.5 Gigabits/second on a single longitudinal mode fiber with 100Km repeater spacing.

The fourth generation of fiber optic systems made use of optical amplifiers as a replacement for repeaters, and utilized wavelength division multiplexing (WDM) to increase data rates. By 1996, transmission of over 11,300Km at a data rate of 5 Gigabits/second had been demonstrated using submarine cables..

Future Trends in Fiber Optics Communication:
Fiber optics communication is definitely the future of data communication. The evolution of fiber optic communication has been driven by advancement in technology and increased demand for fiber optic communication. It is expected to continue into the future, with the development of new and more advanced communication technology. Below are some of the envisioned future trends in fiber optic communication.

A. All Optical Communication Networks :
An all fiber optic communication is envisioned which will be completely in the optical domain, giving rise to an all optical communication network. In such networks, all signals will be processed in the optical domain, without any form of electrical manipulation. Presently, processing and switching of signals take place in the electrical domain, optical signals must first be converted to electrical signal before they can be processed, and routed to their destination. After the processing and routing, the signals are then re-converted to optical signals, which are transmitted over long distances to their destination. This optical to electrical conversion, and vice versa, results in added latency on the network and thus is a limitation to achieving very high data rates.

Another benefit of all optical networks is that there will not be any need to replace the electronics when data rate increases, since all signal processing and routing occurs in the optical domain. However, before this can become a reality, difficulties in optical routing, and wavelength switching has to be solved. Research is currently on going to find and effective solution to these difficulties.

B. Multi-Terabit Optical Network :

Dense wave Division Multiplexing (DWDM) paves the way for multi-terabit transmission. The world-wide need for increased bandwidth availability has led to the interest in developing multi-terabit optical networks. Presently, four terabit networks using 40Gb/s data rare combined with 100 DWDM channels exists. Researchers are looking at achieving even higher bandwidth with 100Gb/s. With the continuous reduction in the cost of fiber optic components, the availability of much greater bandwidth in the future is possible.

C. Intelligent Optical Transmission Network:

Presently, traditional optical networks are not able to adapt to the rapid growth of online data services due to the unpredictability of dynamic allocation of bandwidth, traditional optical networks rely mainly on manual configuration of network connectivity, which is time-consuming, and unable to fully adapt to the demands of the modern network. Intelligent optical network is a future trend in optical network development, and will have the following applications: traffic engineering, dynamic resource route allocation, special control protocols for network management, scalable signaling capabilities, bandwidth on demand, wavelength rental, wavelength wholesale, differentiated services for a variety of Quality of Service levels, and so on. It will take some time before the intelligent optical network can be applied to all levels of the network, It will first be applied in long-haul networks, and gradually be applied to the network edge.

D. Ultra – Long Haul Optical Transmission:

In the area of ultra-long haul optical transmission, the limitations imposed due to imperfection in the transmission medium are subject for research . Cancellation of dispersion effect has prompted researchers to study the potential benefits of soliton propagation. More understanding of the interactions between the electromagnetic light wave and the transmission medium is necessary to proceed towards an infrastructure with the most favorable conditions for a light pulse to propagate.

E. Improvements in Laser Technology:

Another future trend will be the extension of present semiconductor lasers to a wider variety of lasing wavelengths. Shorter wavelength lasers with very high output powers are of interest in some high density optical applications. Presently, laser sources which are spectrally shaped through chirp managing to compensate for chromatic dispersion are available. Chirp managing means that he laser is controlled such that it undergoes a sudden change in its wavelength when firing a pulse, such that the chromatic experienced by the pulse is reduced. There is need to develop instruments to be used to characterize such lasers.

F. Laser Neural Network Nodes:

The laser neural network is an effective option for the realization of optical network nodes. A dedicated hardware configuration working in the optical domain and the use of ultra-fast photonic sections is expected to further improve the capacity and speed of telecommunication networks become more complex in the future, the use of optical laser neural nodes can be an effective solution.

G. Polymer Optic Fibers:
Polymer optic fibers offer many benefits when compared to other data communication solutions such as copper cables, wireless communication system and glass fiber. In comparison with glass optical fibers, polymer optical fibers provide an easy and less expensive processing of optical signals, and are more flexible for plug interconnections. The use of polymer optical fibers as the transmission media for aircrafts is presently under research by different Research and Development groups due to its benefits. Also, in the future, polymer optical fiber will likely displace copper cables for the last mile connection from the telecommunication company's last distribution box and the served end consumer…

H. High – Altitude Platforms:
Presently, optical satellite links and orbit-to-ground links exists, the latter suffering from unfavorable weather conditions. Current research explores optical communication to and from high altitude platforms. High altitude platforms are airships situated above the clouds at heights of 16 to 25 Km, where the unfavorable atmospheric impact on a laser beam is less severe than directly above the ground. As shown in figure 3, optical links between high-altitude platforms, if a high-altitude platform functions as a data relay station.

I. Improvements in Optical Transmitter/Receiver Technology:
In fiber optics communication, it is important to achieve high quality transmission even for optical signals with distorted waveform and low signal to noise ratio during transmission. Research is on-going to develop optical transceivers adopting new and advanced modulation technology, with excellent chromatic dispersion and Optical Signal to Noise Ratio (OSNR) tolerance, which will be suitable for ultra-long haul communication systems. Also, better error correction codes, which are more efficient than the present BCH concatenated codes are envisioned to be available in the nearest future.

J. Improvement in Optical Amplification Technology:
Erbium Doped Amplifier (EDFA) is one of the critical technologies used in optical fiber communication system, In the future, better technologies to enhance EDFA performance will be developed. In order to increase the gain bandwidth of EDFA, better gain equalization technology for high accuracy optical amplification will be developed. Also, in order to achieve a higher output power, and a lower noise figure, high power pumping lasers that possess excellent optical amplification characteristics with outputs of more than +20dBm, and very low noise figure are envisioned to exist in the nearest future.

K. Advancement in Network Configuration of Optical Submarine Systems:
In order to improve the flexibility of network configuration in optical submarine communication systems, it is expected that the development of a technology for configuring the mesh network will be a step in the right direction. Presently, most large scale optical submarine systems adopt the ring configuration. By adopting the optical add/drop multiplexing technology that branches signals in the wavelength domain, it is possible to realize mesh network configuration that directly inter-connects the stations. Research is ongoing, and in the future such network configuration will be common.

L. Glass Fiber Design and Component Miniaturization:

Presently, various impurities are added or remove from the glass fiber to change its light transmitting characteristic. The result is that the speed with light passes along a glass fiber can be controlled, thus allowing for the production of customized glass fibers to meet the specific traffic engineering requirement of a given route. This trend is anticipated to continue in the future, in order to produce more reliable and effective glass fibers. Also, the miniaturization of optical fiber communication components is another trend that is most likely to continue in the future.

CONCLUSION:

The fiber optics communications industry is an ever evolving one the growth experienced by the industry has been enormous this past decade. There is still much to be done to support the need for faster data rates, advanced switching techniques and more intelligent network architectures that can automatically change dynamically in response to traffic patterns and at the same time be cost efficient. The trend is expected to continue in the future as breakthroughs already attained in the laboratory will be extended to practical deployment thereby leading to a new generation in fiber optics communications. The future is bright. Just remember, the information superhighway is paved with glass.

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NANOTECH IMPROVISED LITHIUM ION BATTERY – FUTURE BATTERY FOR NEW GENERATION AUTOMOBILES: A REVIEW PAPER

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ABSTRACT:
Greater emphasis on clean and green transport technologies are because of climate and environment issues and are being well addressed by the adoption of zero emission Electric Vehicles (EVs).EVs being battery driven ensures energy security by replacing fossil fuel resources having an alarming status. Lithium Ion Batteries (LIBs)are the most preferred contenders that offer high energy density, high power density. However, issues like underperformance, fire contagiousness, high cost and slow charging speed of batteries have motivated researchers to further improve these batteries by using nanostructured materials to overcome these hurdles in its applications. Another important performance parameter is the battery SOC (State Of Charge)and its proper estimation by guiding mathematical principles enhances battery life. This review papercompares the various materials used for the fabrication of key components of LIBs and highlights the potential contribution of Nanotechnology to improve different structural and performance parameters of LIBs to make them more efficient and cost effective and finally focus on proper and accurate estimation of SOC of batteries to enhance their life cycle.Opportunities, challenges and latest researchin battery materials/ technology will alsobe discussed in this paper.

Keywords: Electric Vehicles, Lithium ion batteries, nanotechnology, SOC

1. INTRODUCTION:

Global transport emissions are still on an increasing trend despite progress in electrification of vehicles and their introduction. People still prefer to go for more efficient diesel vehicles which outweigh the impact of higher shares of electric vehicle sale globally. The initiative to move away from using fossil fuels as the energy source for transport use is necessary to ensure energy security, environmental conservation and revenue protection. The difficulties in controlling the GHG emissions and the over-dependence of fossil fuels play major roles in shaping the future of transportation. Multiple alternate energy sources are available that can power modern EVs but currently Lithium ion battery (LIB) is the preferred choice as it displays a reasonablyhigh voltage, high power density and high energy density. A major limitation of LIB is that it should be used within suitable voltage and temperature ranges as any violation in terms of operating voltage leads to safety and performance issues. Other challenges in LIB system are higher costs, durability and recharging issues of batteries that promote advancements in technology to ensure more efficiency. Moreover, according to USABC (United States Advanced Battery Consortium) present LIBs do not meet the standards of EVs. So to overcome the energy challenges, investments are being made on nanotechnology research for new and advanced batteries [1][2][3][4]. Further, LIBs require careful monitoring to avoid deterioration of battery as well as situations leading to explosions. SOC of a battery which determines the available capacity of battery is an important parameter that reflects its performance and extends battery life. So accurate SOC estimation of LIB is essential for a successful application in EVs [5]. LIB is a main component integrated in the battery management system (BMS) of an electric vehicle. To

prevent the dangerous situations during overcharging and to improve battery performance considerably BMS of EVs perform the vital operation of SOC estimation. BMS uses battery models and algorithms to determine the state of charge of battery [6].
This paper highlights keytechnological developments in LIB components to improve its performance for transport applications. This paper also discuss various mathematical techniques adopted for accurate SOC estimation of LIBs for EV applications and their recent trends.

2. LIB: A potential energy source for Electric vehicles:

Electric vehicles powered by electricity are advantageous over conventional vehicles for having lower operating costs and no tailpipe emissions.Electric vehicles are expected to increase to 22% of global share by 2030 [7]. There is a wide variety of electric energy sources that can power EVs like batteries, fuel cells, ultra capacitors and ultra-high speed flywheels etc. Among these energy sources, batteries in particular have been developed to power a diverse range of applications in transportation. In this paper we shall focus on Lithium ion batteries as the potential energy sources preferred by automation industry. The success of an Electric vehicle is governed by the efficiency of its energy storage device. John Goodenough [8] created the first lithium-ion batteries in 1980. Commercial lithium-ion batteries such as lithium cobalt oxide (LCO), lithium iron phosphate (LFP), lithium manganese oxide (LMO), lithium nickel manganese cobalt oxide (NMC), lithium nickel cobalt aluminium oxide (NCA), and lithium titanate oxide (LTO) have been widely accepted by electric vehicles in recent years [9]. LIBs have a fabulous combination of high energy and power density that makes it the technology of choice for EVs[10] [11] [12] [13] [14]. Still higher energy storage for delivering higher power for longer span, getting rid of thermal runaway problem [15], higher rate capabilities are some of the expectations for which a lot of research work is going on. New fabrication materials and techniques are being adopted to achieve the success in overcoming the barriers for widespread use of LIBs.

2.1 Battery electrodes:

The electrodes of LIB, both anode and cathode are made of materials that have the ability to be easily intercalated with lithium ions. The electrodes should also have high electrical conductivity so that the LIB can have high charging rates. Moreover the electrode material should remain consistent over a significant number of charge-discharge cycles that will ensure higher battery life and hence low costs. Also higher cell voltage is desired for achieving high energy density. Thus there has always been an effort to keep innovating and improving the existing electrode materials' chemistries to achieve higher operating voltages, high stability, long life and resistant to chemicals [16][17].Lithium cobalt oxide $LiCoO_2$ (~140$mAhg^{-1}$),Lithium transition metal oxides $LiMn_2O_4$(~110$mAhg^{-1}$), and Lithium Nickel Manganese Cobalt Oxide (NMC) $LiNiMnCoO_2$ (~154$mAhg^{-1}$)proved to be good options for battery cathode [18][19][20][13] but Lithium iron phosphate($LiFePO_4$) (~170$mAhg^{-1}$) cathodes with higher specific capacity [21] are more commonly used as they are chemically stable, inherently safe and thermally stable in the operating temperature range of EVs.

Anode of LIB is mostly made up of graphite which is an abundant and low cost material. It has specific capacity of 370 $mAhg^{-1}$. At high charging/discharging current rate Li dendrites are formed which move through the separator and causes internal short circuit and thermal runaway(fire) in the battery. LIBs with graphite anodes are good at low temperatures only whereas the required temperature range for EV application is -30^oC to 60^oC [21]. Hunt for

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suitable anodic material led researchers to use Silicon electrodes, Lithium metal alloys (Li_XSn) [22], (Li_XGa) [23], Transition Metal Oxides (TMO) [24], conversion electrodes [25]. Silicon anode has a theoretical specific capacity (~4200mAhg^{-1}) but has the drawback of large volume expansion of anode during Li insertion. One of the TMO namely TiO_2 anode having specific capacity ~180mAhg^{-1}has the potential to suppress the Li dendrites and hence improves the safety and durability of LIBs. But it has low ionic diffusivity and low conductivity which are its limitations. A promising replacement for graphite is lithium metal, which has more than ten times the capacity of graphite (3860mAhg^{-1}), and would allow for the development of smaller, longer-lasting batteries. Lithium dendrites with large tortuosity often form during lithium plating, and if these dendrites penetrate the membrane separator in the middle of the battery, they can create short-circuits, raising concerns about battery safety. Due tosafety issues and low coulombic efficiency Lithium metal battery research was abandoned in favour of graphite.Li-Sulphur and Li –Air batteries are also seeking much attention due to their high energy density (~500Wh/kg) but Li-S battery suffer by capacity fade problem.Each of these materials specified above have some advantages over the other but overall efficiency, safety, high cost of battery could not be completely met with.

2.2 Battery electrolyte:

The electrolyteis an essential part of the battery which enable Li ions conduction between two electrodes. LIB electrolytes are usually based on an organic solvent with a lithium salt such as ethyl carbonate with $LiPF_6$or $LiBF_4$. But it is a flammable electrolyte so if at any given time the voltage exceeds the voltage window the battery runs the risk of catching fire. Forsafety, development of a non-flammable electrolyte or a constituent that can develop rapidly a solid electrolyte-interface (SEI) layer to prevent plating of Li on a carbon anode is required. Gel polymer electrolyte and solid polymer electrolyte are options for building safer LIBs. Currently solid state batteries with solid electrolytes are close to commercial reality but they suffer from low ionic conductivity at room temperature as compared to organic liquid electrolytes. LIBs suffer thermal runaway issue at temperatures above 130^{0}C [26]. Room Temperature Ionic Liquids (RTILs), a new development, can mitigate the flammability and volatility of organic electrolytes issue

3.Nanotech improvised LIBs:

Nanotechnology deals with materials at nano scale (10^{-9} m) and it can significantly improvethe rate capability and cyclic behaviour of electrode materials of LIB. Nanoparticles are formed through either by breaking down of larger particles or by controlled assembly processes. Nano materials are of special interest for researchers as at such a small size these materials possess distinguished properties and characteristics which have the ability for application in different fields. Nanomaterial has greater chemical reactivity due to much greater surface area to volume ratio as compared to conventional sized materials hence it greatly increases their strength. Quantum effects play an important role at nanoscale in determining the materials' properties and characteristics which result in novel magnetic, electrical and optical behaviours. Nanotechnology finds applications in many spheres like pharmaceutics, advanced materials, energy production and storage [27]. Its recent applications in transportation[28][29] include improving the efficiency of energy generation from renewable resources and energy storage in batteries by providing alternative materials and fabrication methods to produce cost effective batteries. Nanotechnology plays a vital role in battery chemistry and thus increasing the energy storage capacities, energy density, safety and durability of conventional batteries used in EVs[29] by employing Carbon Nano Tubes

(CNTs), fullerenes, quantum dots and nano composites in various components of the battery such as electrodes, catalysts[27] and electrolyte [4][30][31]. Nano composites can also be used to improve energy storage capacities of batteries. Nanotechnology companies used their own proprietary material composition to reduce the risk of the battery catching fire. Nanoscale batteries can be combined together to function as a macro battery such as within a Nano pore battery. Nano batteries which can charge 60 times faster than conventional batteries have been developed using nanotechnology. Batteries that can prevent electrode contact prior to activation have been developed which gives limitless shelf life and longer active life to them. In all nanotechnology enables more energy at less cost [1].

3.1 Nanostructured LIB Electrodes:

Faster intercalation of Li ions can be facilitated by using nanosized materials for electrodes, which offer high surface areas and short diffusion paths, and hence faster storage and delivery of energy. One prominent example is the cathode material of A123 LIBs that use nanosized lithium iron phosphate cathode [15]. Altair nano LIBs that use lithium titanate spinel electrodes do not suffer thermal runaway issue below 250^0C temperature [26].Several other nanomaterials like graphene, carbon coated silicon nanowires [32], CNTs [33], layered nanostructured vanadium oxide (V_2O_5) [34] ,manganese oxide (MnO_2), TiO_2 Nanotubes[35] are used to replace conventional graphite electrodes in batteries to increase the surface area that is accessible to battery electrolyte, low volume expansion during Lithium insertion which enhances the electricity output [36]. Graphene, a single layer of graphite, allow intercalation of Li ions on both sides forming LiC_3 thus doubles its capacity. Silicon nanowires and silicon porous structure overcomes the drawback of large volume expansion of silicon anodes and successfully help in maintaining structural integrity in addition to enhancing cyclic performance.TiO_2 based nanostructured electrodes ensure improved electrochemical performance when applied to lithium ion batteries. TiO_2 NT arrays are one group of nanostructures which basically offer high structural stability and safety during cycling, as these are crucial requirements in many. Hence nanostructured electrodes when used in batteries improve the energy storage that automatically improves battery life [37][27].

3.2 Nanostructured Electrolyte ensures battery safety:

Electrolytes in LIB conduct lithium ions between two electrodes. Instead of conventional liquid electrolytes use of solid electrolytes could ensure high-energy battery chemistries and better safety. A major challenge is to reduce interfacial resistance between the solid electrolyte and lithium based anodes and to increase lithium ion conductivity. Nano structuring of solid electrolytes has proven to improve the lithium ion conductivity. Al_2O_3, SiO_2, or ZrO_2 nanoparticles added to solid polymer gel could significantly enhance the conductivity and storage capacity of the electrolyte[39].

In case of the LIBs with lithium metal as anodes, lithium dendrite are formed across the electrolyte that leads to short circuits and overheating. Separators with nano porous structures can prevent the spreading of dendrites by acting as a mechanical barrier without hindering the ion-transport during charging and discharging cycles. Recently, a nano porous polymer-ceramic composite separator has been reported. This laminated nano porous gamma alumina sheet (pore size of 100 nm) sandwiched between macro porous polymer membranes could prevent the spreading of dendrites and hence cell failure that is caused by short circuits [15].

3.3 Nano sensors:

In each cell of the battery nano sensors are integrated which help to monitor the State Of Charge (SOC) in real time which will be helpful not only for security reason but also be useful to maximize the use of battery [38]. Accurate SOCestimation is one of the most important functions in a battery management system for battery packs used in electrical vehicles.

4. Estimation of State Of Charge of LIBs:

Electric vehicles are always equipped with a battery management system (BMS) to regulate and monitor LIB pack. The BMS has built in battery management hardware and software including algorithms which predict various states, available capacity and power of battery to ensure high efficiency and safety [40]. To estimate SOC, battery model techniques and accurate battery SOC estimation algorithms are currently of extreme importance due to their applications in electrified transportation and energy storage systems [41]. A battery model is required to establish the relationship between internal and external behaviours of battery by mathematical modelling. In any battery model the first step is to accurately identify the parameters of the model for battery state estimation. . A lithium-ion battery model can be classified as an electrochemical model, physical model, equivalent circuit model (ECMs), thermal model, coupled electro-thermal model, and so on. Among them, ECMs are the most commonly used in lithium-ion batteries SOC estimation for EV applications due to their simple model structure. Various ECMs used are the Rint model, Thevenin model, Partnership for a New Generation Vehicle (PNGV) model, nRC model, and FOM (Fractional Order Model). As hundreds of single cells are connected in parallel and series to compose a battery pack and provide energy that can meet the requirements of an application like a smart grid or EV, a whole battery pack model is required [42].

The existing SOC estimation methodologies can be categorized into five groups, which are the direct method, model based method, the adaptive filter algorithm, the learning algorithm and the hybrid algorithm.

4.1 Direct methods :

These are conventional methods which utilize battery characteristics such as voltage, current and temperature and solve some equations to estimate SOC.

(1) Coulomb Counting (CC) method also called Ampere –hour counting method is used to integrate the discharging or charging current to calculate the remaining charge in the battery[43] given by equation

$SOC(k) = SOC(0) - T/C_n \int_0^k [\eta \cdot I(t) - S_d]\, dt$

Where SOC(0) is the initial battery SOC, I(t) is the current at time t, T is the sampling period, C_n is the nominal capacity of the battery, η is the coulombic efficiency, and S_d is the self-discharging rate. This method however has short term accuracy and needs initial SOC value which pose limitations in its widespread use. The average error in estimation of SOC by this method is <$\pm$ 4%.

(2) Open Circuit Voltage (OCV) method is successful in case battery e.m.f and SOC relationship is stable which is not always the case. The relationship is expressed as

$$SOC = f\,(OCV)$$

Temperature and cycle life of battery can change this relationship which is no more reliable [44]. The average error in estimation of SOC by this method is <$\pm$ 5%.

(3) Impedance and internal resistance method is used to describe the intrinsic electric characteristic under any current excitation, if temperature, SOC, and SOH are fixed. Electrochemical impedance spectroscopy (EIS) involve processing of large amount of data and internal resistance changes slowly so not fit for SOC estimation. In general, SOC estimation based on the impedance and internal resistance method are not suitable for use in EVs [45].

4.2 Model based methods:

Model-based methods deploy a battery model with advanced algorithms to estimate the states of a battery from its measured parameters such as voltage, current, and temperature. Electrochemical model and Equivalent circuit model (ECMs) fall under this category which use resistances and RCs to simulate the electrical characteristics for lithium-ion batteries. We can estimate SOC directly through ECM parameter identification or in association with SOC-OCV method. These methods have a high order of accuracy and are suitable for online SOC estimation[46]. However ECMs cannot simulate actual battery voltage under all current values and also have high computational load. The average error in estimation of SOC by this method is $<\pm$ 5%.

4.3 Adaptive Filter Algorithm:

Adaptive techniques combine the direct and model-based methods.The flow chart depicts that a predetermined SOC is taken as input, a battery model selected then calculate output battery voltage and then gain is calculated to update the SOC by comparing measured voltage and model voltage. The adaptive filter algorithm improves the accuracy and robustness of the battery SOC estimation and reduces the noise influence on the battery model. Various adaptive techniques include Kalman filter (KF), a linear state-space model that predicts the current state from the earlier state and updates the current state to converge it to the real value. The average error in estimation of SOC by this method is $<\pm$ 1.76%.

KF is applicable for linear systems only so to take care of non-linear systems Extended Kalman filter (EKF) is devised [47]. A linearization process using Taylor series expansion approximate the nonlinear system with a linear time varying (LTV) system then applied to KF to give EKF. But this method degrades SOC estimation accuracy so Unscented Kalman Filter (UKF) is devised to handle highly nonlinear state space model by applying a discrete-time filtering algorithm[48]. The Particle filter (PF) algorithm is used to estimate the states, which approximate the probability density function of a non-linear system by using the Monte Carlo simulation technique. The $H\infty$ filter is a simple design model with robustness,even in the presence of parameter uncertainties and modelling errors, to restrict the effect of exogenous disturbances on output. However, hysteresis, aging, and temperature effects may deviate the accuracy of the model [49].The filter algorithms based on the equivalent circuit model with fixed model parameters are often used to estimate the battery state. However, the parameters of the equivalent circuit model are often affected by temperature, C-rate, SOC, and battery aging. Some researchers have proposed adaptive filtering (AF) methods for on-line identification of battery model parameters. Practice has proved that these methods are easier to implement in on-line applications. Besides, these methods can help to compensate for parameter values for battery variations and aging. Therefore, some hybrid estimation methods have been proposed to handle these problems.

4.4 Hybrid Methods:

The hybrid algorithm method is composed of two more algorithms. It can improve the efficiency and accuracy of the battery model and avoid the shortcomings of a single algorithm. The hybrid algorithm method not only achieves reliable and effective results, but also reduces the cost of the battery management system. However, this method has a very complex mathematical calculation, which requires a large storage memory and computing power unit. These methods are usually made up of two parts. The first part is used to identify the parameters of the model with recursive least squares (RLS) on-line. The second part is used to estimate the battery state parameters with filter algorithms [50].

4.5 Artificial Intelligence Based Learning Methods:

Recently trend is to estimate battery state using datainstead of using battery models as a lot of improvement has been done in embedded hardware performance. Over the last five years,the inclusion of some probabilistic techniques or artificial intelligence has been incorporated to improve the performance of estimation algorithms. The Artificial Intelligence (AI) based learning approach including artificial neural network (ANN) modelling as well as the support vector machine (SVM) was proposed [51], and could be very accurate depending on the training data. For EVs and HEVs, the trend is now towards the design of intelligent BMS, which involves research into intelligent-adaptive SOC estimation methods.

5. CONCLUSION:

For EVs to offer environmental benefits over conventional vehicles, technological improvements introduced by nanomaterials must be incorporated by the manufacturers in batteries of EVs. Future research will be focused on bringing down cost of nanomaterials for batteries, and making sure these stand up to the requirements of large-scale commercial applications. As nano engines, nanomotors and nanostructure are already developed, it can be widely used in future EVs to render them highly efficient vehicles. Various research efforts on nanotechnology based LIB technology has already led into the production and use of high performance LIBs (Toshiba, A123 Systems, Altair Nano, Next Alternative Inc., etc.). Altair nano LIBs provide a unique feature of recharge at minus 30^0C which is unachievable in conventional LIBs. Sothese ensure excellent cold temperature performance.

For accurate SOC estimation of real-time EV, the battery model must be as simple as possible.ECM is regarded as the most appropriate for online estimation and, based on this type of model the adaptive filter-based and artificial-intelligence-based approaches are presented to estimate the SOC with high precision. From the review, it is clear that adaptive filter-based algorithms are more suitable for EV applications, and those based on artificial intelligence are not suitable for this application due to its intensive computing. In future more attention must be given to develop less complex SOC estimation algorithm having less implementation cost for EV application.

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A STUDY OF HRD MECHANISM IN NATIONALIZE BANK (A CASE STUDY OF PNB BANK)

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ABSTRACT:
Human Resource Development plays vital role in every organization because it is now considered as a part of modern management system. HRD is helpful in development of employee's physical capacities, relationships attitudes, values, knowledge and skills. The success of the Banks depends not only on the satisfaction of their customers but also the satisfaction level of the employees working in the banks. The level of satisfaction of the bank employees can be ensured and enhanced by effective HR-practices adopted by the Banks. The study is aimed at assessing the extent of HRD Practices in Punjab National Bank. For the purpose of the study, primary data is collected from 300 employees of Punjab National Bank from Bilaspur district through a structured questionnaire. The study revealed that the Performance appraisal, Training and Development, career planning, motivation, compensation management and grievance redressal are better in Punjab National Bank.

KEYWORDS: HRD Practices, Punjab national bank, Satisfaction of the Bank Employees

I. INTRODUCTION:

In India, the origin of human resource management can be traced in the 1970s, concern for welfare shifted towards higher efficiency, a change in professional values of human resource managers was visible. During the 1980s due to new technology and other environmental changes, Human Resource Development (HRD) became a major issue. During the 1990s, the overwhelming role of human factor in industry has been realized. Growing awareness about the significance of human side of organisation has led to the development of human resource management as a distinct discipline. Focus on human values and a philosophical approach, are likely to provide this discipline the status of a profession.

Human Resource Development (HRD) is a process of developing skills, competencies, knowledge and attitudes of people in an organization. The people become human resource only when they are competent to perform organizational activities. Therefore, HRD ensures that the organization has such competent human resource to achieve its desired goals and objectives. HRD imparts the required knowledge and skill in them through effective arrangement of training and development programs. HRD is an integral part of Human Resource Management (HRM) which is more concerned with training and development, career planning and development and the organization development. The organization has to understand the dynamics of HR and attempt to cope with changing situation in order to deploy its HR effectively and efficiently. According to **Leonard Nadler,** "Human resource development is a series of organised activities, conducted within a specialised time and designed to produce behavioural changes."

Banks have to understand that the capital and technology-considered to be the most important pillars of banking are replicable, but not human capital, which needs to be viewed as a

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valuable resource for the achievement of competitive advantage. The long-term vision for India's banking system is to transform itself from being a domestic one to the global level may sound far-fetched at present. To take up this industry to the heights of international excellence requires combination of new technologies, better processes of credit and risk appraisal, treasury management, product diversification, internal control, external regulations and human resources at the most. The satisfaction of the employees working in the bank is predominant condition for the success of any bank. The satisfaction of the bank employees can be ensured and enhanced by effective HR-practices adopted by the Banks. An attempt to verify the HR-practices of one among the Punjab National Bank situated in Bilaspur District of Himachal Pradesh.

II Brief History of Punjab National Bank:

PNB is an Indian banking and financial services company .The Head office of PNB in New Delhi, India. It serves over 80 million customer having 6968 across 764 cities. It has 9935 ATM's across branches over the world. PNB India's first Swadeshi bank commenced its operation on April 12, 1895 from Lahore. Lala laj Path Rai played a key role in PNB's birth. When the first branch opened the Rai was its account holder with 2 lakh capital and 20000 its working capital. India's first Prime Minister Jawaharlal Nehru and Gandhi included the other customers of PNB's over the years. After the partition PNB its register its office Lahore to New Delhi. Today, PNB has nearly 7000 branches in India and this bank listed on world's biggest public companies.

III. REVIEW OF LITERATURE:

Sirca et.al.(2012) The aim of the paper is to theoretically and empirically describe the role of HR practices in the area of HR development: training and education, career development, performance management and reward management, on job satisfaction. The study has been conducted on a sample of Slovenian employees (N = 824), from medium size private and public sector organisations. Cluster analysis of participant responses on the HRM practices questionnaire shows four different groups of organisational approaches to HR development that are moderately related to job satisfaction. The results are discussed from four perspectives: individual attitudes, HRM theory, social exchange theory and the psychological climate theory.

Kesti (2012) the article suggests that the employee quality of working life can be measured by working unit collective competencies consisting leadership, team culture and processes. However, these competencies and attributes should be first validated for each organization environment, situation and strategy. This is essential because effective HRD process has to focus on the development of organization specific human drivers of performance. This seems to be vital for generating optimal workplace innovations.

Maier et. al. (2014) this paper proposes a framework for HR development for innovation, to reach the actual performance of an organization. The purpose of this framework is to offer, to all managers, a clear picture of existing HR capacity to innovate and possibilities to develop this capacity in order to improve the organization's capacity for innovation, thus increasing their chances of success in today's highly dynamic and competitive business environment.

Rakesh (2016) in his article highlighted banking sector implement the latest ideas on management regarding development of human resources. Developing the individual or human capacity is an integral element of building capacity and, in fact, capacity building initiatives are now increasingly becoming necessary. Taking the banking industry to the heights of excellence, especially in the present competitive environment, requires a combination of new technologies, better processes of credit and risk appraisal, treasury management, product diversification, internal control and external regulations and, not the least, human resources. Skilled and efficient manpower inventory is now becoming the need of the hour. So banks must start proper HRD practices to their future growth.

IV HRD PRACTICES IN INDIAN BANKS:

In the booming economy and the continuing expansion most of the banks facing challenges to do well and it outlines the fact that, contrary to public perception it is not just the new private sector banks that are doing well. There are few public sector banks are also doing well and got the place in top 10 best performing Indian banks. And it's worth mentioning that these public sector banks have performed so admirably in spite of the fact that they operate with many handicaps, such as strong unions and the inability to offer market salaries and incentives and burdened with extravagant workforce. The secret of success of any company depends on how they treat employees and keep them satisfied. For that they have to design their human resource processes like recruitment, selection, training and development, performance appraisal and other based on employee perspective in order to benefit them.

In India the banking industry is becoming more competitive than ever with private and public sector banking competing each other to perform well. The executives of the bank are now modifying their traditional human resources practice in to innovative human resource practices in order to meet the challenges from other competitive banks. Effective human resource practices relate to Bank's performance by contributing to employee and customer satisfaction, innovation, productivity and development of a favourable organizational working climate in the Banks. In order to verify HR practices adopted by the chosen public sector bank in Bilaspur District, the following functions have been chosen by the author as attributes of HRD practices. Recruitment and selection, Training and development, Performance appraisal, Career development, Motivation, Compensation management, Grievance Handling

V. OBJECTIVES OF THE STUDY:

- To explore the HRD practices in Punjab National Bank of Bilaspur District.
- To find out the satisfaction level of bank officers on HRD practices of their banks.
- To measure the relationship between the demographic characteristics of bank officers and their level of satisfaction on HRD practices.

VI. METHODOLOGY:

Research Design:

The methodology of the study is based on the primary data as well as secondary data. The study depends mainly on the primary data collected through a well-framed and structured questionnaire to elicit the well-considered opinions of the respondents.

Targeted Population & Sample:

Punjab National Bank operating in Bilaspur District has been chosen for the study. In Bilaspur District 21 Punjab National Banks are functioning. 300 officers & non officers of different age group, different gender, different designation and different length of work

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experience have been chosen as respondents for this study by using simple random sampling technique

Source of Data:
Data was collected from staff members of PNB through primary & secondary source of data. Questionnaire & interviews were used.

Data Collection Tool:
Required data had been enumerated from the chosen respondents by using structured schedule. The schedule contained the items related to the demographic variables of the employees and important attributes of HR-practices namely recruitment and selection, training and development, performance appraisal, career development, motivation, compensation management and grievance handling. Descriptive, t test and one way ANOVA were used as a statistical tool for the study.

VII Hypotheses of the Study:

- Gender of the Punjab National bank officers does not influence their satisfaction level on HR-practices.
- There is no significant relationship between the age of the Punjab National Bank officers and their satisfaction level on HR-practices.
- The level of satisfaction of the PNB officers based HR-practices does not differ with respect to their work experience.
- Designation of the Punjab National Bank officers does not have any influence on the satisfaction level on HR-practices.

VIII ANALYSIS AND INTERPRETATIONS:
It is found from the analysis of the data that offices of PNB functioning in Bilaspur district in Himachal Pradesh follow HR-practices. The study has been tested by using the statistical tools‘t’ test and One-Way ANOVA.

TABLE 1: **PROFILE OF THE SAMPLE UNITS**

S.no.	Name of Branches	Number of respondents		
		Male	**Female**	**Total**
1	Hatwar	12	03	15
2	Talyana	14	02	16
3	Barmana	12	03	15
4	Deoth	12	02	14
5	Mehri Kaithla	11	01	12
6	Chhakoh	12	01	13
7	Harlog	11	01	12
8	Jadukuljiar	10	01	11
9	Dabatmatari	11	02	13
10	Rani Kotla	11	02	13
11	Panjgain	12	02	14
12	Bilaspur	25	02	27
13	Ghumarwin	12	02	14
14	Jhandutta	13	02	15

15	Maloh	11	02	13
16	Chandpur	11	02	13
17	Kuthera	13	02	15
18	Majhwar	12	02	14
19	Samoh	11	03	14
20	Namhol	11	03	14
21	Dhamli	11	02	13
	Total	258	42	300

Gender wise classifications

S.no.	Particular	No. of respondents	%age
1	Male	258	86.0
2	Female	42	14.0
	Total	300	100

It is clear from the table that majority of respondents i.e. 86% belongs to the total sample of male category and 14 % under the category of female employees in PNB bank.

Designation

S.no	Particular	No. of Respondents	% age
1	Officers	142	47.33
2	Non-officers	158	52.67
	Total	300	100

It is clear from the table that majority of respondents i.e. 52.67% belongs to the total sample of non-officers category and 47.33% under the category of officers' employees in PNB bank.

Age wise classification

S.no.	Particular	No. of respondents	Percentage
1	20-30	110	36.7
2	31-40	134	44.7
3	41-50	45	15.0
4	51-60	11	3.7
	Total	300	100

It is clear from the table that majority of respondents i.e.44.7% belongs in the age group between 31-40, followed by 36.7% that is in the age group of 20-30 .very few respondents i.e.3.7% belongs to the age group of 51-60 years.

Work experience

S.no.	Particular	No. of respondents	Percentage
1	Up to 5 years	167	55.7
2	6-10 years	89	29.7
3	11-15 years	21	7.0
4	More 15 years	23	7.7
	Total	300	100

It is clear from the table that majority of employees 55.7% having up to 5 years of work experience whereas very few respondents i.e. 7% having up to 11-15 years of work experience.

Hypothesis –I

Ho: Gender & designation of PNB employees does not influence their satisfaction level on HR practices.

HR Practices	**Gender**				**'t' value**	**'p' value**
	Male		**Female**			
	Mean	**SD**	**Mean**	**SD**	-0.57	0.57
	59.77	7.765	60.55	10.098		

The Table 2 indicates that the calculated p value (0. 57) for the variables Gender and HR practices in Punjab National Bank is found to be greater than 0.05 at 5% level of significance. Since the calculated p value for the variables Gender and HR practices is greater than 0.05, the null hypothesis H0 is accepted. Hence, it was concluded that Gender of the Punjab National Bank does not influence their satisfaction level on the HR practices followed in their banks.

HR practices	**Designation**				**t value**	**p value**
	officer		**Non-officer**			
	Mean	**SD**	**Mean**	**SD**		
	14.27	2.54	14.03	2.17	0.89	0.38

The Table 2 indicates that the calculated p value (0. 38) for the Designation and HR practices in Punjab National Bank is found to be greater than 0.05 at 5% level of significance. Since the calculated p value for the designation and HR practices is greater than 0.05, the null hypothesis H0 is accepted. Hence, it was concluded that designation of the Punjab National Bank does not influence their satisfaction level on the HR practices followed in their banks.

Hypothesis – II

Results of One-Way ANOVA for Demographic Characteristics and HR Practices

Age Group	**Mean**
Below 25	11.58
25-35	11.91
35-45	11.40
Above 45	11.27
'F' value	0.75
'p' value	0.52

Work experience	**Mean**
Up to 5 years	25.26
6-10 years	25.76
11-15 years	26.62
More 15 years	25.65
'F' value	1.07
'p' value	0.36

Table 3 shows that the calculated 'p' value for the variables 'Age' and HR-practices is 0.52 which is greater than the value 0.05. So, it is needless to say that H0 is accepted. Results of the One-Way ANOVA reveal that there is no significant difference between the Punjab National Bank of different age group towards their satisfaction on HR practices adopted by their Banks.

Table 3 depicts that the calculated 'p' value (0.36), for the variables "Work Experience and HR practices" is greater than 0.05 at 5% level of significance. Since, the "p" value is greater than 0.05, the H0 is not rejected for the variables work experience and HR practices. Hence, it is concluded that the work experience of the Punjab National Bank does not influence their level of satisfaction on the HR practices adopted in their banks.

IX. CONCLUSION:

The Study carried out in Punjab National Bank branches, Bilaspur District adopt HRD practices discloses various facts. However, the efficiency and the performance of HRD practices differ among banks. Though the bank officers insignificantly differ themselves in the level of satisfaction on HR practices on different functional areas of HRM of their banks, yet all respondents have same level of satisfaction over the HR practices adopted in the banks. Moreover the demographic characteristics Gender, Age, Designation and Work Experience do not influence the level of satisfaction of the Punjab National Bank on the HRD Practices.

X. SUGGESTIONS:

On the basis of the findings of the study, it can be concluded that since the employees are assumed as the inherent force to make the progress of the banks, the managements of these banks should make a concerted effort to provide greater satisfaction that boost the morale of the employees.

Bank to create more awareness among the employees that the knowledge and skill acquired through training to apply on the job.

- PNB Bank to activate HR department which, in turn, takes initiative and interest in organizing training and development programs.
- PNB Bank to organize adequate number of training programs.
- Bank needs to motivate the superiors in developing their subordinates.
- Banks are suggested to make the existing system of evaluation of superiors by subordinates more effective.
- Bank makes the use of the existing performance appraisal system for effective utilization of employees.
- PNB Bank suggested the existing performance appraisal system strong in order to identify the strengths and weaknesses relating the employees' performance.
- The Study suggested giving more importance in giving an opportunity to employees for linking their growth to the future of the organization.
- The study suggested enhancing its efforts to motivate supervisors to guide their subordinates for improved performance.
- It is proposed by the researcher to PNB to strengthen the existing feedback on positive aspects of performance appraisal.

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CONTRIBUTION OF APPLIED SCIENCES IN THE TRANSFORMATION OF BUSINESS MANAGEMENT AND COMMERCE - A 21STCENTURY BUSINESS PERSPECTIVE

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ABSTRACT:
Applied sciences have contributed significantly towards the development of discipline of business management and commerce. The Quantitative,System and Contingencyapproach has changed the business perspective in 20thcentury .The basic applied sciences behind all these was mathematics , statistics , computing etc.The fuel to the fire in 21stcentury is the technology and innovation, which is rapidly making application of existing knowledge to more practical applications resulting into application oriented devices. The key to success in any business with the rapidly evolving technology is analysis. Therefore, field of Analytics has evolved to provide useful insights and help in optimising the business activities, and that's how the Machine Learning and Artificial Intelligence has stepped in and now have a bigger role to play by using algorithms for decision making. Hence, there is a need to redesign and reengineer the business. The tomorrow's successful entrepreneurs, managers have to apply the fundamentals of business management holistically, thoughtfully keeping in view all of the applied sciences relevant or becoming relevant along with the concepts like, Artificial Intelligence , Machine Learning, Internet of Things and Block Chain etc.

KEY WORDS: Artificial Intelligence, Analytics, Block Chain, Internet of Things, Machine Learning.

INTRODUCTION:
Human being is capable of doing and creating so many things, it is well said that, thank God, we do not have wings but we have created an aeroplane which can fly faster than the speed of sound; thank God, we do not have fins, the man has created Ships and under water vehicles. We are exploring ocean as well as space. Space ships are there to travel and under water activities are numerous. We are capable of launching satellites in to the geo- stationary sphere of the earth. Satellite communication is a reality. Now we are exploring Moon and Mars. All these activities have been commercialised up to certain extent.Integrated Applied Sciences and most modern era's evolved disciplines are making things real and virtual as well for the betterment of mankind but it is bringing challenges and threats also.

STATEMENT OF THE PROBLEM:
Integrated Applied Sciences are contributing a lot for the development of existing and new disciplines. Therefore, most of the disciplines are application oriented today. Technology is also converging towards digital technologies. At present the digital technologies evolved may be listed as 5G Networks, Machine Learning and Artificial Intelligence, Augmented Analytics, Smart devices, Robotic Process Automation etc. Any scientific idea or technology which is useful formasses turns in to a business idea. Therefore, there is a focus on the contribution of applied sciences in the transformation of discipline like Business Management and Commerce. Although, the discipline of Business Management and Commerce is considered as a discipline of social science but the developments stated above

has made it a discipline of science and technology oriented in this century. Still, more than eighty years are remaining of this century, the change and adaptability to change is brisk, therefore, we may perceive a big shift in 21st century business,

OBJECTIVES:

1. To identify the applied sciences those have contributed to the Business Management and Commerce.
2. To study the contribution of applied sciences in the transformation of Business Management and Commerce.
3. To study the 21st century business perspective so that quick change may be anticipated for further brisk change,

METHODOLOGY:

The study is to identify the contribution of applied sciences and it focuses on the 21st century business perspective. The methodology to be adopted here is to look at the development of management thought i.e., a theoretical perspective, through secondary sources of information from books, dallies and magazines etc.

SCOPE OF THE STUDY:

The scope of the study is very wide, since it is concerned with different individualdisciplines. In modern terms these disciplines have wider application orientation. This increases its scope manifold.

LIMITATIONS OF THE STUDY:

The study is based on the contribution then we have to look back.But the concepts are emerging quickly. Therefore, a big gap is felt in concepts, their development and emerging patterns. The primary source based study needs time and labour. This is the main limitation of the study.

REVIEW OF LITERATURE:

The development of management thought in its modern term begins with the coming of 20th century. Initially it was considered a Rule of Thumb, then policy oriented-Business/Corporate Policy, Strategy oriented-Strategic, and now known as Advanced Strategic Management.The development of thought took many turns up to the 21st century. It may be perceived that during this century these turns will certainly be more and quick. All those turns were segmented as Classical era, Neo-classical era and Most- modern era.

The classical era's contribution may be spelled as, the concept of formal organisation, scientific management-productivity approach and management process –functional approach. The contributors were mainly from the field of engineering i.e., applied sciences. We may conclude that the genesis of modern management is through the thinkers who were from applied science – may be marked as initial contribution.

During the Neo-classical era the school of thought developed through Human Relations approach, Behavioural and social sciences. The contributors were from the fields of Logic, Ethics, Philosophy, and Social Psychology. This amounted to the contribution of social sciences.

The most modern era begins with the application of Operations Research and Computerised Data Processing techniques. These are mainly the fields of Applied Mathematics. The technical and quantitative sciences resulted in to the development of Modern System Theory (Multidisciplinary and holistic), Contingency theories of organisations. Here, Environmental forces, Technology, Formal Structure, Behavioural forces and Decision Making processes were integrated. All from different fields of knowledge, their applied part became relevant for the development of modern management thought.

There is another most modern way to look at the phenomenon as we term it as Industry0.0, Industry 1.0, Industry 2.0, Industry 3.0 and Industry4.0. These terms also signify change and development of diverse phenomenon relevant to the human life through industrial development. Industry 1.0 identified as Industrial Revolution i.e., mechanisation. Industry 2.0 means Development of Telecommunication, Infrastructure development (Rail-Road, Electricity etc.), transformed society with the fast dissemination of goods, services and ideas. Industry 3.0 begins with the period after World War II followed by the period of technological progress, Information Communication Technologies, Personal Computers, Internet, Pager, Cellular phones etc. Industry 4.0 may have 100% Digitisation, thisperiod iscoming withsmart devices, even smart cities, smart homes,smart factories/ Dark factories so on and so forth. However, as per the Fred W. Riggs's model of Prismatic Society- an intermediate society, India may have to deal with all Industry 1.0,2.0,3.0,simultaneously to achieve Industry 4.0. Because some part of India has not tasted the Industrial Revolution, still deprived of infrastructure, do not know about the benefits of computerisation/digitisation etc.

Whatsoever, is possible till date is due to the application of the pure and social sciences, the integration of applied science,the development of new disciplines from such integrated applies sciences. Behind all these is the basic science and spirit of mathematics.

DISCRIPTION AND EVALUATION OF THE PROBLEM:

The review of literature and the management continuum helped us to identify the disciplines which contributed to bring Business Management and Commerce to its present form and shape. We may begin with the Indian Philosophy which through its scriptures provides us unique insight for the solution of complex problems of family, society, organisations etc. Among other individual or ethnic group we may mention Sumerians, Egyptians, Chinese, Greeks, Socrates, Plato, Alexander the Great, Kautilya, there are hundreds of such names which may be mentioned here.The Ancient, Medieval, Modern and Most-modern period of History may also contribute in many ways. The Business Environment components are Social, Economic, Political, Administrative, Legal, Ethical, Scientific and Technological aspects. These have their own authority and are beyond the control of the business organisation and most volatile in nature. Although, ever changing environmental components but have impact on the business and in turn are being affected. Now there is sufficient evidence to say that right from Anthropology to present day integrated applied science subjects all have contributed a lot towards Business Management and Commerce.

The contribution of each and every subject, Philosophy, Individual and ethnic group cannot be taken or mentioned here because it is not possible to create voluminous research paper; therefore, we will take the present phenomenon relevant to the problem under study.

Now any phase of Business Organisation and Management, Decision-making, Strategic Planning and Controlcan be expressed in quantitative terms for a more exacting analysis. The number of techniques that have evolved with the quantatives and analytics related to or based on computing are going to revolutionise the field of business management and commerce. The phenomenon evolved recently may be mentioned as follows.

1. 5G:

5G is the fifth generation of cellular network technology. It is next to the 4G LTE fourth generation long term evolution. 5G network work on the 600MHz spectrum and 5G tower can transmit signals over one thousand square miles. With the advent and adoption of 5G technology, there will be elevation of the cellular network not only to interconnect people but also to interconnect and control machines. It will be 200 times faster than the 4G LTE network. It will enable new operating models instead of redesign of production line and offer manufacturers to build smart factories. The connectivity and data interoperability in IIoT projects will be possible.

2. Internet of Things (IoT) and Industrial Internet of things (IIoT):

It is a network of things which includes devices, appliances (domestic or industrial), vehicles of various mode of transport, robots and other things capable of computing applications equipped with sensors, electronic gadgets, software, having connectivity with one and all so that there is exchange of data.

It is certain that this technology may influence the areas like Safety, security, convenience, energy efficiency, operational efficiency and agricultural productivity etc. This has made the concept of smart home and smart work place possible. Building Management system for commercial buildings may be drawn and installed. The IoT will influence the present business by facilitating Data Sharing, Remote Work, to prepare skilled workers, Inventory tracking and Management, Customer Relationship Management etc. But the issue of connectivity depends upon the wired, wireless and hybrid network. This will influence the implementation strategies. However, the wired is more reliable in comparison to the wireless or hybrid. Hence, Signal strength shall matter in wireless connectivity.

3. Business Analytics:

The IoT shall generate huge data. Moreover, with the continuous increase of internet and web users and universal mobile computing for the masses led to additional data generation. Business is the interested to get every type of data for their business purposes. Therefore, there arises a field of Analytics. It is a field where there is a process involved in collating, mining, sorting, processing, drawing and studying the data by using statistical tools and techniques and mathematical models for making it purposeful. When the concept is used in business it is called Business Analytics. Here, the aim is to identify which datasets are of use for particular business and then may be used to solve the decision making problems and in turn to increase productivity, revenue and efficiency. The field of Business Analytics is classified as Descriptive Analytics, Diagnostic Analytics, Predictive Analytics and Prescriptive Analytics. Over a period of time different tools have been developed, these may be assessable from the open source platforms named as Metabase, Matomo, OmniSci, SpagoBI, Zeppelin by Apache and Birt. With these tools the businesses can manage their processes and may draw solutions in a relevant and comprehensive way. Analytics is committed to simplify the human life by doing future sales forecasting, demand planning.

However, the challenges for business Analytics may arise due to Poor coordination, Lack of commitment of decision makers, Executive Distrust and Slow Information Maturity.

4. Artificial Intelligence and Machine Learning:

Artificial Intelligence is a technological reality, it is a process through which machines/devices like computers in terms of hardware and in combination with the software simulates human activities including learning and self- correction, problem solving etc.
Machine Learning is a type of Artificial Intelligence, rather a process of applying Artificial Intelligence without explicit programming, so that the machine/device is able to learn automatically.

Now, the Artificial Intelligence is paving the way for more impactful digital business models. It will transform the Enterprise Resource Planning (ERP) by identifying the problems even before they occur, Automate routine tasks with system intelligence and Improve user experience through interaction. The Conversational Gateways has made the omnichannel businesses possible. Adoption of Artificial Intelligence and its implementation require technical literacy of business leaders only then best AI solutions may be sought. Company culture is the detrimental factor in the adoption of AI. The sectors those are attracted towards implementation of AI are retail, banking, healthcare, IT, infrastructure. However, the list may expand further. There is an evolution of a concept of Deep Learning now. It is more specific version of Machine Learning which relies on neural networks to engage in nonlinear reasoning. The neural networks are a form of AI where flow of information resembles the human brain. At this point of time we are not in a position to see the exact impacts of AI-ML technology. But we may foresee the new start-ups, displacement of certain jobs and creation of entirely new one, AI-ML businesses and business applications.

5. Robotic Process Automation (RPA):

It is the technology that allows anyone to configure computer software or a Robot to emulate and integrate the actions of a human interacting within digital systems to execute a business process. It is specialised software and Artificial Intelligence to automate repeatable tasks and business processes using programs called “bots.”

6. The Cloud:

The cloud computing means storing and accessing data and programs over the internet instead of own computer’s hard drive. Fog computing refers to extending cloud computing to the edge of an enterprise’s network. It facilitates the operation of compute, storage and networking services between end devicesand cloud computing data centers. It may connect all the industries and businesses of the world.

7. 3D Printing:

It is also called additive manufacturing, is a process by which physical objects are created by depositing material in layers based on digital model. These processes require software, hardware and materials to work together.If 3D printing get industrialised, any product one may think of could be taken out of 3D printer.

8. Block-chain Technology:

It is a technology to decentralise the web. It is a peer to peer network. It is based on Distributed Ledger Technology which is the outcome of interdisciplinary application of the fields of Accounting (Triple Entry Accounting System) and Cryptography. It is capable of making the processes more democratic, secure, transparent and efficient. Initially this technology was applied to evolve Digital Currency named Bitcoin. However, the technocrats have found and exploring other potential uses of this technology. The business applications of Block Chain are supply chain management/auditing, smart contracts, The sharing economy, crowd funding, governance, prediction markets, protection of intellectual property, Anti-money laundering, Know your customer, land title registration and stock trading etc. The challenges posed by this technology may be enumerated as, an environmental threat due to high carbon emission because of the computational resources used. The technology needs high power, processing and storage. Quantum computing is another threat to the block chain technology because the processing speed of the computers will increase to the extent, that may enable hackers to find loopholes in block chain based networks.

9. Cyber security:

All we know that in computing security is a paramount issue, however, when everything will be digitized and nothing is offline, and then imagine what may happen. Presently, cyber threats are increasing worldwide and India is no exception. This year there were so many major security breaches. Therefore, everyone has to address the issue. This month India is going to establish Defence Cyber Agency (DCA) to counter cyber threats.

A 21st Century Business Perspective:

The emerging technologies cited above evolve a fact that 21st century business activity which will transform the ethos of business.The complete digitisation will bring unprecedented benefits in efficiency and productivity. It will make the businesses more reliable, predictable and scalable. The digital transformation shall amount to the transformation of Business Management and Commerce. The picture may not be that much rosy as we may perceive today. The key to the smart systems is meaningful data/ information. The more data is available, we may have better solutions, if information is available and we are not in a position to process it, it can work counterproductive. Moreover, India is not comparatively ready to adopt such technologies quickly, because these are yet to be fully implemented the world over. Therefore, we have to address the challenges and threats first. A holistic effort is needed at every step.

CONCLUSION:

It is a proven fact that Applied Sciences and Integrated Applied Sciences have transformed so many ideas, concepts, disciplines, and Business Management and Commerce not an exception. The quantitative tools and techniques were more instrumental. There is a big change in the technological environment of the business. The technological environment had further influenced the other components of business environment. Therefore, the influence of the technology will compel the businesses to adopt and cope with the big and brisk change. Skilful and technical savvy workforce will be needed. Robots may be engaged along with the human beings. The coexistence of such machines will require more adjustments with machines because they lack emotions. The organisational culture will take a new shape. More digital technology applications are inevitable. The challenges are to be dealt with the strengths. A 21st century business activity will contain new kind of activities because the technologies and concepts discussed above are getting integrated. For example RPA plus AI plus Analytics, the integration of these three will automate every process. Moreover, if any of the components discussed under Sr. No. 1-9 is missing, the whole process shall stall. All these technologies are capable to enable Industry4.0 enterprise – A Smart Business Entity.

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INFORMATION AND COMMUNICATION TECHNOLOGY AND MATHEMATICS EDUCATION: PRACTICAL CHALLENGES AND FUTURISTIC PERSPECTIVES

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ABSTRACT
Education has been remained the most outstanding development criterion and priority area for the development of any society throughout the world.However, Education becomes dynamic and practical by use of technology in its various domains. Today, almost all of the universities and colleges are promoting the development of practical knowledge and skills within the individuals worldwide.In this regard, the role of digital technologies has been remained the subject of interest for mathematics educators for at least the past three decades. Mathematics consists of basic structure for many fields like physics, business terminologies, and architecture by means of use of calculus, algebra, statistics, and trigonometry. Thus, the responsibility of education system is to acknowledge students with the need and practical use of mathematical knowledge in their life.Technology generates the new avenues for its users especially in the various areas of mathematics education such as control theory, signal processing and cryptography etc as theuse of Information and Communication Technology (ICT) in classrooms develops the visual impact and practical environment within the classroom. In this direction, the present paper is focussing on understanding the use of ICTin mathematics classroom and also exploring the practical challenges that are faced by educators as well as students while teaching and learning mathematics. The paper is also discussing the futuristic perspectives in regard to use of ICT in Mathematics classroom.

KEYWORDS: Mathematics Education,Information and Communication Technology, Knowledge, Teaching and Learning.

INTRODUCTION
The advancement of a nation depends upon the quality and quantity of education received by its people. Education plays a vital role in shaping the future of a nation. Only education can differentiate between animals and men.

Gorge and Polya (2002) had discovered two aims of school education. First is to converting out employable adults who contributes to social and economic development of the country and second is to develop the inner resources of growing child. Lockey has aptly remarked, “Plants are developed by cultivation and men by education”.

The education system needs some tools that make learning in classroom effective for students .This purpose can be resolved by use of some visual tools such as images, videos and visual presentation. The teaching becomes effective as well as student’s interest makes class room learning more studious.

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ICT stands for information and communication technology. It means diverse set of technological tools and resources used to communicate and to create, store and manage information. Since 1970's the digital technologies have been available in school mathematics by the introduction of simple forth calculators. ICT has a great impact on education as well as other aspects. It has both positive and negative impact on students and teachers. Nowadays the role of mathematics is not restricted to purely academic domain but it is widen into domain of technology and industry. Robler, Casting and King, (1988) have analysed that by the introduction of ICT many researchers and funding agencies were led to invest their resources to investigate the possibility of replacing teachers in key instructional role by computers.

Olive and Makar (2010) argued that the connection of mathematical knowledge and mathematical practices can be strengthen by the use of technologies. There are many tools and visual aids that can create interest of students for mathematics rather than regular calculation studies in class room. Mathematics has trigonometry and geometry portions that can be represents more effectively by projection technique for 3D Plane as well as 2D plane.

The Department of Higher Education proposes to closely integrate the critical components into a framework, incorporating the learning and best practices from the similar initiatives at the global level and practical needs of the evolving knowledge society.

The objective of this paper is to study the present status of usage of ICT by teachers during teaching learning process and to investigate the barriers in mathematics classroom. The main aim of this paper is to determine the key area of policies and awareness among teachers towards use of ICT.

MATHEMATICS EDUCATION:

Mathematics is very important part of our life and plays a major role in achieving the aims of education. Mathematics is science of measurement, quantity, magnitude, space and numbers. It has comprehensive applications in the subjects like social sciences, medicine, business and commerce, life, health, physical sciences and technical sciences. As we compare Mathematics with other subjects it is the most important subject because there are lots of techniques and branches for finding the formulae on the problem of various fields in mathematics.According to Napoleon"The progress and the improvement of mathematics are linked to the prosperity of the state."

Mathematics helps children to think creatively, develop his imaginations and think logically. The main goal of mathematics education in schools is the mathematisation of the child's thinking.As we compare Mathematics with other subjects it is the most important subject because there are lots of techniques and branches for finding the formulae on the problem of various fields in mathematics.

Mathematics can be used as a component in many interdisciplinary combinations. The interdisciplinary programmes in mathematics will make the subject more attractive and meaningful. We can propose a number of such interdisciplinary programmes involving mathematics for example Physical science-Mathematics, Commerce-Mathematics, Economics-mathematics, Education-Mathematics etc. We make learning more fruitful and appealing by such interdisciplinary programmes involving mathematics.

Mathematics is used directly or indirectly to fulfil the needs of man such as in business, carpentry, shop keeping, banking, accountancy, weighing, measuring, counting etc. By the study of mathematics an individual can be able to apply his knowledge to a new situation.
In India mathematics is included as compulsory subject right from kindergarten level upto secondary level in school curriculum. Due to modernization the curriculum of mathematics changes time to time in India. So the government has to change the teaching method from chalk and talk method to student centred approach method. This helps the students to think and understand the concepts logically in different way. Ping and Hua(2015) investigated that maximum number of students are not able to solve the basic operations well due to dearth of understanding the concepts. Some factors which affectthe achievement of mathematics are social factors, personal factor, economic factors, psychological factor, etc.

ICT and Mathematics Education:
The use of ICT in the mathematics classroom has long been a topic for consideration by mathematics educators. D.D.Agyei and J.Voogt (2010) analysed some examples of use of ICT in mathematics are portables, graphic calculators and computerized graphing, specialised software, programmable toys or floor robots, spreadsheets and databases etc.Computers, wireless classroom microphones, interactive white boards, digital video on demand, online study tools, LCD projectors etc. can be used as teaching aids in classrooms. Graphic calculators, Geogebra, Dynamic geometry tools, Microsoft mathematics, Auto shapes, Mat lab are the software used for teaching and learning of mathematics. ICT-supported education can develop the acquisition of the knowledge and 21st century skills such as critical thinking, creativity and problem solving. Recent research indicates that the purposeful use of computers in classroom instruction can indeed enhance student outcomes (Archer, 1998; Milheim, 1995). Studies involving computer assisted algebra instruction (Brunner & Sheehan, 1997; O'Callaghan, 1998) and instruction with graphing calculators (Adams, 1997) have similar positive results. The National Council of Teachers of Mathematics (NCTM) concluded that technology is an important tool for teaching and learning process and it is stated by NCTM in Curriculum and Evaluation Standards for School Mathematics (1989) and Principles Standards for School Management.

In those countries where majority of children cannot afford more than one notebook, the country should provide appropriate low cost technology for teaching and learning process.

In 1972 the scheme of Educational Technology was introduced during the IVth plan.Later, in 1984-85 the Computer Literacy and Studies in Schools (CLASS) Project was introduced as a pilot project.

In VIIIth plan (1993-98),this project was considered as a Centrally Sponsored Scheme and its scope is enlarged to provide financial grants to educational institutions.

Muhammad Shahid Farooq (2006) studied the effects of teacher's professional education on students achievements in mathematics. The purpose of the study was to compare the effectiveness of teaching of professionally trained and untrained teachers. Data were collected from four public and private boys and girls high schools record. Four hundred secondary school graduates (Two hundred boys and two hundred girls) taught by trained and untrained teachers of mathematics were selected conveniently. The results of the study supported the fact that the students taught by trained teachers showed better results in

Mathematics and gender has no significant effect on achievement in mathematics.

Practical Challenges in usage of ICT in Mathematics Education:
Many studies have shown a lot of difficulties that teachers experience in the synthesis of ICT in their classrooms. Jones (2004) found a number of barriers for the integration of ICT into lessons such as low confidence among the teachers during integration, dearth of time for integration, technical problems in using software, lack of access to resources etc.

Chong, et al. (2005) has identified six major barriers in a survey to study the barriers preventing the integration and adoption of ICT in teaching mathematics. These barriers are lack of time in school schedule for project involving ICT, insufficient teacher training opportunities for ICT project, lack of knowledge about ways to integrate ICT to enhance the curriculum, difficulty in integrating and using different ICT tools in a Popoola et.al (2018) finds that importance existed in the use of ICT facilities on teaching and learning of mathematics. This showed that ICT plays a major role in teaching and learning of mathematics as a teaching tool. Also, there was sound effect of ICT facilities availability and usage on student's performance in mathematics. This also showed that ICT facilities helps students to have unlimited access to study materials and ability to relate with their correspondent in other institutions across the globe even from the comfort of their rooms and this undoubtedly helps them to improve in their academics. It was equally revealed that there exists significant influence of ICT facilities on teaching of mathematics. This equally showed that lecturers also relied to certain extent on ICT facilities on how to improve on their skills and teaching methods. Lastly, the study revealed that significant effect of ICT facilities in improving the students' learning and knowledge in mathematics existed. This implied that ICT facilities help greatly in expanding the knowledge and understanding of students in mathematics.

Some of the barriers in usage of ICT in mathematics education are lack of confidence among teachers during synthesis, lack of personal access during lesson preparation, lack of ways to integrate ICT to enhance the curriculum and insufficient teacher training opportunities for ICT project.

ICT and Mathematics Education: Futuristic Perspectives:
Ting SengEng (2005) investigated that ICT has small effects on learning outcomes and sometimes negative effects have been found. Qualitative research on how to improve the effective use of ICT through innovative methods, possibly incorporating a variety of ICT tools should be investigated as pilot studies. Such studies should be on going such that feedback can be obtained and methods modified to refine the teaching and learning process.

Lecturers should encourage the students to use internet and to visit cybercafé to solve their mathematics problems. To enhance the skills and lecture method of lecturers, they should be trained and retrained for the benefits of ICT. College management should make sufficient supply of computers into all the departments in the institution and assure that all students have unconditionalapproach to them during lectures. Management should also make wireless internet connection available in the institution and ensure free accessibility for both students and lecturers. Students must be properly orientated on the numerous benefits and opportunity that ICT offers in enhancing their academic performance in mathematics and science in general. (Popoola et. al.2017).

As considering future of mathematics with ICT the major resources is internet for lectures as well as for students .there are lot of websites containing content for different theorems and calculations. As updated form of theorems are also be available on internet. This will enhance the quality of lectures .Students may attend online lecture if they miss them in any reason during study time.

CONCLUSION:

The 21st century is based on science and technology The education system also needs advancement. The use of ICT facilitates modern education system with accuracy and updated knowledge. As concern with mathematics study, lot of new updates are there in mathematics with respect of time. The Indian teacher should be ready to face upcoming challenges for imparting new age education .Teacher has major role between students and new upcoming in mathematics. This is responsibility of them to convey these updated education to their students. They should use ICT for smooth flow of lectures.Efforts must be made by the educationist to change the process of teaching-learning in order to prepare the students to adjust themselves to the society; this could definitely create a new learning environment and information rich society.

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MAGNETO-ROTATORY THERMAL CONVECTION IN RIVLIN-ERICKSEN VISCOELASTIC FLUID HEATED FROM BELOW

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ABSTRACT:
A layer of Rivlin-Ericksen viscoelastic fluid heated from below is considered in the presence of uniform vertical magnetic field and rotation. Following the linearized stability theory and normal mode analysis, the paper mathematically established the condition for characterizing the oscillatory motions which may be neutral or unstable, for rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, opposite to gravity and a constant vertical adverse temperature gradient, are necessarily non-oscillatory, in the regime

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\leq 1 ,$$

where T_A is the Taylor number, Q isthe Chandrasekhar number, p_2 is the magnetic Prandtl number and F is the viscoelasticity parameter. The result is important since it hold for all wave numbers and for horizontal rigid boundaries of infinite extension at the top and bottom of the fluid, and the exact solutions of the problem investigated in closed form, is not obtainable.

KEY WORDS: Thermal convection; Rivlin-Ericksen Fluid; Rotation; Magnetic Field PES; Rayleigh number; Chandrasekhar number: Taylor number.

1. INTRODUCTION:

Stability of a dynamical system is closest to real life, in the sense that realization of a dynamical system depends upon its stability. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors (e.g., Bénard [1], Rayleigh [2], Jeffreys [3]) under different conditions. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [4]. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Bhatia and Steiner [5] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner [6]. They have found that the magnetic field stabilizes a viscoelastic (Maxwell) fluid just as the Newtonian fluid. Sharma [7] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted upon by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. In another study Sharma [8] has studied the stability of a layer of an electrically conducting Oldroyd fluid [9] in the presence of magnetic field and has found that the magnetic field has a stabilizing influence.

There are many viscoelastic fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [9] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen [10] has proposed a theoretical model for such one class of elastic-viscous fluids. Sharma and kumar [11] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar et al. [12] considered effect of rotation and magnetic field on Rivlin-Ericksen viscoelastic fluid and found that rotation has stabilizing effect, where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics.

Pellow and Southwell [13] proved the validity of 'principle of exchange of stability' (PES) for the classical Rayleigh-Bénard convection problem. Banerjee et al [14] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [15] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [16]. However no such result existed for non-Newtonian fluid configurations, in general and for Rivlin-Ericksen viscoelastic fluid configurations in particular. Banyal [17] have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of Rivlin-Ericksen viscoelastic fluids, this paper is an attempts to study Rivlin-Ericksen viscoelastic fluid heated from below in the presence of uniform vertical magnetic field and rotation. It has been established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid in the present configuration, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, the magnetic Prandtl number p_2 and the viscoelasticity parameter F, satisfy the inequality $\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2} \leq 1$. These results hold for all wave numbers with rigid boundaries of infinite horizontal extension at the top and bottom of the fluid.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

Considered an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained.

The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ and a uniform vertical magnetic field $\vec{H}(0,0,H)$.

The equation of motion, continuity, heat conduction, and Maxwells equations governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field and rotation are (Rivlin and Ericksen $[10]$; Chandrasekhar $[4]$ and Kumar et al $[12]$) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_o}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}$$
$$+\frac{\mu_e}{4\pi\rho_o}(\nabla\times\vec{H})\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right), \tag{1}$$

$$\nabla.\vec{q}=0, \tag{2}$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \tag{3}$$

$$\nabla.\vec{H}=0, \tag{4}$$

$$\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{q}+\eta\nabla^2\vec{H}, \tag{5}$$

Where ρ, p, T, ν, ν' and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature, kinematic viscosity, kinematic viscoelasticity and velocity of the fluid, respectively and $\vec{r}(x,y,z)$.

The equation of state for the fluid is

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \tag{6}$$

Where the suffix zero refer to the values at the reference level z = 0. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and α is the coefficient of thermal expansion. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity κ, and electrical resistivity η, are all assumed to be constant.

The initial state is one in which the velocity, density, pressure, and temperature at any point in the fluid are, respectively, given by

$$\vec{q}=(0,0,0),\ \rho=\rho(z),\ p=p(z),\ T=T(z), \tag{7}$$

Assume small perturbations around the basic solution and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$ and $\vec{h}=\left(h_x,h_y,h_z\right)$ denote respectively the perturbations in density ρ, pressure p, temperature T, velocity $\vec{q}(0,0,0)$ and the magnetic field $\vec{H}=(0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0[1-\alpha(T+\theta-T_0)]=\rho-\alpha\rho_0\theta, \text{ i.e. } \delta\rho=-\alpha\rho_0\theta. \tag{8}$$

Then the linearized perturbation equations are

$$\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\vec{g}\,\alpha\theta+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right), \tag{9}$$

$$\nabla.\vec{q}=0, \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \tag{11}$$

$$\nabla.\vec{h}=0, \tag{12}$$

$$\frac{\partial\vec{h}}{\partial t}=\left(\vec{H}.\nabla\right)\vec{q}+\eta\nabla^2\vec{h}. \tag{13}$$

Within the framework of Boussinesq approximation, equations (9) – (13), becomes

$$\frac{\partial}{\partial t}\nabla^2 w=\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^4 w+\frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right)+g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-2\Omega\frac{\partial\varsigma}{\partial z}, \tag{14}$$

$$\frac{\partial\varsigma}{\partial t}=\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\varsigma+2\Omega\frac{\partial w}{\partial z}-\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}, \tag{15}$$

$$\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta \tag{16}$$

$$\frac{\partial h_z}{\partial t}=H\frac{\partial w}{\partial z}+\eta\nabla^2 h_z \tag{17}$$

$$\frac{\partial\xi}{\partial t}=H\frac{\partial\varsigma}{\partial z}+\eta\nabla^2\xi \tag{18}$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and; $\varsigma=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ denote the z-component of vorticity and current density respectively.

3. Normal Mode Analysis

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$[w,\theta,h_z,\varsigma,\xi]=[W(z),\Theta(z),K(z),Z(z),X(z)]\exp\left(ik_x x+ik_y y+nt\right), \tag{19}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^{\ 2} + k_y^{\ 2}\right)^{\frac{1}{2}}$, is the resultant wave number, and n is the growth rate which is, in general, a complex constant.

Using (19), equations (14) – (18), in non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[\left(1+F\sigma\right)\left(D^2 - a^2\right) - \sigma\right]W = Ra^2\Theta + T_A DZ - Q\left(D^2 - a^2\right)DK \, , \tag{20}$$

$$\left[\left(1+F\sigma\right)\left(D^2 - a^2\right) - \sigma\right]Z = -DW - QDX \, , \tag{21}$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W \, , \tag{22}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW \, , \tag{23}$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ \, , \tag{24}$$

Where we have introduced new coordinates (x', y', z') = (x/d , y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $F = \frac{\nu'}{d^2}$, is the Rilvin-Ericksen kinematic viscoelasticity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2}$, is the Taylor number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $Z = \frac{2\Omega d}{\nu}Z_\oplus$, $K = \frac{Hd}{\eta}K_\oplus$, $X = \left(\frac{Hd}{\eta}\right)\left(\frac{2\Omega d}{\nu}\right)X_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the case where both the boundaries are rigid and perfectly conducting and are maintained at constant temperature, then the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (20)--(24), must possess a solution are

W = DW = 0, $\Theta = 0$, Z=0, K=0 and DX = 0 at z = 0 and z = 1. (25)

Equations (20)--(24), along with boundary conditions (25), poses an eigenvalue problem for σ and we wish to Characterize σ_i when $\sigma_r \geq 0$.

We first note that since W and Z satisfy $W(0) = 0 = W(1)$, K(0)=K(1) and $Z(0) = 0 = Z(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz \, ; \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz \text{ and} \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz \, , \tag{26}$$

Further, for $W(0)=0=W(1)$, K(0)=0 =K(1)and $Z(0)=0=Z(1)$, Banerjee et al.$[25]$ have show that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2\, dz \quad \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2\, dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2\, dz, \tag{27}$$

4. MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\} dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz$$

Proof: Multiplying equation (23) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0)=0=K(1)$, it follows that

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\} dz + \sigma_r p_2 \int_0^1 |K|^2\, dz = \text{Real part of} \left\{\int_0^1 K^* DW dz\right\} \leq \left|\int_0^1 K^* DW dz\right|$$

$$\leq \int_0^1 \left|K^* DW\right| dz,$$

$$\leq \int_0^1 \left|K^*\right|\left|DW\right| dz \;\; \leq \int_0^1 |K||DW| dz \leq \left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{28}$$

(Utilizing Cauchy-Schwartz-inequality),
which gives that

$$\int_0^1 |DK|^2\, dz \leq \left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{29}$$

inequality (28) on utilizing inequalities (26) and (29), gives

$$\left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{\pi^2}\left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{30}$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (28) on utilizing (30), give

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right) dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz, \tag{31}$$

This completes the proof of lemma**.**

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2\, dz \leq \frac{1}{\pi^4}\int_0^1 |DW|^2\, dz \text{ and } \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right) dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2\, dz$$

Proof: Multiplying equation (21) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (24) and appropriate boundary conditions (25), it follows that

$$(1+F\sigma_{r)}\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\} dz + \sigma_r \int_0^1 |Z|^2 dz + Q\int_0^1 \left\{|DX|^2 + a^2|X|^2\right\} dz + Qp_2\sigma_r \int_0^1 |X|^2\, dz$$

=Real part of $\left\{\int_0^1 DW^* Zdz\right\} \leq \left|\int_0^1 DW^* Zdz\right|$,

$$\leq \int_0^1 \left|DW^* Z\right| dz \leq \int_0^1 \left|DW^*\right|\left|Z\right| dz,$$

$$= \int_0^1 |DW||Z| dz \leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} , \tag{32}$$

(Utilizing Cauchy-Schwartz-inequality),
which gives that

$$\int_0^1 |DZ|^2 dz \leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{33}$$

inequality (32) on utilizing inequalities (26) and (33), gives

$$\left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \leq \frac{1}{\pi^2}\left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{34}$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (32) on utilizing (34), give

$$\int_0^1 |Z|^2 dz \leq \frac{1}{\pi^4}\int_0^1 |DW|^2 dz \text{ and } \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right) dz \leq \frac{1}{\pi^2}\int_0^1 |DW|^2 dz, \tag{35}$$

This completes the proof of lemma**.**
We prove the following theorems:
Theorem 1: If $R \rangle 0$, $F \rangle 0$, $Q \rangle 0$, $T_A \rangle 0$, $p_1 \rangle 0$, $p_2 \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K, Z, X) of equations (20) – (24), together with boundary conditions (25) is that

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \rangle 1 .$$

Proof: Multiplying equation (20) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$(1+F\sigma)\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz - \sigma\int_0^1 W^*\left(D^2 - a^2\right) W dz$$

$$= Ra^2\int_0^1 W^*\Theta dz + T_A\int_0^1 W^* DZdz - Q\int_0^1 W^* D\left(D^2 - a^2\right) Kdz, \tag{36}$$

Taking complex conjugate on both sides of equation (22), we get

$$\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* = -W^*, \tag{37}$$

Therefore, using (37), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* dz, \tag{38}$$

Also taking complex conjugate on both sides of equation (21), we get

$$(1+F\sigma^*)\left(D^2 - a^2\right)Z^* - \sigma^* Z^* = -DW^* - QDX^*, \tag{39}$$

Therefore, using (39), we get

$$\int_0^1 W^* DZdz = -\int_0^1 DW^* Zdz = (1+F\sigma^*)\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \sigma^*\int_0^1 Z^* Zdz + Q\int_0^1 ZDX^* dz, \quad (40)$$

Integrating by parts the third term on left hand side and using equation (24), and appropriate boundary condition (25), we get

$$\int_0^1 W^* DZdz = (1+F\sigma^*)\int_0^1 Z^*\left(D^2-a^2\right)Zdz - \sigma^*\int_0^1 Z^* Zdz + Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^* dz, \quad (41)$$

Also taking complex conjugate on both sides of equation (23), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^* = -DW^*, \quad (42)$$

Therefore, equation (42), using appropriate boundary condition (25), we get

$$\int_0^1 W^* D\left(D^2-a^2\right)Kdz = -\int_0^1 DW^*\left(D^2-a^2\right)Kdz = \int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz, \quad (43)$$

Substituting (38), (41) and (43), in the right hand side of equation (36), we get

$$(1+F\sigma)\int_0^1 W^*\left(D^2-a^2\right)^2 Wdz - \sigma\int_0^1 W^*\left(D^2-a^2\right)Wdz = -Ra^2\int_0^1 \Theta\left(D^2-a^2-p_1\sigma^*\right)\Theta^* dz$$

$$+T_A(1+F\sigma^*)\int_0^1 Z(D^2-a^2)Z^* dz - T_A\sigma^*\int_0^1 Z^* Zdz + T_A Q\int_0^1 X\left(D^2-a^2-p_2\sigma^*\right)X^* dz$$

$$-Q\int_0^1 K^*\left(D^2-a^2\right)^2 Kdz - Qp_2\sigma^*\int_0^1 K^*\left(D^2-a^2\right)Kdz, \quad (44)$$

Integrating the terms on both sides of equation (44) for an appropriate number of times and making use of the appropriate boundary conditions (25), we get

$$(1+F\sigma)\int_0^1 \left\{\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \sigma\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz = Ra^2\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz$$

$$+Ra^2 p_1\sigma^*\int_0^1 |\Theta|^2 dz - T_A(1+F\sigma^*)\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz - T_A\sigma^*\int_0^1 |Z|^2 dz - T_A Q\int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz$$

$$-T_A Qp_2\sigma\int_0^1 |X|^2 dz - Q\int_0^1 \left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz, \quad (45)$$

now equating imaginary parts on both sides of equation (45), and cancelling $\sigma_i(\neq 0)$ throughout from imaginary part, we get

$$F\int_0^1 \left\{\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz$$

$$= -Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A F\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz + T_A\int_0^1 |Z|^2 dz - T_A Qp_2\int_0^1 |X|^2 dz + Qp_2\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\}dz,$$

(46)

Now R $\rangle$ 0, $p_2\rangle 0$, $p_1\rangle 0$, $Q\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (26), (27), (31) and (35), the equation (46) gives,

$$\left[(1+\pi^2F) - \left(\frac{T_AF}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right)\right]\int_0^1 |DW|^2 dz + I_1\langle 0, \quad (47)$$

Where

$$I_1 = F\int_0^1 \left\{2a^2|DW|^2 + a^4|W|^2\right\}dz + a^2\int_0^1 |W|^2 dz + Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A Q p_2 \int_0^1 |X|^2 dz, \tag{48}$$

Is positive definite, and therefore, we must have

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \rangle 1. \tag{49}$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then } \frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \rangle 1 . \tag{50}$$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 2: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation is that, $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$, where T_A is the Taylor number, Q is the Chandrasekhar number, p_2 is the magnetic Prandtl number and F is the viscoelasticity parameter, when both the boundaries are rigid.

or

The onset of instability in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F, satisfy the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$, when both the bounding surfaces are rigid.

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Theorem 3: If $(W, \Theta, K, Z, X, \sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (15) – (19), with R $\rangle$ 0 and,

$$\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1,$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ is that $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2} + \frac{T_A}{\pi^4} + \frac{Qp_2}{\pi^2}\right\} \leq 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state the above theorem as follow:-

Theorem 4: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field and rotation is that the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F must satisfy the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\rangle 1$, when both the bounding surfaces are rigid

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in Rivlin-Ericksen fluid heated from below i. e. when Q = 0 = T_A. (Sunil et al, 2002).

(ii). Magneto-thermal convection in Rivlin-Ericksen fluid heated from below (T_A=0), if $\frac{1}{(1+\pi^2 F)}\left(\frac{Qp_2}{\pi^2}\right)\le 1$,and when F=0 then we have $\left(\frac{Qp_2}{\pi^2}\right)\le 1$ (Gupta et al, 1986).

(iii). Rotatory-thermal convection in Rivlin-Ericksen fluid heated from below (Q = 0), if $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}\right\}\le 1$.

(iv) When F = 0, then we retrieve the result of Newtonian fluid by (Gupta et al, 1986) in the presence of uniform vertical magnetic field and rotation i. e.

$$\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\le 1$$

5. CONCLUSIONS:

This theorem mathematically established that the onset of instability in a Rivlin-Ericksen fluid in the presence of uniform vertical magnetic field and rotation cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, p_2 the magnetic Prandtl number and the viscoelasticity parameter F satisfy the inequality $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\le 1$, when both the bounding surfaces are rigid.

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having rigid boundaries at the top and bottom of the fluid, in the presence of uniform vertical magnetic field and rotation, parallel to the force field of gravity, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\frac{1}{(1+\pi^2 F)}\left\{\frac{T_A F}{\pi^2}+\frac{T_A}{\pi^4}+\frac{Qp_2}{\pi^2}\right\}\le 1$, and in particular PES is valid.

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LIMITATIONS TO COMPLEX GROWTH RATE IN MAGNETO-ROTATORY THERMOSOLUTAL CONVECTION IN COUPLE-STRESS FLUID IN POROUS MEDIUM

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ABSTRACT:
The thermosolutal instability of a couple-stress fluidVeronis (1965) type configuration acted upon by uniform vertical rotation and magnetic field, heated from below in a porous medium is investigated. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of couple-stress fluid convection with a uniform vertical rotation and magnetic field in porous medium, for the case of rigid boundaries shows that the complex growth rate σ of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle

$$\sigma_r^{\ 2} + \sigma_i^{\ 2} \langle \frac{R_s^{\ 2}\varepsilon^2}{\pi^4}\left[1-\left\{\frac{T_A P_l^{\ 2}}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-2},$$

in the right half of a complex σ-plane, where R_s is the thermosolutal Rayleigh number, Q is the Chandrasekhar number, T_A is the Taylor number, P_l is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in a magneto-rotatory couple-stress fluidof Veronis (1965) type configuration in porous medium heated from below.A similar result is also proved for Stern (1960) type of configuration.

KEY WORDS: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Rayleigh number; Chandrasekhar Number; Taylor number**.**
MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05**.**

1. INTRODUCTION:
A detailed account of the theoretical and experimental study of the onset of thermal instability in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1] and the Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion, except in the external force term. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [2]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can name the food processing industry, the chemical processing industry, solidification, and the centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium.The problem of thermohaline convection in a layer of fluid heated from below and

subjected to a stable salinity gradient has been considered by Veronis [3]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat Tabor and Matz [4]and some Antarctic lakes Shirtcliffe [5]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust Lister [6]. Generally it is accepted that comets consists of a dusty ‘snowball’ of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice - versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context Mc Donnel [7]. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth’s core where the Earth’s mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov [8] and Linden [9] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth’s rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [10] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [11] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge.

The theory of couple-stress fluid has been formulated by Stokes [12]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze - film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded – bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [12], couple-stresses appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [13] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [14] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [15] have considered a couple - stress fluid with suspended particles heated from below. In another study, Sunil et al. [16] have considered a couple- stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [17] have considered the thermal instability of a layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Pellow and Southwell [18] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [19] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [20] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [21]. However no such result existed for non-Newtonian fluid configurations in general andin particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [22] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in porous medium in the presence of uniform rotation and uniform magnetic field has been considered.The present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible couple-stress fluidconfiguration of Veronis[3] type, in porous medium heated from below in the presence of uniform vertical rotation and magnetic field, opposite to force field of gravity, when the bounding surfaces are of infinite horizontal extension, at the top and bottom of the fluid are rigid.A similar characterization theorem is also proved for Stern[23]type of configuration.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta^/\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1.

Let p, ρ, T, C, α, $\alpha^/$, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid (Chandrasekhar[1];Joseph[2];Stokes[12]) are

$$\frac{1}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu^/}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T, \quad (3)$$

$$E^/\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)C=\kappa^/\nabla^2 C, \quad (4)$$

$$\rho = \rho_0\,[1 - \alpha\,(T-T_0) + \alpha^/\,(C-C_0)], \quad (5)$$

Where the suffix zero refers to values at the reference level z = 0 and in writing equation (1), use has been made of Boussinesq approximation.Here E = $\in+(1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^/$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o, C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The magnetic permeability μ_e, the kinematic viscosityν, couple-stress viscosity $\mu^/$, the thermal diffusivity κ and the solute diffusivity $\kappa^/$ are all assumed to be constants.

The Maxwell's equations yield

$$\in \frac{d\vec{H}}{dt} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in \eta\nabla^2 \vec{H}\ , \tag{6}$$

and $\nabla \cdot \vec{H} = 0,$ (7)

where $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \in^{-1} \vec{q}\cdot\nabla$ stands for the convective derivative.

The steady state solution is

$\vec{q}(u,v,w) = (0,0,0)$, $T = T_0 - \beta z$, $C = C_0 - \beta^{/} z$,

$\rho = \rho_0 (1 + \alpha\beta z - \alpha^{/}\beta^{/} z)$. (8)

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, $\delta\rho$, θ, γ , $\vec{h}(h_x, h_y, h_z)$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,0)$ and velocity $\vec{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$\delta\rho = -\rho_0 (\alpha\theta - \alpha^{/}\gamma)$. (9)

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}(\alpha\theta - \alpha^{/}\gamma) - \frac{1}{k_1}\left(\nu - \frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H} + \frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{10}$$

$$\nabla.\vec{q} = 0, \tag{11}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta\ , \tag{12}$$

$$E^{/}\frac{\partial\gamma}{\partial t} = \beta^{/} w + \kappa^{/}\nabla^2\gamma\ , \tag{13}$$

$$\in\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in\eta\nabla^2\vec{h}\ , \tag{14}$$

and $\nabla.\vec{h} = 0.$ (15)

3. NORMAL MODES ANALYSIS:

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$[w, \theta, h_z, \gamma, \varsigma, \xi] = [W(z), \Theta(z), K(z), \Gamma(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt)$, (16)

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k = (\sqrt{k_x^2 + k_y^2})$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively. $W(z), K(z), \Theta(z), Z(z)$ and $X(z)$ are the functions of z only.

Using (16), equations (10)-(15), within the framework of Boussinesq approximations, in the non-dimensional form transform to

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$$\left(D^2 - a^2\right)\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]W = Ra^2\Theta - R_s a^2\Gamma + T_A DZ - Q\left(D^2 - a^2\right)DK , \qquad (17)$$

$$\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]Z = -DW - QDX , \qquad (18)$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W , \qquad (19)$$

$$\left(D^2 - a^2 - E^{/} p_3\sigma\right)\Gamma = -W , \qquad (20)$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW , \qquad (21)$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ , \qquad (22)$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number;, $p_3 = \frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu^{'}/(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $R_s = \frac{g\alpha^{'}\beta^{'}d^4}{\kappa^{'}\nu}$ is the thermosolutal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W = W_{\oplus}$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_{\oplus}$, $\Gamma = \frac{\beta^{'}d^2}{\kappa^{/}}\Gamma_{\oplus}, Z = \frac{2\Omega d}{\nu\varepsilon}Z_{\oplus}$, $K = \frac{Hd}{\varepsilon\eta}K_{\oplus}$, $X = \left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_{\oplus}$ and $D_{\oplus} = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, are perfectly conducting and maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (17)--(22), must possess a solution are

W = 0 =DW =Z=DX =K= $\Theta = \Gamma$, on both the horizontal boundaries, (23)

Equations (17)-(22), along with boundary conditions (23), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, K and Z satisfy $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [24]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz\ ,\quad \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz\ ,\quad \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz\ ,$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \text{ and } \int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz\ , \tag{24}$$

Further, for $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, Banerjee et al. [25] have shown that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz\ ,\quad \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz\ . \tag{25}$$

4. MATHEMATICAL ANALYSIS:

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2 \pi^2 E p_1 |\sigma|} \int_0^1 |DW|^2 dz\ .$$

Proof: Multiplying equation (19) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Θ namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\int_0^1 \left\{ |D\Theta|^2 + a^2 |\Theta|^2 \right\} dz + E\sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of} \left\{ \int_0^1 \Theta^* W dz \right\},$$

$$\leq \left| \int_0^1 \Theta^* W dz \right| \leq \int_0^1 |\Theta^* W| dz \leq \int_0^1 |\Theta^*||W| dz\ ,$$

$$\leq \int_0^1 |\Theta||W| dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{26}$$

(Utilizing Cauchy-Schwartz-inequality),

So that the fact that $\sigma_r \geq 0$, we obtain from the above that

$$a^2 \int_0^1 |\Theta|^2 dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{27}$$

Multiplying equation (19) and its complex conjugate, and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Θ namely $\Theta(0) = 0 = \Theta(1)$, we get

$$\int_0^1 \left| \left(D^2 - a^2\right)\Theta \right|^2 dz + 2Ep_1\sigma_r \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz + E^2 p_1^2 |\sigma|^2 \int_0^1 |\Theta|^2 dz = \int_0^1 |W|^2 dz\ , \tag{28}$$

Since $\sigma_r \geq 0$ therefore the equations (28) give,

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{E^2 p_1^2 |\sigma|^2} \int_0^1 |W|^2 dz \tag{29}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (27) on utilizing (29), gives

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2\pi^2 E p_1 |\sigma|} \int_0^1 |DW|^2 dz \, , \tag{30}$$

This completes the proof of lemma 1.

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^2 E' p_3 |\sigma|} \int_0^1 |DW|^2 dz \; .$$

Proof: Multiplying equation (20) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{ |D\Gamma|^2 + a^2 |\Gamma|^2 \right\} dz + E' \sigma_r p_3 \int_0^1 |\Gamma|^2 dz = \text{Real part of} \left\{ \int_0^1 \Gamma^* W dz \right\},$$

$$\leq \left| \int_0^1 \Gamma^* W dz \right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*| |W| dz \, ,$$

$$\leq \int_0^1 |\Gamma| |W| dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{31}$$

(Utilizing Cauchy-Schwartz-inequality),

So that, since $\sigma_r \geq 0$, we obtain from the above that

And thus, we get

$$a^2 \int_0^1 |\Gamma|^2 dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{32}$$

Multiplying equation (20) and its complex conjugate, and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, we get

$$\int_0^1 \left| \left(D^2 - a^2 \right) \Gamma \right|^2 dz + 2E' p_3 \sigma_r \int_0^1 \left(|D\Gamma|^2 + a^2 |\Gamma|^2 \right) dz + E'^2 p_3{}^2 |\sigma|^2 \int_0^1 |\Gamma|^2 dz = \int_0^1 |W|^2 dz \; , \tag{33}$$

Since $\sigma_r \geq 0$ therefore the equations (33) give,

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{E'^2 p_3{}^2 |\sigma|^2} \int_0^1 |W|^2 dz \tag{34}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (29) on utilizing (30) and (24), gives

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2\pi^2 E' p_3 |\sigma|} \int_0^1 |DW|^2 dz \, , \tag{35}$$

This completes the proof of lemma 2.

Lemma 3: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{ |DK|^2 + a^2 |K|^2 \right\} dz \leq \frac{1}{\pi^2} \int_0^1 |DW|^2 dz$$

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Proof: Multiplying equation (21) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0) = 0 = K(1)$, it follows that

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of} \left\{\int_0^1 K^* DW dz\right\} \le \left|\int_0^1 K^* DW dz\right| \le \int_0^1 |K^* DW| dz,$$

$$\le \int_0^1 |K^*||DW| dz \le \int_0^1 |K||DW| dz \le \left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{36}$$

(Utilizing Cauchy-Schwartz-inequality),
Inequality (36) on utilizing (24), gives

$$\left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \le \frac{1}{\pi^2} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \tag{37}$$

Since $\sigma_r \ge 0$ and $p_2 \rangle 0$, hence inequality (36) on utilizing (37), give

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz \le \frac{1}{\pi^2} \int_0^1 |DW|^2 dz, \tag{38}$$

This completes the proof of lemma 3**.**
Lemma 4: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \le \frac{P_l^2}{(1+\pi^2 F)^2} \int_0^1 |DW|^2 dz \tag{39}$$

Proof: Multiplying equation (18) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (22) and appropriate boundary conditions (23), it follows that

$$\frac{F}{P_l} \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 |Z|^2 dz + Q\int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz + Qp_2\sigma_r \int_0^1 |X|^2 dz$$

$$= \text{Real part of } \left\{\int_0^1 DW^* Z dz\right\} \le \left|\int_0^1 DW^* Z dz\right|,$$

$$\le \int_0^1 |DW^* Z| dz \le \int_0^1 |DW^*||Z| dz,$$

$$= \int_0^1 |DW||Z| dz \le \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}}, \tag{40}$$

(Utilizing Cauchy-Schwartz-inequality),
Utilizing the inequality (24), (40) gives that

$$\frac{(1+\pi^2 F)}{P_l} \int_0^1 |Z|^2 dz \le \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \tag{41}$$

Inequality (40) on utilizing (41), gives

$$\int_0^1 |Z|^2 dz \leq \frac{P_l^2}{(1+\pi^2 F)^2} \int_0^1 |DW|^2 dz , \tag{42}$$

This completes the proof of lemma 4**.**
Now we prove the following theorems:

Theorem 1: If $R\rangle\, 0\,, R_s\rangle 0\;\; F\rangle\, 0,\;\; Q\rangle 0\,,\;\; T_A\rangle\, 0,\;\; P_l\rangle 0\,,\;\; p_1\rangle 0\,,\;\; p_2\rangle 0\,,\;\; \sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, K, Z, X)$ of equations (17) – (22), together with boundary conditions (23) is that

$$|\sigma| \langle \frac{R_s \varepsilon}{\pi^2} \left[1 - \left\{ \frac{T_A P_l^2}{(1+\pi^2 F)^2} + \varepsilon \left(\frac{Q p_2}{\pi^2} \right) \right\} \right]^{-1} .$$

Proof: Multiplying equation (17) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l} \int_0^1 W^* \left(D^2 - a^2\right)^2 W dz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 W^* \left(D^2 - a^2\right) W dz$$

$$= Ra^2 \int_0^1 W^* \Theta dz - R_s a^2 \int_0^1 W^* \Gamma dz + T_A \int_0^1 W^* DZ dz - Q \int W^* D(D^2 - a^2) K dz \;, \tag{43}$$

Taking complex conjugate on both sides of equation (19), we get

$$\left(D^2 - a^2 - E p_1 \sigma^*\right) \Theta^* = -W^* , \tag{44}$$

Therefore, using (44), we get

$$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta \left(D^2 - a^2 - E p_1 \sigma^*\right) \Theta^* dz , \tag{45}$$

Taking complex conjugate on both sides of equation (20), we get

$$\left(D^2 - a^2 - E' p_3 \sigma^*\right) \Gamma^* = -W^* , \tag{46}$$

Therefore, using (46), we get

$$\int_0^1 W^* \Gamma dz = -\int_0^1 \Gamma \left(D^2 - a^2 - E' p_3 \sigma^*\right) \Gamma^* dz , \tag{47}$$

Also taking complex conjugate on both sides of equation (18), we get

$$\frac{F}{P_l} \left(D^2 - a^2\right) Z^* - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l} \right) Z^* + Q D X^* = -D W^* , \tag{48}$$

Therefore, using (48), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \frac{F}{P_l} \int_0^1 Z^* \left(D^2 - a^2\right) Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 Z^* Z dz + Q \int_0^1 Z D X^* dz \;,$$

Integrating by parts the third term on left hand side and using equation (22), and appropriate boundary condition (23), we get

$$\int_0^1 W^* DZ dz = \frac{F}{P_l} \int_0^1 Z^* \left(D^2 - a^2\right) Z dz - \left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 Z^* Z dz + Q \int_0^1 X \left(D^2 - a^2 - p_2 \sigma\right) X^* dz , \tag{49}$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[D^2 - a^2 - p_2\sigma^*\right]K^* = -DW^*, \tag{50}$$

Therefore, equation (50), using appropriate boundary condition (23), we get

$$\int_0^1 W^* D\left(D^2 - a^2\right)Kdz = -\int_0^1 DW^*\left(D^2 - a^2\right)Kdz = \int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right)K^*dz\ , \tag{51}$$

Substituting (45), (73), (49) and (41), in the right hand side of equation (43), we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 Wdz - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 W^*(D^2 - a^2)Wdz = -Ra^2\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz$$

$$+ R_s a^2\int_0^1 \Gamma(D^2 - a^2 - E^/ p_3\sigma^*)\Gamma^* dz + \frac{T_A F}{P_l}\int_0^1 Z(D^2 - a^2)Z^* dz - T_A\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 ZZ^* dz$$

$$+T_A Q\int_0^1 X\left(D^2 - a^2 - p_2\sigma\right)X^* dz - Q\int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right)K^* dz, \tag{52}$$

Integrating the terms on both sides of equation (52) for an appropriate number of times and making use of the appropriate boundary conditions (23), we get

$$\frac{F}{P_l}\int_0^1 \left\{\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz$$

$$= Ra^2\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2 + Ep_1\sigma^*|\Theta|^2\right)dz - R_s a^2\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2 + E^/ p_3\sigma^*|\Gamma|^2\right)dz$$

$$-\frac{T_A F}{P_l}\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz - T_A\left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 |Z|^2 dz - T_A Q\int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz$$

$$-T_A Qp_2\sigma\int_0^1 |X|^2 dz - Q\int_0^1 \left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz, \tag{53}$$

now equating imaginary parts on both sides of equation (53), and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\frac{1}{\varepsilon}\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz = -Ra^2 Ep_1\int_0^1 |\Theta|^2 dz + R_s a^2 E^/ p_3\int_0^1 |\Gamma|^2 dz$$

$$+\frac{T_A}{\varepsilon}\int_0^1 |Z|^2 dz + Qp_2\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz - T_A Qp_2\int_0^1 |X|^2 dz\ , \tag{54}$$

Now R $\rangle$ $0, Q\rangle 0\ \varepsilon\rangle 0\ p_2\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (35), (39) and (42), the equation (54) gives,

$$\frac{1}{\varepsilon}\left[1 - \left\{\frac{P_l^{\,2}T_A}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Qp_2}{(\pi^2}\right) + \varepsilon\left(\frac{R_s}{\pi^2|\sigma|}\right)\right\}\right]\int_0^1 |DW|^2 dz + I_1\langle 0, \tag{55}$$

Where $I_1 = \frac{a^2}{\varepsilon}\int_0^1 |W|^2 dz + Ra^2 Ep_1\int_0^1 |\Theta|^2 dz + T_A Qp_2\int_0^1 |X|^2 dz$, is positive definite.

and therefore , we must have

$$|\sigma|\langle\frac{R_s\varepsilon}{\pi^2}\left[1 - \left\{\frac{T_A P_l^{\,2}}{(1+\pi^2 F)^2} + \varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}\ , \tag{56}$$

Hence, if

$\sigma_r \geq 0$ and $\sigma_i \neq 0$, then $|\sigma| \langle \frac{R_s \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^2}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}$, (57).

And this completes the proof of the theorem.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state and prove a theorem 1 as follow:-

Theorem 2: If R $\langle$ 0 , $R_s \langle 0$ F $\rangle$ 0, Q $\rangle$ 0, $T_A \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution $(W, \Theta, \Gamma, K, Z, X)$ of equations (17) – (22), together with boundary conditions (23) is that

$$|\sigma| \langle \frac{|R| \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^2}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}, \qquad (58)$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (14) –(17) and proceeding exactly as in Theorem 1and utilizing the inequality (30), we get the desired result.

CONCLUSIONS

The inequality (57) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as

$$\sigma_r^2 + \sigma_i^2 \langle \frac{R_s^2 \varepsilon^2}{\pi^4}\left[1-\left\{\frac{T_A P_l^2}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-2},$$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces rigid, in the presence of uniform vertical rotation and magnetic field parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, must lie inside a semi-circle in the right half of the $\sigma_r \sigma_i$ - plane whose centre is at the origin and radius is $\frac{R_s \varepsilon}{\pi^2}\left[1-\left\{\frac{T_A P_l^2}{(1+\pi^2 F)^2}+\varepsilon\left(\frac{Qp_2}{\pi^2}\right)\right\}\right]^{-1}$,where R_s is the thermosolutal Rayleigh number, Q is the Chandrasekhar number, T_A is the Taylor number, P_l is the dimensionless medium permeability of the porous medium, ε is the porosity and F is the couple-stress parameter. The inequality (58) established a similar result for Stern (1960) type of the configuration.

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HALL EFFECT ON MAGNETO-THERMAL STABILITY OF VISCOELASTIC FERROMAGNETIC FLUID SATURATING POROUS MEDIUM

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Abstract:
The effect of Hall Currents on the thermal stability of an electrically conducting ferromagnetic rheological fluid heated from below saturating a porous medium has been investigated. The rheology of the ferromagnetic fluid is described by the Walters'(model B`). The exact solutions of the eigen-value problem for stress- free boundary surfaces are obtained analytically Galerkin technique to find the thermal Rayleigh number for onset of both oscillatory and non- oscillatory modes. It is observed that thermal Rayleigh number decrease with increasing medium permeability and Hall Current, whereas magnetic field and magnetization showing opposite result for both oscillatory and non- oscillatory modes. The principle of exchange of stabilities is not valid. The oscillatory modes are introduced due to viscoelastic parameter F. in oscillatory mode viscoelastic parameter, magnetic field, magnetization as well as Hall current gives stabilizing effect for certain condition, whereas medium permeability showing destabilizing effect. These results have also been shown graphically.

Keywords: Porous medium, Hall current, Galerkin Weighted Residual method, Ferromagnetic Fluids, Viscoelasticity, Magnetic field and Magnetization.

INTRODUCTION

Magnetic fluids which are formed by dispersion of stable non - conducting colloidal suspension of magnetic nanoparticles in a conducting carrier liquid such as heptane's, water or kerosene etc., are known as ferromagnetic fluids. These fluids have some special features like polarization force and body couple which are not found in ordinary fluids. The behavior of these fluids are homogeneously continuum and exhibit various fascinating phenomena. These fluids are not found in nature but are artificially synthesized and find diverse application of ferromagnetic fluids in many science field such as instrumentation, lubrication, printing, vacuum technology, vibration damping, metal recovery, acoustics and medicine, commercial usage of these ferromagnetic fluid includes vacuum feedthroughs of semiconductor (Moskowitz, 1975) ,pressure seals used in compressors and blowers (Brooks, 1955), sealing of rotating shaft in computer disk drives(Bailey, 1983), cooling loudspeakers (Hathaway, 1979) attracted many researchers towards this field. Significant contribution is given by various researchers toward ferromagnetic fluid and their application by taking theoretical and experimental aspects.

Convection stability of ferromagnetic fluid for fluid layer heated from below in the presence of uniform vertical magnetic field has been discussed in detail by Finlayson (1970). He found that there is induction of convection due to temperature and temperature gradient across the layer which leads to magnetization of ferrofluids.Lalas and Carmi (1971) analyzed thermoconductive stability of ferrofluids and proves that linear and energies theories give identical results for stationary convection in ferrofluids. Shliomis (1974) has studied the mechanisms of relaxation of the magnetization of a suspension.

Earlier many authors considered Bernard's convection in non-porous (Siddheshwar, 1993, 1995; Sunil, Sharma and Shandil, 2006; Aggarwal and Prakash, 2009). In recent years researchers concentrate on the convective flow and stability of fluid layer in porous medium which was earlier considered by(Lapwood, 1948) and (Wooding, 1960).

Growing importance of non- Newtonian fluid in geophysical fluid dynamics chemical technology and petroleum industry aroused interest of many researchers toward studies of convective fluid motion in porous medium because of its important application in prediction of ground water movement, in atmospheric physics and recovery of crude oil from porous of rocks. There is vast variety of non- Newtonian fluids. Principle types of non- Newtonian fluid include: couple stress fluids, viscoelastic fluids (Rivlin- Ericksian fluids, Walters' (Model B′) fluid), plastic solids, power –law fluids, time dependent etc.Walters(Walters, 1960) studied the motion of oscillation flow of the viscoelastic liquid in a concentric- sphere elastoviscometer. He concludes that concentric-sphere elastoviscometer and coaxial- cylinder elastoviscometer both discriminant exclusive viscoelastic liquids in same way. Walters(1962) again studied the behaviour of non– Newtonian elasto-viscous fluids at small rates of shear which are characterized by a general linear equation of state. With growing importance of non- Newtonian viscoelastic fluids in science and technology has also been studied by many authors (Sharma and Kumar, 1997; Kumar and Sharma, 2000; Kumar*et al.*, 2004; Ali *et al.*, 2012; Pandey*et al.*, 2016).

If electric current flow through a conductor in magnetic field which reduce conductivity parallel to the electric field and hence, the current is reduced in the direction normal to both electric and magnetic field.This phenomenon in the literature is known as Hall effect.The Hall current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. The effect of Hall current on thermal instability has been studies by Aggarwal in the presence of dusty couple fluids (Aggarwal and Verma, 2016). In this research paper he has conclude that effect of Hall current on thermal convection with dusty couple particle show destabilizing effect and also found that oscillatory modes are produced in the present of Hall current. P. Narayana studied the effect of rotating micro polar fluid and radiation in Hall current (Narayana, 2013). He observed that with increase in the value of Hall current parameter micro – rotation profile decreases. There are many several authors studied the effect of Hall effect on different thermal instability(Gupta, 1967; Raghavachar and Gothandaraman, 1989; Sharma and Thakur, 2000; Sunil *et al.*, 2005; Aggarwal and Verma, 2017).

Motivated by the various application of rheology, medium porosity, Hall current and medium permeability, an attempt has been made to study on the criterion for stability of a layer of ferromagnetic viscoelastic fluid heated from below saturating a porous medium.The purpose of work is to analyze the influence of viscoelasticity in the magneto thermal convective thresholds in ferrofluids in the presence of presence of Hall current and rheology of the ferrofluids is described by the fluid Walters' (model B') saturating a porous medium in ferrofluids. In the present problem, we have studies the effect of Hall current on thermal stability of ferromagnetic fluid heated from below in porous medium in the presence of horizontal magnetic field.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Figure 1: Geometrical Configuration

Consider magnetic nanoparticles suspended in conducting various carrier liquids. This prevents on the porous matrix, which is electrically non-conducting and incompressible. We consider that horizontal magnetic field H_{mf} $(H, 0, 0)$ and uniform vertically downward gravity force ***g*** *(0,0, -g)* of a viscoelastic ferromagnetic porous medium is confined between the planes z = 0 and z = d. The temperatures at the lower and upper boundaries are assumed to be T_l and $T_u$respectively ($T_l > T_u$). Constant temperature difference $\Delta T\ (= \ T_l - T_u)$ is maintain between the boundaries.

The governing basic equations for the flow of an incompressible viscoelastic ferromagnetic fluids saturating a porous medium are

Equation of state

$$\rho_{mf} = \rho_l\left[1-\alpha_{mf}\left(T-T_l\right)\right], \tag{1}$$

Equation of continuity

$$\nabla.\vec{u}_{mf} = 0, \tag{2}$$

Equation of conservation of momentum are given by

$$\frac{\rho_l}{\varepsilon}\frac{D}{Dt^*}\overrightarrow{u_{mf}} = -\nabla p_{mf} + \vec{g}\rho_{mf} + \overrightarrow{M_{mf}}.\nabla\overrightarrow{H_{mf}} - \frac{\rho_l}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{u_{mf}} + \frac{\mu_{me}}{4\pi}\left(\nabla\times\overrightarrow{H_{mf}}\right)\times\overrightarrow{H_{mf}}, \tag{3}$$

where $\frac{D}{Dt^*} = \left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon}\left(\overrightarrow{q_{mf}}.\nabla\right)\right]$ is convective derivative.

Equation of energy

$$E\frac{\partial T}{\partial t^*} + \left(\overrightarrow{u_{mf}}.\nabla\right)T = k_{mf}\nabla^2 T, \tag{4}$$

Where in above equations, p_{mf} is the pressure, ρ_{mf} is density of ferromagnetic fluid, T is the temperature, α_{mf} is the thermal coefficient of expansion, $\vec{g} = (0,0,-g)$ is the gravitational acceleration acting vertically downward; t^* is the time, ρ_l is the ferromagnetic density at a reference temperature $T_l\left(z=0\right)$; $\vec{u}_{mf}=\left(u,v,w\right)$ is the filter velocity at coordinate x,y and z respectively; $\mu_{me}=4\pi\times10^{-7}\,m/A$ is the vacuum magnetic permeability; k_{mf} is thermal diffusivity.

To study traditional ferromagnetic concepts, the relationship between the induced field $\overrightarrow{B_{mf}}$, the external magnetic field, $\overrightarrow{H_{mf}}$ and the intensity of magnetization $\overrightarrow{M_{mf}}$, are given by following equations

$$\overrightarrow{M_{mf}}\times\overrightarrow{H_{mf}}=0 \tag{5}$$

$$\nabla.\overrightarrow{B_{mf}}=0,\nabla\times\overrightarrow{H_{mf}}=0 \tag{6}$$

$$\overrightarrow{B_{mf}}=\mu_0\left(\overrightarrow{H_{mf}}+\overrightarrow{M_{mf}}\right) \tag{7}$$

We superimpose that the magnetization is correspond to the magnetic field, while its magnitude $\left|\vec{M}_{\mathrm{mf}}\right|=M_{\mathrm{mf}}$ dependence on the magnetic field and temperature so that

$$\overrightarrow{M_{mf}}=\frac{H_{\mathrm{mf}}}{H}M_{\mathrm{mf}}\left(H,T\right) \tag{8}$$

In the presence of Hall currents Maxwell's equations reduces to

$$\frac{\partial\overrightarrow{H_{mf}}}{\partial t^*}=\frac{1}{\varepsilon}\left[\nabla\times\left(\overrightarrow{u_{mf}}\times\overrightarrow{H_{mf}}\right)\right]+\eta_{\mathrm{mf}}\nabla^2\overrightarrow{H_{mf}}-\frac{1}{4\pi N_{\mathrm{mf}}e}\nabla\times\left[\left(\nabla\times\overrightarrow{H_{mf}}\right)\times\overrightarrow{H_{mf}}\right] \tag{9}$$

$$\nabla.\overrightarrow{H_{mf}}=0 \tag{10}$$

All the quantities appeared in the above equations are already defined in nomenclature. In process $\overrightarrow{H_{mf}}$ and T are only two thermodynamics variables of $\overrightarrow{M_{mf}}$ magnetization. We assume the first approximation for temperature dependent magnetization $\overrightarrow{M_{mf}}=M_{\mathrm{mf}}\left(T\right)$ are given in the only form of temperature as

$$\frac{\overrightarrow{M_{mf}}}{M_l}=\left[1-\gamma_{mf}\left(T-T_l\right)\right] \tag{11}$$

where M_l represents the magnetization at lower temperature which known as reference temperature with $T=T_l$ at $z=0$, and

$$\gamma_{mf}=\frac{1}{M_l}\left(\frac{\partial\overrightarrow{M_{mf}}}{\partial T}\right)_H.$$

Assume basic state for the given system is $\overrightarrow{u_{mf}}\left(u,v,w\right)=u_b\left(0,0,0\right)=0,\ p_{mf}=p_b\left(z\right),T=T_b\left(z\right)=T_0-\beta\ z,$

$$\rho_{mf}=\rho_0\left(1+\alpha_{mf}\beta\ z\right)=\rho_b\left(z\right),\overrightarrow{M_{mf}}=M_b\left(z\right). \tag{12}$$

Let us consider infinitesimal perturbations around the initial state in the following forms $\overrightarrow{u_{mf}}(u,v,w)=0+\overrightarrow{u_{mf}},T=T_b+\theta,p_{mf}=p_b\left(z\right)+\delta p,\rho_{mf}=\rho_b+\delta\rho,$

$$\overrightarrow{H_{mf}}=H_b\left(H,0,0\right)+\vec{h}\left(h_x,h_y,h_z\right),\overrightarrow{M_{mf}}=M_b\left(z\right)+\delta M \tag{13}$$

Where $\delta\rho,\delta p,\delta M,\theta,h(h_x,h_y,h_z)$ and $u_{mf}(u,v,w)$ denote the perturbations in density, pressure, magnetization, temperature, magnetic field $H_{mf}(H,0,0)$ and filter velocity u_{mf} (zero initially), respectively which are superimposed into the basic state. The small change in $\delta\rho$ density and δM magnetization, caused mainly by the perturbations in concentration γ_{mf} and temperature θ, is given by

$$\frac{\delta\rho}{\rho_0}=-\alpha_{mf}\theta,\frac{\delta M}{M_0}=-\gamma_{mf}\theta \tag{14}$$

Using boussinesq approximation, linearized perturbations equations (1) – (4), (9) and (10) for ferromagnetic fluids are

$$\frac{\rho_0}{\varepsilon}\frac{\partial\overrightarrow{u_{mf}}}{\partial t^*}=-\nabla\delta p-\vec{g}\rho_0\alpha_{mf}\theta-\gamma_{mf}M_0.\nabla\overrightarrow{H_{mf}}\theta-\frac{\rho_0}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{u_{mf}}+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\overrightarrow{H_{mf}}, \tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{16}$$

$$E\left(\frac{\partial}{\partial t^*}-k_{mf}\nabla^2\right)\theta=\beta w, \tag{17}$$

$$\varepsilon\left(\frac{\partial}{\partial t^*}-\eta_{mf}\nabla^2\right)\vec{h}=\left(\overrightarrow{H_{mf}}.\nabla\right)\overrightarrow{u_{mf}}-\frac{\varepsilon}{4\pi N_{mf}e}\nabla\times\left[\left(\nabla\times\vec{h}\right)\times\overrightarrow{H_{mf}}\right], \tag{18}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0. \tag{19}$$

As we know that vector components of $\overrightarrow{u_{mf}}$, $\vec{h}$ and $\overrightarrow{H_{mf}}$ are

$$\overrightarrow{u_{mf}}=(u_x,v_y,w_z),\vec{h}=\left(h_x,h_y,h_z\right)$$

and

$\overrightarrow{H_{mf}}=H\left(H,0,0\right)$. The unknowns $u_x,v_y,w_z,h_x,h_y,h_z,\delta p$ can be reduced to w_z,h_z,φ,ψ by operating on eq. (15) and eq. (18) with $\hat{e}_z.curlcurl$ and using the identity curl operator i.e.,

$curlcurl \equiv graddiv - \nabla^2$ with the aid off eq. (16) and eq. (19),we get suitable result in the followings forms

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\nu-\frac{1}{k_1}\frac{\partial}{\partial t^*}\nu'\right]\nabla^2 w=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(g\alpha_{mf}-\frac{\gamma_{mf}M_0\nabla.\overrightarrow{H_{mf}}}{\rho_0}\right)\theta+\frac{\mu_e\overrightarrow{H_{mf}}}{4\pi\rho_0}\left(\nabla^2\frac{\partial h_z}{\partial x}\right), \quad (20)$$

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\right]\varphi=\frac{\mu_e H_{mf}}{4\pi\rho_0}\left(\frac{\partial\psi}{\partial x}\right), \quad (21)$$

$$\left(\frac{\partial}{\partial t^*}-\eta_{mf}\nabla^2\right)\psi=\frac{H_{mf}}{\varepsilon}\left(\frac{\partial\varphi}{\partial x}\right)+\frac{H_{mf}}{4\pi N_{mf}e}\frac{\partial}{\partial x}\left(\nabla^2 h_z\right), \quad (22)$$

$$\left(\frac{\partial}{\partial t^*}-\eta_{mf}\nabla^2\right)h_z=\frac{H_{mf}}{\varepsilon}\left(\frac{\partial w}{\partial x}\right)-\frac{H_{mf}}{4\pi N_{mf}e}\frac{\partial}{\partial x}\left(\nabla^2\psi\right), \quad (23)$$

where $\varphi=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ is the z-component of vorticity, $\psi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ is the z-component of current density and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is laplacian operator on the horizontal plane.

NORMAL MODE ANALYSIS

Analyzing the arbitrary disturbance into a complete set of normal modes and examine the stability of modes individually. Thus, we assume that perturbation quantities dependence on x, y and t are of the form

$$\left[w,\theta,\psi,\varphi,h_z\right]=\left[W^*(z),\Theta^*(z),X^*(z),Z^*(z),K^*(z)\right]\exp\left(iq_x x+iq_y y+\sigma^* t\right), \quad (24)$$

Where q_x and q_y are horizontal wavenumbers in the x and y-directions, respectively, $k\left[=\left(q_x^2+q_y^2\right)^{\frac{1}{2}}\right]$ is the resultant wave number of the disturbance and σ^* is the growth rate of disturbance (in general, a complex constant). For functions with this dependence on x, y and t, $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=-q^2\right)$ and $\left(\nabla^2=\frac{\partial^2}{\partial z^2}-q^2\right)$.

Using expression (24), linearized the perturbation equations (17), (20)- (23) in non –dimensional form become

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)W^*=-\frac{\alpha_{mf}a^2d^2}{\nu}\left(g-\frac{\gamma_{mf}M_0\nabla H_{mf}}{\rho_0\alpha_{mf}}\right)\Theta^*+\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)K^*, \quad (25)$$

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)Z^*=\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}X*, \quad (26)$$

$$\left(D^2-a^2-p_2\sigma\right)K^* = -\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}W^* + \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}X^*, \tag{27}$$

$$\left(D^2-a^2-p_2\sigma\right)X^* = -\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}Z^* - \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}\left(D^2-a^2\right)K^*, \tag{28}$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta^* = -\frac{\beta d^2}{\kappa}W^*. \tag{29}$$

where $a = qd, q_x = q\cos\theta, \sigma = \frac{\sigma^* d^2}{\nu}, D = \frac{d}{dz}$ and the non- dimensional parameters are $p_1 = \frac{\nu}{\kappa}$ is the thermal Prandtl number, $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number, $P_1 = \frac{k_1}{d^2}$ is the dimensionless medium permeability and $F = \frac{\nu'}{k_1}$ is non – dimension viscoelastic parameter.

Eliminating Θ^* from equation (25) and (29) by using algebraic operation $\left(D^2-a^2-Ep_1\sigma\right)$ with equation (25), one gets

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)-R_f a^2\right]W - \frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)K = 0, \tag{30}$$

also we can rewrite equation (27) as

$$\frac{ik_x Hd^2}{\varepsilon\eta_{mf}}W^* - \frac{ik_x Hd^2}{4\pi N_{mf}\eta_{mf}e}X^* + \left(D^2-a^2-p_2\sigma\right)K^* = 0, \tag{31}$$

Eliminating Z^* by operating equation (28) by $\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)$ and using equation (26), which yields

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2-p_2\sigma\right)-\frac{k_x^2 d^2 Q}{\varepsilon}\right]X^* + \frac{ik_x H}{4\pi N_{mf}\eta_{mf}e}\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)K^* = 0 \tag{32}$$

Method of solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the W, X and K of equations (31)-(33) with the corresponding boundary condition. The appropriate boundary conditions for both bounding surfaces free- free; which transform expression (25) in non- dimensional form

$$W = D^2W = 0, X = DX = 0, \Theta = 0, D^2Z = 0, K = 0 \text{ at } z=0 \text{ and } z=1 \tag{34}$$

Accordingly, the base functions W, X and K are taking in the following way:

$$W=\sum_{n=1}^{N}A_nW_n,\ X=\sum_{n=1}^{N}B_nX_n \text{ and } K=\sum_{n=1}^{N}C_nK_n \tag{35}$$

where $W_n=\left(z^n-2z^{n+2}+z^{n+3}\right), X_n=\left(z^{n+1}-2z^{n+2}+z^{n+3}\right), K_n=\left(z^n-z^{n+1}\right)$.

The trial solutions satisfying the dimensionless boundary conditions A_n, B_n and C_n are unknown coefficients, and $n=1,2,3,........,N.$

Calculating the residual or integrals in the limits from zero to unit, we obtain a set of linear homogeneous equations in the $3N$ unknowns A_n, B_n and C_n which admits a non – trivial solution only if its determinant is equal to zero. Once the determinant is equated to zero, thus it found the characteristic equation of the system in terms of the thermal Rayleigh number R_f , other parameters. For a first approximation, we take N =1; this produces the result

$$\begin{bmatrix} a_{11} & 0 & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{36}$$

where the elements of matrix are

$$a_{11}=\left(\frac{24}{5}+\frac{17}{35}\left(2a^2+Ep_1\sigma\right)+\frac{31}{630}a^2\left(a^2+Ep_1\sigma\right)\right)\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)-\frac{31}{630}R_f a^2,$$

$$a_{13}=-\frac{\iota k_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(4+\frac{2}{5}\left(2a^2+Ep_1\sigma\right)+\frac{17}{420}a^2\left(a^2+Ep_1\sigma\right)\right),$$

$$a_{21}=\frac{11}{1260}\left(\frac{\iota k_x Hd^2}{\varepsilon\eta_{mf}}\right), a_{22}=-\frac{1}{630}\frac{\iota k_x Hd^2}{4\pi N_{mf}e\eta_{mf}},$$

$$a_{23}=-\frac{1}{15}-\frac{1}{140}(a^2+p_2\sigma),$$

$$a_{32}=\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(\frac{-1}{15}-\frac{1}{140}\left(a^2+p_2\sigma\right)\right)-\frac{1}{140}\left(\frac{k_x{}^2Qd^2}{\varepsilon}\right),$$

$$a_{33}=\frac{\iota k_x Hd^2}{4\pi N_{mf}e\eta_{mf}}\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(-\frac{1}{3}-\frac{1}{30}a^2\right)$$

$$R_f=\frac{-\left(\begin{array}{c}56\left(\varepsilon+P_1\sigma-FP_1\varepsilon\sigma\right)M_3M_2+6\left(28+3a^2+3p_2\sigma\right)\left(-\varepsilon+P_1\left(-1+F\varepsilon\right)\sigma\right)M_2M_1\\ -33a^2P_1Q\left(17a^4+168\left(10+Ep_1\sigma\right)+a^2\left(336+17Ep_1\sigma\right)\right)M_1\cos^2\theta\end{array}\right)}{\left(62a^2P_1\varepsilon\left(-28M_3+3\left(28+3a^2+3p_2\sigma\right)M_1\right)\right)} \tag{37}$$

where $M_1=\left(28+3a^2+3p_2\sigma\right)\left(-\varepsilon+P_1\left(-1+F\varepsilon\right)\sigma\right)-3a^2P_1Q\cos^2\theta$,

$M_2=\left(3024+31a^4+306Ep_1\sigma+a^2\left(612+31Ep_1\sigma\right)\right)$,

$M_3 = a^2\left(10+a^2\right)M\left(\varepsilon + P_1\sigma - FP_1\varepsilon\sigma\right)\cos^2\theta$,

Where $R_f\left(=\left[g-\dfrac{\gamma_f M_0 \nabla . H_{mf}}{\rho_0\alpha_f}\right]\dfrac{\alpha_f \beta d^4}{\nu\kappa}\right)$ is the Rayleigh number for ferromagsnetic fluids, $Q\left(=\dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta_{mf}}\right)$ is the Chandrasekhar number, $M\left(=\dfrac{H}{4\pi N_{mf} e\eta_{mf}}\right)^2$ is the Hall parameter and $F=\dfrac{\nu'}{k_1}$ is the viscoelastic number.

Mathematical analysis

Non-oscillatory convection

When instability sets in stationary convection, the marginal state is characterized by putting $\sigma = 0$, in the dispersion relation (37), then the Rayleigh number for the case of stationary convection is given by

$$R_f^{stat} = \frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta + 6\left(28+3a^2\right)M_4M_5\varepsilon + 33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)} \tag{38}$$

Where $M_4 = \left(28+3a^2\right)\varepsilon + 3a^2P_1Q\cos^2\theta$

$M_5 = \left(3024+612a^2+31a^4\right)$

$M_6 = \left(1680+336a^2+17a^4\right)$

which is identical with the result of R_f^{stat} derived by sharma et al.(2017) for stationary convection. For stationary convection R_f^{stat} is independent of viscoelastic parameter since it vanishes with disappearing of σ which show same result as other non- viscoelastic ferromagnetic fluids.

In order to investigate the effects of magnetic field, medium permeability and Hall currents, we examine the behavior of $\dfrac{dR_f}{dQ},\dfrac{dR_f}{dM}$ and $\dfrac{dR_f}{dP_1}$ analytically.

$$\frac{dR_f}{dQ} = \frac{33M_6\cos^2\theta\begin{pmatrix}\left(28+3a^2\right)\left(27a^4P_1^2Q^2\left(3+4\cos[2\theta]+\cos[4\theta]\right)+28\left(10+a^2\right)M\varepsilon^2\left(4a^2+\cos[2\theta]\right)\right)+\\ \left(28+3a^2\right)^2\left(18P_1Q\varepsilon\left(4a^2+\cos[2\theta]\right)+24\left(28+3a^2\right)\varepsilon^2\right)+\\ 168\left(10+a^2\right)a^2MP_1Q\varepsilon\left(3a^2+\cos[2\theta]+a^2\cos[4\theta]\right)\end{pmatrix}}{496\varepsilon\left(3\left(28+3a^2\right)^2\varepsilon+a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2} \tag{39}$$

$$\frac{dR_f}{dM} = \frac{-231a^2 M_6\left(10+a^2\right)Q\cos^4\theta\left(56\varepsilon+3a^2\left(P_1Q+2\varepsilon\right)+3a^2P_1Q\cos\left[2\theta\right]\right)}{31\left(3\left(28+3a^2\right)^2\varepsilon+a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2}, \tag{40}$$

$$\frac{dR_f}{dP_1} = \frac{-\left(\begin{array}{l}9\left(28+3a^2\right)^4 M_5\varepsilon^2+6a^2\left(28+3a^2\right)^2 M_5\varepsilon\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta+\\ a^4M_5\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)^2\cos^4\theta-1386a^6\left(10+a^2\right)M_6MP_1^2Q^2\cos^6\theta\end{array}\right)}{31a^2P_1^2\left(3\left(28+3a^2\right)^2\varepsilon+a^2\left(9\left(28+3a^2\right)P_1Q+28\left(10+a^2\right)M\varepsilon\right)\cos^2\theta\right)^2} \tag{41}$$

It is clear from equations (39) that for stationary convection the magnetic field has a stabilizing effect for all wave numbers. Equation (40) and equation (41) shows that Hall effect and medium Permeability have destabilizing effect on the system.

In order to investigating the analytical effect of magnetization replaced R_f by R_f/π^4in equation (38), which yields

$$R_f^{stat} = \frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta+6\left(28+3a^2\right)M_4M_5\varepsilon+33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)}\frac{\pi^4}{1-\frac{\gamma_{mf}M_0\nabla H}{\rho_0\alpha_{mf}\,g}} \tag{42}$$

To see the effect of magnetization, we examine the behavior of $\frac{dR_f}{dM_0}$ analytically. Equation (42) yields:

$$\frac{dR_f^{stat}}{dM_0} = \frac{\left(56a^2\left(10+a^2\right)MM_5\varepsilon^2\cos^2\theta+6\left(28+3a^2\right)M_4M_5\varepsilon+33a^2M_4M_6P_1Q\cos^2\theta\right)}{\left(186a^2P_1\varepsilon M_4\left(28+3a^2\right)\right)}\frac{\pi^4}{\left(1-\frac{\gamma_{mf}M_0\nabla H}{\rho_0\alpha_{mf}\,g}\right)^2}\frac{\gamma_{mf}\nabla H}{\rho_0\alpha_{mf}\,g} \tag{43}$$

It is clear from equation (43) that the magnetization has stabilizing effect on the system.

Oscillatory convection

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of viscoelastic number parameter F. Equating the imaginary parts of equation (37) (by putting $\sigma = \iota\,\sigma_i$,), we obtain

$$\begin{aligned}&\sigma_i[(A_{14}c_1 - A_{11}c_2) + \sigma_i^2(A_{15}c_1 - A_{14}c_3 - A_{12}c_2 + A_{11}c_4) + \\ &\sigma_i^4(A_{16}c_1 - A_{15}c_3 - A_{13}c_2 + A_{12}c_4) + \sigma_i^6(A_{13}c_4 - A_{16}c_3)] = 0\end{aligned} \tag{44}$$

It is evident from equation (44) that σ_i may be either zero or non- zero, meaning that the modes may be either non- oscillatory or oscillatory and the principle of exchange of stabilities is not satisfied for the problem.

The case of overstability

Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (44) will admit of solutions with σ_i real. For oscillatory motions, the real part of σ *is* zero. Hence, putting $\sigma = \iota\sigma_i$, *in equation* (37) where σ_i is real and is the dimensional frequency; one obtains

$$R_f = \Delta_{\mathrm{Re}al} + i\sigma_i \Delta_{\mathrm{Im}g}, \tag{45}$$

$$\Delta_{\mathrm{Re}al} = \frac{-\begin{pmatrix} A_{11}c_1 + \sigma_i^2\left(A_{12}c_1 - A_{11}c_3 + A_{14}c_2\right) + \sigma_i^4\left(A_{13}c_1 - A_{12}c_3 + A_{15}c_2 - A_{14}c_4\right) + \\ \sigma_i^6\left(-A_{13}c_3 + A_{16}c_2 - A_{15}c_4\right) - \sigma_i^8\left(A_{16}c_4\right) \end{pmatrix}}{\left(c_1 - \sigma_i^2 c_3\right)^2 + \sigma_i^2\left(c_2 - \sigma_i^2 c_4\right)^2}$$

where (46)

$$\Delta_{Img} = [(A_{14}c_1 - A_{11}c_2) + \sigma_i^2(A_{15}c_1 - A_{14}c_3 - A_{12}c_2 + A_{11}c_4) + \sigma_i^4(A_{16}c_1 - A_{15}c_3 - A_{13}c_2 + A_{12}c_4) + \sigma_i^6(A_{13}c_4 - A_{16}c_3)] = 0 \tag{47}$$

Equation (44) implies for $\sigma_i \neq 0$ for oscillatory modes, therefore equation (45) implies that $\Delta_{\mathrm{Im}g} = 0$ which on simplification yields a dispersion relation as

$$[(A_{14}c_1 - A_{11}c_2) + \sigma_i^2(A_{15}c_1 - A_{14}c_3 - A_{12}c_2 + A_{11}c_4) + \sigma_i^4(A_{16}c_1 - A_{15}c_3 - A_{13}c_2 + A_{12}c_4) + \sigma_i^6(A_{13}c_4 - A_{16}c_3)] = 0 \tag{48}$$

Also equation (45) with $R_f^{OSC} = \Delta_{\mathrm{Re}al}$ on simplification gives the thermal Rayleigh number for oscillatory modes as

$$R_f^{OSC} = -\begin{pmatrix} A_{11}c_1 + \sigma_i^2\left(A_{12}c_1 - A_{11}c_3 + A_{14}c_2\right) + \\ \sigma_i^4\left(A_{13}c_1 - A_{12}c_3 + A_{15}c_2 - A_{14}c_4\right) + \\ \sigma_i^6\left(-A_{13}c_3 + A_{16}c_2 - A_{15}c_4\right) - \sigma_i^8\left(A_{16}c_4\right) \end{pmatrix}\left(\left(c_1 - \sigma_i^2 c_3\right)^2 + \sigma_i^2\left(c_2 - \sigma_i^2 c_4\right)^2\right)^{-1} \tag{49}$$

where $A_{11} = A_1 + A_3 + A_6,\ A_{12} = -A_2 + A_4 + A_7,\ A_{13} = A_5,\ A_{14} = B_1 + B_3 + B_6$

$A_{15} = -B_2 + B_4 + B_7,\ A_{16} = -B_5,\ A_1 = 56a^2\left(10 + a^2\right)\varepsilon^2 M\left(3024 + 612a^2 + 31a^4\right)\cos^2\theta$

$A_2 = 56a^2\left(10 + a^2\right)M\left(P_1 - P_1\varepsilon F\right)\left[\left(P_1 - P_1\varepsilon F\right)\left(3024 + 612a^2 + 31a^4\right) + 2\varepsilon E p_1\left(306 + 31a^2\right)\right]\cos^2\theta$

$A_3 = \left(\varepsilon^2\left(28 + 31a^2\right)^2 + 3a^2\varepsilon P_1 Q\left(28 + 31a^2\right)\cos^2\theta\right)\left(3024 + 612a^2 + 31a^4\right),$

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$$A_4 = \left(28\varepsilon + 31a^2\varepsilon + 3a^2P_1Q\cos^2\theta\right)\begin{pmatrix} 3P_1p_2\left(-1+F\varepsilon\right)\left(3024+612a^2+31a^4\right)+ \\ \left(-3p_2\varepsilon + P_1\left(-1+F\varepsilon\right)\left(28+3a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)\end{pmatrix},$$

$$A_5 = 3P_1p_2\left(-1+F\varepsilon\right)\left(-3p_2\varepsilon + P_1\left(-1+F\varepsilon\right)\left(28+31a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)$$

$$+3P_1p_2\left(-1+F\varepsilon\right)\begin{pmatrix} 3P_1p_2\left(-1+F\varepsilon\right)\left(3024+612a^2+31a^4\right)+ \\ \left(-3p_2\varepsilon + P_1\left(-1+F\varepsilon\right)\left(28+3a^2\right)\right)\left(306Ep_1+31Ep_1a^2\right)\end{pmatrix},$$

$$A_6 = 33a^2P_1Q\cos^2\theta\left(17a^4+336a^2+1680\right)\left(\left(28\varepsilon+3a^2\varepsilon\right)+3a^2P_1Q\cos^2\theta\right),$$

$$A_7 = -33a^2P_1Q\cos^2\theta\left(17a^2Ep_1+168Ep_1\right)\left(\left(28\varepsilon P_1+3a^2\varepsilon P_1\right)\left(-1+F\varepsilon\right)+3p_2\varepsilon\right)$$

$$\left(99a^2p_2P_1^2Q\cos^2\theta\left(17a^4+336a^2+1680\right)\left(-1+F\varepsilon\right)\right),$$

$$B_1 = 56a^2\left(10+a^2\right)M\left[2\varepsilon\left(P_1-P_1\varepsilon F\right)\left(3024+612a^2+31a^4\right)+\varepsilon^2Ep_1\left(306+31a^2\right)\right]\cos^2\theta,$$

$$B_2 = 56a^2\left(10+a^2\right)M\left(P_1-P_1\varepsilon F\right)^2Ep_1\left(306+31a^2\right)\cos^2\theta,$$

$$B_3 = -\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)\begin{pmatrix}\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(3024+31a^4+612a^2\right)+ \\ \left(-28\varepsilon-3a^2\varepsilon\right)\left(306Ep_1+31a^2Ep_1\right)\end{pmatrix}+$$

$$\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(-28\varepsilon-3a^2\varepsilon\right)\left(3024+31a^4+612a^2\right),$$

$$B_4 = 3p_2\left(-P_1+P_1\varepsilon F\right)\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)\left(306Ep_1+31a^2Ep_1\right)-$$

$$\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\begin{pmatrix}3p_2\left(-P_1+P_1\varepsilon F\right)\left(3024+31a^4+612a^2\right)+ \\ \left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(306Ep_1+31a^2Ep_1\right)\end{pmatrix}$$

$$-3p_2\left(-P_1+P_1\varepsilon F\right)\begin{pmatrix}\left(-3p_2\varepsilon-\left(P_1-P_1\varepsilon F\right)\left(28+3a^2\right)\right)\left(3024+31a^4+612a^2\right)+ \\ \left(-28\varepsilon-3a^2\varepsilon\right)\left(306Ep_1+31a^2Ep_1\right)\end{pmatrix},$$

$$B_5 = 9p_2^2\left(-P_1+P_1\varepsilon F\right)^2\left(306Ep_1+31a^2Ep_1\right),$$

$$B_6 = 33a^2P_1Q\cos^2\theta\begin{pmatrix}\left(168Ep_1+17a^2Ep_1\right)\left(28\varepsilon+3a^2\varepsilon+3a^2P_1Q\cos^2\theta\right)+ \\ \left(17a^4+336a^2+1680\right)\left(\left(28+3a^2\right)\left(P_1-F\varepsilon P_1\right)+3p_2\varepsilon\right)\end{pmatrix},$$

$$B_7 = 99a^2p_2P_1Q\cos^2\theta\left(168Ep_1+17a^2Ep_1\right)\left(-P_1+F\varepsilon P_1\right),$$

$$c_1 = 62a^2P_1\varepsilon\left(-3\varepsilon(28+3a^2)^2-28a^2(10+a^2)M\varepsilon cos^2\theta-9a^2P_1Q(28+3a^2)cos^2\theta\right),$$

$$c_2 = 62a^2P_1\varepsilon\begin{pmatrix}-18p_2\varepsilon(28+3a^2)+3(28+3a^2)^2\left(-1+F\varepsilon\right)- \\ 28a^2(10+a^2)MP_1cos^2\theta-27a^2p_2P_1Qcos^2\theta\end{pmatrix},$$

$c_3 = 62a^2 P_1 \varepsilon \left(18 p_2 (28+3a^2)\left(-1+F\varepsilon\right) - 27 p_2{}^2 \varepsilon\right)$, $c_4 = 27 p_2{}^2 a^2 P_1 \varepsilon \left(-62+62F\varepsilon\right)$.

Now, the oscillatory neutral solutions of equation (49) are obtained by firstly determine the roots of cubic equation (48) in term of $\sigma_i{}^2$. For overstability σ_i is real, at most one positive root is required of equation (48) for which the critical thermal Rayleigh number for oscillatory modes is obtained for various values of non- dimensional wave number which is seen in table no. 2 and related graphs on overstability convection.

Table 1. Rayleigh number with respect to wave number for stationary convection with M =100, Q = 10, P_1 =1.2, p_1 =7, p_2 =1, $\varepsilon = 0.5$, $\theta = 45^o$, $\alpha_{mf} = 10$, $\gamma_{mf} = 0.5$, $\rho_o = 10$, $\nabla H = 10$.

a	P_1 = 1.0	P_1 = 1.2	P_1 = 1.4	M = 80	M = 120	Q = 20	Q =30	M_0 =10	M_0 =20	M_0 = 30
1	149.613	132.092	120.093	137.656	127.914	187.814	257.868	13559.7	14329.1	15192.3
2	76.8884	72.2329	69.6816	78.173	67.8964	137.835	224.062	7414.42	7835.7	8307.73
3	75.4791	73.6116	73.3578	81.2685	68.0508	161.031	276.888	7555.5	7985.27	8466.31
4	89.4442	89.0194	90.2061	99.2831	81.5768	207.791	365.562	9137.49	9656.67	10238.4
5	111.794	112.55	115.102	126.227	102.637	271.883	483.697	11552.8	12209.2	12944.7
6	140.795	142.736	146.772	160.612	129.785	351.751	629.695	14651.3	15483.8	16416.5

Table 2. Rayleigh number with respect to wave number for overstability convection with M =100, Q =10, P_1 =1.2, p_1 =7, p_2 =1, $\varepsilon = 0.5$, F = 1.005, $\theta = 45^o$, $\alpha_{mf} = 10$, $\gamma_{mf} = 0.5$, $\rho_o = 10$, $\nabla H = 10$.

a	P_1 = 1.0	P_1 = 1.2	P_1 = 1.4	M = 80	M = 120	Q = 20	Q =30	F = 0.5	M_0 =10	M_0 =20	M_0 = 30
0.5	26011.8	20214.4	17789.8	31049	17447.9	22459.3	24787.9	22024.7	2074930	2192830	2324930
1.0	6510.65	6397.78	6308.96	5904.16	6958.65	7189.23	7986.19	7714.82	656707	694020	735829
1.5	5929.21	5901	5877.08	5166.05	6658.11	6651.06	7403.32	7254.79	605808	640229	678797
2.0	6586.15	6595.49	6600.13	5652.66	7550.06	7405.42	8217.5	8214.98	677001	715467	758568
2.5	7874.8	7924.13	7961.86	6714.67	9142.57	8841.33	9759.98	9995.13	813381	859596	911379
3.0	9740.79	9847.73	9933.42	8283.55	11419.9	10906.8	11966.6	12582	1010830	1068260	1132620
3.5	12242.6	12435.4	12592.6	10404.4	14474.2	13667.1	14898.8	16094.5	1276450	1348970	1430240
4.0	15483.7	15801.6	16062.6	13165.5	18445.6	17235	18667.9	20711	1621970	1714130	1817390

NUMERICAL RESULTS AND DISCUSSION

The equations (48) and (49) have been examined numerically using the software Mathematica version- 5.2. we have plotted the variation of Rayleigh number with respect to wavenumber using equation (38) for stationary case and equation (48) and equation(49)for overstability case, for the fixed permissible vales of the dimensionless parameters $M = 100, Q = 10, P_1 = 1.2, p_1 = 7, p_2 = 1, \varepsilon = 0.5, F = 1.005, \theta = 45^o, \alpha_{mf} = 10, \gamma_{mf} = 0.5, \rho_o = 10, \nabla H = 10$ as shown in table 1 and table 2.

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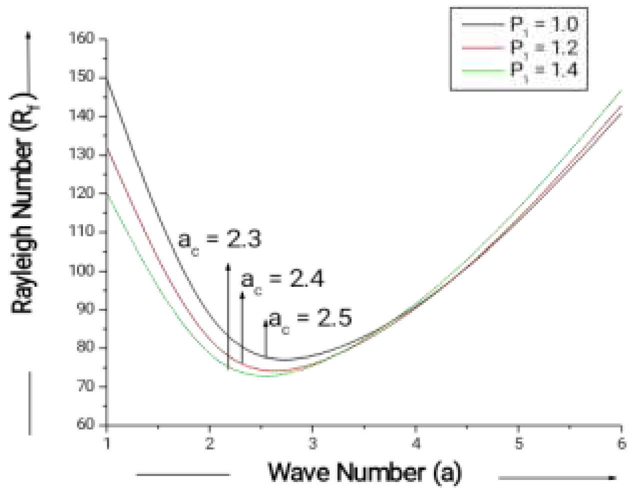

Figure 2: The variation of thermal Rayleigh Number R_f (Stationary) versus wave number **a** for three different values of the medium permeability parameter P_1 = 1.0, 1.2, 1.4.

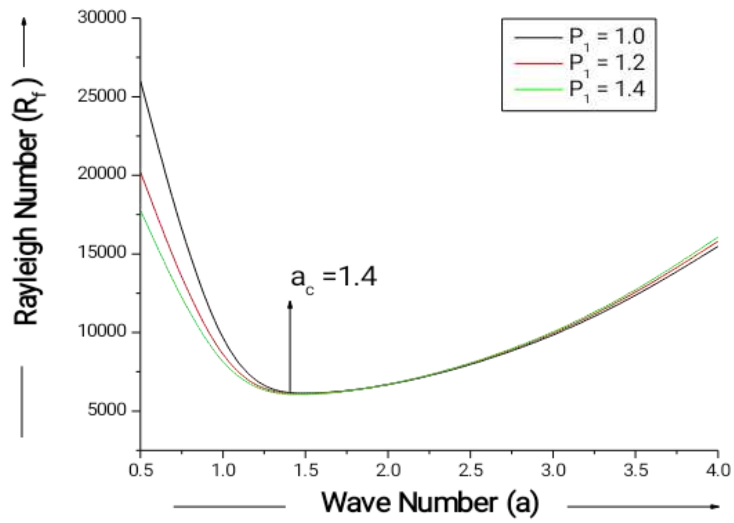

Figure 3: The variation of thermal Rayleigh Number R_f (overstability) versus wave number **a** for three different values of the medium permeability parameter P_1 = 1.0, 1.2, 1.4.

Figures 2 and 3 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of medium permeability parameter $P_1 = 1.0, 1.2, 1.4$. the Rayleigh number deceases with the increase in the parameter P_1 showing thereby the destabilizing effect of medium permeability parameter on the system but for $a \geq 4$ for stationary convection Rayleigh number increase with increase in the parameter P_1 and for the case of overstability wavenumber $a \geq 2$ Rayleigh number increase with increase in medium permeability thereby the stabilizing effect on the system.

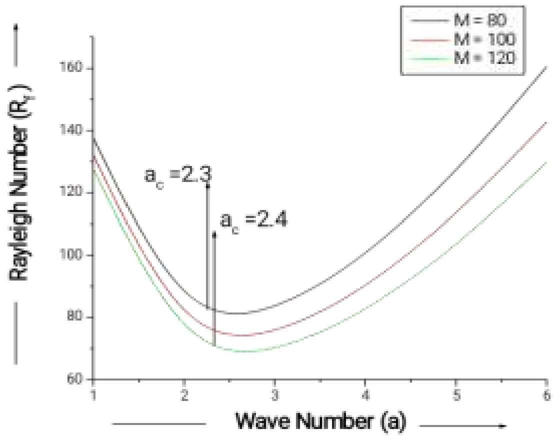

Figure 4: The variation of thermal Rayleigh Number R_f (Stationary) versus wave number **a** for three different values of the Hall current parameter M = 80, 100, 120.

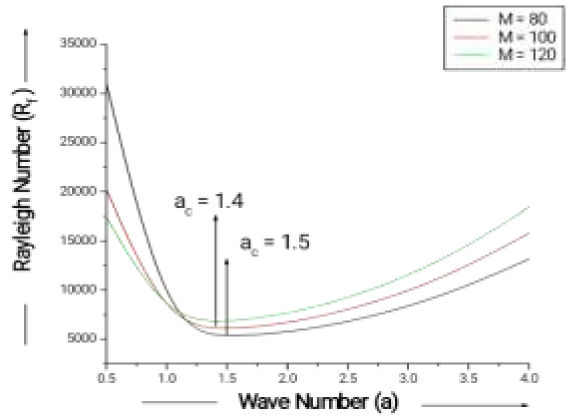

Figure 5: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the Hall current parameter **M** = 80, 100, 120.

Figures 4 and 5 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for various values of Hall Current parameter $M = 80,100,120$. For the case of stationary convection as well as overstability Rayleigh number deceases with the increase in the parameter M showing thereby the destabilizing effect of Hall Current parameter on the system whereas for wave number $a \geq 1.2$ in the case of overstability Rayleigh number increase with increase in the parameter M showing thereby stabilizing effect on the system.

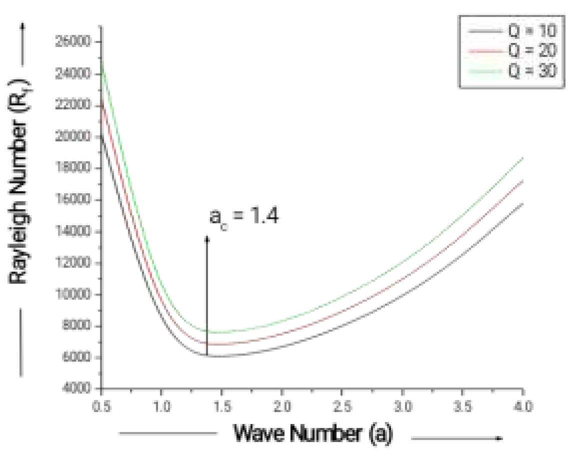

Figure 6: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (Stationary) versus wave number **a** for three different values of the magnetic field

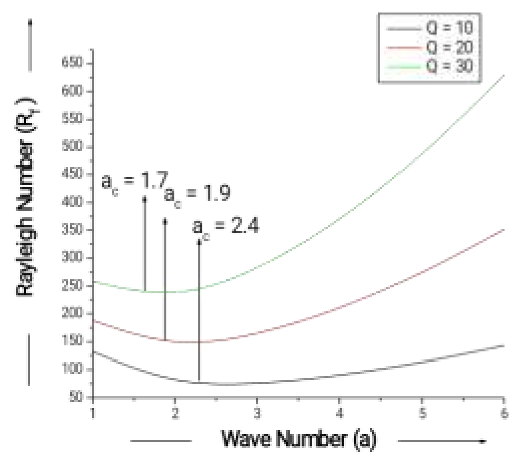

Figure 7: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the magnetic field parameter **Q** = 10, 20, 30.

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Figures 6 and 7 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of magnetic field parameter $Q = 10, 20, 30$. The Rayleigh number increase with the increase in the parameter Q showing thereby the stabilizing effect of magnetic field parameter

on the system.

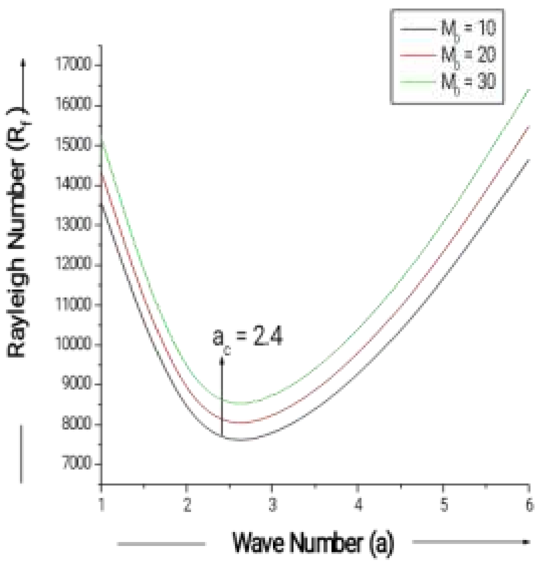

Figure 8: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (Stationary) versus wave number **a** for three different values of the magnetization parameter M_0 = 10, 20, 30.

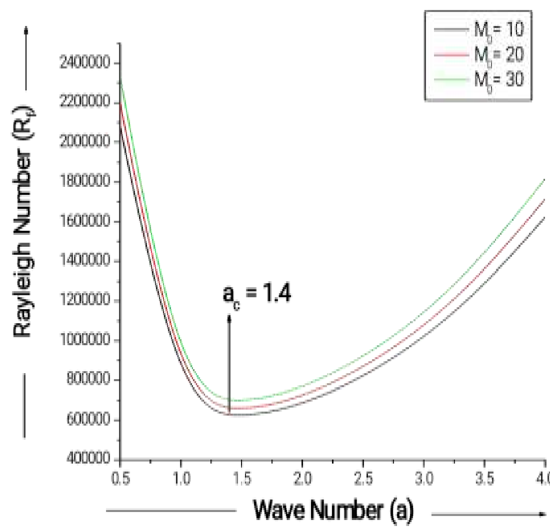

Figure 9: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for three different values of the magnetization parameter M_0 = 10, 20, 30.

Figures 8 and 9 represent the variation of thermal Rayleigh number R_f versus wavenumber a for stationary convection and the case of overstability respectively, for both cases for various values of magnetization parameter $M_0 = 10, 20, 30$. The Rayleigh number increase with the increase in the parameter M_0 showing thereby the stabilizing effect of magnetization parameter on the system.

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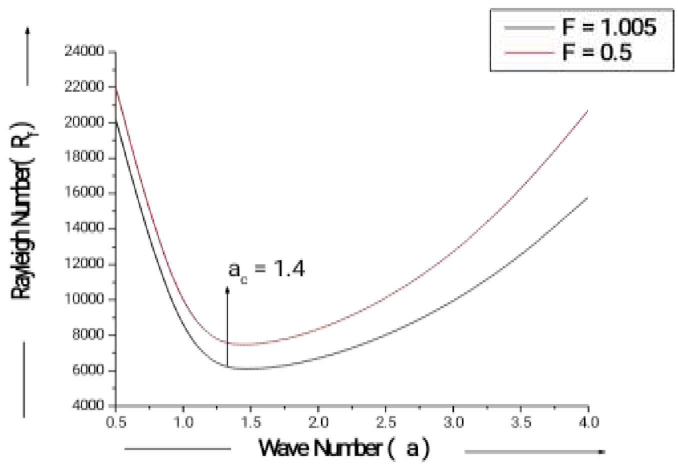

Figure 10: The variation of thermal Rayleigh Number $\mathbf{R_f}$ (overstability) versus wave number **a** for different values of the viscoelastic number parameter F = 0.5, 1.005.

Figure 10 represents the variation of thermal Rayleigh number R_f for the overstability convection with the wave number a for various value of viscoelastic number parameter $F = 0.5, 1.005$. The Rayleigh number increase with increase in the parameter F showing thereby the stabilizing effect of the viscoelastic number parameter on the system. It is also clear from the figure 2- 10 that overstability dominant mode of stability.

9. CONCLUSION

In the present paper, the combined effect of medium permeability, horizontal magnetic field, Hall effect, viscoelastic number and magnetization has been considered on the thermal stability of an elastic-viscous ferromagnetic fluid. The effect of various parameters such as medium permeability, horizontal magnetic field, Hall effect, viscoelastic number and magnetization has been investigated analytically as well as numerically. The main results from the analysis of paper are as follows.

- In order to investigate the effects of medium permeability, horizontal magnetic field, Hall effect and magnetization, we examine the behavior of $\frac{dR_f}{dP_1}, \frac{dR_f}{dQ}, \frac{dR_f}{dM}$ and $\frac{dR_f}{dM_0}$ analytically.
- It is found that magnetic field and magnetization have a stabilizing effect whereas medium permeability and Hall effect have a destabilizing effect on the system. Figures 2, 4, 6 and 8 support the analytic results graphically. The reasons for stabilizing effect of magnetic field and destabilizing effect of Hall currents are accounted by Chandrashekhar(Chandrashekhar, 1961). This is also valid for second- order fluids as well.
- It is found that for overstability convection magnetic field and magnetization as well as viscoelastic number parameter have a stabilizing effect whereas medium permeability and Hall current have a destabilizing effect on the system but for wavenumber $a \geq 2$ Hall

current as well as medium permeability parameter both shows stabilizing effect on the system.

- The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of Hall current (and hence magnetic field), it is valid under certain conditions.

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PROPAGATION OF WAVES IN FIBRE-REINFORCED ANISOTROPIC THERMOELASTIC PLATES

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Abstract

The propagation of waves in a homogeneous fibre-reinforced linear thermoelastic isotropic plate is investigated in the context of couple theory of thermoelasticity. Secular equations for symmetric and antisymmetric wave modes in completely separate terms are derived in closed form and isolated mathematical conditions. It is shown that the purely transverse motion (SH mode), which is not affected by thermal variations, gets decoupled from rest of the motion of wave propagation. The results for isotropic materials theory of thermoelasticity have been obtained as particular case. The special cases, such as short wavelength waves and thin plate waves of the secular equations are also discussed. Finally, the numerical solution is carried out for fibre-reinforced isotropic plate. The dispersion curves for symmetric and antisymmetric wave modes and attenuation coefficient are presented in order to illustrate and compare the theoretical results. The theory and numerical computations are found to be in close agreement.

1. Introduction

A fibre reinforced composite can be designed for all conditions of stresses that may occur and in accordance with the principle of mechanics. The characteristic property of a reinforced composite is that its components act together as single anisotropic units as long as they remain in the elastic condition. The wave propagation in a reinforced medium plays a very interesting role in civil engineering and geophysics. The studies of propagation, reflection and transmission of waves are of a great interest to seismologists. The propagation of waves depends upon the ground vibration and the physical properties of the structure. Thus the role of the propagation of surface waves in a fibre reinforced medium cannot be neglected in any way.

The dynamical problem of propagation of Surface waves in elastic media are of considerable importance in earth quake, engineering and seismology on account of occurrence in the earth’s crust as the earth is made up of different layers. Surface waves have been well recognized in the study of earthquake waves, seismology, geophysics and geodynamics. A good amount of literature is to be found in the standard books of Bullen [1], Ewing *et al.* [2], Rayleigh [3], Love [4], Stoneley [5] and Jeffreys [6] regarding surface waves in classical elasticity.

Most of the previous investigations have been studied the effect of reinforcement. The idea of introducing a continuous self reinforcement at every point of an elastic solid was introduced by Belfied *et al.* [7]. The problem of surface waves in fibre reinforced anisotropic elastic media was discussed by Sengupta and Nath [8]. The elastic moduli for fibre reinforced materials were discussed by Hoshin and Rosen [9]. The propagation of plane waves in a fibre reinforced media is investigated by Chattapadhyay *et al.* [10]. The problem of wave propagation in thermally conducting linear fibre reinforced composite materials is discussed by Singh [11]. Othman and Abbas [12] investigated the effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half space.

The coupling between the strain and temperature fields was first studied by Duhamel [13] who derived equations for the distribution of strains in an elastic medium subjected to temperature gradients. Biot [14] gave a satisfactory derivation of the equation of thermal conductivity – which includes the dilation term – based on thermodynamics of irreversible processes. Sharma [15], Sharma et al. [16], investigated the propagation of thermoelastic waves in homogeneous isotropic plate subjected to stress free and rigid insulated and isothermal conditions in the conventional coupled and generalized theories of thermoelasticity. Sharma and Singh [17] studied the propagation of circularly crested thermoelastic waves in a homogeneous isotropic cylindrical plate subjected to stress free and isothermal conditions. Sharma and Pathania [18] studied the propagation of waves in a homogeneous, transversely isotropic, thermally conducting plate bordered with layers of inviscid liquid or half-space of inviscid liquid on both sides.

For wave propagation in a homogenous medium, the introduction of displacement potentials leads to the decoupling of P, SV and SH motions. On close examination of Sengupta and Nath [8], it is found by Singh [19] that their expressions for the potentials do not satisfy one of the equations of motion. Consequently, most of the result and conclusions of Sengupta and Nath [8] are unacceptable. The method of potentials is not suitable for studying wave propagation in fibre reinforced anisotropic elastic media. Sengupta and Nath [8] apply this method incorrectly.

In the present paper, we have discussed the propagation of waves in a homogeneous fibre-reinforced linear thermoelastic isotropic plate and coupled thermoelastic plate. Secular equations for symmetric and skew-symmetric wave modes are derived in closed form and isolated mathematical conditions. The special cases, such as short wavelength waves and thin plate waves of the secular equations are also discussed. The dispersion curves for symmetric and antisymmetric wave modes and attenuation coefficient of symmetric and skew-symmetric modes.

2. Basic Equations

The constitutive equations for a fibre-reinforced linearly thermoelastic isotropic medium with respect to the reinforcement direction $\vec{a}$ as discussed by (Belfield *et al.* [7])

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) + 2(\mu_L - \mu_T)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta a_k a_m e_{km} a_i a_j - \gamma T\delta_{ij} \tag{1}$$

where σ_{ij} are components of stress; $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ are components of strain; λ, μ_T are elastic parameters; $\alpha, \beta, \mu_L - \mu_T$ are reinforced anisotropic elastic parameters; $\gamma = (3\lambda + 2\mu)\alpha_t$; α_t is thermal expansion coefficient; T is the temperature change; u_i are the displacement vectors

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components and $\vec{a} = (a_1, a_2, a_3)$, where $a_1^2 + a_2^2 + a_3^2 = 1$. If $\vec{a}$ has components that are $(1,0,0)$ so that the preferred direction is the x_1 axis, Eq. (1) simplifies, as given below

$$\sigma_{11} = A_{11}e_{11} + A_{12}e_{22} + A_{12}e_{33} - \gamma T;$$
$$\sigma_{22} = A_{12}e_{11} + A_{22}e_{22} + \lambda e_{33} - \gamma T;$$
$$\sigma_{33} = A_{12}e_{11} + \lambda e_{22} + A_{22}e_{33} - \gamma T;$$
$$\sigma_{23} = 2\mu_T e_{23};$$
$$\sigma_{13} = 2\mu_L e_{13};$$
$$\sigma_{12} = 2\mu_L e_{12}; \tag{2}$$

where $A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta;$
$A_{12} = \alpha + \lambda;$
$A_{22} = \lambda + 2\mu_T.$

The equations of motion in the absence of body forces are

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{3}$$

and equation of heat condition in absence of heat source

$$KT_{,ii} = \rho c_e \dot{T} + \gamma T_0 \dot{u}_{ii} \tag{4}$$

where ρ is mass density; T_0 is the reference temperature; c_e is the specific heat at constant strain; K is thermal conductivity.

3. Formulation of the problem

We consider a homogeneous fibre-reinforced linear thermoelastic isotropic plate of thickness $2d$ initially at uniform temperature T_0. We take origin of the co-ordinate system (x, y, z) on the middle surface of the plate. The $x - z$ plane is chosen to coincide with the middle surface and the y-axis normal to it along the thickness. The surface $y = \pm d$ is subjected to different boundary conditions. We take $x - y$ as the plane of incidence and assume that the solutions are explicitly independent of z but implicit dependence is there so that component w of displacement is non-vanishing.

The basic governing equations for homogeneous fibre-reinforced linear thermoelastic isotropic medium, in the absence of body forces and heat sources(on using Eqs. 1-4), are given by

$$\left(A_{11}\frac{\partial^2}{\partial x^2} + \mu_L \frac{\partial^2}{\partial y^2}\right)u + A_{21}\frac{\partial^2 v}{\partial x \partial y} - \gamma \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$A_{21}\frac{\partial^2 u}{\partial x \partial y} + \left(\mu_L \frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2}\right)v - \gamma \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \tag{6}$$

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$$\left(\mu_L \frac{\partial^2}{\partial x^2} + \mu_T \frac{\partial^2}{\partial y^2}\right) w = \rho \frac{\partial^2 w}{\partial t^2} \tag{7}$$

$$K\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T = \rho c_e \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{8}$$

where $A_{21} = \mu_L + A_{12}$;

For convenience, we define the quantities

$$(x^/, y^/, z^/) = c_1\eta(x,\ y,\ z),\ \ (u^/, v^/, w^/) = c_1\eta(u,\ v,\ w),$$

$$t^/ = c_1^{\,2}\eta\, t\,,\ \theta = \frac{\gamma}{\rho c_1^{\,2}} T\,,\ \sigma_{ij}{}^/ = \frac{\sigma_{ij}}{\mu_T}\,,\ \eta = \frac{\rho c_e}{K},\ c_1^{\,2} = \frac{\mu}{\rho}. \tag{9}$$

Upon introducing quantities (9) in governing Eqs. (5) to (8) we obtain (on suppressing dashes for convenience)

$$\left(h_{11} \frac{\partial^2}{\partial x^2} + h_1 \frac{\partial^2}{\partial y^2}\right) u + h_2 \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{10}$$

$$h_2 \frac{\partial^2 u}{\partial x \partial y} + \left(h_1 \frac{\partial^2}{\partial x^2} + h_{22} \frac{\partial^2}{\partial y^2}\right) v - \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2} \tag{11}$$

$$\varepsilon_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta = \frac{\partial \theta}{\partial t} + \varepsilon_1 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{12}$$

$$\left(h_1 \frac{\partial^2}{\partial x^2} + h_3 \frac{\partial^2}{\partial y^2}\right) w = \frac{\partial^2 w}{\partial t^2} \tag{13}$$

where $h_1 = \dfrac{\mu_L}{\rho c_1^{\,2}}$; $h_2 = \dfrac{A_{21}}{\rho c_1^{\,2}}$; $h_3 = \dfrac{\mu_T}{\rho c_1^{\,2}}$; $h_{11} = \dfrac{A_{11}}{\rho c_1^{\,2}}$; $h_{22} = \dfrac{A_{22}}{\rho c_1^{\,2}}$.

4. Boundary conditions

The surfaces $y = \pm d$ of the plate are assumed to be stress free and thermally insulated or isothermal. Therefore, the non-dimensional boundary conditions to be satisfied are given as

$$\sigma_{xy} = g_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\sigma_{yy} = g_1 \frac{\partial u}{\partial x} + g_2 \frac{\partial v}{\partial y} - g_3\theta$$

$$\theta_{,y} + g_4 H\theta = 0 \tag{14}$$

where H is Biot's heat transfer coefficient, $g_0 = \frac{\mu_L}{\mu_T}, g_1 = \frac{A_{12}}{\mu_T}, g_2 = \frac{A_{22}}{\mu_T}, g_3 = \frac{\rho c_1^2}{\mu_T}$, $g_4 = \frac{1}{c_1 \eta}$.

5. Solution of the problem

We assume solutions of the form

$$(u, \quad v, \quad w, \quad \theta) = (1, \quad V, \quad W, \quad \Theta) U \exp\{i\xi(x \sin\phi + my - ct)\} \tag{15}$$

where $c = \omega/\xi$ is the non-dimensional phase velocity, ω is the frequency and ξ is the wave number. Here ϕ is the angle of inclination of wave normal with y-axis; m is still unknown parameter. Here V, W and Θ respectively, the amplitude ratios of displacements v, w and temperature θ to that of displacement u.

Upon using solutions (15) in Eqs. (10)–(13), we obtain

$$\left(h_1 s^2 + h_3 m^2 - c^2\right) W = 0 \tag{16a}$$

$$h_{11} s^2 + h_1 m^2 - c^2 + smh_2 V + \tau_1 s \Theta = 0 \tag{16b}$$

$$h_2 sm + (h_1 s^2 + h_{22} m^2 - c^2) V + \tau_1 m \Theta = 0 \tag{16c}$$

$$\varepsilon_1 sc + \varepsilon_1 mcV + (\varepsilon_2 s^2 + m^2 - \tau_1 c)\Theta = 0 \tag{16d}$$

where $\tau_1 = i\xi^{-1}$, $s = \sin\phi$.

Eq. (16a) in the above system corresponds to purely transverse wave mode (SH) that decoupled from rest of the motion and is not affected by the thermal variations. The characteristic roots corresponding to Eq. (16a) are given by

$$m_7, m_8 = \pm\sqrt{\frac{c^2 - h_1 s^2}{h_3}}\,. \tag{17}$$

From the rest of Eqs. (16), which corresponds to the coupled longitudinal, shear vertical (SV) and thermal (T-mode) motion, one can obtain

$$\left(m^2 - m_1^2\right)\left(m^2 - m_3^2\right)\left(m^2 - m_5^2\right) = 0 \tag{18}$$

where m_k^2, $k = 1, 3, 5$ are the roots of the equation

$$m^6 + Am^4 + Bm^2 + C = 0 \tag{19}$$

Here the coefficients A, B, C are given by

$$A = -\frac{Jc^2 - Ps^2 + \varepsilon_1 \tau_1 h_1 c + (c\tau_1 - \varepsilon_2) h_1 h_{22}}{h_1 h_{22}};$$

$$B=\frac{\left(c^2-h_{11}s^2\right)\left(c^2-h_1s^2\right)+\left(Jc^2-Ps^2\right)\left(c\tau_1-s^2\varepsilon_2\right)+\varepsilon_1\tau_1c\left[c^2-(h_{11}+h_{22}-2h_2)s^2\right]}{h_1h_{22}}$$

$$C=-\frac{\left(c^2-h_1s^2\right)\left[\left(c\tau_1-s^2\varepsilon_2\right)\left(c^2-h_{11}s^2\right)-\varepsilon_1\tau_1cs^2\right]}{h_1h_{22}}$$

where

$J=h_1+h_{22}$, $P=h_1^{\,2}-h_2^{\,2}+h_{11}h_{22}$

Eq. (19) being cubic in m^2 admits six solutions for m; which also have the property $m_2=-m_1$, $m_4=-m_3$, $m_6=-m_5$. For each m_q; $q=1,2,\cdots,6$ the amplitude ratios V and Θ can be expressed as

$$V_q=\begin{cases}\dfrac{m_qa_q}{s}, & q=1,\ 2,\ 3,\ 4\\[2ex] \dfrac{c^2-h_{11}s^2-h_1m_q^{\,2}-\tau_1s\Theta_q}{sh_2m_q}, & q=5,\ 6\end{cases} \tag{20}$$

$$\Theta_q=\begin{cases}\dfrac{c^2-h_{11}s^2-(h_1+h_2a_q)m_q^{\,2}}{s\tau_1}, & q=1,\ 2,\ 3,\ 4\\[2ex] \dfrac{h_1h_{22}m_q^{\,4}-Pm_q^{\,2}+(c^2-h_1s^2)(c^2-h_{11}s^2)}{s\tau_1\left(c^2-h_1s^2+(h_2-h_{22})m_q^{\,2}\right)}, & q=5,\ 6\end{cases} \tag{21}$$

where

$$a_q=\frac{c^2-h_{11}m_q^{\,2}+(h_2-h_{11})s^2}{c^2-h_1s^2+(h_2-h_{22})m_q^{\,2}},\qquad q=1,2,\cdots,6. \tag{22}$$

Combining Eqs. (20) and (21) with stress–strain–temperature relations we rewrite the formal solution for the displacements, temperature, stresses and temperature gradient as

$$\left(u,\quad v,\quad \theta\right)=\sum_{q=1}^{6}\left(1,\quad V_q,\quad \Theta_q\right)U_q\exp\left\{i\xi\left(xs+m_qy-ct\right)\right\},\qquad -d<y<d \tag{23}$$

$$\left(\sigma_{xy},\quad \sigma_{yy},\quad \theta_{,y}\right)=\sum_{q=1}^{6}i\xi\left(D_{1q},\quad D_{2q},\quad D_{3q}\right)U_q\exp\left\{i\xi\left(xs+m_qy-ct\right)\right\}, \tag{24}$$

Here

$D_{1q}=g_0\left(m_q+sV_q\right),\ q=1,2,\cdots,6$

$D_{2q}=g_1s+g_2m_qV_q-g_3\Theta_q,\ \ q=1,2,\cdots,6$

$$D_{3q} = m_q\,,\ q = 1, 2, \cdots, 6 \tag{25}$$

6. Derivation of the secular equations

By invoking stress free and thermal boundary conditions at plate surfaces $y = \pm d$; we obtain a system of six simultaneous linear equations in amplitudes U_q; $q = 1, 2, \cdots, 6$ as

$$\sum_{q=1}^{6} D_{1q} E_q U_q = 0; \qquad \sum_{q=1}^{6} D_{2q} E_q U_q = 0, \qquad \sum_{q=1}^{6} D_{3q} E_q U_q = 0 \tag{26}$$

where $E_q = \exp(\pm i\xi m_q d)$; $q = 1, 2, \cdots, 6$.

System of Eqs. (26) have a non-trivial solution if the determinant of the coefficient of U_q; $q = 1, 2, \cdots, 6$ vanishes, which leads to a characteristic equation for the propagation of modified guided thermoelastic waves in the plate. We refer such waves as plate waves rather than Lamb waves whose properties were originally derived by Lamb in 1917 for isotropic elastic solids. The characteristic equation for the thermoelastic plate waves in this case, after applying lengthy algebraic reduction and manipulations of the determinant leads to the following secular equations

$$\left(\frac{T_1}{T_5}\right)^{\pm 1} - \frac{D_{13}G_3}{D_{11}G_1}\left(\frac{T_3}{T_5}\right)^{\pm 1} = -\frac{D_{15}G_5}{D_{11}G_1} \tag{27}$$

Here the superscript $+1$ corresponds to skew-symmetric and -1 refers to symmetric modes and $T_k = \tan(\xi\, d\, m_k), k = 1,\ 3,\ 5$

$$G_1 = D_{23}\left(D'_{45} + iD''_{45}T_5^{\pm 1}\right) - D_{25}\left(D'_{43} + iD''_{43}T_3^{\pm 1}\right)$$
$$G_3 = D_{21}\left(D'_{45} + iD''_{45}T_5^{\pm 1}\right) - D_{25}\left(D'_{41} + iD''_{41}T_1^{\pm 1}\right)$$
$$G_5 = D_{21}\left(D'_{43} + iD''_{43}T_3^{\pm 1}\right) - D_{23}\left(D'_{41} + iD''_{41}T_1^{\pm 1}\right)$$

where $D'_{4q} = m_q \Theta_q$, $D''_{4q} = -H\tau_1 g_4 \Theta_q$.

Here

$$G_1 = D_{23}D'_{45} - D_{25}D'_{43}$$
$$G_3 = D_{21}D'_{45} - D_{25}D'_{41}$$
$$G_5 = D_{21}D'_{43} - D_{23}D'_{41} \tag{28}$$

for a stress-free thermally insulated $(H \to 0)$ plate and

$$G_1 = D_{23}D''_{45}T_5^{\pm 1} - D_{25}D''_{43}T_3^{\pm 1}$$
$$G_3 = D_{21}D''_{45}T_5^{\pm 1} - D_{25}D''_{41}T_1^{\pm 1}$$
$$G_5 = D_{21}D''_{43}T_3^{\pm 1} - D_{23}D''_{41}T_1^{\pm 1} \tag{29}$$

for stress-free isothermal $(H \to \infty)$ plate.

The secular Eq. (27) is the transcendental equation, which contains complete information about the phase velocity, wave number and attenuation coefficient of the plate waves. In general, wave number and hence the phase velocity of the waves is complex quantity, therefore the waves are attenuated in space. If we write

$$c^{-1} = V_P^{-1} + i\omega^{-1}Q \tag{30}$$

where V_P and Q are real, the exponent $e^{i\xi(x\sin\phi - ct)}$ in the plane wave solution (15) becomes

$$\frac{i\omega}{V_P}\{x\sin\phi - V_P t\} - Qx\sin\phi .$$

This shows that V_P is the propagation speed and Q the attenuation coefficient of the waves. Upon using Eq. (29) in Eq. (27) the values of V_P and Q for different modes can be obtained.

Also all the results reduce to the theory of coupled thermoelasticity of isotropic materials when the anisotropic parameters $\alpha = 0$, $\beta = 0$, $|\mu_L - \mu_T| = 0$ for the fibre-reinforced medium tend to zero (if necessary writing $\mu_L = (\mu_L - \mu_T) + \mu_T$ and considering $|\mu_L - \mu_T| \to 0$).

7. Discussion of the secular equation

1. Regions of the secular equation

Here depending on whether m_1, m_2, m_3 being purely imaginary or complex, the frequency Eq. (27) is correspondingly altered as follows.

1.1 Region I

When the roots of characteristic Eq. (19) are of type $m_k^{\,2} = -m_k'^{\,2}, k = 1,3,5$; $m_k = im_k'$ is purely imaginary or complex number. This ensures that the superposition of partial waves has the property of ''exponential decay''. In this case the secular equation is written from Eq. (27) by replacing circular tangent functions of m_k, $k = 1,3,5$ with hyperbolic tangent functions of m_k', $k = 1,3,5$:

$$\left(\frac{\tanh(\xi d m_1')}{\tanh(\xi d m_5')}\right)^{\pm 1} - \frac{D_{13}G_3'}{D_{11}G_1'}\left(\frac{\tanh(\xi d m_3')}{\tanh(\xi d m_5')}\right)^{\pm 1} = -\frac{D_{15}G_5'}{D_{11}G_1'} \tag{31}$$

where

$$G_1' = D_{23}\left\{D_{45}' + iD_{45}''\left(\tanh(\xi d m_5')\right)^{\pm 1}\right\} - D_{25}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\}$$
$$G_3' = D_{21}\left\{D_{45}' + iD_{45}''\left(\tanh(\xi d m_5')\right)^{\pm 1}\right\} - D_{25}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\}$$
$$G_5' = D_{21}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\} - D_{23}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\} \tag{32}$$

Here $D_{1q}, D_{2q}, D_{3q}, D_{4q}$; $q = 1,2,\cdots,5$ can be obtained on replacing m_k by im_k'; $k = 1,3,5$ in the corresponding expressions.

Region II

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In case two of the roots of Eq. (19) are of the type $m_k{}^2 = -m_k'{}^2, k = 1,3$; then the frequency equation can be obtained from Eq. (27) by replacing circular tangent functions of m_k; $k = 1,3$ with hyperbolic tangent functions of $m_k', k = 1,3$:

$$\left(\frac{\tanh(\xi d m_1')}{\tan(\xi d m_5)}\right)^{\pm 1} - \frac{D_{13}G_3}{D_{11}G_1}\left(\frac{\tanh(\xi d m_3')}{\tan(\xi d m_5)}\right)^{\pm 1} = -\frac{D_{15}G_5}{D_{11}G_1} \tag{33}$$

where

$$G_1' = D_{23}\left\{D_{45}' + iD_{45}''\left(\tan(\xi d m_5)\right)^{\pm 1}\right\} - D_{25}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\}$$
$$G_3' = D_{21}\left\{D_{45}' + iD_{45}''\left(\tan(\xi d m_5)\right)^{\pm 1}\right\} - D_{25}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\}$$
$$G_5' = D_{21}\left\{D_{43}' + iD_{43}''\left(\tanh(\xi d m_3')\right)^{\pm 1}\right\} - D_{23}\left\{D_{41}' + iD_{41}''\left(\tanh(\xi d m_1')\right)^{\pm 1}\right\} \tag{34}$$

Here $D_{1q}, D_{2q}, D_{3q}, D_{4q}$; $q = 1`,2,\cdots,5$ can be obtained on replacing $m_k$ by $im_k'$; $k = 1`,3$ in the corresponding expressions.

1.2 Region III

In the general case the roots $m_k{}^2$; $k = 1,3,5$ are complex numbers, and then the frequency equation is given by Eq. (27).

Waves of short wavelength

Some information on the asymptotic behavior is obtainable by letting $\xi \to \infty$: If we take $\xi > \frac{\omega}{\sqrt{\mu_L / \rho}}$; it follows that $c < \sqrt{\mu_L / \rho}$ and the roots of characteristic equation lies in region I in this case. Then we replace m_1, m_3 and m_5 in the secular equation by im_1', im_3' and im_5'. Here for

$$\xi \to \infty,\ \frac{\tanh(\xi d m_1')}{\tanh(\xi d m_5')} \to 1 \text{ and } \frac{\tanh(\xi d m_3')}{\tanh(\xi d m_5')} \to 1$$

so that secular Eq. (27) reduces to

$$D_{11}G_1'' - D_{13}G_3'' + D_{15}G_5'' = 0 \tag{35}$$

Here

$$G_1'' = D_{23}\left(D_{45}' - D_{45}''\right) - D_{25}\left(D_{43}' - iD_{43}''\right)$$
$$G_3'' = D_{21}\left(D_{45}' - D_{45}''\right) - D_{25}\left(D_{41}' - D_{41}''\right)$$
$$G_5'' = D_{21}\left(D_{43}' - D_{43}''\right) - D_{23}\left(D_{41}' - D_{41}''\right)$$

Eq.(35) is the similar to as obtained and discussed by Sharma et al.[20] in case of homogeneous transversely isotropic thermoelastic plate with stress-free thermally insulated plate. It is merely Rayleigh surface wave equation. The Rayleigh results enter here since for such small wavelengths, the finite thickness plate appears as semi-infinite medium. Hence vibration energy is transmitted along the surface of the plate.

8. Thin plate results

The thin plate limits are specified by $\xi d << 1$ when the transverse wavelength of the plate is quite large as compared to the thickness of the plate. So for $\xi > \frac{\omega}{\sqrt{\mu_L / \rho}}$; we have $m_k = i m_k'$; $k = 1, 3, 5$. In this case the secular equation can be obtained from Eq. (27) just by replacing circular tangent functions with hyperbolic tangent functions.

9. Numerical result and Discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results for the physical constants as discussed in Singh and Singh [21].

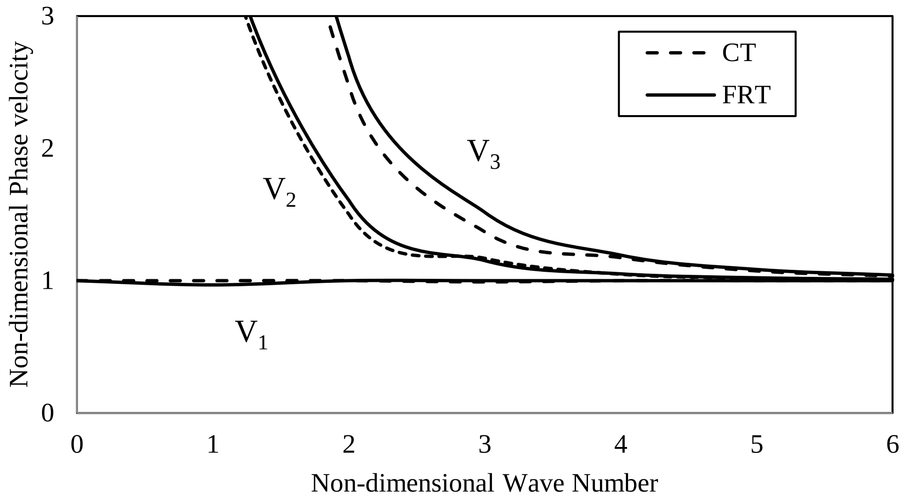
$\lambda = 7.59 \times 10^7\ N \cdot m^{-2}, \mu = 3.86 \times 10^7\ kg \cdot m^{-1} \cdot s^{-2}, \mu_T = 1.89 \times 10^9\ N \cdot m^{-2},$

$\mu_L = 2.45 \times 10^9\ N \cdot m^{-2}, T_0 = 200 K, \alpha = -1.28 \times 10^9\ N \cdot m^{-2}, \beta = 0.32 \times 10^9\ N \cdot m^{-2},$

$\rho = 7800 kg \cdot m^{-3}, \alpha_t = 1.78 \times 10^{-4}\ K^{-1}, c_e = 50 J \cdot kg^{-1} \cdot K^{-1}, K = 10^7\ w \cdot m^{-1} \cdot K^{-1}.$

The complex roots of characteristic equation (19) have been computed with the help of reduced Cardano's method, which are then used in various relevant relations. The secular equation (27) is solved for the phase velocity by using iteration method. The sequence of iteration is made to converge after sampling it over about 100 sample values in order to achieve the desired level of accuracy. The phase velocity, group velocity and attenuation coefficient profiles of first three symmetric and skew-symmetric modes of fibre-reinforced thermoelastic (FRT) and coupled thermoelastic (CT) plate has been computed and corresponding dispersion curves are represented graphically in Fig.1-6 for wave normal inclination $\phi = 90^\circ$ with the axis of symmetry.

Fig. 1: Variations of phase velocity of symmetric modes versus wave number.



Figures 1-2 show the variations of non-dimensional phase velocities of symmetric and skew-symmetric modes $V_i\ (i = 1, 2, 3)$ respectively, versus wave number in the context of FRT and CT theories. The phase velocity of fundamental mode (V_1) of symmetric motion has been observed to be almost dispersionless and whereas it found to be zero at vanishing wave number in case skew symmetric motion of both FRT and CT theories. The phase velocities of higher modes $V_i (i = 2, 3)$ attain quite large values at vanishing wave number which sharply slash down to become steady and asymptotically closer to shear wave velocity at extremely large wave numbers. It is observed that the reinforcement has a significant effect on the distribution of the field quantities. It is noticed that the reinforcement effect has negligibly small effect on the phase velocities at higher wave numbers and the phase velocities of various modes become asymptotic and tend to the Rayleigh wave speed. In such situation a finite thickness plate appears to be half-space and the vibration energy is mainly transmitted through the surface of the plate. Figs. 3-4 present the variations of group velocities $(V_{g_i}, i = 1, 2, 3)$ of symmetric and skew symmetric modes in FRT and CT plates versus non-dimensional wave number respectively. The values of group velocities of various modes are found to be zero at vanishing wave numbers which correspond to the condition of zero energy transmission in horizontal direction. The profiles of group velocity, both in FRT and CT, increase monotonically in the interval $[0, 1]$ and it tends to phase velocity profiles of respective modes at high frequency limits for all modes. Figs. 5-6 show the variations of attenuation coefficient $(Q_i, i = 1, 2, 3)$ for symmetric and skew symmetric modes in FRT and CT plates versus non-dimensional wave number respectively. It is noticed that the attenuation coefficient is small for the mode which have large magnitude of the phase velocity and vice versa. It is also noticed that curves of attenuation coefficient with wave number in each mode is almost reflection (mirror image) of the graph of phase velocity with wave number.

Fig. 2: Variations of phase velocity of skew-symmtric modes versus wave number**.**

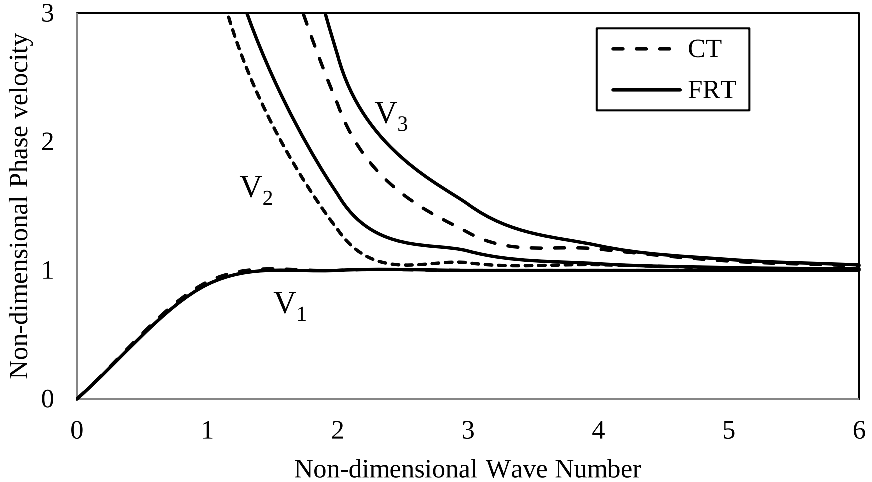


Fig. 3: Variations of group velocity of symmetric modes versus wave number**.**

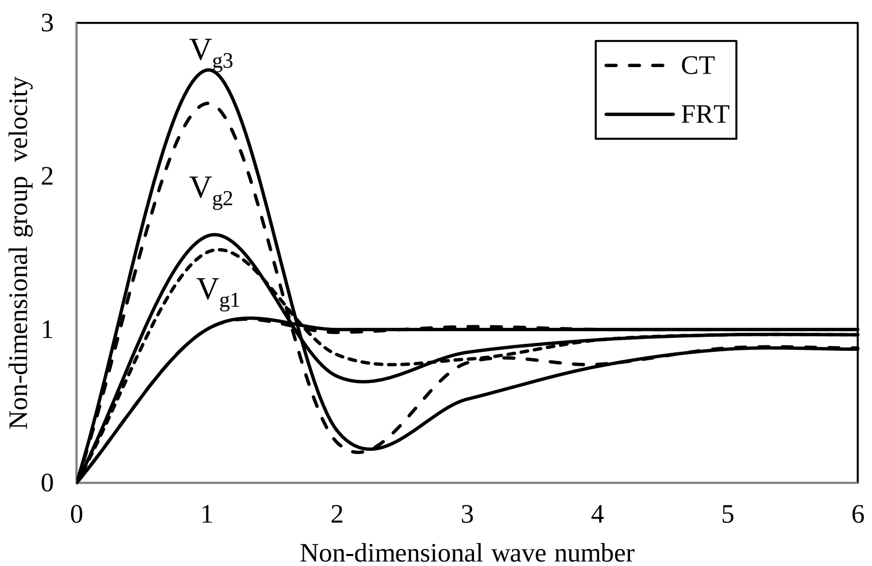


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Fig. 4: Variations of group velocity of skew-symmetric modes versus wave number.

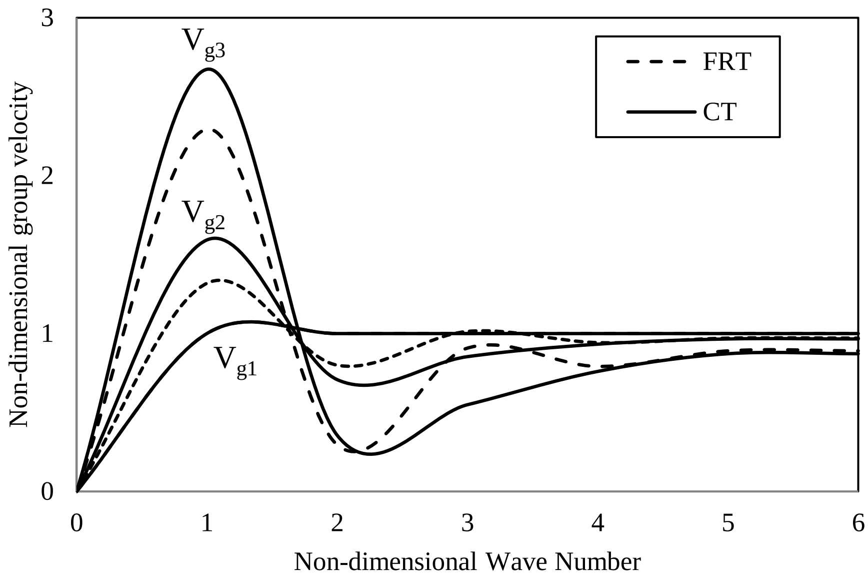


Fig. 5: Variations of attenuation coefficient of symmetric modes versus wave number.

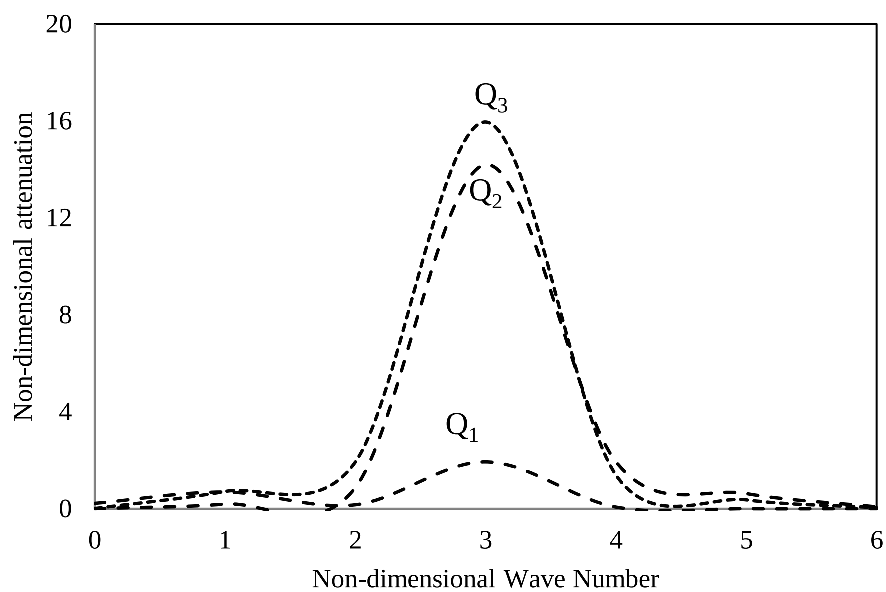
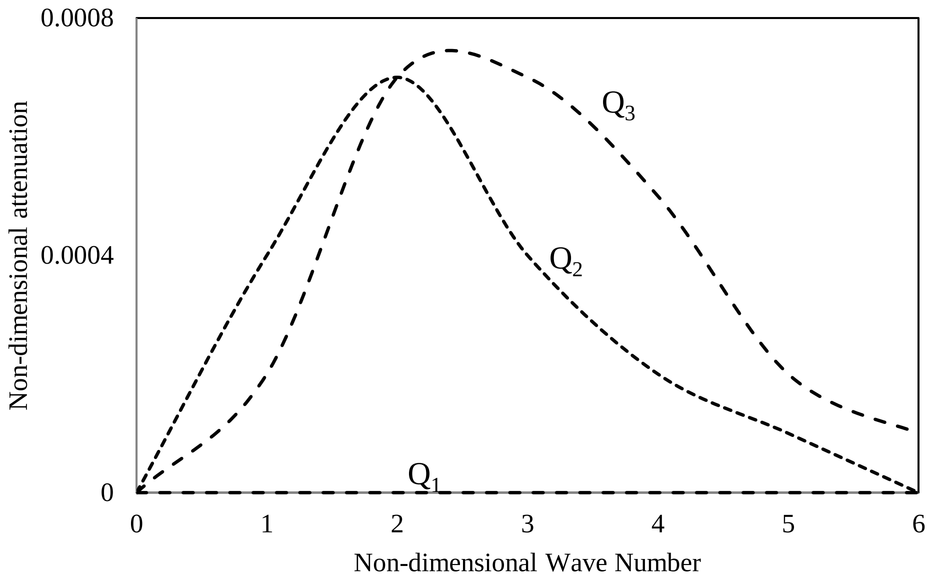


Fig. 6: Variations of attenuation coefficient of skew-symmertic modes versus wave number.



10. CONCLUSIONS:

It is observed that the phase velocity of the acoustic symmetric mode become dispersionless and asymptotic. For increasing wave number this remains closer to the velocity of thermoelastic Lamb waves because in this case the energy transmission takes place mainly along the surface (interface) of the plate. The acoustic skewsymmetric mode has zero velocity at vanishing wave number, which increases to become closer to the velocity of thermoelastic Lamb wave with increasing wave number. The phase velocities of optical modes of propagation attain large values at vanishing wave number, which slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number. The hypothetical results in case of FRT theory here observed to resemble closely to that of CT theory of thermoelasticity for initial two modes namely acoustic and first modes of wave propagation. It is clear from the above investigation that the surface waves in the fibre-reinforced medium are affected by the reinforced parameters. Also all the results reduce to the CT results when the anisotropic parameters for the fibre-reinforced medium tend to zero. We conclude that the Rayleigh wave velocity in a fibre-reinforced elastic medium is considerably higher than the Rayleigh wave velocity in isotropic media.

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EFFECT OF ROTATION ON ELECTROTHERMAL - CONVECTION IN A DIELECTRIC MAXWELLIAN NANOFLUID LAYER SATURATING A DARCY POROUS MEDIUM

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Abstract
In this paper, effect of rotation on electrothermal-convection in a horizontal layer of dielectric rheological nanofluid saturating a porous medium is investigated. The rheological behaviour of nanofluid is described by Maxwell model and for porous medium, Darcy model is employed. The used Boungiorno model for nanofluid incorporates the effects of thermophoresis and Brownian diffusion. The Navier-Stokes equations of motion are modified due to the presence of applied AC electric field and rotation by the inclusion of dielectrophoretic force, Coulomb force and Coriolis force, respectively. By applying linear stability analysis based upon perturbation theory and one-term Galerkin method, we derive the expressions for thermal Rayleigh number for both the cases of stationary convection and the oscillatory motion are derived. The effects of Taylor number, AC electric Rayleigh number, Lewis number, modified diffusivity ratio and nanoparticles Rayleigh number on the onset of stationary and oscillatory modes.

Keywords: Electrohydrodynamics, Nanofluid, Maxwell model, Porous Medium, Rotation.

1. Introduction

Electro-thermo-hydrodynamics (ETH) is an interdisciplinary area of composite communications among the thermal gradient and electric field applied to a horizontal dielectric nanofluid layer. Due to its practical importance, numerous studies have been approved to evaluate the consequence of electric field on natural convection. A temperature gradient applied to a dielectric fluid produces a gradient in the dielectric constant and electrical conductivity. The applied electric force of fluid motions is effective method in getting very helpful interesting results in the cooling of lap-tops and devices of the flight in space, usage to a variety of applications ranging from electro-kinematic assays to electro spray ionization made on nanoscale being used at a large scale in the present era. In dielectric liquids characterized by low values of conductivity, Maxwell's equations reduce to the electro-quasistatic limit. Keeping this information in mind, many envils of the onset of convection instability in a horizontal layer and a vertical temperature gradient have been investigated in the history.

Maxwell (1891) was the first who coined the idea of dispersing solids in fluids. Nanofluids has been formed with a completely different thermophysical properties such as density, specific heat capacity, thermal conductivity, convective heat transfer, thermal difusivity, and viscosity. Commercializing nanofluid is facing challenges due to their poor stability which occurred by

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the interaction between the particles themselves and between the particles and the surrounding liquid. Convection of non-Newtonian fluids in a porous medium is of considerable importance in several applied fields such as oil recovery , food processing, and the spread of contaminants in the environment, and in various processes the chemical and materials industries. Rotation plays important role in the thermal instability of nanofluid layer and has applications in rotating machineries such as nuclear reactors, petroleum industry, biomechanics, automotive industries, power plant cooling systems, computers etc. The effect of rotation on viscous and viscoelastic fluids has been studied by different authors [Chandrasekhar (1961), Vadasz, (1996) , Malashetty et al. (2007), Dhannajay et al.(2011), Agarwal and Bhadauria (2014). Chand et al. (2015), Rana and Agarwal (2015), Govender (2016a, b). Mahajan and Sharma (2018) studied the penetrative convection stimulated by internal heating in magnetic nanofluid saturating a rotating porous medium while detailed review on the instability and convection in rotating porous media has been explained by Vadasz (2019).They found that the rotation parameter has a stabilizing effect on thermal instability of viscous and viscoelastic fluid. Takashima (1976) studied the convective instability in a dielectric fluid under the simultaneous action of rotation and the *AC* electric field whereas the electrohydrodynamic instability of a rotating layer of viscoelastic dielectric regular fluid was studied by Othman (2004). Shivakumara et al. (2013) studied electrohydrodynamic instability of a rotating couple stress dielectric fluid layer while Rana et al. (2016) studied the instability of a viscoelastic fluid saturating a porous medium in electrohydrodynamics. The onset of electroconvection in a dielectric nanofluid saturating a rotating Darcy porous medium has been studied analytically and numerically by Wakif et al. (2018) using realistic boundary conditions and they have shown that the onset of electroconvection can be accelerated on increasing AC electric Rayleigh –Darcy number, the Lewis number and the modified diffusivity ratio.

Recently, Rana et al. (2019) investigated electrohydrodynamic thermal instability of an elastico-viscous nanofluid saturating a porous in the presence of vertical AC electric field and found that couple-stress parameter, medium porosity and Brinkman-Darcy number have stabilizing effect whereas AC electric Raleigh number, Lewis number and modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effect on the system. Sharma et al. (2019) have examined the rheological behavior of electro-hydrodynamics convection in dielectric rotating maxwellian nanofluid layer with vertical AC electric field and have shown that applied electric field has a stabilizing effect on the system. Here, effect of rotation on electrothermal-convection in a dielectric Maxwellian nanofluid layer saturating a Darcy porous medium is studied which include one additional parameter, namely, the Taylor number.

2 Formulation of the Problem and Mathematical Model

An infinitely extending electrically horizontal layer of an incompressible dielectric Shear-Thinning viscoelastic nanofluid for bottom/top-heavy distribution of nanoparticles, heated from below is considered. It is confined between two parallel planes $\tilde{z}=0$ and $\tilde{z}=\tilde{d}$, where temperature and volumetric fraction are kept constant : $\tilde{T}=\tilde{T}_0$ and $\tilde{\varphi}=\tilde{\varphi}_0$ at $\tilde{z}=0$ and $\tilde{T}=\tilde{T}_1$ and $\tilde{\varphi}=\tilde{\varphi}_1$ at $\tilde{z}=\tilde{d}$, $\left(\tilde{T}_0>\tilde{T}_1\right)$. Both the bounding surfaces are assumed to be stress free. The thermophysical properties of nanofluids (viscosity, density, thermal conductivity and specific heat) are taken as constants for the analytical formulation, but these quantities are not constant and strongly depend on the volume fraction of nanoparticles. The nanofluid layer is rotating uniformly about vertical axis with a uniform angular velocity $\tilde{\mathbf{\Omega}}(\mathbf{0,0,\Omega})$ and the acceleration due to gravity $\tilde{\mathbf{g}}(\mathbf{0,0,-g})$ pervade the system, z-axis being taken as vertical.

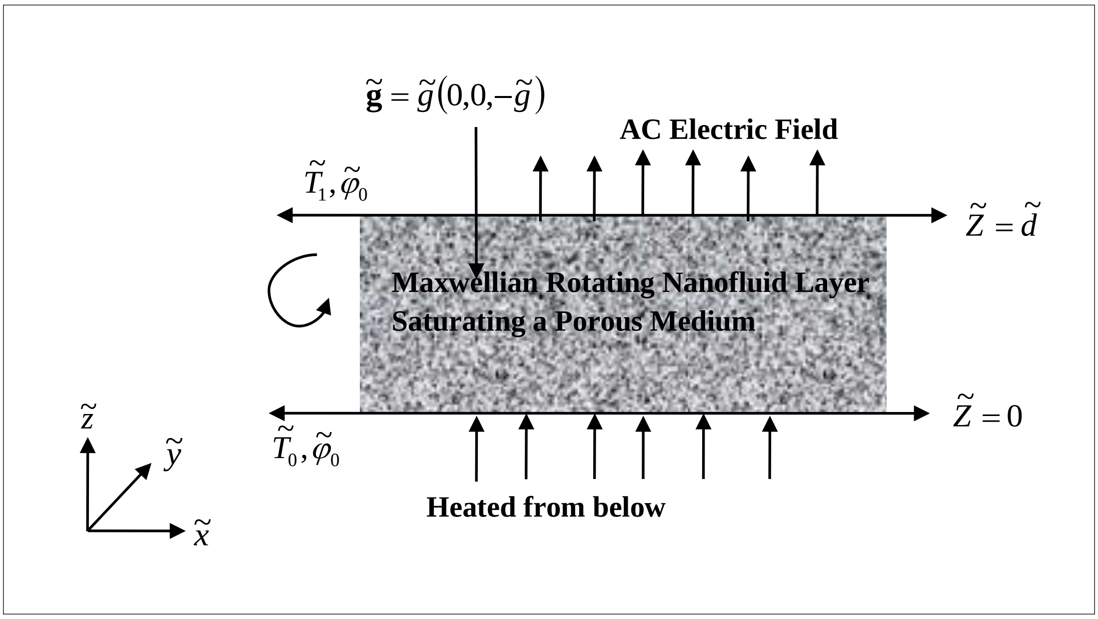

Figure 1: Physical Configuration of the Problem

2.2 Governing Equations

The basic hydrodynamic equations that govern the physical problem using Lapwood (1948), Chandrasekhar (1961), Buongiorno model (2006) are

$$\nabla.\tilde{\mathbf{q}}_{\mathbf{D}} = 0, \tag{1}$$

$$\tilde{\rho}_{nf} = \tilde{\varphi}\tilde{\rho}_{np} + (1-\tilde{\varphi})\tilde{\rho}_{bf}, \tag{2}$$

where $\tilde{\rho}_{np}$ is the density of nanoparticles and $\tilde{\rho}_{bf}$ is the base fluid, respectively. Taking the density of the nanofluid as that of the base fluid, as adopted by Tzou (2008a, 2008b), the specific weight thus becomes

$$\tilde{\rho}_{nf}\tilde{\mathbf{g}} = \left[\tilde{\varphi}\tilde{\rho}_{np} + (1-\tilde{\varphi})\left\{\tilde{\rho}_{nf}\left(1-\tilde{\alpha}\left(\tilde{T}-\tilde{T}_0\right)\right)\right\}\right]\tilde{\mathbf{g}}. \tag{3}$$

where $\tilde{\alpha}$ is the coefficient of thermal expansion.

Due to the Brownian motion and thermophoresis mass flux of the nanoparticles in the base fluid, $\tilde{\mathbf{j}}_{\mathbf{p}} = -\rho_{np}\tilde{D}_B\nabla\tilde{\varphi} - \tilde{\rho}_{np}\dfrac{\tilde{D}_T}{\tilde{T}}\nabla\tilde{T}$. (4)

where $\tilde{D}_B$ is the Brownian diffusion coefficient, given by Einstein-Stokes equation and $\tilde{D}_T$ is the thermophoretic diffusion coefficient of the nanoparticles and are given as

$$\tilde{D}_B = \frac{\tilde{k}_B\tilde{T}}{3\pi\tilde{\mu}_{bf}\tilde{d}_{np}}, \quad \tilde{D}_T = \frac{\tilde{\mu}_{bf}\,0.26\tilde{k}_{bf}}{\tilde{\rho}_{nf}\left(2\tilde{k}_{bf}+\tilde{k}_{np}\right)}\tilde{\varphi}, \tag{5}$$

where $\tilde{k}_B$ is Boltzmans constant, $\tilde{\mu}_{bf}$ is the base fluid viscosity, $\tilde{d}_{np}$ is the diameter of the nanoparticle, $\tilde{\rho}_{bf}$ is the base fluid density , $\tilde{k}_{bf}$ and $\tilde{k}_{np}$ are the thermal conductivities of the base fluid and nanoparticles respectively.

Equations of motion for an incompressible rotating viscoelastic Maxwellian nanofluid saturating a porous medium are

$$\frac{\tilde{\rho}_{bf}}{\tilde{\varepsilon}}\left(1+\tilde{\lambda}_0\frac{\partial}{\partial \tilde{t}}\right)\left[\frac{\partial}{\partial \tilde{t}}+\frac{\tilde{q}_D}{\tilde{\varepsilon}}.\nabla\right]\tilde{\mathbf{q}}_{\mathbf{D}}=\left(1+\tilde{\lambda}_0\frac{\partial}{\partial \tilde{t}}\right)\left[-\nabla\tilde{P}-\tilde{f}_e+\frac{2\tilde{\rho}_{nf}}{\tilde{\varepsilon}}\left(\tilde{\mathbf{q}}_{\mathbf{D}}\times\tilde{\mathbf{\Omega}}\right)+\tilde{\varphi}\tilde{\rho}_{np}+(1-\tilde{\varphi})\tilde{\rho}_{nf}\left\{1-\tilde{\alpha}\left(\tilde{T}-\tilde{T}_1\right)\right\}\tilde{\mathbf{g}}\right]-\frac{\tilde{\mu}}{\tilde{k}_1}\tilde{\mathbf{q}}_{\mathbf{D}}. \tag{6}$$

Where, $\tilde{P}=\tilde{p}-\frac{\tilde{\rho}_{bf}}{2}\left|\tilde{\mathbf{\Omega}}\times\vec{\mathbf{r}}\right|^2$

$$\tilde{\mathbf{f}}_e=\tilde{\rho}_e\tilde{\mathbf{E}}-\frac{1}{2}\tilde{\mathbf{E}}^2\nabla\tilde{K}+\frac{1}{2}\left(\tilde{\rho}_{nf}\frac{\partial\tilde{K}}{\partial\tilde{t}}\tilde{\mathbf{E}}^2\right), \tag{7}$$

The term $\tilde{\rho}_e\tilde{\mathbf{E}}$ is neglected as compared to the di-electrophoretic force term $-\frac{1}{2}\tilde{\mathbf{E}}^2\nabla\tilde{K}$ for dielectric fluids. It is assumed that the dielectric constant , $\tilde{K}$ can be expressed as [Yadav et al. (2015)]

$$\tilde{K}=\tilde{K}_0\left[1-\tilde{\gamma}_0\left(\tilde{T}-\tilde{T}_0\right)\right], \tag{8}$$

$\tilde{\gamma}_0>0$, is the coefficient of the dielectric constant with temperature relative variations, which is assumed to be small $0<\tilde{\gamma}_0\Delta\tilde{T}<<1$.

The modified pressure term using equation (6) is

$$\tilde{P}=\tilde{p}-\frac{1}{2}\left(\tilde{\rho}_{nf}\frac{\partial\tilde{K}}{\partial\tilde{t}}\tilde{\mathbf{E}}^2\right), \tag{9}$$

Assuming free charge density to be very small, the relevant Maxwell equations [Roberts (1969)] are

$$\nabla.\left(\tilde{K}\tilde{\mathbf{E}}\right)=0, \tag{10}$$

$$\nabla\times\tilde{\mathbf{E}}=0, \tag{11}$$

In view of equation (11), $\tilde{\mathbf{E}}$ can be expressed as $\tilde{\mathbf{E}}=-\nabla\tilde{\varphi}$.

The conservation equation for the nanoparticles is

$$\left[\frac{\partial}{\partial\tilde{t}}+\frac{\tilde{\mathbf{q}}_{\mathbf{D}}\cdot\nabla}{\tilde{\varepsilon}}\right]\tilde{\varphi}=-\frac{1}{\tilde{\rho}_{np}}\nabla.\tilde{\mathbf{j}}_{\mathbf{p}}. \tag{12}$$

Here $\tilde{\varphi}$ is the nanoparticle volumetric fraction, $\tilde{\rho}_{np}$ is the density of nanoparticles and $\tilde{\mathbf{j}}_{\mathbf{p}}$, the nanoparticles diffusion mass flux is given by

$$\tilde{\mathbf{j}}_{\mathbf{p}}=-\tilde{\rho}_{np}\tilde{D}_B\nabla\tilde{\varphi}-\tilde{\rho}_{np}\frac{\tilde{D}_T}{\tilde{T}_1}\nabla\tilde{T}, \tag{13}$$

Using the value of $\tilde{\mathbf{j}}_{\mathbf{p}}$ from equation (13) into equations (12), the conservation equation of nanoparticles yields that

$$\left[\frac{\partial}{\partial\tilde{t}}+\frac{\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla}{\tilde{\varepsilon}}\right]\tilde{\varphi}=\tilde{D}_B\nabla^2\tilde{\varphi}+\frac{\tilde{D}_T}{\tilde{T}_1}\nabla^2\tilde{T}. \tag{14}$$

The heat energy equation is

$$\left(\tilde{\rho}_{nf}\tilde{c}\right)_{bm}\left\{\frac{\partial\tilde{T}}{\partial\tilde{t}}+\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{T}\right\}=\tilde{k}_m\nabla^2\tilde{T}+\tilde{\varepsilon}\left(\tilde{\rho}_{nf}\tilde{c}\right)_{np}\left\{\tilde{D}_B\nabla\tilde{\varphi}.\nabla\tilde{T}+\frac{\tilde{D}_T}{\tilde{T}_0}\nabla\tilde{T}.\nabla\tilde{T}\right\}, \tag{15}$$

where $\tilde{c}_{np}$ is the specific heat of the material constituting the nanoparticles. $(\tilde{\rho}_{nf}\tilde{c})_{bm}$ is the effective capacity, $(\tilde{\rho}_{nf}\tilde{c})_{bm}$ is the heat capacity of nanofluid.

We introduce non-dimensional variables as

$$\left(\tilde{x}',\tilde{y}',\tilde{z}'\right)=\left(\frac{\tilde{x},\tilde{y},\tilde{z}}{\tilde{d}}\right),\left(\tilde{u}',\tilde{v}',\tilde{w}'\right)=\left(\frac{\tilde{u},\tilde{v},\tilde{w}}{\tilde{\kappa}_{bm}}\right)\tilde{d},\tilde{t}'=\frac{\tilde{t}\,\tilde{\kappa}_{bm}}{\tilde{\sigma}\tilde{d}^{2}},\tilde{p}'=\frac{\tilde{p}\tilde{k}_1}{\tilde{\mu}\tilde{\kappa}_{bm}}\tilde{d}^{2},$$
$$\tilde{\varphi}'=\frac{\tilde{\varphi}-\tilde{\varphi}_0}{\tilde{\varphi}_1-\tilde{\varphi}_0},\tilde{T}'=\frac{\tilde{T}-\tilde{T}_1}{\tilde{T}_0-\tilde{T}_1},\tilde{K}'=\frac{\tilde{K}}{\tilde{\gamma}_0\tilde{\mathbf{E}}_{\mathbf{0}}\Delta\tilde{T}\tilde{d}},\tilde{V}'=\tilde{\gamma}_0\tilde{\mathbf{E}}_{\mathbf{0}}\tilde{\beta}\tilde{d}\tilde{V}. \tag{16}$$

where $\tilde{\kappa}_{bm}=\dfrac{\tilde{k}_m}{(\tilde{\rho}_{nf}\tilde{c}_{np})_{bf}}$ is thermal diffusivity of the base fluid, $\tilde{\sigma}=\dfrac{(\tilde{\rho}_{nf}\tilde{c}_{np})_{bm}}{(\tilde{\rho}_{nf}\tilde{c}_{np})_{bf}}$ is the thermal capacity ratio, $\tilde{\beta}=\dfrac{\Delta\tilde{T}}{\tilde{d}}$ or $\tilde{\beta}=\dfrac{\tilde{T}_0-\tilde{T}_1}{\tilde{d}}$ is the adverse temperature gradient, $\tilde{E}_0$ is the root mean square value of the electric field at $\tilde{z}=0$ and $\nabla\tilde{T}$ is the temperature difference between the horizontal planes, $\tilde{\gamma}_0$ is the dielectric constant at reference temperature $\tilde{T}_0$ and $\tilde{\varphi}_0$ is a reference value for the nanoparticles volume fraction.

The equations (2)-(15) equations in non-dimensional form after dropping the dashes (') for convenience can be written as

$$\nabla.\tilde{\mathbf{q}}_{\mathbf{D}}=0, \tag{17}$$

$$0=\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\left[-\nabla\tilde{P}-\tilde{\mathbf{f}}_{\mathbf{e}}+\frac{2\tilde{\rho}_{nf}}{\tilde{\varepsilon}}\left(\tilde{\mathbf{q}}_{\mathbf{D}}\times\tilde{\mathbf{\Omega}}\right)+\tilde{\varphi}\tilde{\rho}_{np}+(1-\tilde{\varphi})\tilde{\rho}_{nf}\left\{1-\tilde{\alpha}\left(\tilde{T}-\tilde{T}_1\right)\right\}\tilde{\mathbf{g}}\right]-\frac{\tilde{\mu}}{\tilde{k}_1}\tilde{\mathbf{q}}_{\mathbf{D}} \tag{18}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial\tilde{\varphi}}{\partial\tilde{t}}+\frac{1}{\tilde{\varepsilon}}\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{\varphi}=\frac{1}{\tilde{L}_e}\nabla^2\tilde{\varphi}+\frac{\tilde{N}_A}{\tilde{L}_e}\nabla^2\tilde{T}, \tag{19}$$

$$\frac{\partial\tilde{T}}{\partial\tilde{t}}+\tilde{\mathbf{q}}_{\mathbf{D}}.\nabla\tilde{T}=\nabla^2\tilde{T}+\frac{\tilde{N}_A}{\tilde{L}_e}\nabla\tilde{\varphi}.\nabla\tilde{T}+\frac{\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}\nabla\tilde{T}.\nabla\tilde{T}, \tag{20}$$

$$\nabla^2\tilde{V}=\frac{\partial\tilde{T}}{\partial\tilde{z}}, \tag{21}$$

where the non-dimensional parameters are defined as $\tilde{p}_1=\dfrac{\tilde{\mu}}{\tilde{\rho}_0\tilde{\alpha}}$ (Prandtl number), $\tilde{D}_a=\dfrac{\tilde{k}_1}{\tilde{d}^2}$ (Darcy number), $\tilde{V}_a=\dfrac{\tilde{\varepsilon}\tilde{P}_r}{\tilde{D}_a}$ (Vadasz number), $\tilde{R}_a=\dfrac{\tilde{\rho}_{nf}\tilde{\mathbf{g}}\tilde{\alpha}\tilde{d}\tilde{k}\left(\tilde{T}_0-\tilde{T}_1\right)}{\tilde{\mu}_{bf}\tilde{\kappa}_{bm}}$ (Rayleigh number), $\tilde{R}_{ea}=\dfrac{\tilde{\gamma}_0{}^2\tilde{K}\tilde{\mathbf{E}}_{\mathbf{0}}{}^2\tilde{d}^2\left(\Delta\tilde{T}\right)^2}{\tilde{\mu}\tilde{\kappa}_{bm}}$ (electric Rayleigh number), $\tilde{R}_n=\dfrac{\left(\tilde{\rho}_{np}-\tilde{\rho}_{bf}\right)\left(\tilde{\varphi}_1-\tilde{\varphi}_0\right)\tilde{\mathbf{g}}\tilde{k}_1\tilde{d}}{\tilde{\mu}\tilde{\alpha}}$ (nano particles Rayleigh number), $\tilde{N}_B=\dfrac{\tilde{\rho}_{np}\tilde{c}_{np}}{\left(\tilde{\rho}\tilde{c}\right)_{bf}}\left(\tilde{\varphi}_1-\tilde{\varphi}_0\right)$ (modified particle density increment), $\tilde{L}_e=\dfrac{\tilde{\kappa}_{bm}}{\tilde{D}_B}$ (Lewis number of the nanofluid), $\tilde{N}_A=\dfrac{\tilde{D}_T\left(\tilde{T}_0-\tilde{T}_1\right)}{\tilde{D}_B\tilde{T}_1\left(\tilde{\varphi}_1-\tilde{\varphi}_0\right)}$ (modified diffusivity ratio), $\tilde{R}_m=\dfrac{\left\{\tilde{\rho}_{np}\tilde{\varphi}_0+\tilde{\rho}_{bf}+\left(1-\tilde{\varphi}_0\right)\right\}\tilde{\mathbf{g}}\tilde{k}_1\tilde{d}}{\tilde{\mu}\tilde{\alpha}}$ (basic density Rayleigh number), $\tilde{\lambda}_0 = \frac{\tilde{\lambda}\tilde{\kappa}_{bm}}{\tilde{d}^2}$ (parameter accounting for stress-relaxation time) and $\tilde{T}_a = \frac{4\tilde{\mathbf{\Omega}}^2\tilde{d}^4}{\tilde{\nu}^2}$ (the Taylor number). (22)

Here both the bounding surfaces of the fluid are assumed to be stress-free and the medium adjoining the nanofluid is a perfect conductor, the appropriate boundary conditions are

$$\tilde{w} = \frac{\partial^2\tilde{w}}{\partial z^2} = \tilde{T} = \tilde{\varphi} = \frac{\partial\tilde{\varphi}}{\partial\tilde{z}} = 0 \text{ at } \tilde{z} = 0 \text{ and } \tilde{z} = \tilde{d}. \tag{23}$$

2.3 Basic Solutions

The primary flow representing the basic state is assumed to be quiscent [Kuznetsov and Nield (2009), Nield and Kuznetsov (2009), Sheu (2011), Chand and Rana (2012b), Nield and Kuznetsov (2014), Chand et al. (2014) and Yadav and Kim (2015)], no settling of suspended nanoparticles and is assumed to be stationary . Initially, no motions are present in the nanofluid flow and the physical quantities vary in the vertical direction z-axis only. Therefore, the velocity, pressure, temperature, dielectric constant, electric field, electric potential and nanoparticle volume fraction are given by

$$\tilde{\mathbf{q}}_{\mathbf{D}} = \tilde{\mathbf{q}}_{\mathbf{b}} = 0, \tilde{P} = \tilde{P}_b(\tilde{z}), \tilde{K} = \tilde{K}_b(\tilde{z}), \tilde{T} = \tilde{T}_b(\tilde{z}), \tilde{\varphi} = \tilde{\varphi}_b(\tilde{z}), \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\mathbf{b}}(\tilde{z}), \tilde{V} = \tilde{V}_b(\tilde{z})$$

$$\tilde{T}_b = \tilde{T}_0 - \frac{\Delta\tilde{T}}{\tilde{d}}\tilde{z}, \tilde{\varphi}_b = \tilde{\varphi}_0 + \left(\frac{\tilde{D}_T\Delta\tilde{T}}{\tilde{D}_B\tilde{T}_1\tilde{d}}\right)\tilde{z}, \tilde{K}_b = \tilde{K}_0\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\tilde{z}\right)\hat{k}, \tilde{\mathbf{E}}_{\mathbf{b}} = \frac{\tilde{\mathbf{E}}_{\mathbf{0}}}{\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\tilde{z}\right)}\hat{k}, \tag{24}$$

where subscript 'b' denotes the basic state and $\hat{k}$ is the unit vector along z-axis.
Also we have

$$\tilde{V}_b(\tilde{z}) = -\frac{\tilde{\mathbf{E}}_{\mathbf{0}}\tilde{d}}{\tilde{\gamma}_0\Delta\tilde{T}}\log\left(1 + \frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\right)\hat{k} \quad ,$$

where $\tilde{\mathbf{E}}_{\mathbf{0}} = -\frac{\tilde{V}\left(\frac{\tilde{\gamma}_0\Delta\tilde{T}}{\tilde{d}}\right)}{\log\left(1 + \tilde{\gamma}_0\Delta\tilde{T}\right)}$ is the root mean square value of the intensity of electric field at $\tilde{z} = 0.$

2.4 Perturbation Solutions

Let the primary flow be slightly disturbed from the equilibrium position so as to examine the stability of the perturbed modes with respect to the involved, physical variables by superimposing infinitesimal disturbances to the basic state flow. It is assumed that

$$\begin{aligned} &\tilde{\mathbf{q}}_{\mathbf{D}} = \tilde{\mathbf{q}}_{\mathbf{D}}^*, \tilde{T} = \tilde{T}_b + \tilde{T}^*, \tilde{K} = \tilde{K}_b + \tilde{K}^*, \tilde{P} = \tilde{P}_b + \tilde{P}^*, \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\mathbf{b}} + \tilde{\mathbf{E}}^*, \\ &\tilde{V} = \tilde{V}_b + \tilde{V}^*, \tilde{\varphi} = \tilde{\varphi}_b + \tilde{\varphi}^*, \tilde{\varepsilon} = \tilde{\varepsilon}_b + \tilde{\varepsilon}^* \end{aligned} , \tag{25}$$

where $\tilde{\mathbf{q}}_{\mathbf{D}}^*, \tilde{T}^*, \tilde{K}^*, \tilde{P}^*, \tilde{\mathbf{E}}^*, \tilde{V}^*, \tilde{\varphi}^*$ and $\tilde{\varepsilon}^*$ are the perturbations superimposed into the physical quantities of the equilibrium state.

On substituting these perturbations and using the solutions of primary flow (24) the equations (17)-(21) in the non-dimensional linearized perturbed form using linear theory (neglecting the products and higher orders of perturbed quantities) and Boussinesq approximation yield

$$\left[1+\frac{1}{\tilde{V}_a^*}\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\frac{\partial}{\partial\tilde{t}}\right]\nabla^2\tilde{w}^*-\left\{\begin{array}{l}\tilde{R}_a\nabla^2\tilde{T}^*-\tilde{R}_{ea}\nabla^2\tilde{T}^*+\tilde{R}_n\nabla^2\tilde{\varphi}+\sqrt{\tilde{T}_a^*}\left(\tilde{v}\hat{e}_x-\tilde{u}\hat{e}_y\right)\\+\tilde{R}_{ea}\nabla^2\frac{\partial\tilde{V}^*}{\partial\tilde{z}}\left(1+\tilde{\lambda}_0\frac{\partial}{\partial\tilde{t}}\right)\end{array}\right\}=0\,, \tag{26}$$

$$\frac{\partial\tilde{T}^*}{\partial\tilde{t}}-\tilde{w}^*=\nabla^2\tilde{T}^*+\frac{\tilde{N}_B}{\tilde{L}_e}\left(\frac{\partial\tilde{T}^*}{\partial\tilde{z}}-\frac{\partial\tilde{\varphi}^*}{\partial\tilde{t}}\right)-\frac{2\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}\frac{\partial\tilde{T}^*}{\partial\tilde{z}}\,, \tag{27}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial\tilde{\varphi}^*}{\partial\tilde{t}}+\frac{\tilde{w}^*}{\tilde{\varepsilon}}=\frac{1}{\tilde{L}_e}\nabla^2\tilde{\varphi}^*+\frac{\tilde{N}_A}{\tilde{L}_e}\nabla^2\tilde{T}^*, \tag{28}$$

$$\frac{\partial\tilde{T}^*}{\partial z}-\nabla^2\tilde{V}^*=0\,, \tag{29}$$

Also

$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is a Laplacian operator,

$\nabla_1{}^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

Equation (18) is derived by the use of the identity curlcurl = graddiv - ∇^2.

The boundary conditions (23) , in the non-dimensional form becomes

$$\tilde{w}^*=\frac{\partial^2\tilde{w}^*}{\partial\tilde{z}^2}=\tilde{T}^*=\tilde{\varphi}^*=\frac{\partial\tilde{\phi}^*}{\partial\tilde{z}}=0 \text{ at } \tilde{z}^*=0 \text{ and } \tilde{z}^*=1\,. \tag{30}$$

3 Normal Mode Analysis

Now an arbitrary perturbation is analyzed into a complete set of normal modes and then the stability of each of these modes is examined individually. For the system of equations (26)-(29), the analysis can be made in terms of two–dimensional periodic wave numbers. Thus, we ascribe to the quantities describing the dependence on $\tilde{x},\tilde{y}$ and $\tilde{t}$ of the form $\exp(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t})$, where $\tilde{l}$, $\tilde{m}$ are the wave numbers in the x and y-direction, respectively, and $\tilde{\omega}$ is the growth rate of the disturbances, which in general is a complex constant.

Above consideration allows to suppose that the perturbations quantities $\tilde{w}^*,\tilde{T}^*,\tilde{\varphi}^*$ and $\tilde{V}^*$ are of the form

$$\begin{aligned}&\tilde{w}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=W(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),\\&\tilde{T}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Theta(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),\\&\tilde{\varphi}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Phi(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right),\\&\tilde{V}^*(\tilde{x},\tilde{y},\tilde{z},\tilde{t})=\Psi(\tilde{z})\exp\left(i\tilde{l}\tilde{x}+i\tilde{m}\tilde{y}+\tilde{\omega}\tilde{t}\right).\end{aligned} \tag{31}$$

Using expression (31) and using linear theory, equations (26)-(29), in the linearized form become

$$\left[1+\frac{\tilde{\sigma}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right]^2\left(\tilde{D}^2-a^2\right)W-\left[1+\tilde{\lambda}_0\tilde{\omega}\right]\left[1+\frac{\tilde{\sigma}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right]\left[a^2\tilde{R}_n\Phi-a^2\left(\tilde{R}_a-\tilde{R}_{ea}\right)\Theta-a^2\tilde{R}_{ea}\tilde{D}\Psi\right]$$
$$+\left[1+\tilde{\lambda}_0\tilde{\omega}\right]^2\tilde{T}_a\tilde{D}^2W=0 \tag{32}$$

$$W+\left\{\frac{\tilde{N}_B}{\tilde{L}_e}\tilde{D}+\left(\tilde{D}^2-a^2\right)-2\frac{\tilde{N}_A\tilde{N}_B}{\tilde{L}_e}-\tilde{\omega}\right\}\Theta-\frac{\tilde{N}_B}{\tilde{L}_e}\tilde{D}\Phi=0, \tag{33}$$

$$\frac{W}{\tilde{\varepsilon}}-\frac{\tilde{N}_A}{\tilde{L}_e}\left(\tilde{D}^2-a^2\right)\Theta-\left\{\frac{1}{\tilde{L}_e}\left(D^2-a^2\right)-\frac{\tilde{\omega}}{\tilde{\sigma}}\right\}\Phi=0, \tag{34}$$

$$\left(\tilde{D}^2-a^2\right)\psi=\tilde{D}\Theta \tag{35}$$

where $\tilde{D}\equiv\frac{d}{d\tilde{z}}$ and $a^2=\tilde{l}^2+\tilde{m}^2$ is the dimensionless horizontal wave number.

Using the expression (31), the boundary conditions (30) transform to

$$W=\tilde{D}^2W=\Theta=\Phi=\tilde{D}\Psi=0 \text{ at } \tilde{z}=0 \text{ and } \tilde{z}=1. \tag{36}$$

The set of differential equations (32)–(35) together with the boundary conditions (36) constitute a characteristic value problem for Rayleigh number $\tilde{R}_a$ and given values of the other parameters $\tilde{T}_a, \tilde{\lambda}_0, \tilde{R}_n, \tilde{R}_{ea}, \tilde{\varepsilon}, \tilde{L}_e, \tilde{N}_A$. ., whose solutions ought to be obtained.

4 Linear Stability Convection

Using term Galerkin-technique of first approximation, the trial functions of lowest mode satisfying the boundary conditions (36) are chosen as

$$W=\tilde{A}_1\sin\pi\tilde{z},\ \Theta=\tilde{B}_1\sin\pi\tilde{z},\ \Phi=\tilde{C}_1\sin\pi\tilde{z},\ \Psi=\tilde{D}_1\cos\pi\tilde{z}. \tag{37}$$

Substituting the trial functions given by (37) into equations (32)-(35) and using the orthogonality of trial functions and boundary conditions (36), a system of linear homogeneous equations can be written in matrix form as

$$\begin{bmatrix} \tilde{T}_a\tilde{M}^2\pi^2+\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)^2\left(\pi^2+a^2\right) & -\tilde{M}\left[1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right]a^2\left(\tilde{R}_a+\tilde{R}_{ea}\right) & \tilde{M}\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)a^2\tilde{R}_n & -\tilde{M}\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\tilde{M}\right)\pi a^2\tilde{R}_{ea} \\ 1 & -\left(a^2+\pi^2\right)-\tilde{\omega} & 0 & 0 \\ \frac{1}{\tilde{\varepsilon}} & \frac{\tilde{N}_A\left(a^2+\pi^2\right)}{\tilde{L}_e} & \frac{\left(a^2+\pi^2\right)}{\tilde{L}_e}+\frac{\tilde{\omega}}{\tilde{\varepsilon}} & 0 \\ 0 & -\pi & 0 & -\left(a^2+\pi^2\right) \end{bmatrix}\begin{bmatrix} W \\ \Theta \\ \Phi \\ \Psi \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{38}$$

where $\tilde{M}=\left(1+\tilde{\lambda}_0\tilde{\omega}\right)$.

Using orthogonality, the non-trivial solution of the above matrix requires the vanishing of the determinant of the coefficient matrix in equation (38) which yields eigen –value equation as

$$\tilde{R}_a = -\left(\frac{a^2}{a^2+\pi^2}\right)\tilde{R}_{ea} + \frac{\tilde{T}_a\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\pi^2\left(a^2+\pi^2+\tilde{\omega}\right)}{\left(1+\frac{\tilde{\omega}}{\tilde{V}_a}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)\right)a^2} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e}{\tilde{\sigma}\left(a^2+\pi^2\right)+\tilde{\omega}\tilde{L}_e}\left[\frac{a^2+\pi^2+\tilde{\omega}}{\tilde{\varepsilon}} + \frac{\left(a^2+\pi^2\right)\tilde{N}_A}{\tilde{L}_e}\right]$$
$$+\frac{\left(a^2+\pi^2+\tilde{\omega}\right)}{a^2\left(1+\tilde{\lambda}_0\tilde{\omega}\right)}\left[\left(a^2+\pi^2\right)+\frac{\left(a^2+\pi^2\right)\tilde{\omega}\left(1+\tilde{\lambda}_0\tilde{\omega}\right)}{\tilde{V}_a}\right] \tag{39}$$

Equation (39) is the required dispersion relation accounting for the effect of Lewis number, kinematic visco-elasticity parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio on thermal electro instability in a layer of Maxwell elastico-viscous nanofluid saturating a porous medium under vertical AC electric field. Let us write $\tilde{\omega} = \tilde{\omega}_r + i\tilde{\omega}_i$, where $\tilde{\omega}_r$ and $\tilde{\omega}_i$ are real. For oscillatory motion, the real part of $\tilde{\omega}$ is zero. i.e., $\tilde{\omega}_r = 0, \therefore \tilde{\omega} = i\tilde{\omega}_i \neq 0$.

4.1 Stationary Convection

The stationary motion is characterized by putting $\tilde{\omega} = 0$ in equation (39), one gets

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}} + \tilde{N}_A\right), \tag{40}$$

which expresses the nanofluid Darcy Rayleigh number $\tilde{R}_a$ for stationary convection as a function of the dimensionless wave number a, electric Rayleigh number $\tilde{R}_{ea}$, nanofluid Lewis number $\tilde{L}_e$, modified diffusivity ratio $\tilde{N}_A$, concentration Rayleigh number $\tilde{R}_n$ and medium porosity $\tilde{\varepsilon}$. It is clear from the equation (39) that $\tilde{R}_a$ is independent of stress-relaxation time $\tilde{\lambda}_0$, Vadasz number $\tilde{V}_a$ and ratio of specific heat $\tilde{\sigma}$ for stationary modes, since these vanish with the vanishing of $\tilde{\omega}$.

In the absence of rotation parameter the thermal Rayleigh number is

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}} + \tilde{N}_A\right) = 0, \tag{41}$$

The Darcy Rayleigh number , $\tilde{R}_a^S$ given by equation (41) takes its minimum value by putting $\frac{\partial \tilde{R}_a^S}{\partial a^2} = 0$, we get

$$\frac{a^2.2\left(a^2+\pi^2\right)-\left(a^2+\pi^2\right)^2}{a^4} + \frac{a^2+\pi^2-\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^4} - \left[\frac{\left(a^2+\pi^2\right)-a^2}{\left(a^2+\pi^2\right)^2}\right]\tilde{R}_{ea} = 0, \tag{42}$$

It is clear from equation (42) that the values of critical wave number does not depend on the parameters accounting for nanoparticles, however, depends on $\tilde{R}_{ea}$ only. The effects of various non-dimensional parameters, namely, the electric field $\tilde{R}_{ea}$, the nanofluid Lewis number $\tilde{L}_e$, the modified diffusivity ratio $\tilde{N}_A$, and the concentration Rayleigh number $\tilde{R}_n$ on the stability of stationary modes have been investigated analytically by examining the behavior of $\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_{ea}}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{L}_e}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{N}_A}, \frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_n}$ and $\frac{\partial \tilde{R}_a^S}{\partial \tilde{\varepsilon}}$.

It is depicted from equation (41) that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_{ea}} = -\frac{a^2}{\left(a^2+\pi^2\right)}, \tag{43}$$

which is always negative for all wave numbers, thereby decreasing the Darcy Rayleigh number decreases with increment in $\tilde{R}_{ea}$ (electric Rayleigh number). Thus $\tilde{R}_{ea}$ has always a destabilizing effect on the system.

Equation (41) further yields that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{L}_e} = -\frac{\tilde{R}_n}{\tilde{\varepsilon}}, \tag{44}$$

and

$$\frac{\partial \tilde{R}_a^S}{\partial N_A} = -\tilde{R}_n, \tag{45}$$

It is clear from equation (41) for the bottom-heavy particles (for negative value of $\tilde{R}_n$) both the nanofluid Lewis number $\tilde{L}_e$ and the modified diffusivity ratio $\tilde{N}_A$ stabilize the system for negative value of $\tilde{R}_n$.

Equation (41) also gives that

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{R}_n} = -\left(\tilde{N}_A + \frac{\tilde{L}_e}{\tilde{\varepsilon}}\right), \tag{46}$$

Which is always negative for $\left(\tilde{N}_A + \frac{\tilde{L}_e}{\tilde{\varepsilon}}\right) > 0$, since the value of $\tilde{N}_A$ is taken in the range of -1 to -25 and $\tilde{L}_e$ in the range of 100-400. Here a negative $\tilde{N}_A$, (modified diffusivity ratio), indicates that the density of nanoparticles is larger than that of the base fluid. An increase in negative values of $\tilde{N}_A$ reduces the thermophoresis to push the heavier nanoparticles upwards, which enhances the stabilizing effects of particle distributions.

$$\frac{\partial \tilde{R}_a^S}{\partial \tilde{\varepsilon}} = \frac{\tilde{R}_n \tilde{L}_e}{\tilde{\varepsilon}^2}. \tag{47}$$

The right hand sides of equations (42) and (45) are negative implying there by the AC electric Rayleigh number and nanoparticle Rayleigh number have destabilizing effect of stationary convection. The right hand sides of equations (43) and (44) are negative if $\tilde{R}_n > 0$. Thus, the Lewis number and the diffusivity ratio have destabilizing effect. But if $\tilde{R}_n < 0$, then Lewis number and the diffusivity ratio have stabilizing effect. The right hand side of equation (46) is positive if $\tilde{R}_n > 0$ and is negative if $\tilde{R}_n < 0$. Thus, medium porosity has stabilizing/destabilizing effect on the convection. These results are in good agreement with the results derived by Nield and Kuznetsov (2014), Rana et al. (2015,2016) and Chand et al. (2016).

4.2 Oscillatory Motion

Since $\tilde{R}_a$ is a physical quantity , it must be a real value . Hence it follows from equation that either $\tilde{\omega}_i = 0$ (exchange of stability , steady onset) or $\Delta_2 = 0$ ($\tilde{\omega}_i \neq 0$,overstability, oscillatory onset). Since for overstability, the critical Darcy Rayleigh number for the onset of instability via a state of pure oscillations of increasing amplitude is determined by putting $\tilde{\omega} = i\tilde{\omega}_i$ in equation (39) and after some algebraic simplifications, we get

$$\tilde{R}_a = \Delta_1 + i\tilde{\omega}_i\Delta_2 . \tag{48}$$

where Δ_1 and Δ_2 are given as follows :

$$\Delta_1 = -\left\{\frac{a^2}{(a^2+\pi^2)}\right\}\tilde{R}_{ea} - \frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left[\tilde{\sigma}(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e\right]}{\tilde{\varepsilon}\left[\tilde{\sigma}^2(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e^{\,2}\right]} - \frac{\tilde{\sigma}^2\tilde{R}_n(a^2+\pi^2)^2\tilde{N}_A}{\tilde{\sigma}^2(a^2+\pi^2)^2+\omega^2 L_e^{\,2}}$$
$$+\frac{\left[(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right]}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}\left[(a^2+\pi^2)-\frac{\tilde{\omega}^2\tilde{\lambda}_0}{\tilde{V}_a}(a^2+\pi^2)\right]-\frac{\tilde{\omega}}{\tilde{V}_a}\left[\frac{\tilde{\omega}^2-\tilde{\omega}\tilde{\lambda}_0(a^2+\pi^2)}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}\right](a^2+\pi^2)$$
$$+\frac{1}{a^2\left[\left(1-\frac{\tilde{\lambda}_0\tilde{\omega}^2}{\tilde{V}_a}\right)^2+\frac{\tilde{\omega}^2}{\tilde{V}_a^{\,2}}\right]}\tilde{T}_a\pi^2\left[\left\{(a^2+\pi^2)-\tilde{\lambda}_0\tilde{\omega}^2\right\}\left(1-\frac{\tilde{\lambda}_0\tilde{\omega}^2}{\tilde{V}_a}\right)+\frac{\tilde{\omega}}{\tilde{V}_a}\left\{\tilde{\omega}+\tilde{\lambda}_0\tilde{\omega}(a^2+\pi^2)\right\}\right]. \tag{49}$$

and

$$\Delta_2 = \frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{(a^2+\pi^2)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2)^2+\tilde{\omega}^2\right\}}+\frac{(a^2+\pi^2)\left\{(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}$$
$$-\frac{a^2\tilde{\sigma}^2(a^2+\pi^2)^2\tilde{R}_{ea}-\tilde{\sigma}^2\tilde{R}_n(a^2+\pi^2)^3\tilde{N}_A}{\tilde{\sigma}^2(a^2+\pi^2)^3+\tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)}-\frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e^{\,2}\right\}}$$
$$-\frac{a^2\tilde{\omega}^2\tilde{L}_e^{\,2}\tilde{R}_{ea}}{\tilde{\sigma}^2(a^2+\pi^2)^3+\tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)}-\frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) \tag{50}$$

Equating the real and imaginary parts of equation (48),we have $\tilde{R}_a = \Delta_1$,which gives on simplification, the Darcy Rayleigh number as

$$\tilde{R}_a^{osc} = \frac{(a^2+\pi^2)\left\{(a^2+\pi^2)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2})}-\frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right)-\frac{a^2\tilde{\omega}^2\tilde{L}_e\tilde{R}_{ea}}{\tilde{\sigma}^2(a^2+\pi^2)^3+\tilde{\omega}^2\tilde{L}_e^{\,2}(a^2+\pi^2)}$$
$$+\frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{(a^2+\pi^2)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2)^2+\tilde{\omega}^2\right\}}-\frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2(a^2+\pi^2)^2+\tilde{\omega}^2\tilde{L}_e^{\,2}\right\}}$$
$$-\frac{(a^2+\pi^2)\tilde{\sigma}^2\left\{a^2\tilde{R}_{ea}+(a^2+\pi^2)\tilde{N}_A\tilde{R}_n\right\}}{(a^2+\pi^2)^2\tilde{\sigma}^2+\tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{51}$$

and

$$i\tilde{\omega}_i\Delta_2 = 0 \tag{52}$$

Since for oscillatory modes, $\tilde{\omega}_i \neq 0$, therefore equation (52) implies that $\Delta_2 = 0$, which yields a dispersion relation (relation between $\tilde{\omega}$ and a) as

$$a_1\left(\tilde{\omega}^2\right)^4 + a_2\left(\tilde{\omega}^2\right)^3 + a_3\left(\tilde{\omega}^2\right)^2 + a_4\left(\tilde{\omega}^2\right) + a_5 = 0, \tag{53}$$

where,

$$a_1 = \tilde{\varepsilon} a^2 \tilde{\lambda}_0^{\,4} \tilde{L}_e^{\,2}\left(a^2+\pi^2\right), \tag{54}$$

$$\begin{aligned} a_2 = {} & -\tilde{T}_a \pi^2 \tilde{\varepsilon} \tilde{V}_a^{\,2} \tilde{L}_e^{\,2} \lambda_0^{\,4}\left(a^2+\pi^2\right) + a^2 \tilde{V}_a \tilde{\lambda}_0^{\,4} \tilde{R}_n \tilde{L}_e \tilde{\sigma}\left(a^2+\pi^2\right)\left[-\tilde{\sigma} + \tilde{L}_e + \tilde{\varepsilon}\tilde{N}_A\right] \\ & + 2\tilde{\varepsilon} a^2 \tilde{L}_e^{\,2} \tilde{\lambda}_0\left(a^2+\pi^2\right)^2\left(1-\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{\varepsilon} a^2 \tilde{L}_e^{\,2} \tilde{\lambda}_0^{\,2} \tilde{V}_a\left(a^2+\pi^2\right)\left(1-\tilde{\lambda}_0\right) + \tilde{\varepsilon} a^2 \tilde{\lambda}_0^{\,4} \tilde{\sigma}^2\left(a^2+\pi^2\right)^4, \end{aligned} \tag{55}$$

$$\begin{aligned} a_3 = {} & \tilde{T}_a \pi^2 \tilde{V}_a^{\,3} \tilde{\varepsilon} \tilde{L}_e^{\,2} \tilde{\lambda}_0^{\,2}\left\{1+\tilde{\lambda}_0\left(a^2+\pi^2\right)\right\} - \tilde{T}_a \pi^2 \tilde{V}_a^{\,2} \tilde{\varepsilon} \tilde{\lambda}_0^{\,2}\left(a^2+\pi^2\right)\left\{2\tilde{L}_e^{\,2} + \tilde{\lambda}_0^{\,2}\tilde{\sigma}^2\left(a^2+\pi^2\right)\right\} \\ & + 2a^2 \tilde{\lambda}_0^{\,2} \tilde{V}_a \tilde{R}_n \tilde{L}_e \tilde{\sigma}\left(a^2+\pi^2\right)\left\{-\tilde{\sigma} + \tilde{\lambda}_0\tilde{V}_a\tilde{\sigma} + \tilde{L}_e - \tilde{\lambda}_0\tilde{V}_a\tilde{L}_e + \tilde{\varepsilon}\tilde{N}_A\left(1-\tilde{\lambda}_0\tilde{V}_a\right)\right\} \\ & + 2\tilde{\varepsilon} a^2 \tilde{\lambda}_0^{\,2} \tilde{\sigma}^2\left(a^2+\pi^2\right)^4\left(1-\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{\varepsilon} a^2 \tilde{L}_e^{\,2}\left(a^2+\pi^2\right)\left(1-3\tilde{\lambda}_0 - 2\tilde{\lambda}_0\tilde{V}_a^{\,2} + \tilde{V}_a + 2\tilde{\lambda}_0\tilde{V}_a^{\,2}\right), \\ & + \tilde{\varepsilon} a^2 \tilde{\lambda}_0^{\,2} \tilde{V}_a\left(a^2+\pi^2\right)^2\left(\tilde{V}_a\tilde{L}_e^{\,2} - \tilde{\lambda}_0\tilde{\sigma}^2\right) + \tilde{\varepsilon} a^2 \tilde{\lambda}_0^{\,2} \tilde{V}_a \tilde{\sigma}^2\left(a^2+\pi^2\right)^3 \end{aligned} \tag{56}$$

$$\begin{aligned} a_4 = {} & \tilde{T}_a \pi^2 \tilde{V}_a^{\,2} \tilde{\varepsilon} \tilde{\lambda}_0^{\,2} \tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left(-2+\tilde{\lambda}_0\tilde{V}_a\right) + \tilde{T}_a \pi^2 \tilde{V}_a^{\,2} \tilde{\varepsilon} \tilde{L}_e^{\,2}\left(a^2+\pi^2\right)\left(-1+\tilde{\lambda}_0\tilde{V}_a\right) \\ & + \tilde{T}_a \pi^2 \tilde{V}_a \tilde{\varepsilon} \tilde{\lambda}_0^{\,2} \tilde{\sigma}^2\left(a^2+\pi^2\right) + a^2 \tilde{V}_a \tilde{R}_n \tilde{L}_e \tilde{\sigma}\left(a^2+\pi^2\right)\begin{pmatrix} 2\tilde{\lambda}_0\tilde{V}_a\tilde{\sigma} - \tilde{\lambda}_0\tilde{V}_a\tilde{\sigma} - \tilde{\sigma} - 2\tilde{\lambda}_0\tilde{V}_a\tilde{L}_e \\ + \tilde{\lambda}_0^{\,2}\tilde{V}_a\tilde{L}_e + \tilde{L}_e \end{pmatrix} \\ & + \tilde{\varepsilon} a^2 \tilde{V}_a \tilde{R}_n \tilde{N}_A \tilde{L}_e \tilde{\sigma}\left(a^2+\pi^2\right)\left(-2\tilde{\lambda}_0\tilde{V}_a + \tilde{V}_a^{\,2} + 1\right) + \tilde{\varepsilon} a^2 \tilde{V}_a \tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left(-3\tilde{\lambda}_0 + 2\tilde{V}_a\tilde{\lambda} + 1 - 2\tilde{\lambda}_0^{\,2}\tilde{V}_a\right) \\ & + \tilde{\varepsilon} a^2 \tilde{L}_e^{\,2} \tilde{V}_a^{\,2}\left(a^2+\pi^2\right)^2 + \tilde{\varepsilon} a^2 \tilde{\sigma}^2\left(a^2+\pi^2\right)^4\left(1+\tilde{\lambda}_0^{\,2}\tilde{V}_a^{\,2}\right) \end{aligned} \tag{57}$$

$$\begin{aligned} a_5 = {} & \tilde{T}_a \pi^2 \tilde{V}_a^{\,2} \tilde{\varepsilon} \tilde{\sigma}^2\left(a^2+\pi^2\right)^2\left\{-\left(a^2+\pi^2\right) + \tilde{V}_a + \tilde{\lambda}_0\tilde{V}_a\left(a^2+\pi^2\right)\right\} + a^2 \tilde{V}_a^{\,3} \tilde{R}_n \tilde{L}_e \tilde{\sigma}\left(a^2+\pi^2\right) \\ & \left(-\tilde{\sigma} + \tilde{L}_e + \tilde{\varepsilon}\tilde{N}_A\right) + \tilde{\varepsilon} a^2 \tilde{V}_a^{\,2} \tilde{\sigma}^2\left(a^2+\pi^2\right)^3\left\{\left(a^2+\pi^2\right) + \tilde{V}_a - \tilde{\lambda}_0\tilde{V}_a\right\}. \end{aligned} \tag{58}$$

Equations (51) and (53) are the equations which must be satisfied for the occurrence of overstability for a wave number corresponding to various non-dimensional parameters $\tilde{L}_e, \tilde{V}_a, \tilde{\lambda}_0, \tilde{R}_n, \tilde{\varepsilon}$.

As $\tilde{\omega}$ is real for overstability and at most there must be one change of sign in equation (53) implying thereby at most one positive root of equation (53) for which the critical Darcy Rayleigh number for oscillatory modes is obtained for various values of non-dimensional wave number from equation (51).

It is noteworthy from equation (53) that the existence of oscillatory modes is not (uninfluenced) due to the presence of vertical AC electric field. However, these modes depend on the other non-dimensional parameters accounting for nanoparticles, porous medium and viscoelasticity.

4.3 Validation of Results

In the absence of nanoparticles, rheological parameter and electric field, the equation (53) reduces

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2}, \tag{59}$$

which is confirmation with the earlier result by Lapwood (1948) for classical Newtonian fluids in porous medium. Therefore, due to immersion on nanoparticles in fluid, the parameters $\tilde{N}_A, \tilde{R}_n, \tilde{R}_{ea}, \tilde{V}_a, \tilde{\lambda}_0$ and $\tilde{\sigma}$ are introduced in the expression for the Darcy Rayleigh number, which strongly affect the convection of porous Maxwellian nanofluid layer.

In the absence of electric field that is, $\tilde{R}_{ea}=0$, the equations (51) and (40) reduce to

$$\tilde{R}_a^{osc} = \frac{\left(a^2+\pi^2\right)\left\{\left(a^2+\pi^2\right)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2\left(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2}\right)} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) + \frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{\left(a^2+\pi^2\right)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{\left(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\right)^2+\tilde{\omega}^2\right\}}$$
$$-\frac{\tilde{\sigma}\tilde{R}_n\tilde{L}_e\left\{\tilde{\sigma}\left(a^2+\pi^2\right)^2+\tilde{\omega}^2\tilde{L}_e\right\}}{\tilde{\varepsilon}\left\{\tilde{\sigma}^2\left(a^2+\pi^2\right)^2+\tilde{\omega}^2\tilde{L}_e^{\,2}\right\}} - \frac{\left(a^2+\pi^2\right)\tilde{\sigma}^2\left\{\left(a^2+\pi^2\right)\tilde{N}_A\tilde{R}_n\right\}}{\left(a^2+\pi^2\right)^2\tilde{\sigma}^2+\tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{60}$$

and

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \tilde{R}_n\left(\frac{\tilde{L}_e}{\tilde{\varepsilon}}+\tilde{N}_A\right)=0, \tag{61}$$

which are in good agreement with the earlier results given by Umavathi et al. (2016) (for the limiting case (i.e. $\tilde{\upsilon},\tilde{\eta}\to 1$) for both stress-free boundaries.

When the nanoparticles are not embedded i.e. $\tilde{R}_n=0, \tilde{N}_A=0$, the equation (51) and (40) gives

$$\tilde{R}_a^{osc} = \frac{\left(a^2+\pi^2\right)\left\{\left(a^2+\pi^2\right)+\tilde{\omega}^2\tilde{\lambda}_0\right\}}{a^2\left(1+\tilde{\omega}^2\tilde{\lambda}_0^{\,2}\right)} - \frac{\tilde{\omega}^2}{\tilde{V}_a}\left(\frac{a^2+\pi^2}{a^2}\right) - \frac{a^2\tilde{\omega}^2\tilde{L}_e\tilde{R}_{ea}}{\tilde{\sigma}^2\left(a^2+\pi^2\right)^3+\tilde{\omega}^2\tilde{L}_e^{\,2}\left(a^2+\pi^2\right)}$$
$$+\frac{\tilde{T}_a\pi^2\tilde{V}_a\left\{\left(a^2+\pi^2\right)\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\tilde{V}_a+\tilde{\lambda}_0^{\,2}\tilde{\omega}^4+\tilde{\omega}^2\right\}}{a^2\left\{\left(\tilde{V}_a-\tilde{\lambda}_0\tilde{\omega}^2\right)^2+\tilde{\omega}^2\right\}} - \frac{\left(a^2+\pi^2\right)\tilde{\sigma}^2\left\{a^2\tilde{R}_{ea}\right\}}{\left(a^2+\pi^2\right)^2\tilde{\sigma}^2+\tilde{\omega}^2\tilde{L}_e^{\,2}} \tag{62}$$

and

$$\tilde{R}_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\tilde{T}_a\pi^2\left(a^2+\pi^2\right)}{a^2} - \frac{a^2}{a^2+\pi^2}\tilde{R}_{ea}=0, \tag{63}$$

which agrees with the earlier results of Roberts (1969) in the limiting case.

5 Numerical Results and Discussions

The expressions of thermal Rayleigh number for both the oscillatory and stationary motions are encapsulated in equations (51) and (40) , respectively. The variations of $\tilde{R}_a$ along x-axis , with respect to wave-number a along y-axis have been plotted graphically in figures (1)-(7) and (8)-(14) using equations (40) and (51) for both the cases of stationary and oscillatory modes, respectively. The experimental values and the fixed permissible values of the dimensionless parameters $\tilde{R}_n = -0.1, \tilde{R}_{ea} = 20, \tilde{L}_e = 500, \tilde{N}_A = -5, \tilde{V}_a = 0.1, \tilde{\lambda}_0 = 0.5, \tilde{T}_a = 10$ for bottom heavy distribution and $\tilde{R}_n = 0.1, \tilde{R}_{ea} = 20, \tilde{L}_e = 500, \tilde{N}_A = 5, \tilde{V}_a = 0.1, \tilde{\lambda}_0 = 0.5, \tilde{T}_a = 10$ for top-heavy distribution are used to investigate the effects of various involved parameters on the system numerically by using MATHEMATICA software version - 5.2.

The effects of different values of $\tilde{R}_n$ are illustrated in figures 2 and 3 and stationary convection w.r. t. wave number a. It is depicted from the graphs that $\tilde{R}_n$ destabilizing the stationary modes and stabilizing the oscillatory modes for both bottom/top-heavy distribution of nanoparticles. The effect of $\tilde{N}_A$ for both bottom/top-heavy distribution of nanoparticles on the onset of stationary modes is displayed in figure 4 and 8 respectively.. The figures show that the $\tilde{N}_A$ does not affect the stability of the system both type of distribution of nanoparticles. In figure 5, the variation of $\tilde{R}_a$ has been plotted versus wave number a for three different values of $\tilde{R}_{ea}$. It is observed from the figure that $\tilde{R}_{ea}$ has slight destabilizing effect for both stationary and oscillatory modes for bottom-heavy nanoparticles distributions. Figure 6: illustrates the effect of Lewis number on the stationary modes. It is depicted from the graphs that $\tilde{L}_e$ number has stabilizing effect implying thereby postponement on the onset of stationary convection. The influence of the Lewis number on neutral curves is shown on the graphs. It is observed that the effect of $\tilde{L}_e$ is stabilizing for stationary modes ; whereas this effect is slightly destabilizing for oscillatory mode. The critical wave number for stationary and oscillatory modes are 5.9 and 3.1, respectively.Negative / positive values of $\tilde{R}_n$ and $\tilde{N}_A$ represents a bottom/top-heavy distribution of nanoparticles.

Figure 7: shows the variation of the thermal Rayleigh-Darcy number with respect to wave number for different values of the Taylor number $\tilde{T}_a$. It is observed from the graphs that there is an increment in $\tilde{R}_a^S$ with an increment in $\tilde{T}_a$ for stationary modes, while a reverse effect is observed for oscillatory modes. This happens so because the presence of the Coriolis forces due to rotation of nanofluids allows to minimize the effect of buoyancy forces on the onset of electroconvection is dielectric nanofluids. Hence, $\tilde{T}_a$ has a stabilizing influence on stationary modes for the dielectric viscoelastic nanofluids for bottom-heavy distribution of nanoparticles. The effect of $\tilde{R}_n$ for both bottom-/top-heavy nanoparticles distribution are illustrated in figures 9 and 10 repectively. It is observed from the figure 9 that $\tilde{R}_a$ takes very large value in the range of $0 < a \le 25$ and there is uninfluenced effect of $\tilde{R}_n$ on $\tilde{R}_a$ in this regime. For $a > 2.5$, $\tilde{R}_a$ decreases with increase in $\tilde{R}_n$ implying thereby a destabilizing effect of $\tilde{R}_n$ on $\tilde{R}_a$. Figure 10 shows that $\tilde{R}_n$ has no significant effect on the oscillatory modes in the range $0 < a \le 3.1$ and for $a > 3.1$, $\tilde{R}_n$ destabilize the system.

Uninfluenced effects of $\tilde{N}_A$ for both bottom/top- heavy nanoparticles distributions are clear from figure 11. Vadasz number is shown in the figure 12. From figure 12 we see that there is a destabilizing effect on the physical system. The effect of $\tilde{T}_a$ accounting for angular velocity of the viscoelastic nanofluids is shown in figure 13. It is depicted from the figure 13 that there is a decrement in $\tilde{R}_a^{OSC}$ with increment in $\tilde{T}_a$, thereby destabilizing the oscillatory modes which is contrary to that of the stationary mode. The effect of $\tilde{L}_e$ on the oscillatory modes is displayed in figure 14. It is again found that Lewis number $\tilde{L}_e$ does not show significant effect in the range $0 < a \leq 3.1$. For $a > 3.1$, Lewis number $\tilde{L}_e$ shows slight destabilizing effect.

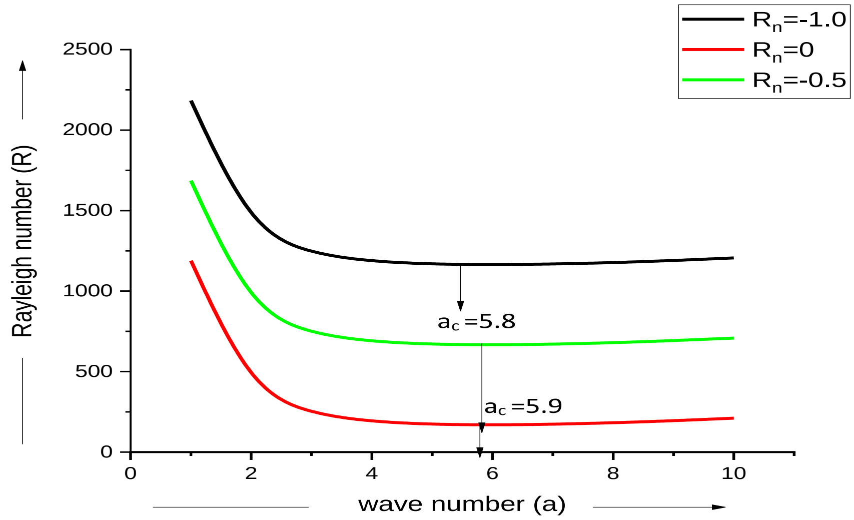

Figure 2: $\tilde{R}_a$ as a function of a for different negative values of $\tilde{R}_n$.

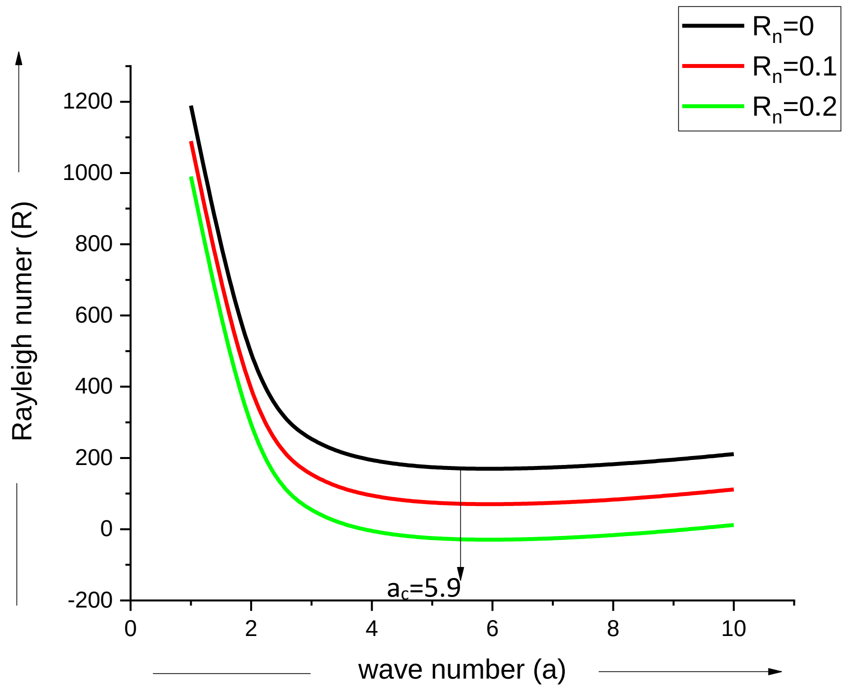

Figure 3: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{R}_n$.

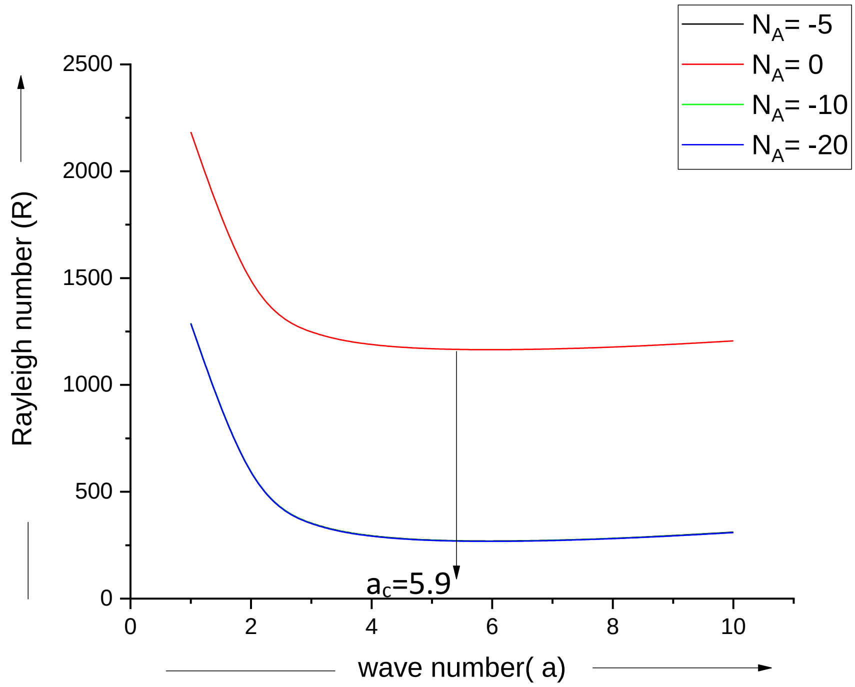

Figure 4: $\tilde{R}_a$ as a function of a for different negative values of $\tilde{N}_A$.

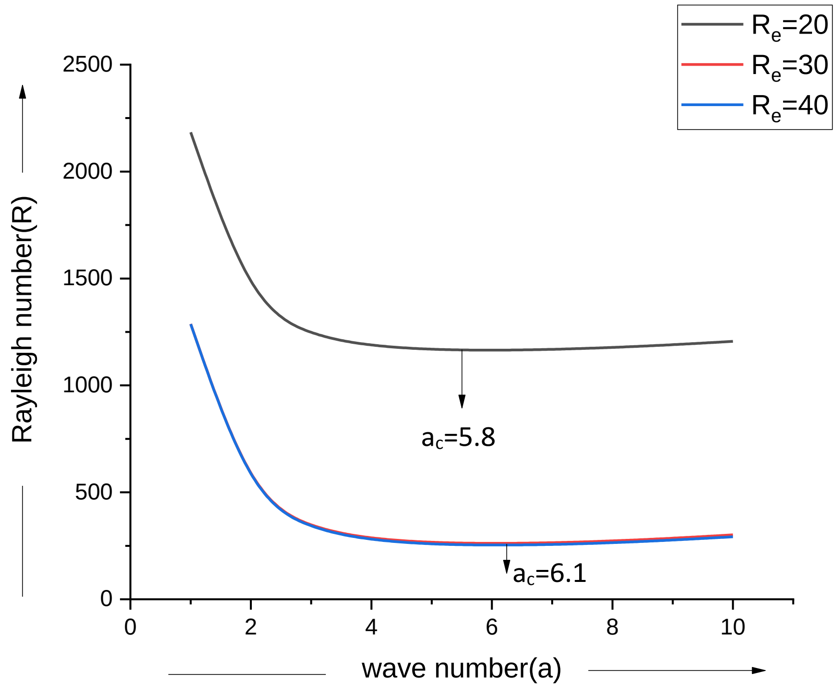

Figure 5: $\tilde{R}_a$ as a function of a for positive values of $\tilde{R}_{ea}$.

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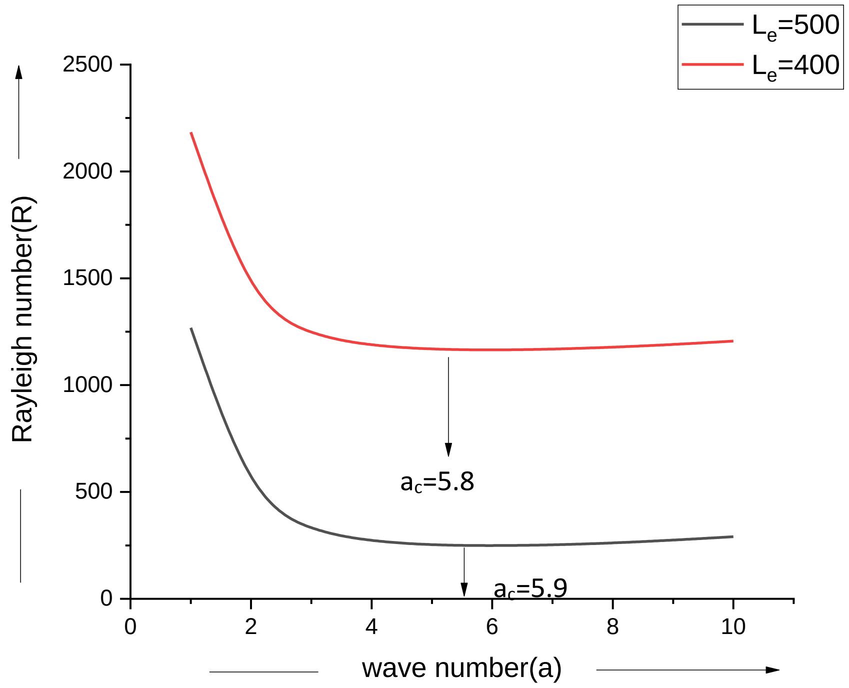

FIGURE: 6 $\tilde{R}_a$ as a function of a for different positive values of $\tilde{L}_e$.

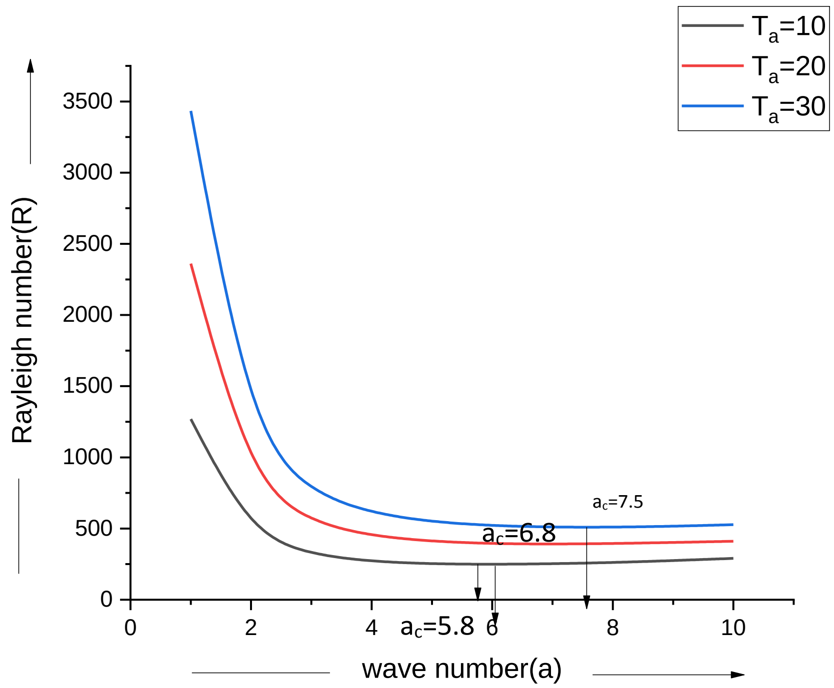

Figure 7: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{T}_a$.

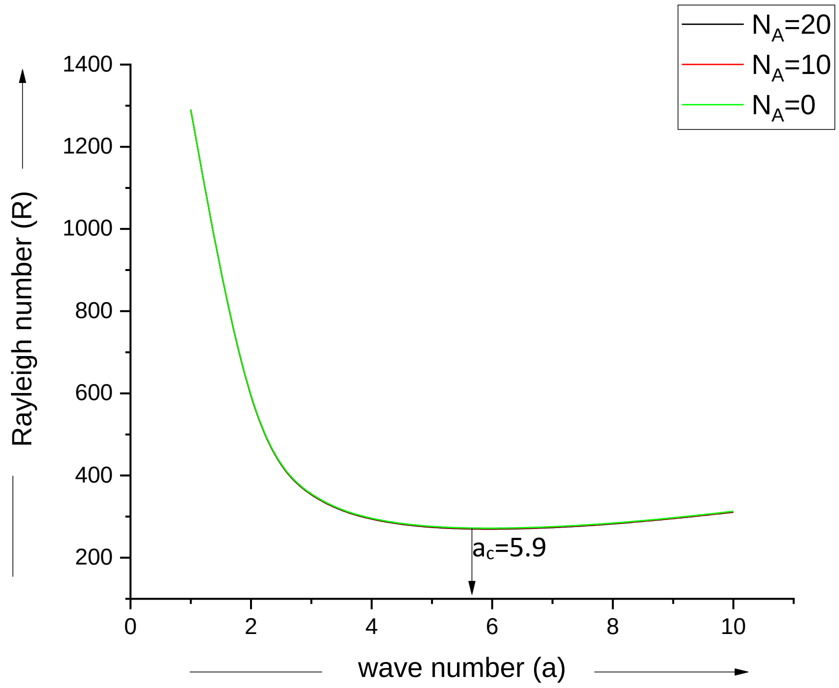

Figure 8: $\tilde{R}_a$ as a function of a for different values of $\tilde{N}_A$.

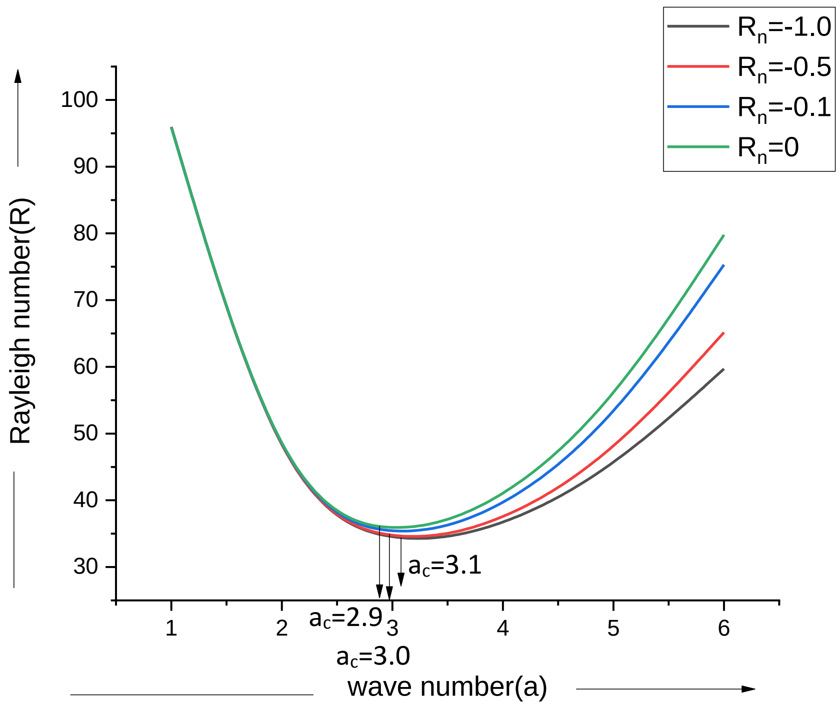

Figure 9: $\tilde{R}_a^{osc}$ as a function of a for different values of $\tilde{R}_n$.

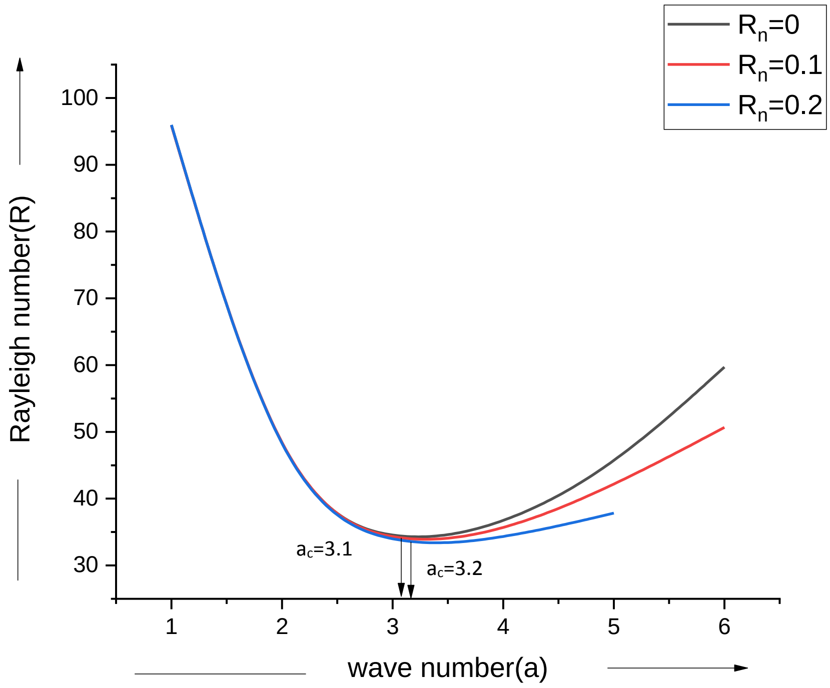

Figure 10: $\tilde{R}_a$ as a function of a for different positive values of $\tilde{R}_n$.

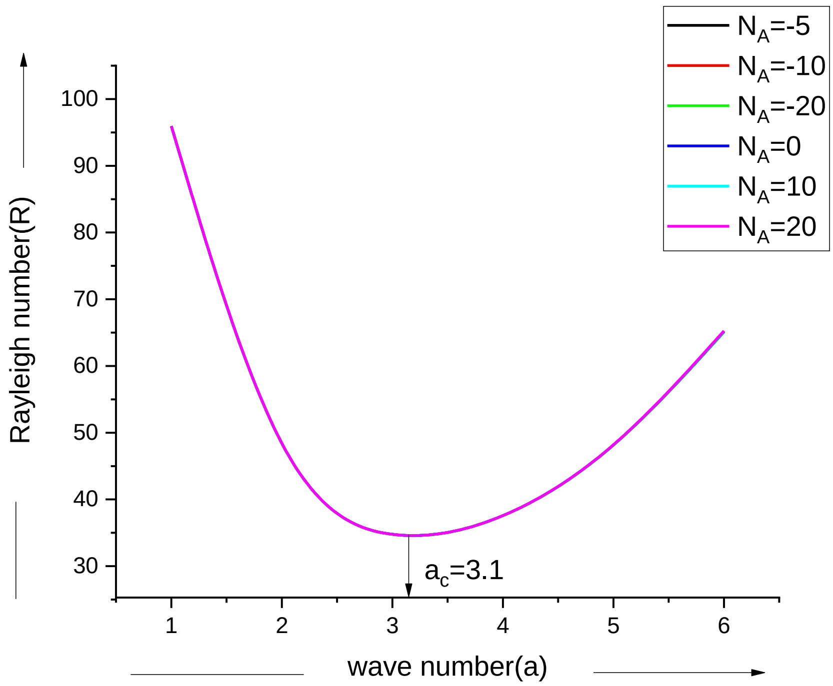

Figure 11: $\tilde{R}_a$ as a function of a for different values of $\tilde{N}_A$.

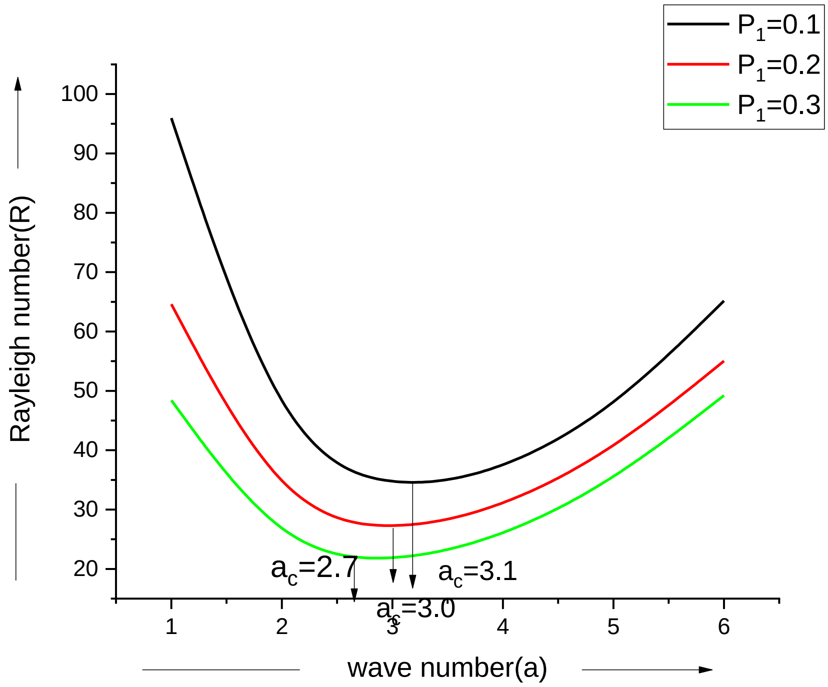

Figure 12: $\tilde{R}_a$ as a function a of for different values of $\tilde{V}_a$.

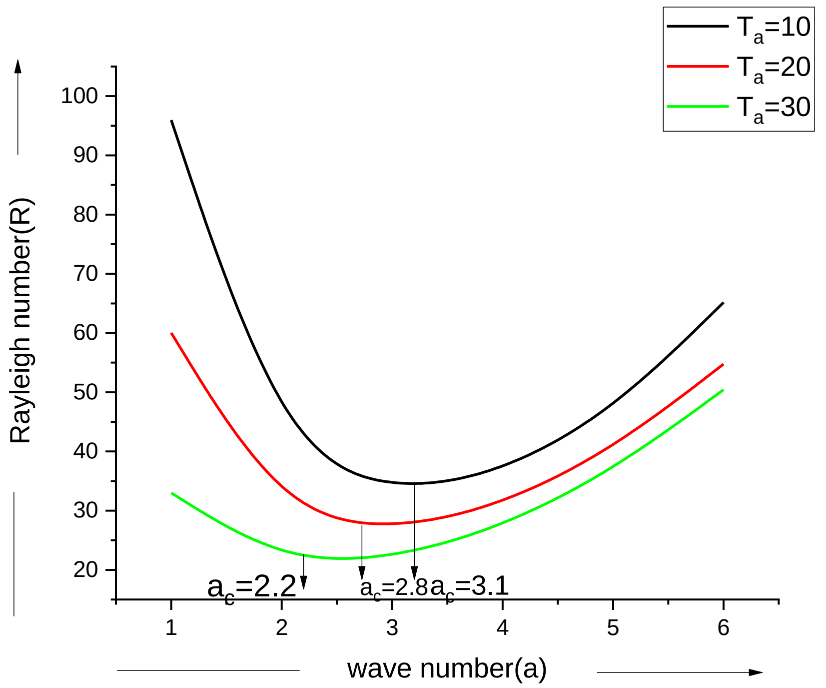

Figure 13: $\tilde{R}_a$ as a function of a for different values of $\tilde{T}_a$

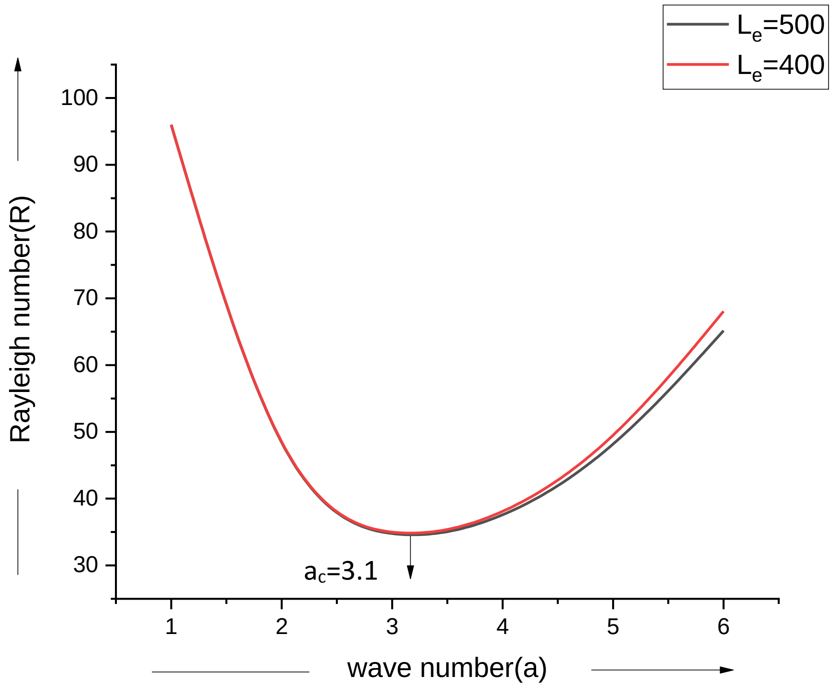

Figure 14: $\tilde{R}_a$ as a function of a for different values of $\tilde{L}_e$.

6 CONCLUSIONS:

The effect of rotation on electrothermal-convection in a dielectric Maxwellian nanofluid layer saturating a Darcy porous medium is studied. The main important results are as under:

- The concentration Rayleigh number $\tilde{R}_n$ destabilize the stationary modes and stabilize the oscillatory modes for both bottom / top - heavy nanoparticles distribution.
- The effect of modified diffusivity ratio $\tilde{N}_A$ is uninfluenced for both the stationary and oscillatory modes.
- Taylor number $\tilde{T}_a$ and Lewis number $\tilde{L}_e$ stabilizes/destabilizes the stationary / oscillatory modes.
- The effect of electric Rayleigh number $\tilde{R}_{ea}$ is very slight on both the stationary and oscillatory modes.
- The Vadasz number accounting for Darcy number destabilizes the oscillatory modes.

Interestingly, the occurrence of oscillatory modes is possible for top-heavy distribution of nanoparticles in the simultaneous presence of electric field and rotation, which was not the case in the absence of rotation and for Newtonian nanofluids.

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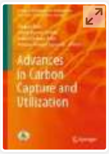

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Progresses in Bioenergy Generation from CO_2: Mitigating the Climate Change

Tanvi Sharma, Reva Bhardwaj, Rupali Bhardwaj, Anand Giri, Deepak Pant & Ashok Kumar Nadda

Chapter | First Online: 27 July 2021

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Abstract

In these days, the interest in vitality has quickly expanded a direct result of the monetary development around the world.

Tanvi Sharma, Reva Bhardwaj and Rupali Bhardwaj have equal contribution to this manuscript.

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HANDICRAFTS OF RAJASTHAN: LOOKING THROUGH THE LENS OF UTILITARIAN PURPOSE, THE CONSTRUCTION AND ART OF POTTERY MAKING

Authored By

Dr. Nirupama Singh
Central University of Punjab, Bathinda (Punjab)

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HANDICRAFTS OF RAJASTHAN: LOOKING THROUGH THE LENS OF UTILITARIAN PURPOSE, THE CONSTRUCTION AND ART OF POTTERY MAKING

Dr. Nirupama Singh Central University of Punjab, Bathinda (Punjab)

Abstract:
India has always been known as a land of diversity and rich heritage culture. Every province in India has its exclusive provincial art and skills that are passed down from generation to generation. This indigenous knowledge is deeply entrenched in the Indian culture. Artists in each generation have shaped the finest works out of existing material. Rajasthan exposes artisan traditions that have stood the test of time. The region has a rich variety of handicrafts and many of which exhibit their unique identity. Among them, pottery is a very attractive and fascinating handicraft due to its varied shapes and sizes. Pottery is used in the process of making different forms of kitchenware and home decor.
Pottery is the entire procedure and product of making different varieties of objects with substances like clay and ceramic, that are then heated at high degrees so to provide them with a durable form. Clay-based pottery is mostly used to make earthenware and stoneware. The term pottery is also used for the places where such wares are made by potters. There are different variations and varieties of pottery are Blue Pottery, Kagzi Pottery, Bikaneri Pottery, Black Pottery, and Pokhran Pottery. Rajasthan has always been the distinction of having a variety of categories in the handicraft of this art. Jaipur, Alwar, Bikaner, Sawai Madhupur, and Jaisalmer are the main places for the manufacturing of products or artefacts. Artisans are skilled manual workers who are engaged in craft production for home décor. They earn their livelihood by these creations of art and craft. The majority of people show interest on a large scale to buy these products these days due to their appearance and utility.
The present paper focuses on four different types of Pottery practised in Rajasthan and their process of making through survey-based information. It also throws light on the utilization of pottery products and their construction skills.
Keywords: *Artist, Construction, Handicrafts, Pottery, Rajasthan, Utility.*

Introduction:
History has been evident of the fact that handicrafts form a cultural peculiarity for a civilisation. The craftsmanship of people reflects the excellence they carry in their work and also highlights various cultural aspects. Handicrafts are nothing but human creations, they require manual excellence, passion, and love to be in their best forms. The art varies in its quality and nature. The term handicrafts refer to the products that belong to a specific community that is manufactured by human labour using traditional techniques. The process of manufacturing and selling handicrafts serves the purpose of employment that fulfil the daily needs of a family. This way the art helps in the socio-economic development of
The north-western state of India- Rajasthan reveals artisan traditions that have stood the test of time. The region has a rich variety of handicrafts and many of which exhibit their unique identity. Among them, pottery is a very attractive and fascinating handicraft due to its variety in shapes and forms.
The oldest evidence of pottery has traced its roots to the Indian sub-continent where a few sites in the Ganga Basin have unique signs of pottery from the Mesolithic ages. It is known that the place that traditionally used clay in pottery making and some architectural forms were the Balluchi district of Mehrgarh (site is located on the principal route between what is now Afghanistan and the Indus Valley) which is the oldest agricultural settlement in the Indian subcontinent Agriculture-based Neolithic settlements. Pottery craft was first innovated in China around 20000 years ago. Since traditional times, pottery and its origins have been an important pillar of cultural history. Pottery as an art was not limited to one location or area but was spreading to distant lands. Therefore, art grew gradually and evolved with certain necessities with the changing economic conditions. The process of making an artefact starts from forming shapes with clay which are then dried in sun and finally heated to give a durable state, hence the word pottery refers to the products made out of clay. Clay works and functions as the primary ingredient that forms the fundamental in all pottery. The primary material

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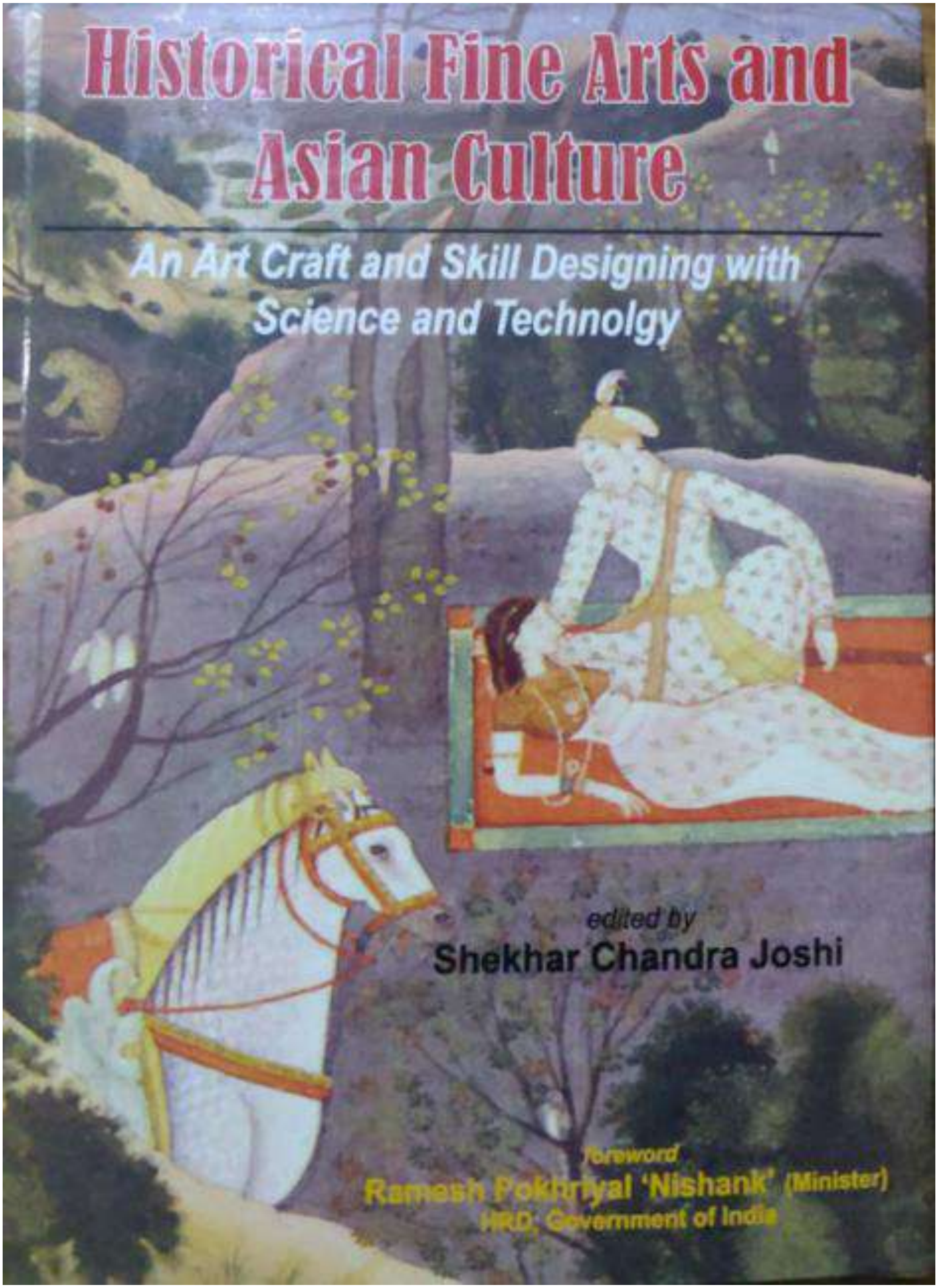
THE ENTIRE HISTORICAL PERSPECTIVE AND STYLISTIC FEATURES OF MAURYAN SCULPTURES

The era of Mauryan period is marked by an impressive development in the Indian sculptures.Mauryan sculpture is the art of the Mauryan Empire which ruled mostly in the northern India,with its peak between 322-232 BCE, decline after Ashoka the great.Mauryan art epitomized an important evolution in Indian art from use of wood to stone. The Mauryan period is generally regarded as most indigenous and exclusive art of the north Indian for the major religious groups.Thestatues of the Mauryan period shows a new lively and sensual quality. These sculptures were made up of sandstone with application of shiny polish, which adds to their beauty and sculptures became more realistic and naturalistic in character. This paper throws light on the entire history and main stylistic features of the Mauryan sculptures.

Keywords: Chandragupta, Mauryan, Polish, Religious, Sculptures

Mauryan sculpture is the art of the Mauryan Empire which ruled mostly in the northern India and manifest by an impressive development in the field of Indian sculpture. In the 4th century B.C., Nanda kings ruled Magadha dynasty and this dynasty was the most powerful kingdom of the north. A Brahman minister called Chanakya also known as Kautilya, trained a young man i.e.Chandragupta Maurya. Chandragupta organized his own army and overthrew the Nanda king Dhana Nand in 322 BCE. Therefore, Chandragupta Maurya is supposed to be the first king. With the coming of Mauryans for the first time almost entire Indian sub-continent comes under the power of monachary.This is significant for the numerous reasons most of which take shape during the time of Ashoka. The founder of the Mauryan dynasty is Chandragupta (324-297 BCE.) followed by Bindusara Maurya (297-273 BCE.) and then Ashoka (268-232 BCE.).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Chapter – 12

A CHRONOLOGICAL STUDY OF JHUNJHUNU FRESCOES

NIRUPAMA SINGH

The fresco culture in India dates back to many centuries however frescoes first develop in the ancient world and continued to be popular through the Renaissance but in Jhunjhunu district, it started at the end of the seventeenth century. Jhunjhunu fresco tradition was basically started by the Marwari families at the peak, were the trading community of Rajasthan. This tradition tells the tales of the enterprising Marwari merchants of the district, whose meteoric rise and enormous wealth was fueled by the artistic talent of the itinerant painters. Later the ingenious skills of the locals came to the fore as painters become expert in fresco paintings. Fresco is a technique in which water based pigments are applied to wet surface and the dry powdered colors. When colors mixed with it, water penetrates the surface and became permanent part of the wall. This technique is also known as *buno* fresco or true fresco to distinguish it from fresco *secco* or dry fresco (painting on dry plaster). This tradition bestowed the region with an artistic wealth not to be seen anywhere else in the world.

At the beginning, the fresco paintings done under the *buono* technique in which painting done on wet surface and pigments were made from locally available materials. It was drawn only on the upper parts of the buildings like on a domed ceiling of temples or *havelis*. Thereafter, from 18^{th} century onwards, the usage of fresco paintings had spread to the guest rooms of the *havelis* and by 19^{th} century saw its peak. Also during the 18^{th} and 19^{th} centuries, the practice to build decorative paintings was rampant amongst the nobles and the wealthier ones, and both strived to outdo each other in embellishing their resident *havelis*. Grand decorations with fresco paintings had become a status symbol. One such trader Dulichand Kakraniya from Chirawa, lived a rich and fascinating life and was known to build his own enterpeise in erstwhile Calcutta, trading with the British while maintaining great friendship with the Kings of Rajputana. He erected his fully adorned *haveli* with intricate frescoes, inviting guests and acquaintances over for sojourn and merriment.

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झुन्झुनूं क्षेत्र की हवेलियों के भित्ति-चित्रों का संयोजन

डॉ. निरुपमा सिंह

असिस्टेन्ट प्रोफेसर
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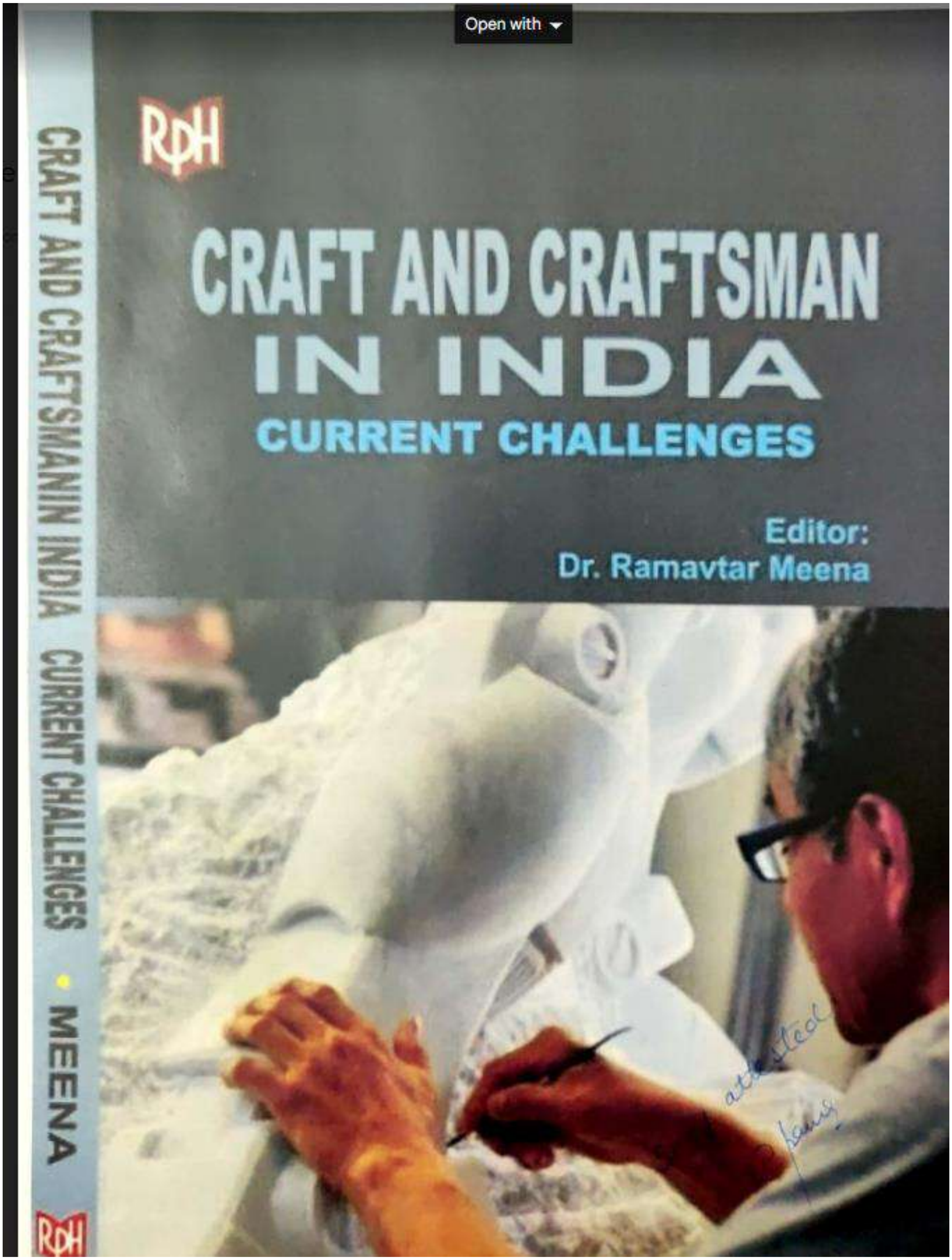
जयपुर

झुन्झुनू क्षेत्र राजस्थान में ही नहीं बल्कि पूरे भारत में भित्ति-चित्रों का महत्वपूर्ण और प्रसिद्ध स्थान रहा है। झुन्झुनू राजस्थान प्रांत में एक जिला है यह क्षेत्र जुझार सिंह नेहरा के नाम पर सन् १७३० में बसाया गया था। इस शहर के छोटे से छोटे गाँव की हवेली भी यहाँ चित्रित की हुई मिलेगी। यह क्षेत्र जयपुर राज्य की एक बड़ी और महत्वपूर्ण निजामत थी। यहाँ के सामन्त जयपुर राज्य और मुगल दरबार से अच्छे सम्बन्धों के कारण, इस राज्य के वासी व्यापार और अपने कारोबार की सफलता की ओर ले गए। इसी प्रकार से ये लोग बड़े-बड़े सेठ और धनी व्यक्तियों की गिंटी में शामिल हुए। झुन्झुनू जिले के नवलगढ़, महनसर, अलसीसर, मडावा, बगड़, रामगढ़, सूरजगढ़, उदयपुरवाटी जैसे अनेक सथान पर हवेलियों का निर्माण बड़े-बड़े सेठ साहुकारों तथा धनी व्यक्तियों ने अपने निवास के लिए करवाया। ये हवेलियाँ गोयनका, सिंघानियाँ, पोद्दार, मोदी, बिरला, पीरामल, डालमिया, मोरारका जैसे देश के बड़े उद्योगपति घरानों की हैं और ये विशाल हवेलियां बहुमंजिला हैं। जिनके बाहरी दोनों ओर चबूतरे बनें हैं तथा इनके दरवाजे चौखट लकड़ी के बने हुए हैं। जिन बारीक और कलात्मक खुदाई से सुसज्जित हैं। इनकी दीवारों पर का हर एक हिस्सा नयनाभिराम रंग-बिरंगे आकर्षक भित्तिचित्र एवं वास्तुकला मन मोह लेते हैं। इन भित्ति-चित्रों पर विविध रूप, रंग और विषयों को साकार करने की चेष्टा की गई है। जैसे खेती करते हुए किसान से लेकर युद्ध करते सेनानी तक, रामायण की कथाओं से लेकर महाभारत के विनाश तक, देवी-देवताओं से लेकर ऋषि-मुनियों तक, पीर बाबा से लेकर ईसा तक, बैलगाड़ी से लेकर रेलगाड़ी, हवाई जहाज तक आदि विषयों पर चित्र बने हुए हैं। इन भित्ति-चित्रों में धार्मिक विषयों के चित्र, नायिका भेद तथा रागमाला के चित्र, प्रतीकात्मक विषयों पर चित्र, राजदरबार से सम्बन्धित विषयों पर चित्र, लोक जीवन, पशु-पक्षी आदि के चित्र चित्रित किए गए हैं। कलाकारों की कल्पना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8

An Evaluative study on Wooden Craft of Ramgarh

Dr. Nirupama Singh*

Rajasthan is among the richest state in the country as far as the field of art and craft is concerned. In concept of quality and workmanships, the handicrafts of Rajasthan are incomparable from others. The entire Shekhawati region radiates with the wonderful wood carvings adorn in buildings which were constructed by the *marwaris* merchants, these buildings are *haveli*, temples, well, cenotaphs, etc.

Wood carving is a highly developed art form in Ramgarh .Ramgarh is a city and now municipality which comes under Sikar district of Shekhawati region in Rajasthan which is famous for its intricately carved wooden doors, windows and furniture. Creativity of wooden crafts is observed in the *havelis*, temples even in all buildings. These buildings are usually similar to *haveli* in respect to their architecture point of view ,which are built mostly by the *Marwaris* of this area in the late 18th century. Buildings of Ramgarh range from a single courtyard house form to an assemblage of multiple courtyards. Majority of these have one or :wo courtyards. A higher status of the owner or an increase in family members resulted in an increase in either the scale of the building or the number of courtyards. These *Marwari* merchants attained a remarkable level of economic prosperity and were responsible for creating the bulk of the built heritage during the mid 18th century till 1830s. They employed artist very huge amount

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Research Paper - Visual Art

A Studyofdepiction of Horse In Contemporary Indian Paintings (With Special Reference To Jaipur District)

* Ms. Shreya Patodia ** Dr.Nirupama Singh

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ABSTRACT

Dignified and classy, horses have been a popular muse for the artists throughout history of India. A formal and symbolic representation of a horse is observable acrossIndia's long history and was painted in different styles and mediums. Horses have an important place among the animals in Indian mythology, that's why they are admired by the common people also. There is an importance of horse in Vastu Shastra and Feng-Shui. We Indians are spiritual and continue to have faith in our ancient epics and beliefs which gives artists an important reason to paint horses now days.

This paper will investigate the possible reasons behind the depiction ofhorses in contemporary Indian paintings with special reference to Jaipur district. To achieve a general view to this subject importance ofhorse in Indian mythology and dreams are talked.

Keywords: Contemporary, Horses, Jaipur, Mythology, Painting.

Introduction:

The journey of Indian paintings is centuries old and Indian paintingshave developed and shaped in a beautiful manner expressingeach era ofIndian culture. India is a land of cultural and natural diversity. This is apparently manifested through art and literature. The Indian state Rajasthan is the land of magnificent legends of passion, love and bravery. Rajasthan's role in the amplification of Indian art has been very significant Jaipur is the capitalof Rajasthan. Rajput painting, also known as Rajasthani painting, emerged and developed in the royal courts of Rajasthan in India. EachRajputana kingdom developed a specific style, but with certain common features. Rajput paintings depict a number of themes, events of epics like the Ramayana. Many paintings were also done on the walls of palaces, inner chambers of the dwellings of Shekhawati.

Horse As An Element of Indian Painting:

From theages, horses have been depicted in various manners with different subject matters in different styles. Horses have an important place among the animals in Indian mythology, that's why they are admired by the common people and are given important places as God's vehicles, Demi-Gods and Demons. The mythical creatures with the composite appearance of horse in Indian mythology are:

Ashvin Kumaras:

They are the divine doctors, the twin devas of Ayurvedic medicine. They aredepicted as humans with the heads of horses.

Farasi Bahari:

They are magical green water horses that live at thebottom ofthe Indian Ocean, shown as a horse in its forepart, with a coiling, scaly, fish-like hindquarter.

Hayagriva: Also spelt Hayagreeva, is a horse-headed avatar of the Lord Vishnu in Hinduism.

Keshi: Keshi is the horse-demon, killed by Krishna (incarnation oflord Vishnu).

Tumburu: He is a horse faced Ghandarva, a celestial musician.

Uchchaihshravas:

It is a seven-headed flying horse that was obtained during the churning of the milk ocean.

These are some examples of the association of horses in Indian mythology that are part of the still going and preserved traditions.

According to Vastu Shastra, horses come with exuberance, and their constant presence around us would subsequently driveus to keep ourselves energized and focused too.

Some beliefs associated with horse according to Vastu Shastra and Feng- Shuiare:

- For growth in professional life, it is advised to hang the painting of seven running horses or keep the statueof horse in the office or home.
- The horseshoe is considered very lucky and used to be hungin many homes to protect from negative energies and attract good fortune for the family residing inside.
- The another belief is putting the horseshoe in a black cloth, placed in your granary. The household will never face the loss for grains and food ever.
- There are many other beliefs like this these which the people admire and use to follow in order to have a healthy and wealthy life according to their preferences.

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Manifestation of Imperative Characters and Events Related to Rama's Life on the Frescoed Walls of Jhunjhunu District

– Dr. Nirupama Singh

Indian Fresco paintings are renowned worldwide for their beauty, finesse and unimpeachable action. Fig.1. Battle scene of Rama and Ravana, Shardul Singh ChhatriThe fresco culture in India dates back from many centuries. Several of the earliest examples from the second century B.C. onwards survive at caves of Ajanta, Bagh, Badami and elsewhere. The techniques varied from one area to another, depending primarily on the raw materials available. In Jhunjhunu district, it started at the end of the seventeenth century. Jhunjhunu fresco tradition was basically started by the Marwari rich families of this region. There are about more than 500 havelis, cenotaphs and temples in Jhunjhunu with fine-looking fresco paintings on the walls. The havelis began to come in to existence after the last quarter of the eighteenth century. They are creations of the rich merchants built between 1780 to 1900-35. The beautiful frescoed walls do not only feast the eyes of the visitors, but they also mirror the contemporary life of the Jhunjhunu people, the ways, the customs, the faiths and beliefs of the people. This tradition tells the tales of the enterprising Marwari merchants of the district, whose meteoric rise and enormous wealth was fueled by the artistic talent of the itinerant painters. Later the ingenious skills of the locals came to the fore as painters become expert in fresco paintings and bestowed the region with an artistic wealth not to be seen anywhere else in the world.

The technique employed for the Jhunjhunu frescoes were elaborate and similar to the Italian frescoes techniques developed around the fourteenth century. As the ultimate symbol of the business class opulence, the Marwaris commissioned artists to paint those buildings each of them covered inside out with colourful frescoes kept alive for almost 300 years. These artists are called Chejara in local language. They prepared colours from locally available materials and done fresco under the supervision of the owner of the buildings.

The subject matters for the fresco paintings have changed over a period of time. In the earlier days at the time of its inception, mythology dominated the themes of frescoes, which shows the local legends, animal, portraits, hunting and wrestling. But a century later there was a change in the

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अनुक्रमणिका

शिल्प प्रवाह

संपादन

डॉ. सुरेन्द्र कुमार

Pustak Bharati
Toronto, Canada

शिल्प प्रवाह

हमारी कलाओं को भावाभिव्यक्ति के लिए सर्वोत्तम व सरलतम् साधन माना गया है। कला के द्वारा भावनात्मक अभिव्यक्ति से मनःस्थिति का प्रदर्शन स्वतः ही हो जाता है। कलाओं का लक्ष्य भी आनंदात्मक मनोरंजन के साथ मानव का सर्वांगीण विकास जैसे–शारीरिक, मानसिक, सामाजिक, सांस्कृतिक व नैतिक विकास कर उसे कलाप्रेमी, स्वावलम्बी एवं विवेकशील बनाना होता है। आज हर मानव मानसिक तनाव से अधिक दुःखी और परेशान है। ऐसी स्थिति में ये सभी अनेकानेक प्रदर्शनकारी कलाएँ शुद्ध मनोरंजन कर उसकी मानसिक थकान मिटाने और नई ऊर्जा को संचारित करनें में निरंतर सहायक सिद्ध हो रही हैं। भारतीय सांस्कृतिक परंपरा और अनेकानेक कलाओं पर केन्द्रित इस पुस्तक को साहित्यिक माध्यम से कला जगत में नए आयामों और विकासात्मक परिवर्तनों को समझने, संवर्धित रखने और जानने की दिशा में एक सार्थक प्रयास है। इन कलाओं की शैलीगत विशेषताओं को सहेजने और सांस्कृतिक मूल्यों को संरक्षित रखने के लिए आत्मचिंतन, आत्ममंथन और आत्मविश्लेषण करने और उसके विभिन्न अनछुए पहलुओं को भी उजागर करने पर इस पुस्तक में विचार किया गया है। इस कृति को राष्ट्रीय स्तर के विभिन्न कला साधकों और युवा संगीत, ललित और साहित्य शोधार्थियों के अमृत विचारों एवं शोध आलेखों के नवीनतम चिंतन द्वारा शोभायमान किया गया हैं जो निश्चित रूप से भारत के विभिन्न कलाओं, कला प्रेमियों, कला सेवियों और कला शिक्षार्थियों के लिए सार्थक सिद्ध होगा।

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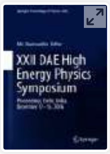

XXII DAE High Energy Physics Symposium pp 251–256

Quark-Lepton Complementarity Model Based Predictions for θ_{23}^{PMNS} with Neutrino Mass Hierarchy

Gazal Sharma, Shankita Bhardwaj, B. C. Chauhan & Surender Verma

Conference paper | First Online: 24 May 2018

Part of the Springer Proceedings in Physics book series (SPPHY,volume 203)

Abstract

After the successful investigation and confirmation of non zero θ_{13}^{PMNS} by various experiments, we are standing at a square where we still encounter a number of issues, which are to be settled. In this paper, we have extended our recent work towards a precise prediction of the θ_{23}^{PMNS} mixing angle, taking into account the neutrino mass hierarchy. We parameterize the non-trivial correlation between quark (CKM) and lepton (PMNS) mixing matrices in quark-lepton complementarity (QLC) model as $V_c = U_{CKM}.\psi.U_{PMNS}$, where ψ is a diagonal phase matrix. Monte Carlo simulations are used to estimate the texture of V_c and compare the results with the standard Tri-Bi-Maximal (TBM) and Bi-Maximal (BM) structures of neutrino mixing matrix. We have predicted the value of θ_{23}^{PMNS} for normal and inverted neutrino mass hierarchies. The value of θ_{23}^{PMNS} obtained for two cases are about 1.3σ away from each other, implying the better precision can give us a strong hint for the type of neutrino mass hierarchy.

Keywords

Neutrinos **Tri-Bi-Maximal** **Bi-Maximal** **Quark-Lepton complementarity**

Chapter

Recent Advances in Enzymatic Conversion of Carbon Dioxide into Value-Added Product

July 2021
DOI:10.1007/978-981-16-0638-0_14
In book: Advances in Carbon Capture and Utilization (pp.313-326)

Authors:

Anand Giri
Suman Chauhan
Tanvi Sharma
Ashok Kumar Nadda
Jaypee University of Information Technology

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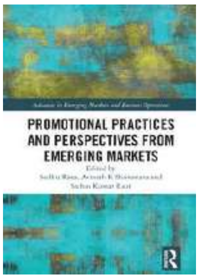
Abstract

Nowadays, climate change and global warming are major environmental issue due to continuous increase to atmospheric carbon dioxide concentration and vast growth in industrialization (Yaashikaa et al. in J CO2 Utiliz 33:131–147, 2019).

... After 35 • C, CA activity starts to decline which could be due to the denaturation of CA at high temperatures. Likewise, the optimum temperature for CA purified from C. freundii, Trachurus trachurus, and P. fragi was found to be 37, 30, and 35 • C (Caglayan et al., 2019; Giri et al., 2021; Sharma et al., 2009). In contrast, the alkali-thermotolerant CA

Chapter



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ABSTRACT

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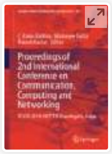

Proceedings of 2nd International Conference on Communication, Computing and Networking pp 63–71

An Identity-Based Authentication Framework for Big Data Security

Vinod Kumar, Musheer Ahmad & Pankaj Kumar

Conference paper | First Online: 08 September 2018

Part of the Lecture Notes in Networks and Systems book series (LNNS,volume 46)

Abstract

Big data raises a robust need for a network structure with the ability to support information retrieval and sharing. Many companies start to supply of big data services for Internet users and at the same time, these services also bring sever all security issues. Presently, the great part of big data set provides digital identity for users to use their services. At the moment, most of these systems use asymmetric and conventional public key cryptography (PKC) to give mutual authentication data security. However, existing authentication schemes trust commonly on the centralized servers to offer record and facilitation facilities for information retrieval. Pairing-free identity-based cryptography has some pull characteristics that gives the idea to healthy requirements in that scenario. The presented scheme is carried out in three phases and are as follows: initialization phase, registration phase and mutual authentication, and session key agreement phase. Detailed security analyzes have been made to authenticate big data server and user. Further, the paper has the resistance to possible attacks in this environment.

Keywords

Big data **Elliptic curve cryptography** **MapReduce**

Authentication and security

Anamika Prakashan

Proceedings of the XXIV DAE-BRNS High Energy Physics Symposium, Jatni, India pp 931–934

Formation of Marginally Trapped Surfaces in Gravitational Collapse

Suresh C. Jaryal & Ayan Chatterjee

Conference paper | First Online: 06 October 2022

Part of the Springer Proceedings in Physics book series (SPPHY,volume 277)

Abstract

Using a combination of analytical and numerical techniques, we study the formation and time evolution of collapsing shells, spherically symmetric marginally trapped tubes, as well as the event horizon. Depending on the mass function, density, and the velocity profile, there can be situations where these marginally trapped surfaces becomes space-like, time-like, or null.

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Orientation effects in the fusion of ^{28}Si+^{28}Si system using SEDF

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Introduction

The study of orientation effects is significant as it gives the most favourable configuration for the nuclei to fuse. For this, we have extended the semiclassical extended Thomas-Fermi approach of Skryme energy density formalism (SEDF) for deformed and oriented nuclei, although quadrupole deformation only. The orientation and deformations dependence in SEDF is included through the nuclear density. Here, two parameter Fermi density is used as nuclear density and is extended to deformed nuclei by using deformed nuclear radius and surface diffuseness of ref. [1]. These parameters for spherical nuclei are taken from the earlier work of one of us [2]. The nuclear proximity potential is obtained in slab approximation and overlap of density distribution is considered in sudden approximation. The total interaction potential is obtained by adding deformed Coulomb and centrifugal terms explicitly to the proximity part. We fixed the orientation of the one of the interacting nucleus and varied the orientation of the other in steps of 15° so as to obtain the maximum barrier height and the minimum interaction radius, a configuration favoured for hot fusion [3]. The characteristic properties of the total interaction potential obtained for this configuration is used in Hill-Wheeler approximation, an alternate to the one dimensional solution to the Schrödinger wave equation with appropriate boundary conditions, for a given partial wave. The transmission probability is obtained in parabolic approximation and the total fusion cross-section is calculated by adding partial waves up to certain maximum value ℓ_{max}.

For comparing our results, we have chosen ^{28}Si+^{28}Si system for which fusion evaporation residues were measured in a recent experiment [4] over center of mass-energy (E_{cm}) range $\simeq$ 31-39 MeV and coupled channel (CC) calculations were implemented to reproduce the observed fusion data. In an another experiment [5] of same group (a year ago) over E$_{cm}$ $\simeq 22-30$ MeV, the CC calculations were done by adjusting the parameters of the interaction potential. Similarly, for some other older experiments [6–8] over E_{cm} range ($\simeq 30-220$ MeV), Esbensen, et al. [9] has calibrated the potential parameters within the CC calculation. Now, with these above mentioned experiments the fusion cross-section data for ^{28}Si+^{28}Si system is available for over a wide range of energy both well below and quite above the barrier but there is no unique choice of interaction potential available in literature for the calculations of fusion cross-section. So, here we have achieved suitable interaction potential for the fusion process by varying nuclear orientations.

Methodology

The nuclear potential, $V_N(R)$ in semiclassical extended Thomas-Fermi approach of SEDF using slab approximation (see Ref. [2]) is,

$$V_N(R) = 2\pi\bar{R}\int_{s_0}^{\infty}\left[H(\rho,\tau,\vec{J})-\sum_{i=1}^{2}H_i(\rho_i,\tau_i,\vec{J_i})\right]dz$$

where $\bar{R}$ is the mean curvature radius for deformed nuclei, $H(\rho,\tau,\vec{J})$ is Skyrme Hamiltonian density and $\rho(=\sum_i\rho_i)$, $\tau(=\sum_i\tau_i)$, $\vec{J}(=\sum_i\vec{J_i})$, are nuclear, kinetic energy and spin-orbit densities respectively for composite system, $i=1,2$ for the two interacting nuclei. The two parameter Fermi density distribution in slab approximation for axially symmetric deformed and oriented nuclei is

$$\rho_i(z_i,\alpha_i,T) = \rho_{0i}(T)\left[1+\exp\left(\frac{z_i-R_i(\alpha_i,T)}{a_i(\alpha_i,T)}\right)\right]^{-1} \quad (1)$$

where $R_i(\alpha_i,T)$ and $a_i(\alpha_i,T)$ are nuclear radii and surface diffuseness parameters for deformed, oriented and coplanar nuclei of [1], respectively. The angle α_i is the angle between nuclear symmetry axis and radius vector, and are uniquely defined for each nuclei after satisfying the minimisation condition [3] for the separation between the interacting surfaces, for a fixed orientation. The nuclear radii and surface diffuseness parameters $R_{0i}(T)$ and a_{i0}(T) are taken from the earlier work of one of us

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[2] and λ(=2) for axially symmetric quadrupole deformation. The total interaction potential $V_T(R)$ is obtained by adding Coulomb $V_C(R)$ and centrifugal terms $V_\ell(R)(= \hbar^2\ell(\ell+1)/2\mu R^2)$ to nuclear proximity $V_N(R)$. The characteristic properties of this potential is used in Hill-Wheeler approximation [10] to calculate the fusion cross-section, as

$$\sigma_{HW} = = \frac{\pi\hbar^2}{2\mu E_{cm}} \sum_{\ell=0}^{\ell_{max}} (2\ell+1)\frac{e^x}{1+e^x} \quad (2)$$

where $x = 2\pi\{V_B(\ell) - E_{cm}\}/\hbar\omega_\ell$, $V_{B\ell}$ is barrier height, $R_{B\ell}$ is barrier position and μ is the reduced mass of interacting nuclei and $\ell_{max} = 38\hbar$ [9].

Calculations and results

The total interaction potential is obtained by taking various orientation of target and projectile at $\ell = 0$. The orientations of projectile and target are varied in steps of 15° to obtained an orientation which gives maximum barrier height at $\ell = 0$. Fig. 1 shows this variation of V_B with projectile orientation θ_1 for fixed orientation of target θ_2, at different target orientations θ_2. This figure clearly shows that the V_B first increase, become maximum and then decreases when $\theta_1 \to 90°$. Out of all possible orientations, the orientation (θ_1, θ_2 =30°, 30°) gives maximum V_B and minimum value of R_B, a favourable configuration for the fusion reaction. This is also clear from the Table I, where the orientation effect on other potential characteristics ($V_B, R_B, \hbar\omega_0$) is given. The characteristic properties of the potential for (30°, 30°) configuration are used to calculate fusion cross-section with HW approximation and partial waves are considered up to $38\hbar$ as per ref. [9]. Fig. 2, shows the calculated fusion excitation function for ^{28}Si+^{28}Si system over $E_{cm} \simeq 22-229$ MeV at (30°, 30°) orientation and is compared with the observed data [4–8]. It clear from the figure that (30°, 30°) configuration is the favoured configuration for the fusion process and the number of partial waves required to be included is $38\hbar$.

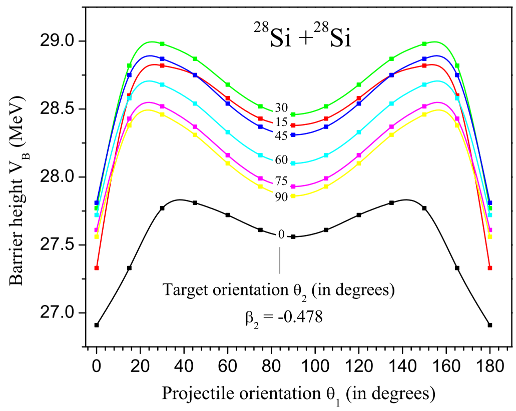

FIG. 1: The interaction potential barrier V_B as a function projectile orientation angle θ_1 at different target orientations angle θ_2.

Orientations:(θ_1, θ_2)	V_B(MeV)	R_B(fm)	$\hbar\omega_0$(MeV)
Spherical	28.23	9.05	2.87
(0°, 30°), (180°, 30°)	27.77	9.14	2.90
(15°, 30°), (165°, 30°)	28.82	8.76	2.85
(30°, 30°), (150°, 30°)	28.98	8.73	2.81
(45°, 30°), (135°, 30°)	28.87	8.81	2.78
(60°, 30°), (120°, 30°)	28.68	8.90	2.75
(75°, 30°), (105°, 30°)	28.52	8.98	2.75
(90°, 30°),(90°, 30°)	28.46	9.01	2.72

TABLE I: The characteristic properties of the total interaction potential for various orientations of projectile at fixed target $\theta_2 = 30°$ for $\ell = 0$.

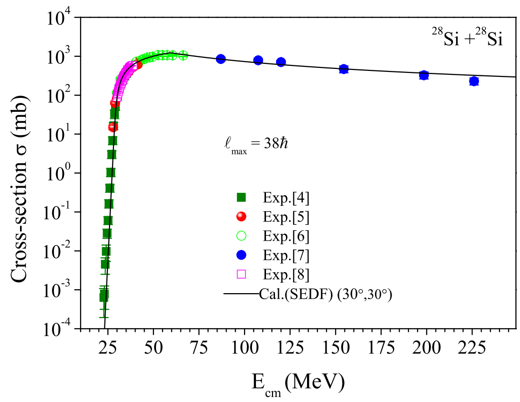

FIG. 2: Fusion excitation function for ^{28}Si+^{28}Si system for most favour configuration for hot fusion compared with experimental data [4–8] over a wide E_{cm} range.

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Isospin effects on the decay of $^{118,122,134}Ba^*$ isotopes

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Introduction

One of the importance of studying the nuclear reaction is to study the effects of isospin degree of freedom on reaction mechanism that is the effect of the incoming channel N/Z ratio on the outgoing fragments in a given channel. The effect of N/Z ratio of the compound nucleus is studied here by considering $^{118,122,134}Ba^*$ isotopes formed in the reaction of $^{78,82,86}Kr+^{40,40,48}Ca$ at centre of mass energies, E_{cm} =264.41, 268.85 and 308.06 MeV, which corresponds to 10 MeV/nucleon energy [1]. The study is done by using dynamical cluster decay model, DCM ([2] and references there in), where $\overline{\Delta R}$ is the only parameter of the model and is fixed arbitrarily at 1.01 fm for all isotopes of barium. Out of all possible fragments combination, the outgoing channel contains only the energetically favoured combination. In the calculations of decay cross-sections, the number of partial waves to be included are obtained by plotting the preformation probability P_0 or σ as a function of angular momentum ℓ and from the plot it is found that at a particular value of ℓ, below critical angular momentum ℓ_c, the $P_0 \to 0$ or $\sigma \to 0$. This angular momentum is called ℓ_{max} and gives the number of partial waves to be included in the decay cross-section. It is well established that the compound nucleus formed carries large angular momentum and is in excited state. The de-excitation of the CN takes place with the emission of both light particles (LPs: Z$\leq$ 2) and intermediate mass fragments (IMFs: 2 < Z $\leq$ 15, here). Here, in this work we have calculated the decay cross-section for both LPs and IMFs and a comparison is made between the three isotopes of Ba under study.

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Formalism

The decay cross-section defined by DCM ([2] and references there in) is,

$$\sigma(E_{cm},\ell) = \frac{\pi\hbar^2}{2\mu E_{cm}} \sum_{\ell=0}^{\ell_c} (2\ell+1) P_0 P \qquad (1)$$

where, P_0 is the preformation probability related to η-motion and is obtained from the solution of stationary Schrödinger equation in η-coordinate with temperature dependent collective fragmentation potentials (which include sum of binding energies, proximity [3], Coulomb and centrifugal potential terms) at a fixed $R = R_a = C_t(\eta,T) + \overline{\Delta R}(T)$, given as

$$P_0(A_i) = |\psi(\eta(A_i))|^2 \sqrt{B_{\eta\eta}} \left(\frac{2}{A}\right) \qquad (2)$$

where ($C_t = \sum C_i$, the Süssman central radii) $i = 1, 2$ for two out going fragments, the penetrability P, referring to R-motion, is WKB integral solved analytically with R_a and R_b as the first and second turning point, satisfying $V(R_a) = V(R_b) = Q_{eff}$, the effective Q-value for outgoing fragments, given as

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_b} \{2\mu[V(R) - Q_{eff}]\}^{1/2} dR\right] \qquad (3)$$

The critical angular momentum ℓ_c in terms of incident energy E_{cm}, reduced mass μ and the first turning point R_a of the entrance channel η_{in} is given by

$$\ell_c = \frac{R_a}{\hbar}\sqrt{2\mu(E_{cm} - V(R_a, \eta_{in}, \ell = 0))} \qquad (4)$$

with μ as the reduced mass and η_{in} as the entrance channel mass asymmetry.

Calculations and results

Barium isotopes considered for the study of isospin effects are $^{118,122,134}Ba^*$ andP_0, P and decay cross-section are the quantities on effects have been observed. The calculations have been done at fixed $\overline{\Delta R}$ =1.01 fm and the ℓ_c-values to the corresponding center of mass energies E_{cm} =264.41, 268.85 and 308.06 MeV are 153, 158 and 191$\hbar$. But, the cross-sections have been added up to the respective ℓ_{max} values i.e up to 76, 90 and 168 $\hbar$, because beyond ℓ_{max} the decay cross-section drops to zero. The Q_{in}-values are shown in Fig.2.

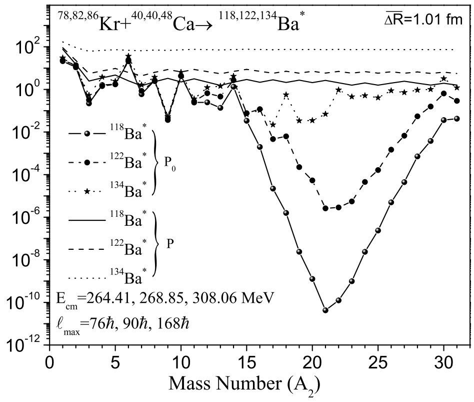

FIG. 1: The P_0 (shown by line plus symbol type) and P (shown by line types) are calculated by using DCM for the energetically favoured LPs and IMFs emitted from the decay of hot and rotating compound systems $^{118,122,134}Ba^*$ at centre of mass energies, E_{cm} =264.41, 268.85 and 308.06 MeV at $\overline{\Delta R}$ =1.01 fm.

Fig. 1, shows the P_0 and P for particles of mass A_2 = 1 - 31, which correspond to Z up to 15, emitted from the isotope of Ba^* compound system. From Fig. 1, we observe the following: (i) the P_0 for outgoing fragments from $^{118,122}Ba^*$ below mass $A_2 \leq 14$ is large compared to fragments of $A_2 > 14$ and (ii) both the P_0 and P increases with increase in N/Z ratio. (iii) the relative change for fragments of $A_2 > 14$ is more then for the fragments of $A_2 \leq 14$. As N/Z→ 1.39286, the P_0 for all the fragments comes almost with in an order of 2. In other words, the two windows of ($A_2 = 1 - 14$ and $A_2 = 15 - 31$) observed in case of $^{118,122}Ba^*$ almost disappears for $^{134}Ba^*$ (iv) the magnitude of P for $A_2 = 1 - 2$ is large by one order compare to the fragments of $A_2 > 2$ and hence the decay cross-section for $A_2 = 1 - 2$ is comparatively large (see Fig. 2) and for other observations for the decays cross-sections remains same as for P$_0$, see Fig. 2. Also, Figs.1 and 2,

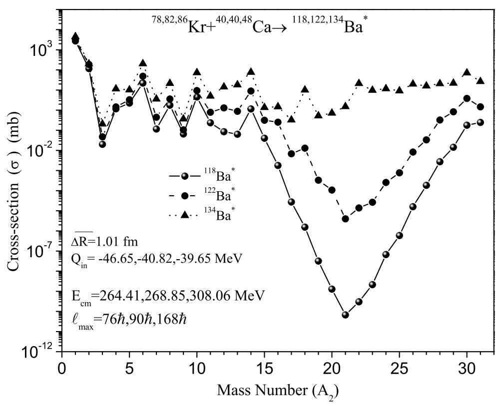

FIG. 2: Same as Fig.1, but for cross section σ.

shows that the magnitude as well as structure of $P_0(A_2)$ and $\sigma(A_2)$ does not change much up to $A_2 = 14$, while for fragments $A_2 > 14$ there is considerable change with increase in N/Z ratio. So, we conclude that the two windows observed for $^{118,122}Ba^*$ disappear for $^{134}Ba*$ which is equivalent to the fact that with increase in N/Z-ratio there is comparatively large increase in the decay probabilities of mass range $A_2 = 15 - 31$ than for masses $A_2 \leq 15$.

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Chapter

Bioremediation of Metals from Lithium-Ion Battery (LIB) Waste

January 2018
DOI:10.1007/978-981-10-7413-4_14
In book: Waste Bioremediation (pp.265-278)

Authors:

Tenzin Dolker
Central University of Himachal Pradesh

Deepak Pant
Central University of Himachal Pradesh

Abstract

Technological advancement has greatly increased the demand for newer lithium-ion batteries (LIBs) due to the more use of advanced energy storage devices like electric vehicles, consumer electronics, renewable energy storage, backup power, medical devices. The existing methods for metal recovery from LIB recycling involved: (i) aqueous stream-based limited recycling using the liquid stream mixed with soda ash with the hammer mill and shaker table (ii) supercritical CO2-based recycling of cathode and anode (iii) pyro- and hydrometallurgical processes. Microbial participation for the recovery of metals from waste (LIBs) was found to be an attractive method due to its environmental-friendly approaches. Lysinibacillus, Micrococcus, Sporosarcina, Empedobacter, Barrientosiimonas, Lysinibacillus, Paenibacillus, Bacillus, Acidithiobacillus are among the species involved in the recycling of metal form LIB.

Chapter

Bioremediation Techniques for E-waste Management

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Authors:

Deepak Pant
Central University of Himachal Pradesh

Anand Giri

Varun Dhiman

Abstract

Bioremediation or microbial cooperation can improve the e-waste management process in a greener way. Every management strategy is concentrated upon the organic and inorganic portion of the e-waste. Organic part consists of variety of thermo and thermosetting plastic with the presence of halogenated material. Microbes are involved in the process of dehalogenation in many ways. Microbes can manage the leaching of inorganic portion of e-waste which consists of both metallic and nonmetallic components.

ut the Book

Is homogeneity of cultures possible, nay advisable, in a heterogeneous world? The ology aims to study the anger and angst of native cultures while battling the so-called sed forces of a materialistic society. The essays included in the anthology not only deal accounts of such conflicts but a study of the literature born out of the after-effects of enging the boundaries of civilisations and cultures. Comprising essays by stalwarts in field of aboriginal studies such as Prof Bruce E. Johansen (University of Nebraska), Prof Thomas (University of Western Australia), and Prof Anand Mahanand (English and ign Languages University), the book is an attempt to bring together not only factual ils of how native cultures are eroded, but also the perils of such research, alongwith native cultures have resisted these burgeoning globalising forces to protect, if not their ure in actuality at least, a memory of it through literature.

ut the Authors

KBS Krishna is currently working as an Assistant Professor in the Department of English European Languages, Central University of Himachal Pradesh, Dharamshala. He did his , MPhil, and PhD from the English and Foreign Languages University (formerly CIEFL), erabad. His doctoral dissertation was on the depiction of cities in hardboiled detective on. He has given invited lectures for undergraduate and post graduate students, and arch scholars. He has served as a subject expert and presenter in NME-ICT e-content elopment Program (EMMRC, Hyderabad). He is the Chief Editor of the Spring Magazine English Literature (www.springmagazine.net) and Editor of Bharatiya Prajna Journal w. indianstudies .net). His poems and short stories have been published in ous international journals and are part of anthologies. His current research interests are modern studies and detective fiction.

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Homogeneity in Heterogeneity

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Bounds on Sterile Neutrino Component in the Solar Neutrino Flux

Govind Singh, Ashish Sharma, Gazal Sharma, Shankita Bhardwaj, Surender Verma & B. C. Chauhan

Conference paper | First Online: 24 May 2018

Part of the Springer Proceedings in Physics book series (SPPHY,volume 203)

Abstract

Solar neutrinos studies have played a crucial role in the development of neutrino physics. It took decades to understand the mysterious nature of neutrinos and to identify a leading solution to the Solar Neutrino Problem (SNP). The mystery of the missing neutrinos deepened as subsequent experiments were performed. The energy spectrum of solar neutrinos, as predicted by standard solar models, is seen by different experiments as they are sensitive to different energy ranges. More than 98 $\%$ of the calculated standard model solar neutrino flux lies below 1MeV. The rare Boron-8 neutrino flux is the high energy tail of solar neutrinos for which statistically significant measurements have been made so far, but this is the tip of the iceberg. As such, the study of low energy neutrinos can give us better understanding and showcase the possibility of species other than three active neutrino flavours, mainly the sterile neutrinos in solar neutrino flux. In the light of latest data available from various solar neutrino experiments including Borexino and KamLAND Solar phase we derive, in a model independent way, the bounds on sterile neutrino component in the solar neutrino flux. We update the limits on the sterile neutrino component and compare them with the previous results obtained using data from SNO Solar Salt Phase and Super-Kamiokande experiments. We retrieve the upper bounds existing in the literature and present the more stringent bounds on the sterile neutrino component.

Keywords

Sterile neutrino **Solar neutrino flux** **Solar Neutrino Problem**

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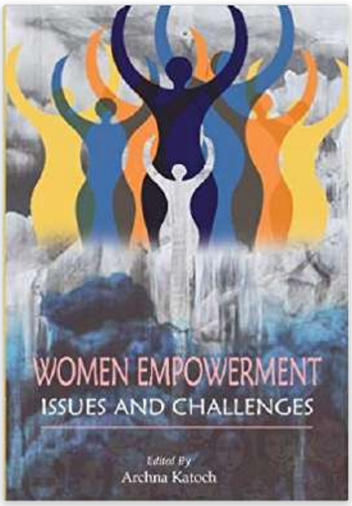


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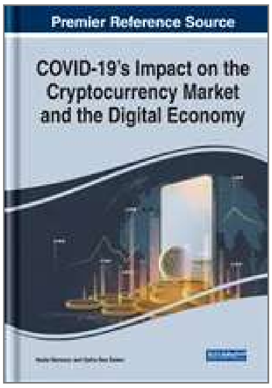

Coalescing Skills of Gig Players and Fervor of Entrepreneurial Leaders to Provide Resilience Strategies During Global Economic Crises

Manpreet Arora (Central University of Himachal Pradesh, India) and Roshan Lal Sharma (Central University of Himachal Pradesh, India)

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Pages: 23
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Abstract

The COVID-19 pandemic has caused diverse sorts of disruption across the globe. The pandemic has drastically impacted the economies of almost every country of the world. The international economic scenario is full of despair as the entrepreneurs and business leaders find it hard to come to terms with the extent of damage caused by the pandemic. In view of largely prevalent gloom and despair, it is imperative that certain resilience strategies are worked out so that global economic crisis can be stemmed from further escalation. The gig economy has been viewed as a powerful resilience mechanism to tide over the economic crisis caused the world over by COVID-19. Entrepreneurial leadership can also make significant difference in providing a paddle-push to the pandemic-struck world by reactivating the engines of economic growth. The nature of this chapter is qualitative, and it seeks to theoretically work out certain strategies that can help various economies of the world to stand up and be resilient in the face of complex challenges that the pandemic has thrown before us.

Chapter Preview

Top

Introduction

When in March 2020, WHO characterized COVID-19 as pandemic, no one had imagined that it might take such a devastating turn affecting all sectors of economy in more than 220 economies across the world. Global economy and financial markets received a powerful blow and disruption could be seen in spheres of production and supply chain, health, tourism, consumer goods, transport, and education leading to high level of loss of livelihoods and employment. Many countries had to impose lockdowns/ curfews/ restrictions to be opened up gradually in a phased manner so that the economic activity could resume. It certainly did gain pace in some countries like USA, Italy, UK, Germany, etc. that were worst hit at one point in time during the first wave of Covid-19 pandemic. On the other hand, developing economies like India and Brazil, saw the largest GDP contraction during 2020-2021. Moreover, India has undergone the second wave of the pandemic which has proved to be deadlier than the first one. It is the second time that the lockdowns/curfews have been imposed here for more than a month. Several South Indian states are now in the grip of the third wave of COVID-19 pandemic. There is a sharp decline in national income as the service sector has totally collapsed. The health system has given in and there is a huge stress on supply chains along with the trade tensions with China that have increased the agony as it had been the largest trading partner of India for a long time. The demand has significantly dropped in almost all nations where lockdowns were imposed.

The COVID-19 pandemic initially did not seem alarming despite the fact that the WHO had declared it as pandemic without any time lapse. But after a point, it spread like wild fire infecting and killing millions of people across the globe. The first world nations at one point were the most adversely affected. The businesses collapsed and economies were hit badly. Out of sheer panic, lockdowns were announced and restrictions were imposed. This pandemic has proven to be unprecedented owing to the life loss as well as the loss of livelihoods and economic damage.

The economic crises faced by the world due to COVID-19 are showing deep and long-lasting effects (Laborde, Martin & Vos, 2021). The interesting fact is the global economic downturn is not caused by economic factors but non-economic ones which have caused such a crisis across the world. Economies are trying hard to revive after the second wave of the pandemic but start-ups and medium-scale organizations which lack liquidity suffered a severe jolt and quite a few of them closed down. On the other hand, many jobs were laid off and only those who were having some technical, digital or specialized skills, could come up with small start ups or ventures and could survive properly. In fact, creativity and skills recued

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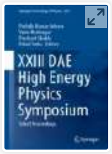

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CP Phase Analysis Using Quark-Lepton Complementarity Model in $3+1$ Scenario

Gazal Sharma, B. C. Chauhan & Surender Verma

Conference paper | First Online: 19 May 2021

Part of the Springer Proceedings in Physics book series (SPPHY,volume 261)

Abstract

The existence of sterile neutrino is revolutionizing Physics from the smallest to the largest scale. After the recent reports from the MiniBooNE experiment at Fermi lab that observed far more ν_e appearance data than expected, suggesting the possible existence of the fourth generation of another generation of neutrinos. These results would provide challenges for the Standard Model of Particle Physics if it is confirmed in future experiments and will have imperative implications on cosmology and astroparticle physics. Also, this will require new neutrino mass models to accommodate these degrees of freedom. With respect to that, the current work is just the extension of our recent work toward the CP phase analysis of Quark-Lepton Complementarity (QLC) model in $3+1$ scenario. The parametrization of non-trivial correlation between CKM_4 and $PMNS_4$ using Monte Carlo Simulation is used to estimate the texture of V_{m_4}. We have predicted the numerical ranges for sterile neutrino parameters and also investigated the values for Dirac CP violation phases and the CP re-phasing invariants using the model in $3+1$ scenario. The consequences of the model are the predictions for CP-Violating phase invariant *J*, Dirac phase ϕ, and the sterile neutrino parameters. The results obtained numerically and analytically this paper stood in good agreement with the experimental data. The results of this work would be very important in view of future sterile neutrino experiments.

Keywords

Neutrino mixing angles   **Quark-lepton complementarity**   **CP violation**

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SOCIO-ECONOMIC PERSPECTIVES

A Quarterly Refereed Journal

LIVELIHOOD PROSPECTS: A CRITICAL REVIEW OF STREET VENDING IN DELHI

Shabab Ahmad*, Prof. Darshan Singh**

This paper most vividly tries to inculcate the ways and mechanism of corruption in the lowest ladder of income generating peoples that is 'Street Vendors.' Giving bribe may get institutionalized pattern of behavior in our communities and it may no longer seem as problem. This paper also tries to give hypotheses

Keywords : Livelihood, Street Vendors, Bribe, Corruption, Entrepreneurs etc.

Introduction

If a person don't have anything and go to anywhere, two things always remain with the poor vendors, viz. Streets and Sky. Streets from where they can earn their basic livelihood, secondly, Sky, from which they never fear from getting removed. Now when these street vendors clearly knows that they are not going to lose anything as they are being positioned in the lower most ladder of the economic deprivation, gives the immense courage and strength to become Entrepreneur. Where risk taking factor is one of the most important quality for becoming successful entrepreneur. Consequently, it can be interpreted as 'every Street Vendor is a born entrepreneur.' Bribe is the form of corruption in which an individual gives money to stakeholder, authoritarian, chairperson, or any person who are responsible to doing work of an individual. The bribe is considered as crime under the laws to prevent corruption like such as prevention of corruption Act, 1988 and IPC (Indian penal code) 1860 and its section 166, section 171B, section 409, section 169 Specifically the prevention of corruption Act 1988 has deal bribery cases of street vending under section 11 and central vigilance commission, CBI (central bureau of investigation) and state anti-corruption bureau these three authorities have dealing such case of bribery and controlling over the corruption. Even once the introduction of varied legislations like Street trafficker (Protection of keep and Regulation of Street Vending) Act, 2014 and formulation of National Policy on Street Vendors, several of them face the matter in obtaining their license issued. Alternative downside includes insecurity at their geographic point, insecurity of earning, constant eviction threats, fines and harassment by traffic policemen, eviction by police, eviction by Municipalities co-operation, money stealing by gangster or local criminal, eviction without notice or relocation during national events etc.

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** *Senior Professor, Chairman- Department of Social Work, Kurukshetra University, Kurukshetra. Haryana.*

If we see that street vendors as small entrepreneur so that they are continuously trying to connect with main streamline of market illegally by giving bribe to local authorities cause of fear of eviction from main market or main streamline. It is showing that government is making new world for street vendor through evicted from main market, how the world you suppose to be as a market for street vendor partially different from sustaining market and placed them newly established vending zone where supply power is low and earning capacity is not much more then the main market. It help to understanding the concept of privatization of market as firstly local authorities focusing on eviction of street vendors mainly by streamlining in order to increase the market value and its shops which also increase the production/ product supply. When they started fight against the eviction than local authorities along with local organization have systematically evicted the street vendor from main market by providing vending zone. It may be possible they will be evicted again when their market able to sustain and making profit than the privatization will started to constructed shop and mall at their vending zone. Eviction from main market is not a solution of street vending, government needs to make a plan for connect street vendors to main market, and help them to sustain their business by providing platform without bribe. Bribe system is a game of money in the market and also to earn money. There are all possible chances in a single place to take bribe for eviction, and also to take bribe not for eviction, similarly to gives bribe to eviction of street vendor, and also giving bribe not for eviction from the main market.

Research Methodology used: The researcher managed to collect data from 319 Street Vendors through Purposive sampling method. But all together 279 Respondent agreed to give their response in the interview

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SOCIO-ECONOMIC PERSPECTIVES

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SCOPE OF SOCIAL WORK INTERVENTIONS WITH THE STREET VENDORS OF DELHI

Shabab Ahmad*

Abstract

Everyone is surrounded with numerous problems. So does the Street Vendors of the Delhi. Social Work discipline is enabling profession. An exploratory study was conducted among the Street Vendors of the Delhi to understand the scope of Social Work intervention among the street vendors. Basic view was to understand the common problems of the Street Vendors and suggest some basic scope of Social Work Intervention strategies that can be effective for ameliorating the problems of the Street Vendors.

Keywords: Social Work, Street Vendors, Problems, Scope, Interventions etc.

Introduction: Living in a pluralistic society where every individual has the right to work and earn their livelihood in the most dignified manner. The societies were government takes care of its citizen and promotes the engaged of its citizen in the fruitful employment and respectful work. The definition of Social Work, "it is a professional activity of helping individuals, group, or communities to enhance or restore their capacity for social functioning and creating a societal condition favorable to that goal." (National Association of Social Work, 1973; p: 43). "All social workers are obliged to advocate in social, political arenas to achieve an equitable distribution of the community's physical, economic, and social resources for social justice under the profession's ethical code." (NASW, 2008). The professional social worker is the individual who believe in building the capacities and capabilities, restoration of impaired capacities, so that the client can either build their capacities to overcome such problems and shortcomings. From time immemorial, street hawking, squatting and vending have been an integral part of Indian economy.

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The unorganised sector or the informal sector is the backbone of any country's economic system. As per the Economic Survey 2007-08, India's workforce comprises nearly 93 per cent in the unorganised segment; also, as per the National Association of Street Vendors of India (NASVI), almost 29 per cent of the Street Vendors of the country almost contribute 60 percent of the Gross Domestic Product (GDP).

Research Methodology: In order to study the Street Vendors of Delhi as an exploratory research method, an Interview Schedule was used to collect the quantitative data from the field; it aims at looking into the overall vending process and also the conditions of decent work. A Sample Size of Three Hundred and Nineteen (319) from various areas of the North and South Delhi was selected for the Study. These areas where majority of the street vendors are selected were Lajpat Nagar, Nehru Place, Chandni Chawk, Pul Mithai, Pili Kothi, Nahar Patri, where the large numbers of Street Vendors are seen.

Findings: In this perspective of Social Work, and most particularly in terms of Social Action, the passage of any law, or the enactment of certain legislation does not guarantees its solution when it come to implementation part. So the real test of success and failure lies in its grass root implementation. Consequently, The Street Vending Act 2014, passed for safeguarding the interest of the Street Vendors, but real test lies on its implementation. Since more than five years has passed but even the Town Vending Committee (TVC) functionality in its spirit and soul are jeopardized, what to talk about the future development of the vendors. As per the response from the Street Vendors one of the one of the major hindrances to their growth and development was corruption as they pay 'hafta or vasuli' to local policemen, MCD people Traffic policeman or sometimes to the shop owners where they vend. Some Street Vendors also reported to be suffering with health problems and drug addiction. Consequently the role of Social Worker now becomes more significant. Social Worker can play a greater role in mitigating the problems faced by the Street Vendors. As per the Morales & Sheafor (2011) the client can be anybody ranges from individuals, families, groups, communities neighborhoods or any units of the society, helping those units to interact more effectively and efficiently to achieve the humanistic goals. The different Social Work intervention strategies for the Street Vendors can be illustrated through the table listed below:-

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A Study of Women and Anti-Liquor Movements in India

Shabab Ahmad
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Abstract: *Liquor is commonly used among adults. Statistics show persons with liquor consumption are more prone to violent behavior. It does not only affect its user, rather it indirectly affects the other family members and the whole society itself. In the report of (National Crime Records Bureau, 2015)state that alcoholism plays a major role in 70-85% of offenses against women. According toWHO reports, around 30% of the total population of India consumed alcohol sprits and the per capita consumption of alcohol has increased from 1.6 to 2.2 liters in a decade which is one of fastest increase in the world in 2012.*

With the increase in affected population due to liquor abuse, it has become a political issue. With the social action by the affected group of people, this has transformed into a movement- 'anti-liquor movement' when women from the different region came together and act against the cause. They form the community who gather around in a large group with other supportive organization, armed with large sticks, they walk around inspecting the place and destroy liquor shop. This acts as the movement which spread over most of the states of India such as Andhra Pradesh, Gujarat, Bihar, Haryana, Maharashtra, Kerala, Uttar Pradesh etc. Through to this movement, women have demanded to enforce a complete ban on sale and consumption of liquor from the government.

This paper focuses on the economic, social, and political dimensions of nation's development with its strengths, weakness, and aftereffects of movements started by women against liquor in India.

Key Words: *liquor, women, violence, alcoholism, anti-liquor movement, social life, social function.*

1. Introduction:

Modernization is going on in India due to which many social problems arise like alcoholism. It creates new other social problems as domestic violence, rape, murder, family breakdown,etc. to the family and especially women. In the family, woman play vital role to give formal and informal education to children, provide information about culture and help to balance the personality development and carries out social regulation by developing the personality. Women play three primary roles in the family as a wife, housewife, and mother when it extent these roles it became a part of the changing value system. Alcoholism createsobstacles to these roles and affects the marital relationship, regulative functioning, socialization, and functioning of the society.

According to World Health Organisation (WHO), around 30% of the total population of India consume alcohol & spirits, and the per capita consumption of alcohol has increased from 1.6 to 2.2 liter in four decades which prove to be the fastest increase in the world in 2012.(Jena, 2015)

For economic and political reasons, the government has not taken any serious action to ban it, which is suitable for women. When people started to die due to alcohol or liquor, so it is extending the women's role and influence them to raise their voice against it.

Chakrapani (1994)states that "Increase awareness, education, and effort of government resulted in some change of attitude 'among women and on women'

Similarly, in this movement majority of activists are housewives, and they are also the victim of domestic violence and harassment due to alcohol. Political parties and women organizations have strengthened the anti-liquor movement of women by providing awareness and education with their rights and they have been able to work together to resolve this issue and to make sure that they are able to intervene in the system.

The movement demand complete ban on sale and consumption of liquor by the governmer.

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2. The objective of study:

- To study about why movement started by women.
- To find the role of the women in the anti-liquor movement.
- To know the challenges faced by women in this movement.

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Vindya Bharti - July 2018

A STUDY OF STREET VENDORS: CASE STUDIES FROM DELHI

Shabab Ahmad

Abstract: This paper most vividly tries to carve out the bitter truth about the street vending. Sitting on the lowest ladder of the economic pyramid, these poor street vendors were subjected to different kinds of maltreatment. This study simply describes the street vendors of Delhi. This research is based on Ethnographic studies, while some unstructured interview guide has been used to extract the natural conditions of the problem faced by the street vendors.

Key Words: Street Vendors, maltreatment, municipal, police, tehbazari, bribe, etc.

Introduction:

Street hawking in Delhi is regulated by the urban local bodies -the Municipal Corporation of Delhi, the New Delhi Municipal Council and the Delhi Cantonment Board. However, for apparent purposes of removing obstruction, even the police can exercise their jurisdiction over them. In spite of various Supreme court judgements and the enactment of street vendors act.

Street Hawkers outside Safdarjung Hospital

There are 161 hawkers on this 500 m stretch. About 44 per centof them sell eatables (chole-bhaturewallahs, paanwallahs, tea-sellers, fruitand vegetable sellers) and the rest sell articles (clothes, books, CDs and others) or services (barbers, cobblers, even ear-waxcleaners!). Of them, only 17 hold the tehbazari permit (The tehbazari denotes that in lieu of a fee of Rs. 120 for 4ft×3ft and Rs. 240 for6ft×4ft the hawker is granted license by the municipal authorities-New Delhi Municipal Corporation in this case-to hawk goods). That makes the remaining 90 per cent of the hawkers illegal and unlicensed.

A little description of the area will help understand the situation better. The stretch is immediately adjacent to two of the most prominent hospitals in Delhi - the Safdarjung hospital and the All India Institute of Medical Sciences. The stretch also has two busstandsand is at the corner of a prominent junction of wide roads.

Apart from the hundreds of visitors to the hospitals, many office-goers and people change buses here en route. Given the size of "captive "customer base here, hawkers do try to avail of its benefit. It is veryevident from the difference in the goods sold at different points. Whileeatables are available at both the hospital gates and the bus stands, fastfood set-ups like chole-bhatureor tea stalls are more prominent at thehospital gates, and lighter snacks like peanuts or fruits are availablemore at the bus-stands, for commuters. In fact, this stretch has 13peanut-sellers in all. The other articles are more for the convenienceof visitors to hospitals - booklets on pregnancy, ceramic pot-pans, etc.Moreover, being government hospitals, most of the customers to thesestalls are from the poor and the lower middle classes.

In the course of our survey, we realised that the respondents wereunderstandably paranoid about divulging information, especially thepayments made to the various authorities for continuance of theirhawking activity. In fact, after much persuasion, most of themmentioned amounts that others paid and denied that they too paid theirown. The reasons given by them were: 'I am an upright citizen' or'because of my long tenure in the market, the police dont force me'.We initially believed in their stories till we realised the explanationswere being repeated time and again. Nevertheless, the following issuesemerged in the course of our survey of about 35 hawkers.

The players: There are three principal players in the "hawk-system" here, apart from the hawkers, the municipal authorities, the police, and the pradhanwho acts as an interface between the hawkers and the authorities. Each hawker pays about Rs. 200-500 monthly to the pradhan, who then forwards the amount collected, to the police and the municipal authorities.

The payments:The amount to be paid depends on various factors like space occupied, profitability of enterprise, "good" relations with the authorities and time period in the market. A "successful" hawker(in this case, usually the fast-food hawkers) on a 6 ft×4 ft rehri pays between Rs. 800-1000 per month which may go upto Rs. 1500 for the most profitable ones. A new entrant would pay about Rs. 1000 for a3ft×2ft space.

The raids: The frequency of the raids varies. The municipal authorities usually carry out raids 2-3 times a day! But then, in a stretch of three days, a raid might happen only once. However, the raids dramatically lessen during election time as was the case when we carried out the survey. Raids may result in either eviction or challenging(levying a penalty). Eviction involves physical dislocation, where the hawkers' items are seized and taken away in the municipal authority's van. Hawkers have to retrieve the goods from the municipal godown later, after payment of requisite fine. Though physical eviction is the norm, sometimes challansare issued to the hawkers without

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confiscating their goods. Interestingly, the hawkers look forward to challans because they later serve as a proof of their existence and tenure in the market. Often, in the past, in Delhi, legality has been extended to the hawkers on submission of these challans.

The nexus: What troubled us most was why raids, especially physical evictions, take place when the authorities are paid their due payments. We found that since the municipal authorities have to do their "duty", the only concession they give the hawkers is enough time to pack their goods and flee. Thus, when on orders from above, the municipal officials prepare to raid the place, the pradhan is informed beforehand. Immediately thereafter, the hawkers pack their goods and hide. If caught, the officials do not spare them. Usually, when the municipal van is in sight, shouts of "committee kigaadi" warn hawkers down the street. Often there is loss of goods in the hurry to flee and hide. When a particular hawker refuses to pay thepradhan's'fee', he is identified by the pradhanand reported to the NDMC official; his goods damaged. The same happens to hisneighbour to make it seem like fair play. Thus, the charade continues.

Recovery of goods: Rarely do hawkers try to get their goods back from the municipal godown because the baledaars(those who pack the goods into the municipal vans) often pocket most of the seized stuff like watches, clothes, purses etc. in the process of packing. Often the baledaars release the goods on the spot to the hawker, for a price, when senior officials are not watching. Further loss of goods happens at the godown. So, it is not worth paying the requisite fine amount, which is at least Rs. 500. Then there is the wait of at least 7-15 days. Hence the hawkers prefer to buy their goods again because of the loss of income for the days spent to recover the goods. Thus for the tea hawker, the loss of his utensils and food (like bread and biscuits)would be far less than the amount to be paid as penalty and the time required for recovery of his goods.

The police: The police "control" the entry of hawkers onto the stretch, which is mediated by the pradhan. In fact, while speaking to different hawkers across the city, it became more evident that no hawker can have a rehri-patri anywhere in the city without paying the police official and the municipal authority of that area, a particular monthly amount. However, the hawkers do not mind paying the police as long as they are allowed to carry on their trade undisturbed. In fact, there were not many reports of police picking up goods and not paying for them, at least on this stretch. Usually, they pay the cost price for whatever they buy, but they routinely "expect" gifts during festivals. Once in a while an odd policeman takes a book from a bookseller, reads and then returns it.

Hawkers are ready to pay for legality: To ward off the harassment of municipal raids which affects their business adversely, the hawkers are willing to pay the government a monthly rent. The non-food-item sellers are ready to pay to the tune of Rs. 300-500 at least, and the amount scales to a high of Rs. 1000-1500 in the case of tea-stall owners and fast-food stalls. It must be mentioned here that a few of the food hawkers are even ready to buy the site they occupy for about Rs. 20,000, indicating the economic value of the space and their activity.

The average incomes: Daily incomes range from Rs. 60 for a peanut seller, about Rs. 100-150 for a clothes seller and over Rs. 150 for fast food sellers.

Street Hawkers in Central Market, Lajpat Nagar

The total number of unlicensed hawkers in this 124m×91mmarket (we covered only the main block of the market) under the jurisdiction of the Municipal Corporation of Delhi is about 355. Thisnumber does not include the pucca shops in the market. It only includes the rehri-patriwallahs(stationary hawkers) and the ambulatoryhawkers. The puccashops are assumed to be licensed, though for many, the claim is suspect. Moreover, almost all of them have encroached on the space outside their shop for display and sale of goods. So much so, that initially we mistook these goods to be those of the hawkers. Also, quite a few shops have sublet a portion of their premises to a key-maker/duplicator who sits in a corner of the shop.

About 25 per cent of the hawkers are involved in selling eatables as diverse as dry fruits and hot sweet-potatoes apart from the usualchat-wallahs. Compared to the 45 per cent of hawkers selling eatableson the AIIMS stretch, the number of hawkers selling them here is less because of the presence of a substantial number of puccaand licensed eateries in the market. There is about 19 hot sweet-potato sellers onthe periphery of the market, because obviously no pucca shop would sell this stuff. Since demand for hot sweet-potatoes is evidently high and capital input for their sale is quite low, it is a niche market that is filled by the hawkers. The other three-fourth of the hawkers are involved in selling articles like clothes, jewellery, purses etc., and providing services like applying mehendi. In fact, the market has 30mehendiwallahs. About 105 hawkers sell clothes/sweaters/bed sheets. An important point to note here is that at least one-tenth of the hawkers in the market do not occupy any space to display their goods - the items they sellare slung across their shoulders or draped on their arms. They walk in an approximate radius of about 12ft displaying their goods and trying to catch the attention of prospective customers. The opportunity to make money in this market of limited space is hard to miss.

In contrast to the AIIMS market stretch, the clientele here is evidently upmarket and consists mostly of upper and middle income groups who visit both the permanent stalls and the hawkers, though the lower and middle income groups mostly frequent the hawkers.

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AN INTERNATIONAL BILINGUAL PEER REVIEWED REFEREED RESEARCH JOURNAL

"A CROSS-SECTIONAL STUDY ON THE LIFE OF RAG-PICKERS IN CHETRU VILLAGE OF DHARAMSHALA: A GRASS-ROOT REALITY"

Dr. Shabab Ahmad *
Ms.Ankita Thakur**

ABSTRACT

Himachal Pradesh is somehow known for its clean environment and natural beauty. Dharamshala, being a smart city under PM Narendra Modi's flagship Mission. This is not a secret that for a clean Environment Rag-pickers are one of the very important assets who pays a lot of hard work to keep our cities clean and still are the most unrecognized people who face a lot of difficulties in their profession. Present study is done to understand their life conditions of the Rag-pickers and their issues. The Rag-pickers live in Chetru village, Dharamshala which is around 8 Km away from main city. Present paper highlights their socio-economic profile, reasons of their working and migration, working conditions, occupational health and stigma towards Rag-pickers. Information in present paper is collected through the interview schedule which include both open and close ended questions.

Keywords: Challenges, Stigma, Stereotyping, Occupational diseases, Rag-pickers, Socio-economic, Migration

* Assistant Professor, Department of Social Work, Central University of Himachal Pradesh
** Master of Social Work, Central University of Himachal Pradesh

Introduction

Dharamshala is selected as one of the hundred Indian cities to be improved as a smart city under PM Narendra Modi's flagship mission of Smart Cities. On 19 January 2017, Dharamshala became the second capital of the state. As per Census 2011, the total population of Himachal Pradesh only 0.02% of population lives in Dharamshala. Census also says that there are no slums existed in Dharamsala.

Dharamshala is also known for refugee area for Tibetans. It is strange that the same city unable to provide place for some families because they are illegal occupants, even they had been played role of building the city as a smart city. Families described in above statement are the Migrant Families. Migrant become vulnerable because they don't have proper place to stay and they remain jobless in the city. These helpless people deploy themselves in Rag-picking or other class (iv) jobs. There are many other poor families in the city people are jobless and uneducated and unable to maintain their family so they are engaged in rag picking. Migrated people are homeless; jobless they start with Rag-picking in order to live. Rag-pickers are highly stigmatized work. Although Rag-pickers plays very important role to protect our environment from waste and keep it clean through their Rag-picking activity, also it helps to raise economy of the country. However they are socially disadvantaged discriminated and harassed by the public.

The Present study is made to understand the life

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AN INTERNATIONAL BILINGUAL PEER REVIEWED REFEREED RESEARCH JOURNAL

THE MIGRATION OF AGRICULTURE LABOUR INTO STREET VENDING: A STRUGGLE OF SUSTAINABLE LIVELIHOOD

❒ Shabab Ahmad *

Abstract

This paper most vividly tries to comprehend and compute the challenges and problem faced due to shifting of occupation from 'Agriculture' to Street Vending, in the urban hub of India, Delhi. What was the reason of occupational changes that they adopted? What was their present situation after they got shifted from agricultural (rural) to urban land? This paper tries to give snapshot of the conditions and situations of Street Vendors.

Hence, In order to comprehend this situation, various research methods were used. This exploratory research was done through an interview schedule was constructed through the technique of purposive research sample. Researcher also comprehended the complex living situation of the target population by becoming participant observer of the Street Vendors. In order to investigate their agricultural occupation among Street Vendors, a Sample Size of Three Hundred and Nineteen (319) from various areas of the North and South Delhi was selected for the Study. These areas where majority of the street vendors are selected were Lajpat Nagar, Nehru Place, Chandni Chawk, Pul Mithai, Pili Kothi, Nahar Patri.

Key words: Agriculture, Street vendors, Occupation, Shifting, Livelihood etc.

* Assistant Professor, Central University of Himachal Pradesh

Introduction:

India is basically agricultural country from time immortal. Agriculture supports Twenty Three percent of GDP and employed almost fifty eight percent of workforces of India (2001 census). Any change in the occupation must be comprehended with utmost urgency. Since it will effect economically to any country whose major economy is agricultural based. In the pursuit of better livelihood each individual tries to maximize its profit. Consequently, migration is the universal phenomenon. Even today, due to massive migration of rural unemployed youths to the urban areas in search of dignified work is the major source of urban unemployment. Most of them do not match the skills that are required in fast

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Certificate of Publication

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ISSN 2320 - 9283

Dr. Shabab Ahmad
Assistant Professor, Department of Social Work,
Central University of Himachal Pradesh, H.P., India

Dear Dr. Ahmad

This is to certify that **Dr. Shabab Ahmad** has published a paper entitled **"A cross-sectional study of Street Vendors of Kangra: A grass-root reality"** which has been published in **"International Journal for Social Development, Vol. 8 Issue 1, Jan. - Mar., (2020).".**

Please find enclosed herewith a complimentry copy of the International Journal and four reprint of **"International Journal for Social Development, Vol. 8 Issue 1, Jan. - Mar., (2020)".**

I am extremely grateful to you for contributing an article for the International Journal and shall look forward to get the same co-operation in future also.

you are requested to acknowledge the receipt of the International Journal.

Thanking you,

Yours sincerely

(Dr. Uday Narayan Singh)
Managing Editor

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- International Journal for Social Development (Q)
- Journal for Social Development (Q)
- Journal for Social Reality (Q)

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Original Article

Dimension Reduction of Subjective Motivational Values toward Child Gender Tool Tested in Women of Reproductive Age from a Hospital in Rural Area of Himachal Pradesh, India

Dinesh Kumar[1], Shabab Ahmad[2], Chirag Goel[3], Avi Kumar Bansal[4], Shripad Patil[5]

[1]Associate Professor, Department of Community Medicine, Dr. Rajendra Prasad Government Medical College, Kangra, [2]Assistant Professor, Department of Social Work, School of Social Sciences, Central University of Himachal Pradesh, Dharamsala, [3]Scientist-C, Model Rural Health Research Unit, Una, Himachal Pradesh, [4]Scientist-E, [5]Director, National JALMA Institute of Leprosy and Other Mycobacterial Diseases, Agra, Uttar Pradesh, India

Abstract

Background: A novel subjective Motivational Value toward Child Gender (MVCG) tool was developed using the theoretical construct of 10 motivational domains described by Shalom H Schwartz. **Objective:** The study aimed to summarize the pattern of correlations of (MVCG) in women of reproductive age in Himachal Pradesh, India. **Methods:** A cross-sectional study was conducted from October 2018 to November 2019 among a sample of 355 women. Required data were collected through an interviewer-administered questionnaire. Maximum likelihood exploratory factor analysis (EFA) with oblique rotation was done with Bartlett's test sphericity and Kaiser-Meyer-Olkin test. **Results:** A total of 28 (53.8%) questions loaded on eight factors explaining maximum variance (68.7%). Reliability analysis of these questions, with high loadings on extracted factors, of the questionnaire, observed with poor Cronbach's alpha of 0.61 and intraclass cluster coefficient (ICC) 0.49. However, selected domains such as tradition, power, achievement, self-direction, and benevolence were observed with a good Cronbach's alpha and ICC. **Conclusion:** MCVG is novel tool in its kind with well scalable properties in measuring subjective motivational values towards child gender. After EFA, total questions across 10 domains reduced from 52 to 28, across 8 domains, loaded on 8 factors with good reliability and agreement.

Key words: Factor analysis, gender, motivation, sex ratio, social values

Introduction

Male child preference is a predominant reason for the declining sex ratio at birth (SRB) in India and observed to be the area of concern varying across states.[1-4] Gender bias and resulted sex preference generally arises due to the embedded social values of an individual, these need to be measured for shaping social interventions.[5,6] Various individual and cultural factors influence subjective values and decision-making processes.[7,8] Extensive literature review was carried out and no evidence was found wherein application of motivation values was done to the formulation of the questionnaire, therefore, subjective Motivational Value towards Child Gender (MVCG) tool of 52 items across 10 motivational domains based on the theoretical construct by Shalom H Schwartz was developed.[9,10] The construct has not been tested in the Indian context and not been applied among women to measure values toward the gender of the child. However, it was understood and hypothesized that the mentioned motivational domains can explain a lot about the pattern of values among women towards the gender of the child.[11,12]

These motivational values are described in 10 domains; (a) Power: Social status and prestige, control or dominance over people and resources; (b) Achievement: Personal success through demonstrating competence according to social standards; (c) Hedonism: Pleasure and sensuous gratification for oneself; (d) Stimulation: Excitement, novelty,

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CHAPTER 10

CHAOS AMID CHAOS: A CRITICAL DEBATE FOR STREET VENDORS – STRUGGLE AND ADAPTATION POST COVID ERA

Shabab Ahmad

ABSTRACT

Indian economy is based on the unorganised sector. The mayhem created to the daily earner on the streets of Delhi. Once it was considered that the street vending was one of the easiest occupations for the survival of the people living in the lowest ladder in the economy. Anyone who does not have any means of earning can espouse vending as profession as it has societal sanction and acceptability in terms of diversity and universalistic point of view. In search of better livelihood, people used to migrate from rural India to urban metros like Delhi. The analysis of the pre-conditions of the street vendors will help us to have a glimpse of prior COVID-19 situation in the region. Now, this chapter tries to debate on the kind of adaptability that street vendors needed post COVID-19 era. An informal interview conducted with the street vendors help us to comprehend their preparedness for their adaptability to survive. This chapter also tries to discuss some of the programmes and policies that can be a lifeline for the street vendors during the post COVID-19 era.

Keywords: COVID-19; street vendors; occupation; adaptation; livelihood; post COVID-19; Town Vending Committee

A New Social Street Economy: An Effect of The COVID-19 Pandemic
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Social Problems in Contemporary India

Social Work Perspective

Editor

Dr. Naseem Ahmad Khan
Assistant Professor
Department of Social Work
Aligarh Muslim University
Aligarh

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NGOs in India
Issues and Concern

Shabab Ahmad

Abstract

The achievements and success of NGOs in various fields is praiseworthy but the excellent work done by them in specific areas is no doubt a tremendous task that has helped to meet the changing needs of the social system. However, in spite of its achievements the functioning and working of the Non-governmental Organization are always in scanner. Now and then media keeps on reporting about the credibility and existence of the non profit organization. NGOs are encountering different difficulties which differ from society to society, region to region. Consequently, an effort has been made to get a comprehensive feedback from different NGOs in India; also this paper tries to find out some of the common problems confronted by the NGOs. Research methodology used is quantitative as well as qualitative study. A survey was conducted in different NGOs, were participant observation and ethno methodology was used.

Key words: NGOs, Issues, Concern, Social System and Problems.

Introduction

NGOs are those grassroots level organizations, where people come together and develop an attitude and behaviour to work against the common contemporary problems, commonly faced by these people lives in that particular community. In society more than seven members are included to get it registered but to register a Trust Two or more members can form it.

In Indian context NGOs are those organizations that have some kinds of established base, and are private, non-profit, self-governing, voluntary in nature, and registered with the government (Nandedkar 1987). But sometime it is controversial that some NGOs due to its altruistic ideas, beliefs and self-reliant (due to self-financing nature) do not want to get themselves registered

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CHAPTER-8

A Case Study of De-Addiction Centre (Apna Ghar) Prayas Bhawan Dharamshala

Shabab Ahmad

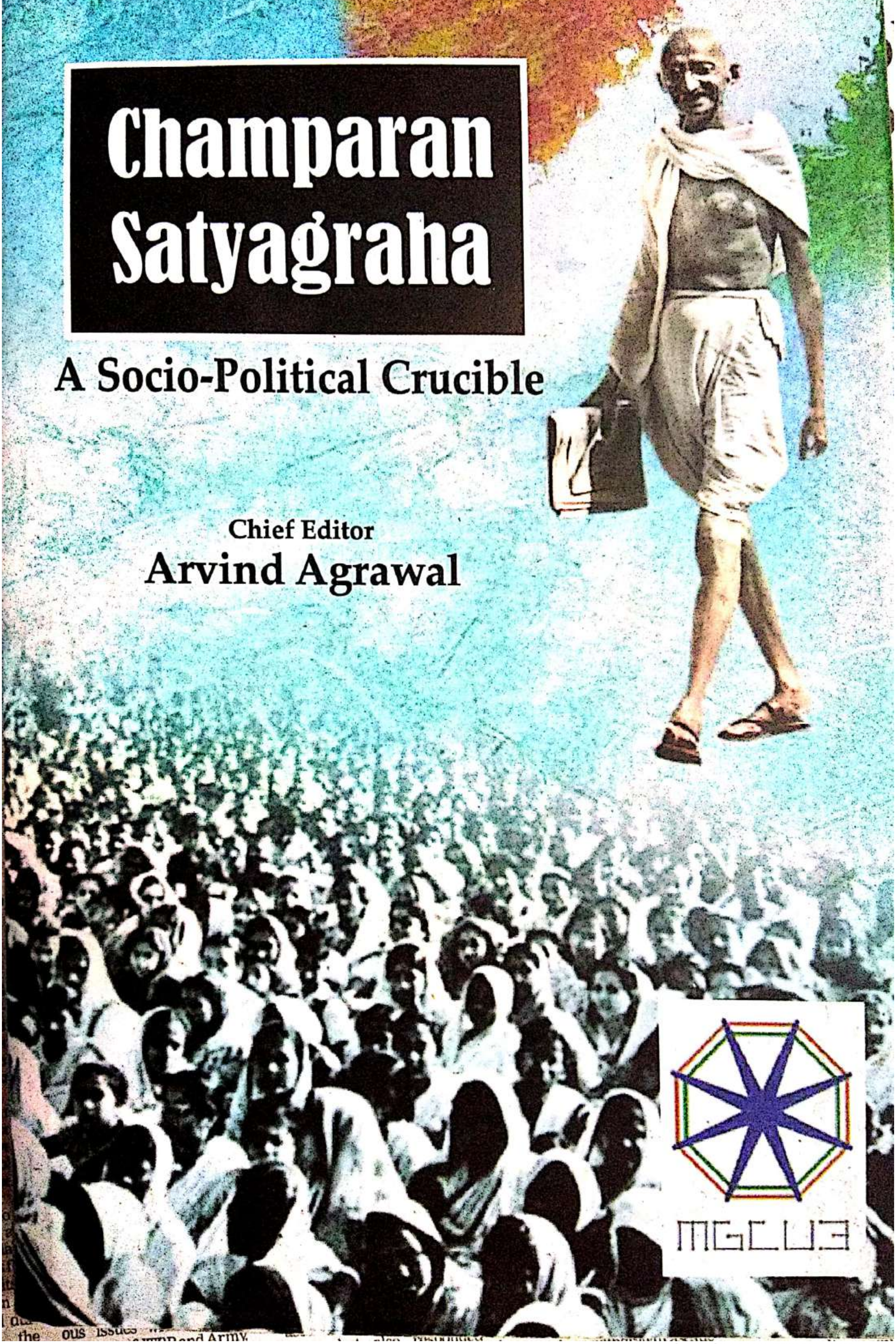
Kangra District Red Cross Society, Dharamshala is running a De-addiction Centre (Apna Ghar) in Prayas Bhawan Dharamshala (Treatment and Rehabilitation Centre for addicts) since January 2002 under the Scheme for Prevention of Alcoholism & Substance (Drug) Abuse sponsored by Ministry of Social Justice & Empowerment, Govt. of India, New Delhi. Kangra Distt. Red Cross Society was receiving 70% budget from the Ministry and 30% NGO share was being paid by the Society till the year 2013-2014. The centre is having 15 bedded indoor facilities for detoxification & Rehabilitation programme (Counselling) minimum for one month extendable on person to person health and recovery condition, OPD facility, group counselling session, family counselling session, individual counseling session, Relapse Prevention Therapy Session, follow up etc. The centre remains fully occupied by the patients all the time from almost all districts of Himachal Pradesh as well as from neighbouring state i.e. Punjab. Patients are very less from Rajasthan and Jammu and Kashmir.

Champaran Satyagraha

A Socio-Political Crucible

Chief Editor

Arvind Agrawal



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Champaran Satyagraha: A Social Work Perspective

Rashmita Ray & Asutosh Pradhan

Introduction

Champaran Satyagraha has been a landmark in the history of the making of an independent India. It was a successful experiment of a social movement that became a germplasm or mother of many more such social movements that contributed to the achieving of independence of India from British rule.

The setting on which this article is based is the Champaran District of North Bihar and dates back to the early twentieth century (1914-1917) when Gandhiji returned from South Africa in 1914 and tried his method of Satyagraha on a mass scale for the first time on Indian soil.

Champaran was the hub with regard to indigo plantation and manufacturing of indigo and the growers were tenant peasants also referred to as *ryots*. Majority of the owners of the factories for processing of indigo were Europeans or British. The land originally belonged to the *Zamindars* and was leased out to the factory owners on long term basis or such land that belonged to the Bettiah Raj. Lease of land in early nineteenth century went to the hands of the Europeans who engaged in indigo and sugarcane cultivation and towards the last quarter of the nineteenth century the takeover of land from Indian Lessees by Europeans became complete. The tenants or *ryots* were forced to cultivate indigo as per the *Tinkathia* system. According to this


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"Where the mind is without fear and the head is held high …": The Challenge posed by Rabindranath Tagore for Social Work Professionals

Rashmita Ray[1]
Asutosh Pradhan[2]
Sasmita Patel[3]

Abstract

Rabindranath Tagore has been an inspiration to the millions of India and the entire world. His writings have inspired individuals and communities, the marginalized and downtrodden to come out of their miseries and making a journey leading to a life of dignity and self-respect. An indicative attempt is made to reinterpret one of his famous poem "where the mind is without fear…" in the present Indian context and from a social work perspective. This article attempts to touch upon issues related to community development in terms of empowerment, patterns of discrimination and social divide, appropriate methods of social work, ethical and reflective practice, and culminates on suggestions for social capital formation and effective social work practice.

Keywords: Rabindranath Tagore, dignity, community development, social work, social capital

Introduction

Rabindranath Tagore has been an inspiration to the millions of India and the entire world. His writings have inspired individuals and communities, the marginalized and downtrodden to come out of their miseries and in making a journey to leading a life of dignity and with self-respect. This article makes a sincere attempt to mirror Tagore's understanding of people for leading a life of dignity, righteousness and free from all kinds of oppression. In the process, an effort has been made to contextualize Tagore's ideology in the context of social work practice in India. Tagore's songs have inspired all

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3 *Assistant Professor, Department of Social Work, Visva-Bharati, Santiiketan, West Bengal.*

and sundry – from the layman to the greats. An indicative attempt is made as to how professionals can interpret the writings of Tagore, as also other great writers and poets, in aligning their roles to achieve the demands of the profession and drawing inspiration for a spirited and concerted effort to uplifting people from suffering and in the creation of a more just and humane society. Understanding how literature motivates the individual and how it is reflexive of the individual's society is something that ought to be given more attention and consideration in social work teaching and practice training. This article attempts to touch upon issues related to community development in terms of empowerment, patterns of discrimination and social divide, appropriate methods of social work, ethical and reflective practice, and culminating on suggestions for social capital formation and effective practice. The outcome of this attempt would possibly add to the indigenous literature of social work practice in India.

Being in the Land of Tagore forces us to reflect on the role and impact literature can have in the lives of individuals and society. Literature, in our view, has both an *Active* and a ***Passive*** role. *Active* in terms of engaging the reader or listener in framing ideas and concepts about the world in which he lives. It helps in the development of his own ideologies which he fondly aspires to live and relive. From this ideology stems individual motivation, action, and engagement. There are numerous examples, as to how the writings of Gandhi, Vinoba, Tagore and others influenced individuals to take part in the Indian Independence movement. Ambedkar's writings have helped in the creation and expansion of the Dalit Movement against caste discrimination. Many a social activist's decision to become so has been influenced by the writings of such great figures.

The second, which connotes to the ***passive*** or ***reflexive*** role, refers to a thing observed rather than thing done. Literature acts as a beacon light to society showing it the importance of ideologies, thought and action. It reveals *what* people thought during a particular time and *how* they thought about it. It guides the individual in developing an understanding as to how a society functioned and why it functioned that way. The writings of/on Gandhi again reveals how people had a belief and trust in non-violence as a means of achieving national goals.

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A STUDY OF STREET VENDORS: CASE STUDIES FROM DELHI

Shabab Ahmad

Abstract: This paper most vividly tries to carve out the bitter truth about the street vending. Sitting on the lowest ladder of the economic pyramid, these poor street vendors were subjected to different kinds of maltreatment. This study simply describes the street vendors of Delhi. This research is based on Ethnographic studies, while some unstructured interview guide has been used to extract the natural conditions of the problem faced by the street vendors.

Key Words: Street Vendors, maltreatment, municipal, police, tehbazari, bribe, etc.

Introduction:

Street hawking in Delhi is regulated by the urban local bodies -the Municipal Corporation of Delhi, the New Delhi Municipal Council and the Delhi Cantonment Board. However, for apparent purposes of removing obstruction, even the police can exercise their jurisdiction over them. In spite of various Supreme court judgements and the enactment of street vendors act.

Street Hawkers outside Safdarjung Hospital

There are 161 hawkers on this 500 m stretch. About 44 per centof them sell eatables (chole-bhaturewallahs, paanwallahs, tea-sellers, fruitand vegetable sellers) and the rest sell articles (clothes, books, CDs and others) or services (barbers, cobblers, even ear-waxcleaners!). Of them, only 17 hold the tehbazari permit (The tehbazari denotes that in lieu of a fee of Rs. 120 for 4ft×3ft and Rs. 240 for6ft×4ft the hawker is granted license by the municipal authorities-New Delhi Municipal Corporation in this case-to hawk goods). That makes the remaining 90 per cent of the hawkers illegal and unlicensed.

A little description of the area will help understand the situation better. The stretch is immediately adjacent to two of the most prominent hospitals in Delhi - the Safdarjung hospital and the All India Institute of Medical Sciences. The stretch also has two busstandsand is at the corner of a prominent junction of wide roads.

Apart from the hundreds of visitors to the hospitals, many office-goers and people change buses here en route. Given the size of "captive "customer base here, hawkers do try to avail of its benefit. It is veryevident from the difference in the goods sold at different points. Whileeatables are available at both the hospital gates and the bus stands, fastfood set-ups like chole-bhatureor tea stalls are more prominent at thehospital gates, and lighter snacks like peanuts or fruits are availablemore at the bus-stands, for commuters. In fact, this stretch has 13peanut-sellers in all. The other articles are more for the convenienceof visitors to hospitals - booklets on pregnancy, ceramic pot-pans, etc.Moreover, being government hospitals, most of the customers to thesestalls are from the poor and the lower middle classes.

In the course of our survey, we realised that th respondents wereunderstandably paranoid about divulgin information, especially thepayments made to the vario authorities for continuance of theirhawking activity. In fa after much persuasion, most of themmentioned amou that others paid and denied that they too paid theirov The reasons given by them were: 'I am an upright citiz or'because of my long tenure in the market, the police d force me'.We initially believed in their stories till realised the explanationswere being repeated time again. Nevertheless, the following issuesemerged in course of our survey of about 35 hawkers.

The players: There are three principal playe the "hawk-system" here, apart from the hawkers municipal authorities, the police, and the pradhanwh as an interface between the hawkers and the autho Each hawker pays about Rs. 200-500 monthly t pradhan, who then forwards the amount collected, police and the municipal authorities.

The payments:The amount to be paid depe various factors like space occupied, profitabil enterprise, "good" relations with the authorities a period in the market. A "successful" hawker(in th usually the fast-food hawkers) on a 6 ft×4 ft teh between Rs. 800-1000 per month which may go u 1500 for the most profitable ones. A new entran pay about Rs. 1000 for a3ft×2ft space.

The raids: The frequency of the raids va municipal authorities usually carry out raids 2-3 day! But then, in a stretch of three days, a raid migh only once. However, the raids dramatically lesse election time as was the case when we carried survey. Raids may result in either evic challenging(levying a penalty). Eviction involves dislocation, where the hawkers' items are seized away in the municipal authority's van. Hawker retrieve the goods from the municipal godown payment of requisite fine. Though physical evict norm, sometimes challansare issued to the hawkers

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Social Work Intervention

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Centre, University of Himachal Pradesh
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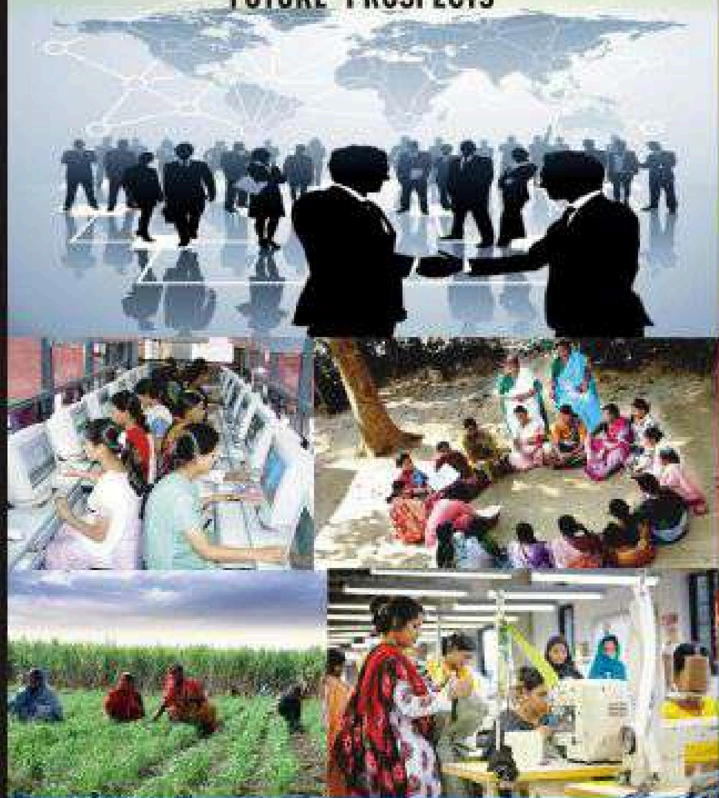

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Dr. Nittam Chandel

Engaged Governance and Women Empowerment in India : Future Prospects

The present book contains papers and case studies presented by various seminarians in the University Grants Commission's (UGC) Sponsored National Seminar on Engaged Governance and Women Empowerment in India: Future Prospects held on 17-18 December, 2016 in SV Govt. College Ghumarwin, Himachal Pradesh. The focus of the book is on engaged governance and empowerment of women in India'. In fact, the concept of engaged governance has been gaining enormous popularity in recent times, both in academic discourse and actual practice, all over the world. It links people more directly to the decision-making processes of a state in a manner that does not by pass the institutions of representational democracy but complements it. Today, it has 'emerged as an effective strategy for empowering women'. Women Empowerment is a process of developing ability or potential of women so that they can think and act freely, make their choices and control their lives; and thereby, help reducing discrimination and exploitation of them. It brings about upliftment of women in social, economic and political spheres in which they can play an equal role at par with men. For centuries, women have been slave of the oppressive patriarchal order that offers them fewer rights and lower social status as compared to men. This widespread discrimination and exploitation of women has evoked the need for empowerment of women. In spite of constitutional guarantees, enactment of laws, initiations of social reforms, efforts by the government through different schemes and programmes and U.N.O.'s directives, the equal status of women in India is still not achieved to meet the desired goals even after 70 years of Independence. Hence, this book is not only useful in the academic discourse but also a valuable one for further research and academic investigation in the discipline.

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Social Problems in Contemporary India

Social Work Perspective

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A cross-sectional study of Street Vendors of Kangra: A grass-root reality

Dr. Shabab Ahmad
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Abstract

Street vendors are an essential part of the unorganized sector. Street vendors include all those who sell goods or services at public places with a temporary built structure. Since, they are unable to get regular jobs in the remuneration formal field due to their low level of education and skills. They strive to solve their livelihood problems through their own small financial resources. They are the main distribution channels for a large variety of daily consumption products such as fruits vendors, vegetables sellers, and readymade garments hawkers, shoes mending cobblers, household gadgets, stationery, newsletters, and magazines and so on. Probably, they are the most neglected lot of the societies and consequently, gathered less attention from academicians, mainstream media, policy makers, etc. This paper gives a brief snapshot of their working conditions on the streets.

Keywords: *Street Vendors, Livelihood, Occupation, Challenges, Issues*

Introduction

Street vendors are generally destitute group and they are too far from mainstream life, they are subject to exploited time to time by the administration, so in order to fill this gap Central Government and State Government providing the several Schemes and programs for their enhancement.[1] Generally street vendors Section includes the hawkers such as fruits, chanabhatura, juice, fast food vendors etc. street vendors may be migrants or local it include both (Bhowmik, 2012).

There is some important reason to study their current Socio-economic status. What they are getting from Governmental Schemes and Programs, are they getting benefit of street vendors act 2014 and they were supported or harassed by the administration/government.[2] In order to know the status of their linkage with financial institutions and Hospitals.[3] To know the status of their children about the education and also generates the awareness regarding education (Bhowmik, 2014).

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Micro level response to Covid-19 catastrophe in India: A Social Worker's reflection on the Pandemic

Shabab Ahmad,
Assistant Professor,
Department of Social Work,
Central University of Himachal Pradesh, Dharamsala, India.

Abstract: This paper tries to carve out niche of short ethnomethodical research, experienced by a Social Worker during the Covid-19 pandemic. This narration enumerates disorder, distraction, and discoveries to cope this novel situation. However, it has also offered moments of optimism and insight as Social Workers also witness acts of courage and cooperation. It has given us raison d'être to take a break, amidst the chaos, and take account of it. Thus, the role of professionals such as doctors, Medical Social Worker, Teachers, becomes very crucial. Consequently, this research enumerates and exhibits the value of spontaneous practices linked with qualitative inquisitions.

IndexTerms – **Covid-19, pandemic, social worker, reflection, distraction, discoveries.**

I. Introduction

The advent of the pandemic Covid-19 has given us the opportunity to retrospect each every aspects of life. The problems to the human worlds are not new. Problem is bound to occur. The history had witnessed human sacrifices, sometimes man made, sometimes natural. It is the response to the problem that makes us different. The preexisting social problems and unable to deal with those problem are like self-suicidal. Things go even worst when 'we together' (thesis & anti-thesis) aggravate the problem in spite of having real intension to solve it. With the disappearance of the disease, what really important was the attitude, opinion and perspective to deal with the disease, of course the technically matters the most. Illustratively, the low death rate in Japan, Taiwan, Singapore and South Korea is due to their living habits such as greetings from distance, sneezing etiquette, wearing mask etc.

The pre conditions that existed before the arrival of Covid-19 in India should be analyzed which made things go bad to worst while dealing with the disease. Just before the advent of corona virus, India was struggling with the massive protest against Citizen Amendment Act (CAA) & National Register of Citizen (NRC). The problem caused as the section of minority community feels insecure under the rule of majority backed and ruled government. The health parameters and social conformation that had been designed in the various societies till date seems to be useless to some extent. Some countries earlier considered themselves 'developed'; some in the process of 'developing' and some unfortunates are looked down as underdeveloped countries. Considering the case of Italy where they boast upon some world ranking on providing better Health conditions to its citizen as compare to the rest of the world. The small virus has made these developed nations to kneel downed, including United States of America and England. In India this pandemic was invaded mainly by the countries like United Kingdom, United States of America, Thailand, Philippines, Italy, Canada, Indonesia, and Myanmar. The pandemic not only arose Health issues throughout the world but also Social, Economical, Political and Religious issues as well. The human world was not ready for pandemic, neither the Social Work discipline. Social Work disciplines always strive for giving best social services to the people. Apart from understanding the various problems that arose out of recent Pandemic Covid-19 reflecting upon the problems from Social Work intervention techniques may help the frontline workers and it will support them. This essay tries to carve out the intervention areas of the social work.

The pandemic which was a health issues, now also has sigficantly become, Social, Economical, and Political as well as religious problem in India. 1. Social- recently those who recovered from the infection were subjected to social stigma by their own communities. 2. Economical- In last midyear, Indian Government had already withdrawn a huge sum of money from its central bank of India which was kept to encounter the emergency situation. The lockdown proves havoc on the daily wage earners and migrants workers too. 3. Political- The non transparency in showing the numbers of Covid-19 cases due to vested interest. Power to know the exact cause of death. But lacking the use of wisdom to use the power to curve this pandemic. 4. Religious- Prof. Y N. Harari believes that the widespread of hatred between different segments may be the greater problem than the disease itself. Jamaat , victimization of minorities in the false accusation of Covid-19. Tibetan died after returning from the United States Social Stigma has caused aggravated the pain; consequently; a case of Muslim man hanged himself despite being tested negative. Since he returned from the international Islamic congregation popularly known as Tablelique Jamaat of Markaj Nizammudin. Where media made much hyped that he has to suffer intense social discrimination. Now the uninterrupted running of the media, targeting a particular minority community to the extent that insecurity and fear loams around them. Forcing the individual to go for quarantine despite being healthy by the administration. And, eventually their own community where he lived. Eventually hatred rise to the extent that some hospitals denied admission to the Muslims, some segregated Muslims patients from Non-Muslims. Now, reflecting on this scenario, few questions are to be answered. Can there may be any social work intervention. If yes, what it could be? Before Social Work interventions, diagnosis of the problem should be done.

II. Psychological Problems

Confinements may leads to psycho social problems too. Infodemic- The influx of too much information on particular issue or any situation. This creates the individual to dilute, delusion and sometimes diversion from the basic knowledge of the problem This may happened in the case of cronavirus itself. Sometimes the influx of unwanted and undesirable knowledge aggravates the

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Original Article

Dimension Reduction of Subjective Motivational Values toward Child Gender Tool Tested in Women of Reproductive Age from a Hospital in Rural Area of Himachal Pradesh, India

[illegible]

Abstract

[illegible]

Introduction

[illegible]

and challenge in life; (e) Self-direction: Independent thought and action-choosing, creating, exploring; (f) Universalism: Understanding, appreciation, tolerance, and protection for the welfare of all people and for nature; (g) Benevolence: Preservation and enhancement of the welfare of people with whom one is in frequent personal contact; (h) Tradition: Respect, commitment, and acceptance of the customs and ideas that traditional culture or religion provide the self; (i) Conformity: Restraint of actions, inclinations, and impulses likely to upset or harm others and violate social expectations or norms; and (j) Security: Safety, harmony, and stability of society, of relationships, and of self.

All domains reflect four major areas: self-enhancement by power, achievement, and hedonism; openness to change by stimulation and self-direction; self-transcendence by universalism and benevolence; and conservation by security, tradition, and conformity.[8] In this context, the present study aimed to extract relatively independent coherent subsets consistent with the theoretical construct. Specific objectives of the study were to identify the pattern of correlations and domain structure of a subjective MVCG tool in reproductive-age women in the rural area of Himachal Pradesh, India.

Materials and Methods

This was a cross-sectional observational study carried out in villages of Haroli health block of district Una, Himachal Pradesh from October 2018 to November 2019. Women within the age group from 18 to 34 years, with last children of <1 year of age, resident of the rural area, with formal education, willing to spare some time for the interview, and with informed consent were selected for the study. Women living with issues related to mental health and admitted or to be admitted to hospital with an illness were excluded from the study. The early phase of reproductive age (18–34 years) was chosen as participants engage and contribute significantly in household activities and can share their experiences towards childbirth without any recall bias. Convenience sampling was followed to recruit the subjects. A total of 370 women were recruited from the civil hospital of rural area, attending for health care and a total of 15 women refused to participate. Thus, finally, a sample of 355 that could be studied was considered to be good for factor analysis as guided by Comrey and Lee.[9]

After obtaining informed consent, MVCG tool with 43 questions was administered by the interviewer where the statement was narrated and the respondent was asked to quantify the response on a Likert scale; 1 = completely disagree, 2 = disagree, 3 = agree, 4 = partially agree, and 5 = completely agree. Questions were asked about their approach toward "the event" which was considered childbirth for an interviewed woman.

To develop the questionnaire based on the theoretical construct, a group of a public health expert (1), social science expert (1), and women from local villages (2) was constituted. Exercise for questionnaire formulation was carried out among technical experts of the group. Independently, women from the village were consulted for gathering their opinion about subjective comprehensibility and applicability of questions in concerned settings. Over 3 months, iterative discussions were carried out to discuss and finalize; motivational values, value-specific questions, and method of scoring (quantification) for questions. Firstly, a consensus was built on the utility of motivational values for the assessment of the parental value system toward the gender of a child. Furthermore, the type of questions and its method of scoring was discussed. The value-specific questions were shared with the group to avoid redundancy of responses and restructuring the statement of questions. Finally, in meeting with the women members, finalization of a number and type of questions for each motivational value was done. The questionnaire was translated into Hindi and then back to English and was decided to be interviewer-administered.[10] It was decided to ask questions in the form of statements that will be measured in the Likert scale from score 1 to 5. It was chosen as questions were of declarative in nature and considered to be more definitive response rather than no and unvarying response. Although, the extent of granularity is debated evidence have showed that 5-7point scale ensures reliability and validity, so recommended often for use.[11] Distribution of responses for each question was also studied and significant difference from normality was assessed with Chi-square test at 5.0% level of significance.

As a dimension reduction process, principal axis factor analysis was done to extract and analyze factors. It was chosen as the factor analysis method takes measurement errors into account [illegible]

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CHILDHOOD CANCER IN INDIA: SOCIAL WORK PERSPECTIVE*

BY

DR. ASHISH NAG♣

Associate Professor, Department of Tourism & Travel Management, Central University of Himachal Pradesh, India

SHISHIR SHARMA

Research Scholar, Department of Social Work, Central University of Himachal Pradesh, India

DR. SHVETA SHARMA

Assistant Professor, Department of Social Work, Central University of Himachal Pradesh, India

ABSTRACT

The childhood cancer affects all societies, gender and persists in almost all socio-economic groups. The feeling of loss and grief has deep correlations with the disease of cancer and initiates even before the disclosure of the disease. Childhood cancer is different from that of cancer form witnessed in adults. Cankids... Kidscan a national level agency working on childhood cancer documents stated that every year around 300,000 children in world are diagnosed with cancer. At such an early age which is full of liveliness, dreams and craziness when a child is detected with cancer, his whole life becomes a battle for survival. Satyanarayana, Asthana & Labani (2014) pens down that In India the cancer is ninth major contributing reason for deaths among children between the age group of 5-14. More than eighty percent of incidences of cancers are occurring in developing economies and also include low income economies. The present attempt focuses in describing the pattern of childhood cancer forms and also substantiates the relevance of Social work in Cancer Care.

KEYWORDS

Childhood, Grief, Gender, Correlation.

INTRODUCTION

In scientific terms, cancer may be represented as a phenomenon in which there is

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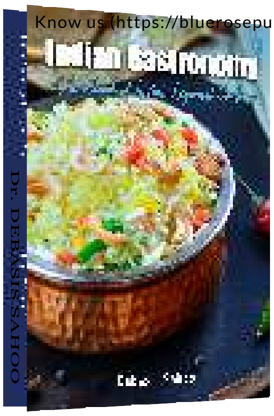
♣ Correspondence Author



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Indian Gastronomy

by: Dr. DEBASIS SAHOO (https://bluerosepublishers.com/author-book/dr-debasis-sahoo/)

Indian cuisine went through a series of evolvements in the past thousands of years and what we saw today is a rich mixture of ancient regional flavours and the subtle influence of international food culture. Despite the above cultural ramification, several communities from different regions of the country are still adhering to the ancient food culture and strictly following the culinary practices as inherited from their ancestors. Hence this book is an attempt to accommodate all such regional gastronomic aspects of the country under one roof. Initially, a brief discussion about the culinary aspects of the state/ region/ community was given for a better understanding of the ingredients, cooking methods & serving procedures, followed by traditional recipes of the region.

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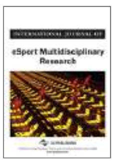

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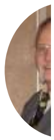

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- *John Radke,*

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META

Role of surface diffuseness and the coupling of relative motion with intrinsic motion in fusion of negative and positive Q-value systems

Jamwal, Ankita (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India)); Sharma, Shailja Mohini (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India)); Kushmakshi (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India)); Verma, Dalip Singh (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India)), E-mail: dsverma@cuhimachal.ac.in

Proceedings of the DAE-BRNS symposium on nuclear physics. V. 61

2016

Abstract

[en] It is well established that fusion cross-sections at sub-barrier energies may be enhanced by several orders of magnitude as compared to the prediction of one dimensional model due to the coupling of relative motion to the nuclear intrinsic degree of freedom. The coupling leads to the modification of the potential energy term and hence the enhancement in the fusion cross-section at below barrier energies. The potential modification can also be done by changing the parameters of the potential used. In this paper, we have calculated the fusion cross-section for four quadrupole deformed systems "3"6S+"6"4Ni, "3"6S+"4"8Ca, "1"6O+"1"8"2W and "1"6O+"1"4"4Nd by considering the coupling of low-lying 2"+ vibrational states, due to the addition of single quadrupole surface vibrations to all systems i.e. the target as well as the projectile, using CCFULL code. Also, the surface diffuseness parameter 'a' of Woods-Saxon potential has been adjusted to fit the observed data for the cases where coupling is not reproducing the data

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Source	John, Bency V.; Dutta, Dipanwita; Saxena, Alok (Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai (India)) (eds.); Board of Research in Nuclear Sciences, Department of Atomic Energy, Mumbai (India); 1160 p; Dec 2016; p. 602-603; 61. DAE-BRNS symposium on nuclear physics; Kolkata (India); 5-9 Dec 2016; 9 refs., 2 figs.
Record Type	Book
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Country of publication	India
Descriptors (DEI)	ANGULAR MOMENTUM, NICKEL 64 TARGET, OXYGEN 16 REACTIONS, Q-VALUE, TOTAL CROSS SECTIONS, VIBRATIONAL STATES, WOODS-SAXON POTENTIAL
Descriptors (DEC)	CROSS SECTIONS, ENERGY, ENERGY LEVELS, EXCITED STATES, HEAVY ION REACTIONS, NUCLEAR POTENTIAL, NUCLEAR REACTIONS, POTENTIALS, TARGETS
Language	English
Reference Number	48040223
Related Record	48039927
INIS Volume	48
INIS Issue	18

META

Angular momentum effects in the fusion of "2"8Si+"2"8Si system

Choudhary, Atul (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India)); Verma, Dalip Singh (Department of Physics and Astronomical Science, Central University of Himachal Pradesh, Dharamshala (India))

Proceedings of the DAE-BRNS symposium on nuclear physics. V. 61

2016

Abstract

[en] In the heavy ion fusion reactions the interaction potential plays an important role as it provides the characteristics like barrier height, barrier position and barrier width in the calculations of fusion cross section. This means different types of interaction potential gives different fusion cross sections or potential parameters are predicted w.r.t the experimental data. In the literature, number of formalism for the calculation of fusion cross sections assumes that the potential barrier position and width is independent of angular momentum (ℓ). However, all the three potential characteristics are ℓ-dependent and are used in the calculation the fusion cross section for a positive Q-value system, "2"8Si+"2"8Si (Q = 10.9 MeV) and is compared with the recently measured fusion cross section

Primary Subject	NUCLEAR PHYSICS AND RADIATION PHYSICS (S73)
Source	John, Bency V.; Dutta, Dipanwita; Saxena, Alok (Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai (India)) (eds.); Board of Research in Nuclear Sciences, Department of Atomic Energy, Mumbai (India); 1160 p; Dec 2016; p. 478-479; 61. DAE-BRNS symposium on nuclear physics; Kolkata (India); 5-9 Dec 2016; 6 refs., 2 figs.
Record Type	Book
Literature Type	Conference
Country of publication	India
Descriptors (DEI)	ANGULAR MOMENTUM, MEV RANGE 10-100, Q-VALUE, SILICON 28 REACTIONS, SILICON 28 TARGET, SKYRME POTENTIAL, TOTAL CROSS SECTIONS
Descriptors (DEC)	CROSS SECTIONS, ENERGY, ENERGY RANGE, HEAVY ION REACTIONS, MEV RANGE, NUCLEAR REACTIONS, NUCLEON-NUCLEON POTENTIAL, POTENTIALS, TARGETS
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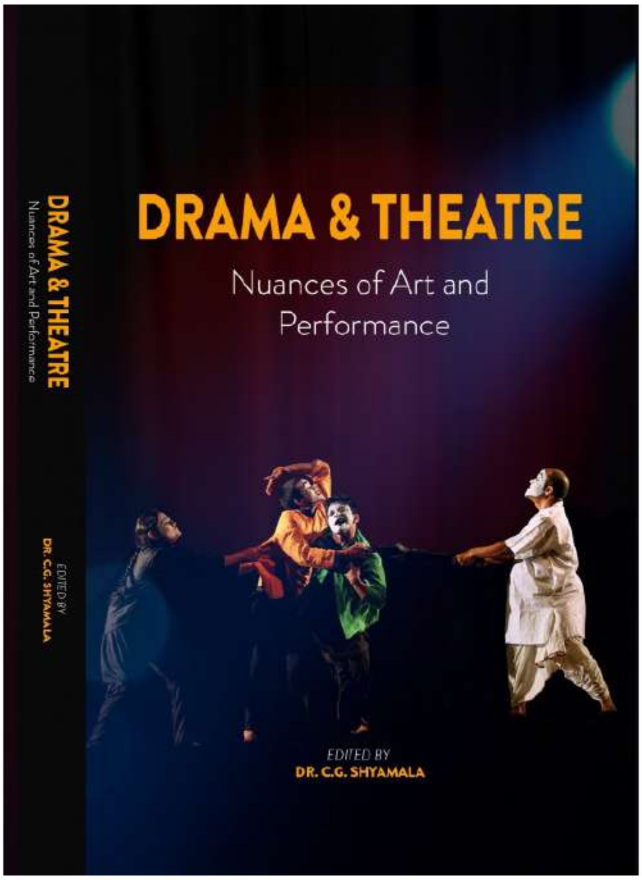
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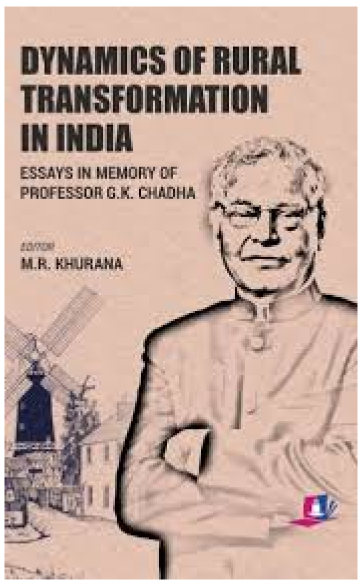

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Description

This book, a collection of 16 essays from colleagues, friends, admirers and students of Professor G.K. Chadha, commemorates his work and contribution to Agricultural Economics, including his role as an institution builder and miscellany of other wide-ranging areas. Professor G.K. Chadha was the President of South Asian University when he left for his heavenly abode. He was founder of the South Asian University, author of over 100 research articles published in journals of national and international repute, and of as many as 16 books on the various facets of rural transformation. Widely acclaimed as a well-established academician and serious researcher for over forty years, Professor Chadha's contribution to the study of rural transformation in India is indeed legendary. The book encompasses in its fold the mainspring of idea of those who have had the good fortune of being in close affinity of Professor Chadha one way or the other, and, as such, were also influenced and inspired by his exemplary scholarship. A notable feature of this volume is that the various included contributions provide a number of policy insights into a wide variety of topics of interest to the planners and policymakers involved in the task of rural transformation in India and elsewhere, and are written by the foremost scholars who have carved out a niche for themselves in their respective areas of specialisation. Taken together, these essays, while giving a sense of some of the key issues and aspects of the dynamics of rural transformation in India, seek to highlight the need to protect the interest of rural masses, in general, and those at the lower rung of the socio-economic ladder, in particular, in the Indian countryside as also elsewhere in situations similar to it. The book also includes the editor's memorial tribute to Professor G.K. Chadha which throws light not only on his personal traits but also on his remarkable academic career, including his roles as CACP Chairman, Vice- Chancellor, Jawaharlal Nehru University, New Delhi, The CEO of South Asian University, New Delhi and finally, its President. The editor's personal reminiscences of Professor G.K. Chadha, in particular, help the reader understand why he was so highly respected and admired and why his sudden death dealt a distressing blow to those associated with him directly or indirectly.

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MILID Yearbook 2018/2019

A collaboration between UNESCO UNITWIN Cooperation Programme on Media and Information Literacy and Intercultural Dialogue, UNESCO Chair on Media and Information Literacy for Quality Journalism, the Autonomous University of Barcelona (UAB) and the Arab Academy for Science, Technology and Maritime Transport, Egypt.

Media and information literacy in critical times: Re-imagining learning and information environments

Edited by José Manuel Pérez Tornero, Guillermo Orozco and Esther Hamburger

Autonomous University of Barcelona, University of São Paulo, Tsinghua University, Cairo University, Temple University, University of the West Indies, Queensland University of Technology, Sidi Mohamed Ben Abdellah University

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Media and information literacy in critical times: Re-imagining learning and information environments

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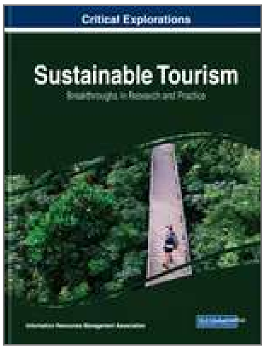
Empowering women and ensuring gender equality in India through Media and Information Literacy approach

I V Malhan
Professor, DLIS, CUHP, Dharamshala

Archna Katoch
Assistant Professor, Department of Journalism & Creative Writing,
Central University of Himachal Pradesh, India

Informing, educating, and empowering women through access to information is an essential pathway for progress in the United Nations' fifth Sustainable Development Goal (SDG) to "achieve gender equality". Access to information is helping women to realize their potential to do better and to stand up for their rights and freedoms. This new awareness facilitates social change in the patriarchal, conventional, and orthodox mindset of society. The current study presents some cases, particularly from the field of agriculture, which demonstrates how access to information and the use of knowledge helps to empower women and improve their economic situation. It also describes how their growing presence on social media is making Indian women more communicative, helping them to overcome the social taboos and unleash their power. It portrays how media and information literacy (MIL), and its different stakeholders, can change the information environment ubiquitously providing new opportunities to women to access relevant information, education, as well as acquire skills, interact, collaborate, network, and have their voices heard; in turn, this leads to their educational, social, economic, health, and political empowerment. MIL competency helps in self-awareness and lifelong learning and in increasing the informational level of women. It thus facilitates their empowerment, promotes gender equality and helps women to realize and stand up for their rights. This paper thus makes a plea for integrating MIL training in women's education. However, factors such as illiteracy, poverty, poor technical skills due to limited access to technology, inadequate infrastructure, lack of appropriate content in local languages, the high cost of connectivity, prejudiced and inequitable social norms, are some of the aspects that may affect women's access to information.

Keywords: Women's empowerment, Gender equality, Social change.



From Earning Profits to Sustainability: A Critical Evaluation of CSR Initiatives in Tourism Sector

Manpreet Arora (Central University of Himchal Pradesh, India) and Sandeep Kumar Walia (Maharaja Agrasen University, India)

Source Title: Sustainable Tourism: Breakthroughs in Research and Practice (/book/sustainable-tourism-breakthroughs-research-practice/207674)

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Abstract

In India the potential of tourism sector is considerable in terms of earning foreign exchange. That is why the government has initiated many programmes and schemes to attract foreign tourists. It has also given a rise to the allied industries to earn profits and diversify may it be hotel industry or aviation sector. As the means of communication are developing and the countries are becoming more open and globalised the tourism industry has got new impetus and dimensions. The spread of education and the development of internet have widened the scope of this industry. This new trend has made the tourism industry a very potential industry in many terms for a developing nation like India. But on the same hand it has posed a great threat to the natural resources also and the stakeholders involved in this sector has started responding to such initiatives through CSR measures. For measuring such initiatives no specific parameters have been defined and they are subjective to a great extent. Therefore a need arises to analyse such initiatives and their growing need for creating sustainability.

Chapter Preview

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Introduction

Tourism sector worldwide contributes for around 6-7 per cent of global employment (direct and indirect) and 5 percent of global income as per the United Nations World Tourism Organization (UNWTO), Tourism Highlights 2012 edition. It is one of the major generators of employment across the world and women account for 70 per cent of the workforce in the travel and tourism industry. Hence it generates more inclusive growth than other sectors. World wide it is discussed and is an issue of concern that whatever we do we should analyse its impact on its stakeholders. The stakeholders may be any, government, public, shareholders or even our own mother nature. India considers tourism to be a “Industry of Potential”. Whatever is invested in this industry gets good return but on the same hand whatever we invest in this industry costs our environment, depletes our natural resources, may pose threat to the heritage sites, endanger flora and fauna, may be harmful to various species of animals etc if such investments are not well thought of and well considered.

In India we have plenty of cultural attractions and healthy climatic conditions for various types of tourists throughout the year. It is perhaps only country where tourists can get every type of climate to enjoy. Therefore the potential of this sector of earning profits is reasonably quite high.

Economic growth and development has always been a precedence in developing nations which is indispensable for the continued existence. Development and growth should be in such a way that it is not only plateful for the present purpose but also creates payback and opportunities for the future generations.

For the conversion of development into sustainable development it is necessary to pay attention on the issues of sustainable development. Sustainability and Corporate Social Responsibility (CSR) are interconnected to each other in such a way that it is difficult to achieve sustainability without CSR which is true for tourism sector too. Sustainability can not be achieved if we do not keep in mind our actions which have direct or indirect effect on the resources to be used and left for future generations. Human has always been selfish in using the natural resources as well as using the man made things in such a fashion that the effect of the products on society at large is always neglected. But now a change is coming over and sustainability issue is gaining momentum. Sustainable development has been defined as a tactic of fulfilling the needs of present without compromising the ability as well as the capacity of future generations to meet their own needs in many aspects. Social, economical and environment issues are of major concern for the achievement of sustainable development as all these factors have direct bearing and a direct linkage with one another. Development is concept of bringing change within the lives of people by fulfilling their basic needs and in India Tourism sector is giving plenty of opportunities to the masses for

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Contents

Gender Equality for Sustainable Development: #Breakthebias

Archna Katoch

Introduction

The Former UN Secretary-General Ban Ki-moon once said in his message on International Women's Day 2016, "We have shattered so many glass ceilings, we created a carpet of shards. Now we are sweeping away the assumptions and bias of the past so women can advance across new frontiers." Presently, women are contributing and taking part in every sphere like education, business, politics, science and technology, media, sports, art etc. They are not only working at grass root level but also participating in decision-making. Women are the shining beacons of hope, and have displayed exemplary dedication in their respective fields.

This year the UN theme for International Women's Day on March 8, 2022 is "Gender equality today for a sustainable tomorrow" with the campaign theme: #BreakTheBias. This day endeavours to applaud all the successful women for the courage and determination they have shown to reach their destination against all odds, and functioning to make a more sustainable future. Therefore, it is a day to celebrate women's social, economic, cultural and political achievements, raising awareness against bias, and taking action to accelerate gender equality and create a better gender-neutral society. Women empowerment is the creation of a social environment, where women can make independent decisions on their personal development as well as shine as equals in society.

There are remarkable women in Indian history, across ideologies, who have shown extraordinary strength and courage to fight their battles

हेमराज बंसल

जन्म : २४ मार्च १९८३ ; गाँव- घियाल, नम्होल, जिला बिलासपुर, हिमाचल प्रदेश !

शिक्षा : एमए, एम.फिल, पीएचडी (अंग्रेजी), हि.प्र.विश्वविद्यालय, शिमला !

आलोचनात्मक पुस्तकें

- *मैपिंग डायस्पोरा आइडेंटिटीस: इंडिया एंड बियॉन्ड* (अनामिका पब्लिशर्स, २०१७; सह-सम्पादित, अंग्रेजी भाषा)
- *होमोजेनाईटी इन हेटरोजेनाईटीजेनाईटी: मेमोरी, कल्चर, एंड रेजिस्टेंस इन एबोरिजिनल कल्चरज़ फ्रॉम अराउंड द वर्ल्ड* (ऑथर्सप्रेस, २०१८; सह-संपादित, अंग्रेजी भाषा)

काव्य-संग्रह

- *द साइलेंट क्राई* (काव्य-संग्रह, अंग्रेजी भाषा, प्रकाशाधीन)

शोध-पत्र:

लगभग बीस से अधिक शोध पत्र विभिन्न राष्ट्रीय एवं अन्तर्राष्ट्रीय पत्रिकाओं और पुस्तकों में प्रकाशित (हाल ही में पाल्ग्रेव मेकमिलन, लन्दन द्वारा प्रकाशित पुस्तक में लेख प्रकाशित हुआ है)

सम्मान

- बेस्ट रिसर्च पेपर अवार्ड ऑन *शेक्सपियर'स कपल ऑफ़ प्लेज़* (२०१५)

सम्प्रति : हिमाचल प्रदेश केंद्रीय विश्वविद्यालय, धर्मशाला में अंग्रेजी विभाग में सहायक प्रोफ़ेसर के पद पर कार्यरत हैं !

इन कविताओं में निहित वेदना का प्रेरणा स्त्रोत कोई और नहीं बल्कि यथार्थ की पृष्ठभूमि पर प्रतिकूल परिस्थितियों में गुजर-बसर करते लोग हैं ! एक बूढ़ी माँ की वेदना हो या फिर कर्ज़ में डूबे आत्महत्या करने वाले किसान की विवश्ता; हिंसा और साम्प्रदायिक दंगो के दर्द में पिस रहे आम इन्सान की बेचैनी हो या फिर एक अबोध बालक ही घबराहट; एक बूढ़े पेड़ की चिंता हो इस धरती के लिए या आदमी की खत्म होती इंसानियत, इस पुस्तक में शामिल कवितायें जीवन के उस हर बिंदु को छूने का प्रयास करती हैं जहाँ हर चीज़ हांफती नज़र आती है सांसो के लिये ! जीवन की विषम एवं हृदय को विदार कर देने वाली अनुभूतियाँ झकझोर देती हैं अंतर्चेतना को और छोड़ जाती हैं कई प्रश्न जिनका उत्तर हमें ढूंढने की जरूरत है !

हाँफती ज़िन्दगी पूछ रही है कुछ ऐसे ही प्रश्न जो शायद पूछे जाने चाहियें ! कविताओं में प्रयुक्त तलवार की धार से भी पैनें शब्द कुछ ऐसे जख्म छोड़ जाते हैं जिन्हें भरने के लिये हमें फिर से इंसान बनना पड़ेगा तभी शायद ज़िन्दगी निखरेगी और लगेगी सुंदर और भी !

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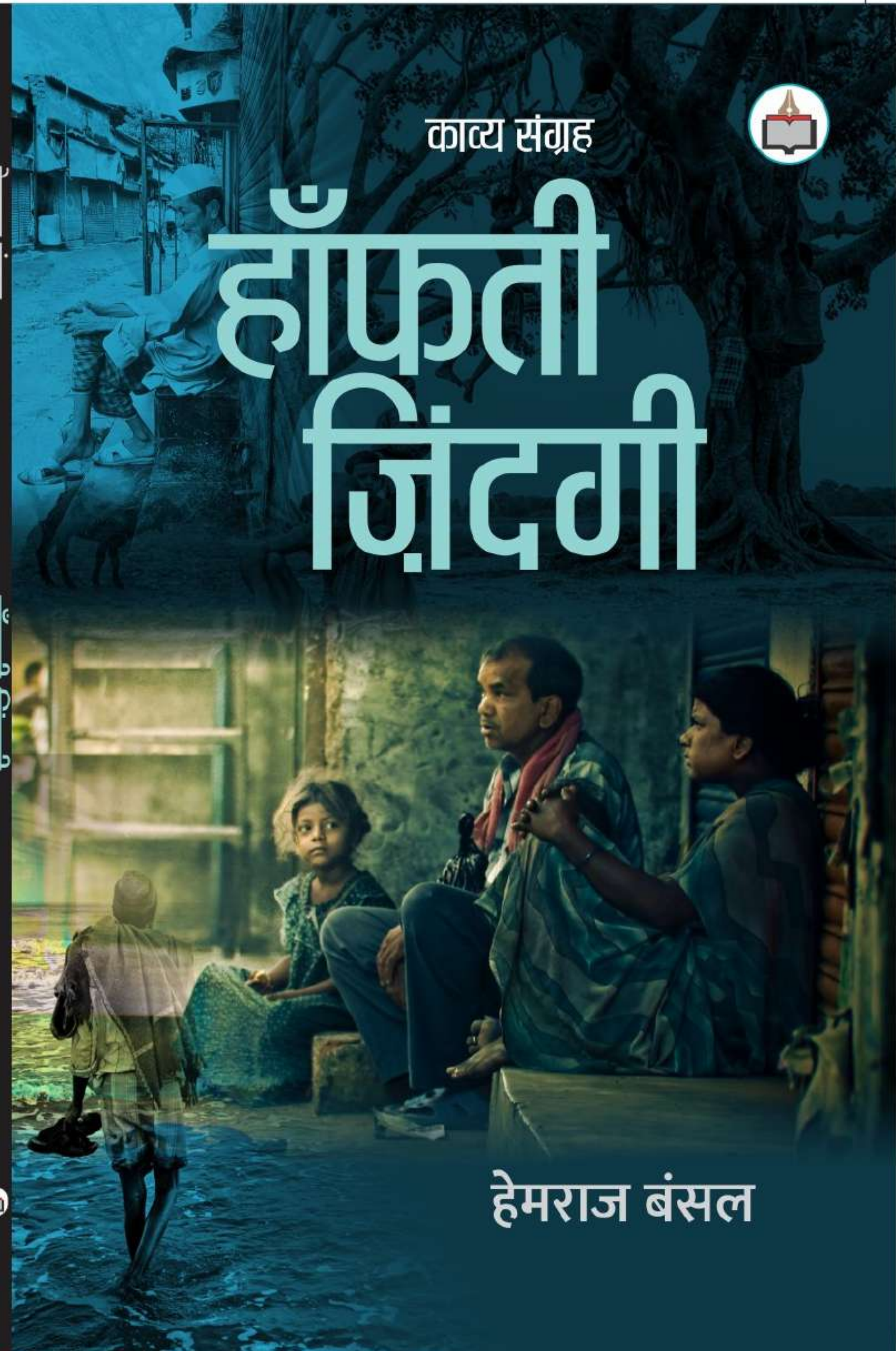

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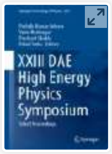

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DAEBRNS 2018, HEPS 2018: **XXIII DAE High Energy Physics Symposium** pp 1001–1005

Implications of Non-unitarity on θ_{23}, Neutrino Mass Hierarchy and *CP*-Violation Discovery Reach in Neutrino Oscillation Experiments

Surender Verma & Shankita Bhardwaj

Conference paper | First Online: 19 May 2021

Part of the Springer Proceedings in Physics book series (SPPHY,volume 261)

Abstract

We have studied the effect of θ_{23} octant, neutrino mass hierarchy and *CP* violation discovery reach on the sensitivity of non-unitarity in short baseline (SBL) experiments using minimal unitarity violation (MUV) scheme. We find that the θ_{23} octant has distinguishing implications towards the sensitivities of non-unitary parameter $|\rho_{\mu\tau}|$ with normal and inverted mass hierarchies. Using *CP*-fraction formalism, we find the possibilities to distinguish the *CP*-violating effects due to unitary (δ) and non-unitary $(\omega_{\mu\tau})$ *CP* phases.

Keywords

Neutrino oscillation **[spsdollar2dollarsps] Violation** **Non-unitarity**

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Covid-19 Pandemic and Economic Development pp 125–138

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Employment Participation of Rural Women in Punjab: A Need for Policy Retrospect

Ashapurna Baruah & Indervir Singh

Chapter | First Online: 28 September 2021

Abstract

Economic policy in India is governed by the neoclassical growth theory as growth being an answer to all problems. This has led to the design of policies that focuses on the expansion of economic activity in urban areas. The underlying assumption is that economic activity in rural areas will gear up with trickling down of benefits sourced in urban areas. This oversimplified understanding of the economy has led to a lopsided development where the gap between rural and urban areas has been widening. Employment of rural women is one of the biggest casualties of this growth policy. Rural women in Punjab, one of the most developed states, have been bearing the brunt of current economic policies. The prevailing gender norms restrict women from travelling outside their villages for work. While men migrate or travel to urban areas for work, women in rural areas have largely become unemployed or underemployed, given the dearth of employment in rural areas. Findings of the primary survey, conducted in rural areas of Punjab, show that the majority of women are willing to work if work is available within the village. The growth policies focussed on urban areas has not only led to adverse employment outcome for these women, but also has kept a large share of workforce out of productive employment. COVID-19 pandemic has further hurt the employment prospect of women. The initial trends suggest that women's employment is more severely hit than their male counterparts. The governments are increasingly using the same set of policies to fight the economic fallout of the pandemic. Such policies are unlikely to benefit women in rural areas. The pandemic has further increased their cost of travel. In addition to the cost of breaking social norms, they have to risk the infection to reach urban areas for work. The increased restrictions on public transport have increased the monetary cost of travel. In such a scenario, women are likely to find the cost of travelling to urban areas for work more than the expected earnings. The economic policy, for an inclusive development, must focus on creating infrastructure and businesses, especially those which can employ women, to create jobs in rural areas. Such a policy will not only create better employment opportunities in rural areas but will also lead to higher growth and faster recovery.

Kash, Pankh Hamaare Hote

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Description

डॉ- प्रिया शर्मा की ये कविताएँ बच्चों के सरल चित्त पर समय की चेतावनियाँ लाल-हरे रंग में झिलमिलाती चलने वाली हँसती-बोलती सी कविताएँ हैं। ‘फास्ट फूड’ या ‘ब्यूटी पार्लर’ या ‘कविताएँ न पढ़ने की प्रवृत्ति’ बाल जगत् में भी प्रवेश पा गए हैं। एक से एक रोचक विज्ञापन मायानगर के ऐयार की तरह टीवी पर आते हैं और फास्ट फूड, श्रृंगार प्रसाधन आदि के प्रति बाल-मन में भी ऐसा आकर्षण भर देते हैं कि घर का पौष्टिक और संतुलित आहार उन्हें अखाद्य लगने लगता है और नहाया-धोया, सादा-सा चेहरा भी
‘ब्यूटी पार्लर’ का प्रत्याशी! गुड़ियानुमा वे ही मम्मियाँ अच्छी लगने लगती हैं जो ‘सुपर बायर्स’ होंμमहाखरीदारμरंगीन पैकेटों में जहर खरीदकर घर लाने वाले। परीकथा के जंगलों में भी उन्हें ‘ब्यूटी पार्लर’ चाहिए।
आंतरिक सौंदर्य, शांति, सौहार्द और प्रेम से भरे सहकारितामूलक जीवन की प्रेरणा सहज ही मन में जगाने वाले ये बालगीत इसलिए भी महत्त्वपूर्ण हैं कि पर्यावरण की चुनौतियों का सामना करते हुए पशु-पक्षी और वनस्पति जगत् से तादात्म्य रखने की उमंग ये मन में भरते हैं। पशु-पक्षी, वनस्पति और बच्चेμये ही युद्ध और आतंक की छाया में पल रहे इस अतिशय भौतिक युद्ध का सच्चा वैकल्पिक प्रतिपक्ष रचेंगे इन कविताओं के दम से।
-अनामिका

Related products

Chapter

Landfill biodegradation process and leachate

January 2018
DOI:10.1007/978-981-10-7413-4_12
In book: Waste Bioremediation (pp.233-247)

Authors:

Rajkumar Joshi
DST Delhi

Deepak Pant

Abstract

Increasing landfill sites leads to serious environmental problems due to the generation of leachates. Leachates are the potentially hazardous waste from landfill sites and consist of many organic and inorganic, dissolved suspended, biodegradable or non-biodegradable matters. In this chapter, we are discussing various process involved during landfill biodegradation, generation of leachate and factor affecting biodegradation process. Microbes played an important role in the biodegradation and bioremediation of landfill waste and leachates. The different kind of microbes used organic material present in landfill and leachate for their metabolic activities and resulted in the production of gases like CH_4 and CO_2. The biodegradation processes of waste material depend on the various factors including characteristics of waste materials, moisture content, temperature, pH and the presence of inhibitors.

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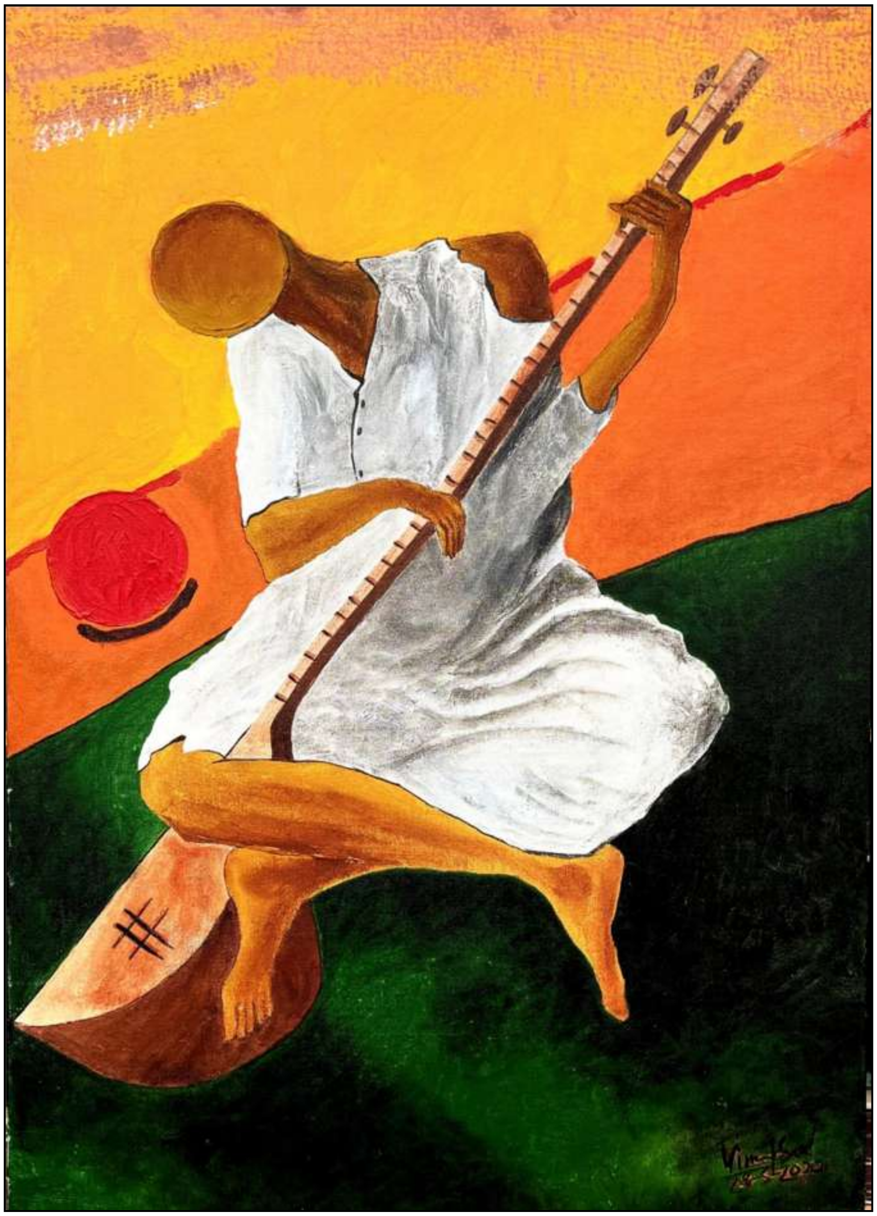

Painting by Vivek Sood Kuthiala

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BY WAY OF INTRODUCTION

PRAVEEN KUMAR & JANESH KAPOOR

It has been an exciting journey for the English Teachers' Forum, Himachal Pradesh ever since it came into existence in 2018, and the present anthology is the culmination of the collaborative spirit that has always been the hallmark of this academic body. Covid-19, which swept through the world in 2020 and held its ferocious sway for almost two years before subsiding to some extent, gave us an opportunity to explore alternative ways to remain creatively and academically engaged. And thus, emerged the idea of holding online story-reading sessions, featuring short stories written by our revered teachers and colleagues. While a few of them are already established names, having been the recipients of numerous literary awards for their outstanding contribution to the world of literature, it took a bit of persuasion to convince others to come on board, and let their dormant creative spark come alive and brighten the world. And once they decided to take the plunge, they came up with remarkable stories in their very first attempt. In order to make the reading sessions more rewarding it was decided to have a discussant for each presented story. This gave the attendees an opportunity to enjoy the creative pieces along with their brilliant critiques, making the whole experience exhilarating and rewarding.

The anthology has a total of twenty-two stories, each having a unique flavour, writing style, linguistic idiom, rhythm, thematic concerns, and emotional intensity. While "Cleansing the sacred 'Kund'" has a beautiful amalgam of history, legend and fiction, "The Party" is a woman's narrative, which captures the complexities of the fragile protagonist's travails and emotional trauma; "Women" as the title itself suggests revolves around women, three to be precise, whose unfulfilled desires for love and sexual gratification leave a vacuum in their lives; "Mook Prem" brings a fresh perspective on the complexities of human relations, especially when the parties involved are a step child and her parents, people's perceptions of such relationships, and the possibilities of the margin of error between the general perception and reality; "Twelve Years Later" highlights the vexing issue of eve-teasing and its insensitive handling by family, friends, teachers, and society, leaving the targeted individual scared and scarred for life; "Terror" is a study in prejudice, which often stems from social conditioning and goes a long way in sowing the seeds of distrust, fear, hatred, and unfounded antipathy, leaving in its trail a weakened social fabric; "The Tear" is a melancholic tale portraying sensitively the plight of a poor family which, like so many other families, was confined to their crowded, cramped space called home during the Corona times. But despite the many challenges the family faces at each step, the characters still retain optimism, probably the greatest asset the poor possess in abundance, and this is what brightens up an otherwise sad narrative; "Apocalypse" is a dark tale of brutalization that the traumatized, emotionally drained daughter has to face at the hands of her father; "A Snow Trail", replete with childhood innocence, charm, fun and bonhomie, takes a disastrous turn when a few individuals dash the fond memories of the narrator's beautiful past with their indifference and nonchalance; "Radhe Radhe" deals with existential questions, the void one feels when one fails to have a single meaningful, loving relationship with a fellow human being, and the challenges one encounters when one refuses to succumb to social stereotypes; "Mama" is about an innocent child who doesn't understand societal norms and the pressures they exert on an his mother, making her life miserable. He fails to understand why his widowed mother can't live like any other woman – wearing jewels and makeup; "The Hole in the Wall" highlights the sad reality of child labour. Compelled to live a life of poverty and squalor, the child protagonist deals with the harshness of life by immersing himself in fantasy, but his fantasy world doesn't hold for long and comes crashing down towards the end of the story; "A Girl as Old as Thirty" raises a very pertinent issue relating to the social stigma and mental anguish that a girl and her parents have to contend with if she crosses a socially-acceptable age bar for marriage; "And Then the River Flowed" is a self-reflexive tale about the creative struggles every writer undergoes at some point in life, and the possibilities of evolution that may lie hidden in a state of temporary stagnation; "Chhatri" deals with the identity crisis of the female protagonist who leads a loveless life and is treated by everyone only as a body and not a living human being. The lost soul however, sees the possibility of a resurrection of her identity when an empathetic human being makes an earnest attempt to touch her soul without trampling on her body; "Why Fear When I am Here" affirms the value of faith in our lives. God is not only merciful but also benevolent, and anyone who puts in sincere efforts in one's personal and professional life is bound to receive God's love and grace; "Billiaan" is a brilliant story which articulates the issues of gender, patriarchy, authority, erosion of authority, inter-generational power dynamics, the complexities of family structure, and irrational fears; "Nirvana" as the name itself suggests, focuses on the themes of self-realization, spiritual enlightenment, and the power of intuition in reaching a higher state of consciousness; "Balance" is a painful tale revolving around gender discrimination, family expectations, especially from the daughter-in-laws, petty family feuds that sometimes get amplified leading to such tragic consequences as suicide, and the eventual disruption such extreme steps cause in the lives of their loved ones; "The Mad Mystic in Heaven" is an unusual tale that challenges the hierarchical structure in which the gods and goddesses and human beings are believed to exist. It explores the possibility of a mortal human transcending his earthly limits through rigorous purification and refinement of consciousness vis-à-vis the fallibility of gods/goddess in certain contexts. It may also be read as a political allegory; "Bimla 'Paagal'" is a sad tale of a woman who is used and betrayed by her own partner, and left to rot in this uncaring world. The world is full of Bimlaas who end up losing their dignity and sanity in the face of ruthless exploitation, eventually dying of neglect and heartbreak; and finally, "Nanga Aadmi" stands out as an intense narrative of trauma and helplessness the voiceless poor are fated to experience. It is a scathing attack on the hypocrisy of the political class and the existence of soul-crushing social disparities that tend to annihilate one's self-esteem.

English Teachers' Forum, as the name itself suggests, comprises teachers of English, but in order to accommodate linguistic preferences of the contributors, offer space to varied literary voices, and make the writings available to a more diverse audience, we invited entries both in English and Hindi. The collection even features three translated stories, viz. "Women", "A Girl as Old as Thirty", and "Terror" -- written originally in Hindi by three prominent literary voices of Himachal Pradesh and translated into English by equally illustrious academics. The whole idea of having a bilingual collection augmented by story-by-story critical analysis, and select live responses of the audience has positioned the anthology in a unique space. We are hopeful that the readers will find this anthology academically and aesthetically pleasing for its freshness, variety, intellectual rigour, and appeal to finer human emotions.

1

CLEANSING THE SACRED 'KUND'

Abhyudita Gautam

The erstwhile dominion kingdom of Devgarh or the Devgarh Valley as it was popularly called was known for the extensive agricultural lands that cultivated vegetables and fruits on the sprawling plain fields. While traveling on the straight road that ran through them, one could feel a refreshing whiff of the fragrance effusing from fruits laden trees in the mango and litchi groves, on both the sides. Devgarh Valley was known far and wide for its productivity, was adorned with green houses, dotted with modern homes and irrigated by the countless water canals. The soil was fertile, and was believed to be so as it had been irrigated by the sacred waters of the holy 'Kund'.

The famous 'Kund' of the Devgarh Valley, came into being when Queen Rukmani of the Devgarh State had laid down her life for procuring water for her kingdom. The head priest of Devgarh had dreamt that only the sacrifice of a member of the Royal family could quench the thirst of this parched kingdom. The valley had not witnessed rains for years altogether making it a drought prone area where one could only see fissured land stretched till far away.

Heeding to the divine dream of the priest, it was declared that the Queen had herself stepped forward for this inimitable act of sacrifice. The King was the head of the kingdom; the sons were too young; so, the Queen decided to lay down her own life for the sake of the kingdom.

The people from all around Devgarh, had gathered to witness this great feat of their Rani, as she bid farewell to her sobbing sons, wailing husband and weeping women as the onlookers attended the complete burial ceremony. Queen Rukmani was buried alive in the wall, which was converted into a temple where the 'Kund' presently existed, brimming with the sacred waters.

The temple priests had begun a 'yagya' simultaneously, making sacred offerings to the consecrated fire and chanting mantras, so that the sacrifice of the Queen may be accepted by the Gods, leading to prosperity and affluence of the kingdom.

After the Queen was buried completely, it poured down heavily for hours, as if the skies shed tears for this brave, beautiful and blessed Queen. Devgarh Valley was resonating with the sounds of the people chanting the Queen Rukmani's name. The clouds thundered and the dried streams overflowed with rainwater.

The day after her sacrifice, the people of the village saw water trickling from the foot of a small hill, just near her burial site. The news reached the King's ears and he too came to the mentioned spot to witness this miracle. Within two days, a large pond or 'Kund' was formed; filled with pure and sacred waters coming from an unknown source deep beneath the lake.

The 'Kund's' formation was as enchanting a news as was the spotting of the Gold Fish in the 'Kund' that day. This Gold Fish was gliding smoothly in the lake waters. The priests declared that this was no one else but the Queen's incarnate and this fish would remain immortal till eternity. The Gold Fish was a symbol of prosperity that was brought by the Queen to the Kingdom. The King's subjects considered this as a holy omen. The tale of the 'Kund' and the Golden Fish now travelled far and wide, as pilgrims thronged to this place to bathe in the sacred waters of the 'Kund' and also to have a glimpse of the beautiful Gold Fish as well.

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The source of the 'Kund' was believed to be underground and perennial. A small channel was directed from the 'Kund' to the fields which allowed the sacred waters to run into the soil and make it more fertile. The land was barren before Queen Rukmani's sacrifice; people starved as the land produced little or no crops.

However, as the modern agricultural methods were adopted, the waters of the 'Kund' became polluted and contaminated with the chemical residues. People misused and abused the waters of the 'Kund'. The flowers and other worship materials were strewn all over the surface of the water. The land was cultivated with the modern techniques, fertilizers were used and hybrid seeds were sown to increase the produce. With the building of the dam, the waters were diverted into the fields.

The productivity increased but so did the diseases. People of the valley now complained of sickness that could not be cured even by the sacred waters of the 'Kund'; earlier its waters served as a remedy for almost every health issue that the people faced.

The Golden Fish too, had disappeared, or in the priest's words, "had disappeared as the people had polluted the waters"; the translucent waters had turned blurry with the dirt and filth. The priests of the temple had even urged the elected representatives to clean the 'Kund' but their requests fell on deaf ears.

Years went by and the valley prospered economically, but the people suffered. They were entrapped in the vicious circle of unending medical problems and treatments. To their respite a wing of the renowned chain of hospitals, The Swasth Charitable Hospitals, which was known for providing the best health services worldwide, had been set up in the Devgarh Valley. The doctors of this hospital were individuals who wanted to serve the needy and the underprivileged and wished to offer their services in the rural and interior places. Most of them had retired or had dedicated their lives for the poor and the sick.

Karmanya

A young Doctor Dr. Karmanya, a British citizen, had recently joined the Swasth Hospital of the Devgarh valley. He had gained his education from London and was working in a renowned hospital when he decided to quit his job and come back to India when he heard that the Swastha Hospital was being opened up for the people of his own village. This was an opportunity for him to come back to his roots. He had never been to India but had heard about his homeland from his parents and grandparents.

Karmanya had just shifted to a rented accommodation near the hospital, beginning a new life, surrounded by the village homes and the residents who lived around his house. The home that he had moved in was the old house of the erstwhile Royal family who had shifted abroad years ago. People were glad to know that the old and dilapidated bungalow that had lain vacant for years now had a new tenant. Even though people had spread rumours about the same being haunted, they chose to keep mum regarding this.

After settling down in his new home, Karmanya got to work in the Hospital during the day and in the evenings went for leisurely strolls and casual walks around the valley. The caretaker of his house was a village man named Chet Ram, whose earlier generations had been employed to look after the royal house for years. Chet Ram would accompany Karmanya during the walks, often introducing him to the villagers who would look at the handsome Karmanya in awe. He was a kind and humble soul who would talk to the children who met him in the fields, greeted the oldies of the village sitting in the front yards of their homes and fed the street dogs who wagged their tails welcoming the new resident of their area.

One day, Chet Ram narrated the story of the 'Kund' and the Gold Fish to Karmanya and told him how the Gold Fish had disappeared and the 'Kund' waters were now polluted drastically. Karmanya was enthralled by the story and decided to visit the temple and the "Kund' the very next day. On reaching, he looked at the 'Kund', which was a lovely turquoise blue in colour but the basin of the 'Kund' that was once visible, could not be traced anymore. Karmanya was mesmerized by the small turquoise coloured pond that had such an intriguing legend behind it. But he was sad to know that the Gold Fish could not be seen in the 'Kund' anymore.

He visited the temple and paid obeisance to the idol of Queen Rukmani that was installed at the raised platform at the extreme end of the huge Temple Hall. Karmanya looked at the idol and wondered how the Queen could gather the courage to lay down her life for the sake of her kingdom, leaving her young sons... and her family behind. What a selfless act of sacrifice!

Curious, Karmanya spoke to the priest to know more about the reason due to which the beautiful blue waters of the 'Kund' were now blurred with muck and filth. The priest apprised Karmanya about the situation: how Queen Rukmani had made the supreme sacrifice of her life to bring water to this kingdom and how it got contaminated with the adoption of new farming techniques. "People have developed shortsightedness, basking in the warmth of the financial boon brought in by the farming techniques, not observing the rise in health problems", he said.

Karmanya was well aware that the Swastha Charitable Hospitals was a British chain of hospitals that are opened only at places where the rise in the diseases and number of patients have shown a sharp increase. As he got to know about in London that one such branch has been set up in his ancestral area, he decided to come back and serve his village. He took a deep breath and came to the conclusion that it was not the cure or treatment being provided by the hospitals but the prevention of diseases which had to be worked upon.

Karmanya decided that to take steps for the cleansing, conservation and rejuvenation of this sacred turquoise coloured Pond, it was necessary to bring in the machineries required for the flushing of the dirty water and scrubbing the basin. Karmanya spoke to the environmentalists and the NGOs who were involved in river cleaning projects. They assured Karmanya of providing the machines and the technicians who would operate them. The Villagers also came forward to help Karmanya in this endeavour.

The news of Karmanya's efforts reached the government and it decided to take up the matter of the cleansing of the 'Kund'. It was named 'Kund Purification Project'. It would look into the setting up of pumping stations, recycling plants for collecting and reusing the flowers to make colour dyes and cleaning the 'Kund's basin. The people of Devgarh and the state were elated on the news. The staff of the Swastha Hospital too lauded the efforts made by Karmanya and donated for this cause generously.

The work began. The priests of the temple too started a 'Yagya' simultaneously for the successful completion of the project. But, the next day, the project work got disrupted as one of the cleansing machines developed a snag. It could be corrected only by diving into the lake and repairing it. Karmanya, decided to take the plunge into the water and, after a while, the machine restarted. People cheered. But Karmanya didn't appear again.

The Devgarh Valley was lashed with rains again after years as it resonated with the sounds of Karmanya's name.

The lake waters had engulfed Karmanya. His body wasn't found even after the lake was cleaned. The project ended the next day. However, the search operation to locate him was unsuccessful.

It was later confirmed that Karmanya Singh was the great grandson of same Queen Rukmani of the Devgarh Valley, who had sacrificed her life years ago to bring the water to the lands….

The family had shifted to London years ago. It was further revealed that his parents had passed away long back and he was their only surviving son. He had gone back to India as was wished by his parents before their death. The house that he lived in was his own ancestral house….

The priest of the temple wasn't surprised to learn about Karmanya's identity. This time he had dreamt that only a male member of the Royal family would bring fortune and good health to Devgarh again.

The project was completed in a few days. The waters of 'Kund' looked clear blue. The people vowed to adopt organic farming on their lands, and refrain from using chemicals.

One fine day, a young girl who was playing around the turquoise coloured 'Kund', saw a Gold Fish dashing through the waters. She called her friends and the priest too came rushing to watch this marvel, the Gold Fish. By the evening, everyone gathered around the 'Kund'. The long lost Gold Fish had reappeared!

However, this time there was not just one but two fishes. There was another Silver Fish, following the Gold Fish around the pellucid waters of the 'Kund', as people looked delightedly at the two symbols of prosperity and good health in the sacred "Kund'.

The priest looked and smiled.

Rukmani and Karmanya would live till eternity in their heaven, the 'Sacred Kund'.

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Review of "Cleansing the Sacred 'Kund'"

HEM RAJ BANSAL

"Cleansing the Sacred 'Kund'" stems from the inner desire to clean the sacred places. At a time when there are cleanliness drives such as the "National Mission for Clean Ganga" or "Namami Gange" and a romantic search for the mythical Saraswati, the story comes up well in time to resonate with water purification projects and especially sacred water sites. History and literature abound in such examples where human sacrifice would result in the magical appearance of water. Angry deities' pacification invited human sacrifice and the propitiation would result in divine favour or in avoiding divine retribution. In Japan, the legends talk about hitobashira ('human pillar') in which virgin girls were buried alive to safeguard the building against enemy attacks or natural calamities.

The story is a fictionalised version of the Rukmani Kund and traces the idea of human sacrifice. Folk literature of various places is abounding with legends of varied sorts and the story of Rukmani's sacrifice for making the otherwise drought hit area fertile is very moving. The story presents Rukmani herself proffering her body to quench the thirst of the entire valley of Devgarh, a fictionalized name for Barsand in Bilaspur. An unquestioning faith is shown in the priest whose mystical/divine vision makes it incumbent on Rukmani to become the victim. However, the societal/patriarchal expectations from a woman were so high that it would have been thought improper had the son or husband offered themselves for the sacrifice. Rukmani complies with the notions of gender performance through this act. Moreover, the idea of a loyal wife and a doting mother also drive home the feministic concerns of the story.

Though the human sacrifice in the story is personal, the concern remains largely philanthropic as it is for the community. The shedding of tears by the family as well as the grief-stricken people is very telling and touching. The sudden appearance of the Golden Fish in the sacred water of the kund after the sacrifice of Rukmani is viewed as a sign of prosperity and immortality is associated with the fish as it is seen as the incarnation of the queen Rukmani herself.

The author also takes a dig at the modern man's unquestioning faith. The scattering of flowers or other puja/worshipping material and the shifting away from organic farming to an inorganic one does not only cause water pollution but also in the bourgeoning of the diseases. The healing power of the water no longer offers any salvation to the diseased or the sufferer. Ironically though the valley becomes fertile, the people begin to fall ill and suffer from many health problems. The water loses its divine/cleansing/healing power due to the negative human intervention.

Karmanya as a young British educated doctor feels drawn towards his roots and chooses to serve his people than live his life lavishly. His act of leaving a job in London and serving his village and area folk becomes an act of communal sacrifice. As a third generation diasporan, it is quite touching that he had formed a picture of "imaginary homelands" through the stories that he had heard from his parents and grandparents. Interestingly, the pull of the homeland is very strong in him and hence makes him stick to the service of the people.

Chet Ram acts as the missing link between Karmanya and his ancestors. While Chet Ram's narrating the tale behind the formation of the Rukmani Kund fills Karmanya with awe, the disappearance of the "Golden Fish" equally disturbs him. Feeling the need to cure not the patients first but to breed out the root cause of the rising number of patients makes him restless. Cleaning the water, planting the trees, or working with the NGOs for the conservation of water becomes his top priority. As the Kund Purification project sets off, a snag in the machinery makes Karmanya dive into the water but never to return back.

As it unfolds further, Karmnya's sacrifice does not only lead to the cleansing of the sacred kund again but also the reappearance of the golden fish. The "Golden Fish" in the story symbolizes prosperity, fertility and happiness. This arrival of the golden fish is invariably linked with the purification of both water and mind. The author drives home the idea that unless due attention is given to the conservation of environment, humans will have to face the consequences.

Though from the perspective of tourism, sacred rivers or lakes become places of paramount significance, the possibility of cleaning the lesser-known water resources is also the need of the hour whether they have any mythological bearing or not. So, cleanliness drives are a must for every lake, river or pond irrespective of any sacred/mythological significance.

In the legend, it is Rukmani, a woman who sacrifices herself for the welfare of the people. However, in the story a male of the same family performs the sacrifice. In the legend, the king had sacrificed his daughter-in-law but, in the story, it is the queen herself, King's wife who does so. By doing so, the author subverts the gender binaries and sets the balance right. In the legend, the king's offering of his daughter-in-law appears to be a more sagacious choice for him than that of his son. However, the story balances that gap.

If we look at the story, when the machinery breaks down, it is doctor Karmanya who plunges into the water for repairing the snag and not engineer Karmanya. It seems unconvincing to have a doctor repair the machine; an engineer would have been a better choice because a doctor treats disease of the body and not the snag of the machines. Though rationally it seems incompatible with the skills of Karmanya, on the level of beliefs/devotion/spirituality, it sounds convincing as a selfless act of human sacrifice for the larger benefit of the public. The name symbolism in the story makes it more convincing as the name 'Karmanya' implies a person who is duty-bound. Similarly, the name Rukmani also signifies someone "who is adorned with gold" and it speaks for the appearance of the Golden Fish in the pond after Rukmani's sacrifice.

Abhyudita's story, thus, does not only revive an old legend but also sensitizes about the need to keep the water resources clean. The dumping of the puja material at the sacred water sites requires timely intervention. Disposal of ritual residues should be in an environment-friendly way and Dispose Well is such an organization which treats leftovers of the rituals in a way that respects the sentiments of devotees and also does not harm the environment.

Audience's Live Response

Namrata Pathania: A wonderful mixture of history and fiction…really interesting story Abhyudita Ma'am.

Anupama Singh: Though I had heard tit-bit of this legend, but today the whole story was narrated marvellously and the fiction nearing reality was amalgamated in a magnificent manner…and the way you presented was wonderful Abhyudita Ma'am.

Sangeeta Singh: The story highlights the pull towards our roots and legends. Beautifully written and narrated Abhyudita Ma'am.

Gitanjali Mahendra: I enjoyed the story, Abhyudita. The analysis was very good, Hemraj sir.

Rajan Kaushal: Beautiful story and wonderful analysis.

2

THE PARTY

ANIL WILSON

There was a sudden spurt of pain in her side. She stopped for a while to give respite to its insistence. Another one, she thought, and soon every inch of her body would be shrieking for attention. Well, she thought, at her age what else could she expect? And she went on with her work. It was Christmas Eve, and she had so much to do. Though her eyesight and hearing were failing her, her sense of smell was just about holding out. It told her that the cake was ready. Another Christmas cake, another year gone by. She stopped her painful movements to give room to the rush of memories: "Vy, I would have married you, if for nothing else, just for the wonderful cake you bake," her husband, Kelly, had said to her one Christmas Eve, perhaps three decades ago. One loses count of years when one is so old. Things become dim and hazy. Yet the remark still stood out so very clearly in her mind. She remembered it because he had looked so handsome in his new blue suit. Yes, the new blue suit -- she remembered it so well. The whole scene connected with that suit was etched in her mind.

She shook herself off those memories. There was no time for reminiscences with company expected soon to spend the Christmas Eve with her. After so many lonely Christmas evenings, today, once again, there would be laughter and gaiety in her house. She had to hurry. There still was so much to be done even though all the foodstuff was prepared. Everything had to be in place in time and she had no one to help her. Kelly used to be a great help right to his last day, especially during the Christmas week when he helped her with all the preparation. But on that bygone Christmas Eve she had not allowed him to work with her. "Your new suit will be ruined," she had told him and then blew out her anger on her work in the kitchen.

"Perhaps it was my mistake," she thought. "I had no business to be where I was not expected to be. Then I would not have seen him in that compromising position with her.... What the eye sees not, the heart does not grieve over." She smiled to herself: “Yes, she had forgiven him, but not till after....”

"O well, I better get the China out," she thought. It would take some effort as it was in the lowest shelf of the cupboard and she had great difficulty bending down.

"An interesting side-light of that dreadful episode," she always thought, "was the discovery that though she had been married for so many years, she had never imagined that love making was such a ludicrous affair to an onlooker." She moved about, dragging herself from one place to another, making sure that everything was in its place, making ever so slight adjustments in the bric-a-brac around the room. She arranged the chairs around the dining table.

"How many people would there be tonight," she asked herself. So many times, since morning, she had asked herself that same question and had always forgotten the answer she had given herself. Yes, there were the old Mr. and Mrs. Shaw. Of course, not so old as she herself was but just as lonely, or perhaps even more than her. The Shaws had each other to talk to, but only when they were in a state of truce which was not often.

She could go on chatting for hours with Kelly telling him about the aches and pains she felt and what the doctor said about them. She would tell Frank, her son, what a naughty little boy he had been when he was a child, intent upon breaking everything in the house. Even today when anything slipped through her stiff fingers and broke, she thought of poor Frankie to put the blame on. She never tired of telling him how smart he looked in the photograph taken in his air-force uniform, which had reached her a day after his plane had been shot down.

She turned around and shuffled into the bed-room. "Must take out the cotton table-sheet," she said to herself, "these new-fangled plastic ones were no doubt very functional but for a proper Christmas party it had to be the traditional white cotton." She opened a drawer and after carefully rummaging inside it, took out a well washed, well ironed white sheet. Satisfied that this was what she wanted, she made her way slowly back to the dining-cum-drawing room.

Yes, there were also Raghu and Suriya. She had known that it would not be easy for her to make them accept her invitation. They were still young; their daughter Indira, had just started college. What, with all the commercial establishments trying to cash in on the Christmas spirit with their candle-light dine-and dance affairs, why should they accept her invitation? To a simple party? But she made up her mind to make them spend this evening with her. They could certainly spare one

Christmas Eve for her-- they had so many more to come. As for herself, she imagined that Kelly and Frank were already making fun of the way she stuck on to her creaking bones. So, before extending the invitation to Raghu and Suriya, she took them back to the days when they had just got married -- a runaway marriage, an elopement. Those were the days of the all too often communal flare-ups. Both the major communities were looking for the runaways of each other's communities. A terrible time she had had, hiding Raghu and Suriya in her small, cramped store-room for a whole fortnight. After that they escaped to a hill-station for three months while tempers cooled off back home. Exactly nine months later Suriya had given birth to a daughter. Vy had fondly hoped that they would name the child after her because she was sure that the child was conceived in her store-room. But no, they named her after the prime minister, perhaps because she was a symbol of secularism. Now when she reminded them of those 'store-room-days', she found them to be visibly uncomfortable. Moments of youthful passion leave embarrassed memories. And so, when she invited them, they just could not bring themselves to refuse her. She giggled to herself. Yes, she could be crafty when she wanted to. But Indira would not come. She had come rushing into her apartment: "Granny, I can't come, I just can't come. O please, please do try to understand, it is so important to me, you know." She had been invited to a party where that famous singer -- what was his name, was performing. "It is a chance of a life-time, granny, I can't miss it." And Vy understood. There would be many more Christmas Evening for Raghu and Suriya, but for Indira, at her age, Christmas came just once a year. That is how it had been for her when she was young. She was pretty, yes. Some even said very pretty. There certainly was no shortage of admirers. But Kelly was the terribly jealous type. And she loved him too much to see him disturbed. She knew what jealousy could do. Many years later it was she who was consumed by jealousy. She had put a burning iron on the trousers of his blue suit because they reminded her of that terrible afternoon. No, that did not satisfy her. So, in her own way she decided to get even with him -- she went to bed with Lenny. It was not an earth-shattering experience and she had not cared to repeat it. In any case, it had calmed her down and made her forgive him. Many a time she had wondered: was that the only time Kelly had been unfaithful to her? There was no way of knowing. Strange, she thought, two people live with each other so closely, so intimately, and yet all they know about each other is what they choose to reveal to each other.

Yes, Lenny was coming too. He was a bachelor once again -- one dead and one divorced. Perhaps he was on the lookout for a third -- no lecher like an old lecher! Yet, he had that very likable quality: always jovial, always lively, always making things interesting, and so very helpful. He had been a very wonderful friend, always there when he was needed. His usual joke was that he had put all his savings in a wager; that Vy would outlive him and that he would wait for her in the hereafter to collect his winnings. At their age death was no longer what one had to go through. At least it was so for her. Not macabre or frightening. It was more of a matter-of-fact detail Frank and Kelly were waiting for her on the other side, and getting a trifle impatient.

She decided to sit down and rest for a while. Then began to wonder if she had put the beer in the fridge. Come summer, come winter, Raghu drank only beer and that too after it had been properly chilled. She stood up, waited for the pain to pause before she pushed ahead. It was such a nuisance, this forgetfulness. Took so much of her energy to do the things she had already done. Mr. Shaw would perhaps have some too, but after a lot of cajoling and after appealing to his wife to let the Christmas spirit make her indulgent. Lenny would have rum, to keep his toes warm, as he said. She had a bottle, saved up from God knows how many years. Kelly had not been very fond of it; it went right to his head. So, other than very special occasions, he generally avoided having any strong drinks. She walked back to her chair and very slowly lowered herself into it. Nothing to do now but wait. It was getting cold; she must get a hot-water bottle if she was to be comfortable. The room was sufficiently warm but her old fingers would refuse to move if they were not kept warm. She got up again. The new surge of pain wants to outdo all the others, she thought with a grimace. As she switched on the electric kettle, she decided to unbolt the door as very often she did not hear the doorbell. By the time she had done this, the pain became unbearable. She sat down. With old age even the pains often lost their sharp edge. This one, however, reminded her of her youth: of the time when her son Frank was born. The pain and the pleasure -- the physical pain almost unbearable; the emotional pleasure made it bearable. The final burst of pain -- just too much, shooting through her body, pulling at every muscle, every joint, straining at the very roots of her being -- and then as he was born, the sudden release, the immediate relaxation.

The electric kettle gave a shrill, piercing whistle -- inviting, demanding attention. But Lenny had lost his bet. She was suddenly free of all aches and pains. The wait of Frank and Kelly was over.

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Narrated by Kulbhushan Sharma

The Aggregate of Memories: A Reading of Prof. Anil Wilson's "The Party"

ANU JAIDEV

"The Party" by Anil Wilson is a short, short story. It is a paean to Papa Hemingway in terms of its masterful selective detailing of the ice-bergian kind: the simple elegance of the enunciated narrative rests on the gravity of the submerged mass.

At the first reading, one is struck by the self-assured felicity of the narrative. It is relaxed, open, confident, comfortable and disarming. At the structural and formal level, the story takes so much in its stride without faltering (a lifetime, to be precise); even as it presents the reader with the travails and troubles of its rather fragile and reluctant protagonist.

The text offers an interesting take on the narrative's engendering of the character with the character's engendering of the narrative. It is a woman's narrative, and hence always connected and connecting with others around her. The individuation of her perceptions is articulated as precisely that, it is not allowed to be the isolated individuation of her experiences. In that it comes as close to being a 'female text' as it can.

In the following critical reading of the story, I shall simultaneously utilize perspectives from critical literary gerontology, gender studies, and the discourse of the body and literary temporality. Gender studies evaluate constructions of masculinity and femininity in literary texts. Critical gerontology evaluates how age, aging and ageism forms part of literary narratives, and indeed how much of literature celebrates youth and ability and thus perpetuates a rather warped worldview.

Simone de Beauvoir in her book *Old Age* (1972) talks of breaking the 'conspiracy of silence' around what she calls this 'forbidden subject': the subject of aging and how it is almost never part of social discourse. Diana Wallace in her study "Literary Portrayals of Ageing" suggests that literary representations of older people are important in that both shape as well as have the potential to counter perceptions about age and ageing. The othering and indeed the erasing of the old could be countered through sensitive portrayals of old age and aging. The proverbial 'foreign country with an unknown language' (May Sarton) after all is the country which most eventually come to inhabit. It is important to imaginatively engage with the fact of our own ageing. It is also important to recognize the subjectivity of those who are already 'older' in order to understand the ways in which age and ageing are culturally constructed.

The discourse of the body is important in that the body is not necessarily intra or extra discursive, but in that it is quite literally the site of the exercise of power. Discourse is not merely a matter of reflection or contemplation, it is also em-bodied.

The narrative hinges on memories of Christmas Eve parties and cakes, past and present, converging on a single visual trigger of a memory: Kenny's blue suit and its attendant associations.

Vy is the reluctant protagonist and subject of the story and the narratorial voice is closely aligned with her. Vy, in the moment of the story, is old. She reminisces while she waits for friends to arrive for her Christmas Eve party. The story is an aggregate of her reminiscences, birthing through her in cold print, with her dying. Vy's name has an almost Beckettian play upon 'why' which ties in with her reluctance to deal with her husband's transgression as well as her son's passing. She'd rather not deal with either. The interior monologue of the character is captured here in terms of a series of seemingly random thoughts and memories. The aggregation of memories ranges from the ordinary and every-day to the life- altering and traumatic. The nature of the recall is what it is because it cannot be and is not, shared. It is simply a churning in the mind, not meant to be said aloud. It remains, by definition, un-avowable.

And yet, the imprint of what she has undergone is singed into the pathologies she experiences in her mind as well as in her body. The churning is never just a contemplation in the mind: it registers in the body as well. There is amnesia, and failing sight and hearing loss, and the difficulty in movement. There's also the stabbing, shooting, debilitating pain, which can immobilize her. And yet, there is one sense that is intact: the sense of smell. She can revel in the aroma of the cake she is baking as well as catch some wafting odours from her past. She can decide not to focus her hearing and even obfuscate her vision; but smells are ambient-- and inescapable.

Vy inhabits her past and present simultaneously as the narrative stretches back and forth in temporalities. From the twilight of her life, she looks back on triumphs and failures of her life with a degree of composure. There's a matter-of-factness about her, an understated quiet confidence, which does her credit. She's not the trailblazing feminist, nor the wizened old dame, but just an aging woman, managing her everyday existence.

Negotiating the aging self is not easy. But Vy demonstrates that while faculties diminish and fail, there's still the inexplicable life-force that wills people to live. The debilitating pain erupting in her side dictates her slightest movements. It is ironically, both a marker of life and death in the story. She needs to take a break ever so often. Ordinary tasks like taking out the china and finding that special cotton table-sheet are difficult for her, but she carries on.

Memories of earlier Christmas Eve parties inform the need for this one. Do bear in mind that this party comes after many, many lonely Christmases for her. The cake's aroma perfuming her house brings back memories from years long past. The past inhabits the present, those who have passed on have a lingering after-life, not only in terms of the forever after, but in terms of the memories that survive them. The memories evoked here are not all pleasant ones: in fact, the text makes it absolutely clear that there is no such thing as an unencumbered clean and pleasant memory.

Memories galore. Kelly, the handsome husband in his new blue suit is also the adulterous betrayer out of it. Vy, does not confront Kelly, (even though she walks in on him when he is with the woman she doesn't deign to name.) She slots the incident in the category of the "ludicrous".

"Perhaps it was my mistake," she thought. "I had no business to be where I was not expected to be. Then I would not have seen him in that compromising position with her.... What the eye sees not, the heart does not grieve over." She smiled to herself: "Yes, she had forgiven him, but not till after...."

The story turns the power/knowledge paradigm on its head. Knowledge is not seen as empowering here, she wishes to erase what she has witnessed. Or more accurately, remove herself from the scene of the incident. She gets mad; then she gets even. Eventually however, she is able to take the transgressions great and small, endured and committed, in her shuffling stride.

Memories of Frankie, her son, are similarly evoked. Frankie's special picture in his Air Force uniform that Vy has memoried, is connected with the news of his plane being shot down. And yet, that is not what she is prepared to focus on. For her it is as if Frankie is just around. In her mind still a boisterous boy, whose antics can be blamed for anything she drops inadvertently. Memories are enmeshed with the pleasant and unpleasant bits, and people don't get to pick and choose. No matter how hard the need to erase and forget.

It could be argued that her husband's transgression must be central to her personal crisis. To which one would say that the story does not read that way. One is not trying to downplay the significance of the transgression, merely to put it in perspective. Of course, she is not happy about it. The several whys bother Vy too much and not enough. In the midst of all these questions her nonchalance betrays a rather underwhelming realization.

She realizes that adultery for her has been incidental: and perhaps for Kelly too. Keeping faith and betraying trust usually involves the same individuals. None of it is pretty, but there are more serious griefs to deal with, like the passing away of an only child. Like the incapacitated, weak, aging body that can still outweigh every other emotional concern.

With age, however, comes at least the necessary deference and regard of others which can be quite useful. She can strategically bring up the fact of Raghu and Suriya's elopement to her advantage and their embarrassment. Power gleaned from knowledge is what can be exercised on others. The self is far too fragile, and any ruthless scrutiny renders it collateral damage: wisdom indeed leads a bitter life. Hence the yearning to revert to a state of innocence. "What the eye sees not, the heart does not grieve over."

And yet, the grief here is not inordinate. Significantly, the narrative pitches Vy's experiences not as catastrophes frozen in time but as moments in the great transition that is life. The narrative actually manages to cast a cold eye on life, on death, and moves on. It is, (to borrow what Virginia Woolf said of *Middlemarch*), a text for grown-ups.

It is interesting to observe how Vy negotiates her identity as the betrayed (and betraying) wife, the grieving mother, the sheltering elder, the honorary grand-mother and the long-term friend. There's no earth-shattering drama here, either in Vy's experiences or in her responses to them, and that, precisely, is the point. There is a quiet dignity in the lives of women -- and men, despite their failures and shortcomings. Not every life culminates into an amplified swan song, and that is all right. Vy is here. Rickety joints, aching limbs et al., but she is here. And that is enough.

The sundry narratives that occupy her mind coalesce together in the one moment in which she reminisces her untold, unshared story and lets it die with/in her with the final pang of pain. The moment of her death is ironically also the moment of the story's closure and birthing. The feminist triumph of the story, one would argue, lies precisely in its refusal to cast the moment of adultery as the defining experience for Vy. It's just one difficult bit in a long and full life.

George Eliot makes an observation in *Middlemarch*: "[In youth] each crisis seems final. Simply because it is new." A lifetime of crises and triumphs teaches Vy resilience, even in the face of the finality ushered in by an overwhelming bout of pain.

As she switched on the electric kettle, she decided to unbolt the door as very often she did not hear the doorbell. By the time she had done this, the pain became unbearable. She sat down. With old age even the pains often lost their sharp edge. This one, however, reminded her of her youth: of the time when her son Frank was born. The pain and the pleasure -- the physical pain almost unbearable; the emotional pleasure made it bearable. The final burst of pain -- just too much, shooting through her body, pulling at every muscle, every joint, straining at the very roots of her being -- and then as he was born, the sudden release, the immediate relaxation.

The electric kettle gave a shrill, piercing whistle -- inviting, demanding attention. But Lenny had lost his bet. She was suddenly free of all aches and pains. The wait of Frank and Kelly was over.

Vy drops out, the narrator picks the threads and closes and seals the warp and weft. The different country too ultimately is no country for old men -- or women on a permanent basis. Vy however, manages the Yeatsian ambition of being gathered in the artifice of eternity by becoming her own story.

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Audience's Live Response

Irene Rattan: Such a mesmerising story by Wilson sir. Excellent storyline.

Mahi Yogesh: A beautiful story by Sir Wilson capturing passions and emotions so ingeniously. Fine lovely details - store room days, people living together know about each other only what they choose to tell each other. I would want to read the story. Thank you for reading it out, Kulbhushan.

Savita J.B. Singh: Interesting story! Characters were well etched...

Kamayani Vashisht: Your analysis brings to the story and its understanding, so much more!

Namrata Tiku: Such a nuanced and layered analysis Anu!

3

WOMEN

Arun Bharati

"Chachi... O.. Chachi. i. i..." Kamli called out as she saw Dwarka walking on the narrow path in the distance. Dwarka stopped suddenly. Shading her eyes from the rays of early morning sun, she looked back and said:

"So you're up! Be quick, or we'll be returning in the scorching heat."

"Just wait for a moment," said Kamli and disappeared into her house. Dwarka put aside the sickle and the rope she had in her hand and sat down on a small boulder on the upper side of the track.

The sun had arisen. Its rays had spread all over the mountain peaks in the distance. Most of the women folk of the village leave their homes in groups at this time to collect green fodder and they return before the sun comes high up.

Dwarka woke up rather late today. Her brother-in-law's son Bishnu had come to visit her the previous night. He had come after a long time and both of them kept talking late into the night. By the time she woke up in the morning most of the women had already left for collecting green fodder. She hurriedly washed her face and milked the cows. Picking up her sickle and rope, she glanced towards Kamli's door and found it closed. Kamli must have already left, she thought and she closed her own door and stepped out.

A loner by nature, she was not very friendly with other women of her village. Her sister-in-law had already died and her old brother-in-law lived separately with his three children. But the children loved Dwarka as their mother. Bishnu was particularly fond of his Chachi.

Kamli too addressed Dwarka as her Chachi. Dwarka had no child of her own. She was married at the age of thirteen. Her husband was nearly fifty. By the time she could understand the meaning of marriage the husband died.

Three or four terraced fields, two cows, two small mud-plastered rooms and a few brass utensils — her life was largely spent in looking after these. Her brother-in-law and his wife lived separately with their two sons, Bishnu and Karma, and a daughter, Meera. Her sister-in-law died two years after Meera's birth and the kids started looking for a mother in Dwarka. Dwarka too found a purpose for living in these kids.

Last year Bishnu had gone to the town and found a job. Karma had taken charge of cultivating the fields. Meera got married this year. Whenever Meera visited her parental home, she spent most of her time with her Chachi. The conjugal bliss seemed to have completely transformed her. Youth seemed to spill out of her like a branch of *Kachnaar* laden with flowers, attracting and tempting all who saw her.

Dwarka never got time to think of what youth really was. Even though she was nearly thirty-five she felt at times that her body was aflame with desire, and her nerves tickling. Her body would feel heavy. She felt like riding a swing and singing songs of love. But she would check herself. It's a sin for a widow even to think of all this. How could a widow do what was proper for a happily married woman? She must not.... With great difficulty she would suppress the ache in her heart.

As she sat there, she started thinking, 'Is the age of thirteen an age to be married off. And after all what was the difference between my father and my husband? Only of the bodies and not of age.' She thought of her drunken husband and how he used to pound her body every night. Her tender body could hardly bear this. How his mouth smelt of the cheap country liquor--- oh! Good, he died.

"Let's go, Chachi," Kamli suddenly surprised Dwarka.

"Yes, yes, let' go. You took so long."

"Oh Chachi, just made myself a cup of tea, that took some time. Anyway, we still have enough time. I've heard Bishnu also came last night?"

"Yes, even I got late last evening because of him. He stayed for long narrating his experiences in the town. Oh, he has changed a great deal. No longer the old thin and lanky Bishnu. He has turned into a sturdy young man." Bishnu's copper-like face appeared before her eyes, and his trimmed thin brown moustache, his neatly done hair and his muscular body and the biceps of his arms.

"How long will he be here?"

"Ten-fifteen days. You can surely meet him."

"What has he brought for you, Chachi, from the city?"

"For me? Oh, he's crazy! You know what he has brought for me — a suit, bright red, bangles and a ribbon, that too red. And he said, "Chachi, now you won't lack anything. I have started earning now. I was moved to tears and I smiled. With great effort I made him understand that this colour only a *suhagin* can wear, one whose husband is alive. I asked him to give those things to Meera."

By this time they had reached the pasture and started cutting grass for fodder.

Kamli was Dwarka's daughter-in-law in distant relations. Her young husband had died last year in a truck accident. Kamli became a widow in the prime of her youth. She remained in a state of shock for some time. There was no one in the family. No brother-in-law or sister-in-law to take care of her. She had no child either in the five years of her married life. Suddenly her life had become totally barren.

Bishnu was five years younger than Kamli's husband. When she got married Bishnu was thirteen or fourteen. Clad in his button-less shirt and striped underpants, barefoot, he used to stare at her constantly. One day Kamli's husband said, as he saw Bishnu staring thus, "Say, how do you like her?" Without saying a word Bishnu ran away, holding his lose underpants, and sniffing his running nose. Kamli laughed heartily.

"Why? What makes you laugh like this?"

"Nothing, Chachi. I just remembered an incident about Bishnu."

"About Bishnu? What's it?"

"When my husband was alive Bishnu used to come to our house and stare at me constantly as if I were some strange creature. One day he asked Bishnu, "So, do you like her?" And Bishnu just ran away. Since then till the day my husband met with the accident Bishnu has never come to our house. It's almost a year since I last saw him."

"He has grown up into a handsome youth. Tall, stout and muscular, just like your husband." Kamli felt a pang in her heart. She threw her sickle aside and, burying her head in her knees, she started sobbing.

"Aye, are you gone mad? Just look at me. I have been living in widowhood for the past twenty years. This is all the will of God. As He wishes! Come, don't be mad, let's go. The sun is getting hot. Let's hurry up."

Kamli wiped her eyes and heaved a deep sigh. Suddenly she felt empty within, and lifeless. Listlessly she got up and started cutting grass.

"Sarasti...! O.... Sarasti!", Dwarka was tired calling out to her. "I wonder where the girl has vanished," Dwarka mumbled to herself.

Suddenly she saw Bishnu with a pitcher of water on his shoulders coming from the natural spring of water. His body was shining like copper in the morning sun. He was wearing vest and underpants. And she was surprised to see Sarasti follow him with a pitcher on her head as well. Bishnu stopped and whispered something to Sarasti as she came closer. Dwarka couldn't hear anything but she started observing them carefully.

All of a sudden Bishnu placed his pitcher on the boundary of a field. Then he lifted Sarasti's pitcher also to place it on the boundary. Like a happy doe Sarasti romped into the fields of maize. Bishnu also ran after her. Some commotion for a while, then quietly both of them disappeared.

Dwarka was standing at a height so she could see it all. She rubbed her eyes, then she saw — the pitchers were there on the boundary.

"So, that is why this girl is not to be seen these days! She started musing. 'Her husband was right, poor fellow. Her conduct is not proper. But she got spoilt after marriage only. They say her husband was impotent. That's why with her brother-in-law... what wild passion! Yes, the flame of passion will indeed consume!.... But no! In her own family, oh God! Brother-in-law is like brother... but... but..."

She was stunned. She felt as if she was being engulfed by invisible flames from all sides. Her body was aflame, nay scorched. Her legs were trembling, her fists clenched, her palms sweating. Bishnu... Sarasti... fire.... Twenty years of widowhood suddenly vanished. The blood running in her veins suddenly became that of a sixteen-year one. Hot like a furnace — boiling. Steam... fog... sparks. Her eyes closed. Her breath sounded like bellows. As if she had stopped suddenly in the middle of a steep climb.

"*Are'* Chachi, what's the matter? What are you muttering right in the morning?"

She was startled by Kamli's voice.

"That... there... Bishnu... Sarasti? No, but you..." She didn't know what to say. In confusion she stared blankly at Kamli.

"What's wrong with you Chachi? Your face is so red... eyes red... are you all right?"

Kamli inspected her from top to toe but she couldn't understand anything. Dwarka felt as if her eyes were piercing her.

"No, nothing. I was just looking for Sarasti." Last evening, she had said that she would have her morning tea with me. God knows where she has disappeared so early in the morning. There is still no sign of her."

"But, Bishnu and Sarasti? You were muttering something about Bishnu and Sarasti?

"Oh, yes, Bishnu also takes his morning tea with me," Dwarka flicked the side of her shirt as if an ant were creeping on her. Suddenly she saw Sarasti heading towards her.

"So you have come? I have been waiting for you for quite some time."

"You know Chachi, I had to go to get water first."

"Has Bishnu left?"

The pitcher on Sarasti's head shook a little. Some water spilled out and drenched her face.

"Yes... he went away a long time back."

Kamli looked suspiciously at Sarasti. Dwarka felt guilty for having asked about Bishnu. Sarasti was staring vacantly at the desolate path.

"Chachi, may I ask you something?"

"What?"

"How did you manage to live so many years without a husband?"

"What's wrong with you?"

"No, please tell me."

"You seem to have gone mad."

"Chachi, tell me, did you never miss him?"

"What would I miss? I was thirteen when I got married and I was hardly fourteen when my husband died."

"Then?"

"Then what? I became a widow. A widow, for whom it is a sin to talk to a man even in her dreams. But you...."

Dwarka looked with surprise at Kamli whose eyes were fixed somewhere far away. Her face was colourless. There was a sound at the door. Dwarka was distracted.

"Chachi, O Chachi!"

"Who's there? O, Bishnu!"

"Yes, Chachi."

Bishnu came in. He was carrying a big bag.

"Let me have a cup of tea, Chachi. Oh! Kamli Bhabhi!" Bishnu was embarrassed to find Kamli around. Kamli's eyes almost got glued to Bishnu's muscular body. She desired to run her fingers on his biceps. His broad chest, hairy...

Kamli's eyes were scanning his body. Bishnu nervously looked away. He dare not look straight into her eyes.

"When are you leaving?" Dwarka asked, blowing into her *Chulha*.

"Right now. Just dropped in to have a cup of tea with you."

"So when do you come next? Do come on Holi this time."

"All right."

There was silence in the room. The crackling of the logs in the *chulha*, Kamli's eyes, Dwarka's blowing into the fire, and Bishnu's aimlessly looking out continued till the tea was ready.

"Take care of yourself in *pardes*," instructed Dwarka as she offered him a cup of tea. Kamli too longed to say something to Bishnu by way of advice but failed to utter a word.

Bishnu quickly gulped his tea.

"All right Chachi, take care. Do tell Karma if you need anything. He'll write to me." Dwarka's eyes were filled with tears. Bishnu bent down and touched her feet, Dwarka blessed him caressing his head. Looking towards Kamli, Bishnu folded his hands.

Kamli and Dwarka kept on looking at Bishnu as long as they could see him moving away on the narrow path amidst the tall pine trees. Sarasti too kept looking at him from her courtyard.

As Bishnu slowly disappeared in the distance the three women felt an aching void possessing them.

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Translated from Hindi by Pankaj K. Singh

Narrated by Leena vaidya

Critical Analysis of the Story "Women"

SUNAINA JAIN

> When men are oppressed, it is a tragedy, when women are oppressed, it is a tradition."
>
> -- *Bernadette Mosala*

The status of widows, especially in rural India, has been miserable as they form the most vulnerable group. Despite efforts done in the direction of widow remarriage and rehabilitation, the ground reality of widows being pushed to the margins of the society and deprived of chances of living a normal life, has not changed much. The story "Women" traverses the lives of three women – Dwarka, Kamli and Sarasti and underpins women's oppression due to circumscribed gender roles assigned to them. Two of these women are young widows and the story captures their travails. In subtle ways, it brings out the repressed sexual desires of these women due to societal impositions.

The story, told in third-person narration, portrays characters that live in the present but their past informs and shapes their lives. The story begins with a dialogue between two women – Dwarka and Kamli and the readers are subsequently apprised of their quotidian lives. The rural setting of the story gives a glimpse of the daily struggles of women as they eke out their living by working in their small farms and carry out mundane household chores.

Dwarka is a thirty-five-year-old childless woman who became a widow at the age of fourteen. Her husband was a middle-aged man who died just a year after marriage. His death caused Dwarka a great deal of troubles as she had to survive all alone in the world but at the same time, this tragedy was also a blessing in disguise as it was a loveless and incompatible marriage in which her husband considered her no more than a commodity to be used to satisfy his sexual urges. The story highlights exploitation of women in forced child marriages from which there is no escape route for them. Such child widows passed their entire lives in oblivion as they were considered pariahs by the society and their widowhood was seen as curse and their dark shadow could taint others' lives too. Dwarka, in the story, becomes a loner. She, being childless and widow, remains aloof and lives in her cocoon. Her only respite from everyday drudgery is the affection of her brother-in-law's children, especially Bishnu and Meera, who look for motherly tenderness and love in Dwarka and Dwarka, in turn, showers her unconditional love on them. However, it is strange that a robust youth like Bishnu (who works in the city) remains unaware of the prevailing norms of the society and brings red suit, bangles and ribbon, all of which are insignia of married women. Dwarka, like other young women, wants to enjoy the pleasures of youth, and "her body was aflame with desire, and her nerves tickling. Her body would feel heavy. She felt like riding a swing and singing songs of love. But she would check herself. It's a sin for a widow even to think of all this." Despite the fact, Dwarka is not able to subvert codified gender roles imposed on her; she questions the absurdity of her matchless marriage with a man of her father's age and in fact, feels relieved at the death of his drunkard and ruthless husband.

The story portrays another widow Kamli, Dwarka's distant daughter-in-law, who became a widow in the prime of her youth, as her husband was killed in a truck accident. The readers get to know through Kamli's dialogue that Bishnu is four or five years younger than her dead husband and as a young boy, he often showed childish impetuousness by staring at Kamli in awe. As he has grown into a sturdy young man, Kamli gets attracted to his muscular body but being a widow, her longing for love and sexual gratification cannot be made manifest as defying traditional gender norms has no impunity for women.

The story introduces another woman Sarasti but unlike Dwarka and Kamli, she does not believe in suppressing her youthful urges and breaks the code of conduct by being romantically involved with her unmarried brother-in-law. She has another reason to violate the code as her husband is impotent and cannot satisfy her bodily desires. However, because of society's hypocritical standards, questioning of man's virility is a sin but labelling a woman for seeking love outside marriage as characterless or branding her as barren, is justified. The storywriter shows Bishnu and Sarasti indulging in love game as Dwarka becomes a mute spectator – aghast, shocked and flabbergasted. "Her body was aflame, nay scorched. Her legs were trembling, her fists clenched, her palms sweating. Bishnu... Sarasti... fire...." But this shock is then metamorphosed into her own burning desire for love which she has to keep under check as Kamli enters the scene and she stands transfixed, but does not betray her thoughts to Kamli.

The story is a sensitive portrayal of the repressed desires of women for love and sexual gratification, but whereas Dwarka and Kamli buttress the same inside the fort of the societal restrictions, Sarasti's desires are fueled by Bishnu's reciprocation. The two pitchers resting on the boundary of a field have been used ingeniously by the storywriter to convey the sexual thirst of young lovers and also the crossing of boundaries as they make a transgression to satiate their desire for love and physical intimacy. The story gathers weight and gravitas through oblique thematic hints thrown into it and like Ernest Hemingway's Iceberg technique, deeper issues concerning rights of women are highlighted. The title of the story is generic but it serves the purpose as the women characters in the story are not just individuals but also representatives of women in general who face subjugation at various levels.

Audiences' Live Response

Rajan Kaushal: Wonderful story and superbly narrated.

Kulbhushan Sharma: Quite evocative and beautiful story Pankaj Ma'am. I am reminded of Arun Bharti ji.

Gitanjali Mahendra: Leena you narrated the story so well. Ma'am Pankaj is a brilliant translator.

Kamayani Vashisht: That the story has commendable merit is evident in the fact that Ma'am Pankaj chose it for translation...

Anu Jaidev: Pankaj ma'am, beautifully nuanced story. It completely reverses the gaze and how! A very fine and seamless translation. Respect, always!

4

मूक प्रेम

दीपा चौहान

'मम्मा! मम्मा! देखो पुराने घर में रहने के लिये कौन आया है?'

मेरी सात साल की बेटी हाँफते हुए घर में आई और मेरी अंगुली पकड़कर मुझे पुराने घर की ओर ले जाने लगी।

'अरे बाबा! छोड़ ना बेटा। क्या है? क्यों शोर मचा रही हो?'

'मम्मा, मेरे स्कूल में पढ़ने वाला विक्रांत अपने परिवार के साथ पुराने घर में रहने के लिये आ रहा है।' बाल सुलभ जिज्ञासा और उत्साह से भरी स्वीटी मुझे पुराने घर की ओर ले जाने लगी।

'कौन विक्रांत... अच्छा!'

मुझे याद आया ये शान्ता बहन का परिवार है। ससुर जी ने शान्ता बहन को कुछ समय के लिये पुराने घर में रहने की इजाजत दी थी। पिताजी बहुत ही दानी प्रवृति के पुरुष थे। अपनी पुश्तैनी ज़मीन से बहुत सारी ज़मीन धर्मार्थ एवं सेवार्थ कार्यों जैसे शिवालय, धर्मशाला, चिकित्सालय एवं पाठशाला के लिये दान कर चुके थे। कोई भी व्यक्ति कुछ मांगने आता तो उनका यही प्रयास रहता कि उसकी हर संभव मदद करें। वो अक्सर कहते थे 'देने से कभी कमी नहीं आती।' अब चूँकि उनके चारों बेटे अपने नये एवं अलग घरों मे रहते थे तो पुश्तैनी मकान खाली रहता था। शान्ता बहन के पिता कभी ससुरजी के पास काम करते थे और क्योंकि वो आदमी बहुत नेक और इमानदार थे तो पिताजी भी उन्हे महत्व देते थे और अक्सर ही उनकी आर्थिक तौर पर मदद किया करते थे। उन्हीं की बेटी शान्ता बहन अपने परिवार के साथ आज यहाँ रहने के लिये आयी थी। वो पिताजी को चाचा-चाचा कह कर कुछ समय के लिये पुराने घर में रहने की इजाजत मांगने लगी। उसका कहना था कि जैसे ही रहने की दूसरी व्यवस्था हो जायेगी, वो घर खाली कर देंगे। अब क्योंकि घर बहुत बड़ा था और पिताजी का दिल भी, तो घर का एक हिस्सा बिना किसी किराये के उन्हे रहने के लिये दे दिया गया।

सामने देखा तो गाड़ी से सामान उतारा जा रहा था। बलवन्त भाई मेरी तरफ आये और कुशल-मंगल पूछने लगे। ' बहन जी, आपके बाल बच्चे ठीक हैं? भाई साहब कैसे हैं?'

'सब ठीक हैं भाई। शान्ता बहन दिख नहीं रही।'

'लो ये आ गयी।'

'अरे भाभी नमस्ते। कैसे हो तुम लोग?', शान्ता बहन बोली।

'हम ठीक हैं। और तुम लोग?'

'बस क्या बतायें। अब आपसे तो कुछ छुपा नहीं है। एक तो बड़ा परिवार ऊपर से कमाई का साधन नहीं, इसीलिए यहाँ आये हैं कि कुछ काम काज शुरु करें और बेटे को भी स्कूल बस से आने में परेशानी होती थी, तो वो समस्या भी हल हो गई।'

'तुम्हारे पास कितने बच्चे हैं?'

'एक ही बेटा है।' शान्ता बहन ने गर्व से बताया।

तभी मैने देखा नौ साल की सहमी सी एक लड़की चार-पांच साल के लड़के को ले कर आ रही है। लेकिन लड़का है कि उसकी बात मानना तो दूर उसे गाली देकर उस पर चिल्ला रहा है।

'क्यों री सोनी इसको क्यूं रूला रही है?' शान्ता ने गुस्से में कहा।

'मम्मी ये मिट्टी में कपड़े गंदे कर रहा था, मैंने मना किया तो जिद कर रहा है।'

'कपडे गन्दे हो गये तो क्या, धुल जायेंगे। अब तू जल्दी चल आज बहुत काम है।' शान्ता ने गुस्से से कहा।

'ये लड़की कौन है?' मैंने पूछा।

'ये? सोनी, इनकी पहली पत्नी की बेटी।' शान्ता ने मुँह बना कर जवाब दिया।

'नमस्ते।' लड़की ने हाथ जोड़ कर बड़ी तमीज़ से कहा।

'नमस्ते बेटा।'

'अच्छा, भाभी अब घर भी व्यवस्थित करना है। अब तो मिलते रहेंगे।'

शान्ता बहन बलवन्त सिंह की दूसरी पत्नी थी। उनकी पहली पत्नी बच्ची के जन्म के दो साल बाद गुजर गई थी। पत्नी गुजरी तो बच्ची की परवरिश का सवाल खड़ा हो गया और क्योंकि प्रश्न पुत्र-लालसा का भी था। तो कुल मिला कर दोनों समस्याओं का एक ही हल निकला, पुनर्विवाह! इस तरह शान्ता बहन और बलवन्त सिंह का विवाह हुआ। स्वभाव से

तेजतर्रार शान्ता का व्यव्हार बच्ची के साथ प्रारंभ से ही अच्छा नहीं था। बच्ची भी सौतेली माँ के सामने डरी सहमी रहती। बलवन्त भाई बहुत सीधे और पत्नी से दबकर रहने वाले पुरुष थे। पत्नी के सामने कुछ बोल जायें या उसकी बात को पलट दें, ऐसी हिम्मत वो अपने सात साल के वैवाहिक जीवन मे कभी नहीं जुटा पाये थे।

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शान्ता बहन और उसके परिवार को हमारे घर में रहते एक महिने से ज्यादा वक़्त हो चुका था और अब ये बात किसी से छिपी नहीं थी कि वो सोनी के साथ दुर्व्यवहार करती थी। लेकिन इस सब में हैरानी की बात ये थी कि बलवन्त सिंह भी सोनी को बुरी तरह मारते थे। शान्ता बहन बड़ी चालाकी से बलवन्त भाई से सोनी की शिकायतें करती और उसे मार पड़वाती।

जाड़े की शाम थी, सुबह से बारिश हो रही थी। आज और दिन की अपेक्षा ठंड बहुत अधिक थी। रुम हीटर से ही कमरे में आराम मिल रहा था। रजाई से निकलने का मन नहीं हो रहा था। तभी मामी-मामी की आवाज सुनाई दी। दरवाजे पर जा कर देखा तो सामने सोनी थी। उसने सूती फ्रॉक और आधी बांह का पतला स्वेटर पहना था। पैरों में हवाई चप्पल थी।

'अरे बेटा तुम्हें ठंड लग जायेगी। अंदर आ जाओ।'

'मामी, पापा और मम्मी जी कहीं गये हुए हैं। उन्होंने कहा था कि अगर चार बजे तक वो घर नहीं पहुँचे तो मैं आप के पास चली जाऊँ। वो आ जायेंगे तो घर चली जाऊंगी।'

'अच्छा! तुम्हारी मम्मी जाते वक़्त मिली थी, मुझे लेकिन मुझे तो कुछ नहीं बताया उन्होंने। हो सकता है भूल गई हों। कोई बात नहीं बेटा, तुम अन्दर आ जाओ।'

'जी मामी!'

'स्वीटी अन्दर वाले कमरे मे अपने टॉयज़ से खेल रही है। तुम भी उसके साथ खेल लो।'

सोनी को देखकर स्वीटी बहुत खुश हुई और उसे अपने टॉयज़ दिखाने लगी। मैं किचिन में स्नैक्स बनाने लग गई और उनकी बातचीत भी सुनने लगी।

'अरे दीदी, आओ मैं आपको अपनी डॉलस दिखाती हूँ। ये मेरी फ्रेंड्स हैं। ये पिंकी है, ये मिनी, ये एलिस और ये सबसे प्यारी सिन्ड्रेला है। इन सबका नाम दुकान वाले ने बार्बी रखा था। देखो ना सबका एक ही नाम।'

'अरे वाह बहुत सुंदर नाम रखे हैं तुमने।' सोनी ने गुड़ियों को देखकर कहा।

स्वीटी बड़े गर्व से अपने खिलौनों का खजाना दिखा रही थी। इतनी बड़ी-बड़ी और प्यारी गुड़ियों को देखकर सोनी के चहरे पर मुस्कान बिखर गयी। वो जिस प्यार से गुड़ियों को सहला रही थी उस से ऐसा लग रहा था जैसे वो वैसी गुड़ियों को पाने का सपना रखती हो।

आखिर दुनिया में कौन सी ऐसी बच्ची होगी जिसका मन गुड़िया को देख कर उसे पाने के लिये लालायित नहीं होगा, और जिसने तो खिलौनों का अभाव देखा हो उसकी प्रसन्नता का तो कोई पारावार ही नहीं रहता।

'अरे वाह स्वीटी, तुम्हारी गुड़िया तो बहुत प्यारी हैं!'

'देखो इनके साथ कॉम भी मिली थी और ये शूज़। ये बाजू आगे पीछे भी करती है। रात को ये सोती भी हैं, आंखें बंद करके।'

'अरे वाह सिन्ड्रेला तो सचमुच सबसे प्यारी है। जैसे परियों की दुनिया से आई हो।'

'ये बहुत दूर से लाये हैं पापा। दिल्ली से। गुड़ वाले गांव से। वहाँ बड़े-बड़े मॉल होते हैं। डी एफ एल मॉल है ना, वहाँ से लाये हैं सिन्ड्रेला को। मुझे तो टी वी वाली सिन्ड्रेला भी बहुत अच्छी लगती है। इसलिये इसका नाम सिन्ड्रेला रखा है मैंने।'

'सिन्ड्रेला के बाल तो बिल्कुल अंग्रेजों जैसे हैं।'

'हाँ तो, सिन्ड्रेला भी तो अंग्रेज थी, वो लंदन में रहती थी।'

'अच्छा!'

दोनो बच्चियाँ बातचीत में पूरी तरह मशगूल थी। सोनी को ऐसे खेलता देख मुझे अच्छा लग रहा था क्यूंकि अधिकांशत मैंने उसे घर का काम करते ही देखा था। जिस उम्र मे लड़कियाँ गुड़ियों से खेलती हैं उस उम्र मे उत्तरदायित्वों के अतिरेक से उसके कोमल हाथ बरतन मान्जने, झाड़ू लगाने और कपड़े धोने से अपनी कोमलता खो बैठे थे। जो उम्र परियों और राजकुमारियों की कहानियों से भरी होती हैं उसमे सौतेली माँ के होने से ऐसी किसी भी मनमोहक वस्तु या स्वप्निल जहाँ के लिये कोई स्थान नहीं रह गया था। ऐसा लग रहा था जैसे वो पहली बार किसी गुड़िया को अपने हाथों मे लेकर इतने पास से देख रही थी। मुझसे रहा नहीं गया मैने पूछ ही लिया,' बेटा तुम्हारे पास भी कोई गुड़िया है?'

'नहीं मामी', बच्ची ने संकोच से कहा, 'हमारे घर पर कार, बन्दूक वाले खिलौने ही आते हैं।'

'तुम्हे गुड़िया पसंद है?'

'जी!', बच्ची ने झिझकते हुए कहा।

'मम्मा आपने मेरे स्नैक्स बना दिये? मुझे भूख लगी है। 'स्वीटी बीच में बोल पड़ी।

'अभी लाती हूँ।'

मैं दो प्लेटस में स्प्रिंग रोलस ले आयी।

'सोनी दीदी मेरी मम्मा बहुत अच्छे स्नैक्स बनाती हैं। मम्मा से अच्छे स्नैक्स कोई नहीं बना सकता।'

'लो सोनी बेटा ये तुम्हारे लिये।' मैने प्लेट सोनी की तरफ बढ़ा दी।

'नहीं, मामी मुझे भूख नहीं है।'

'अरे ये ज्यादा नहीं है बेटा।'

'दीदी आप भी खाओ, बहुत टेस्टी हैं।' स्वीटी ने कहा।

'मामी नहीं मुझे नहीं खाने।' सोनी इन्कार मे सर हिलाती रही।

'क्या हुआ? खा लो बेटा थोड़ा सा।' मैंने ज़ोर देकर कहा।

'नहीं मामी।'

'मम्मी डांटती है क्या?' मैने पूछा मगर सोनी चुप रही।

'अच्छा तो घर ले जाकर खा लेना भाई के साथ। ऐसे ठीक है?'

'जी।' बच्ची अभी भी कुछ संशय में थी।

छह बज चुके थे। तभी शान्ता बहन की आवाज सुनाई दी। वो सोनी को बुला रही थी।

'अरे मम्मी जी आ गई। ओह! खेल-खेल मे भूल ही गई थी। मैं बहुत लेट हो गई।' मासूम सी बच्ची के चेहरे पर डर के भाव साफ दिख रहे थे।

जैसे ही सोनी घर पहुंची शान्ता बहन की उसको लगाई जाने वाली डांट सुनाई दे रही थी। वो उस पर बहुत गुस्सा कर रही थी जिसे सुन कर मुझे मन ही मन उस पर बहुत तरस आया। बेचारी बिन माँ की मासूम बच्ची।

रात के साढे नौ बज रहे थे। मैं स्वीटी को सुला रही थीं। सर्दियों की रात में आवाजें और साफ सुनायी देती हैं। शान्ता बहन फिर किसी बात पर सोनी पर नाराज हो रही थी। 'मम्मा! ' स्वीटी की आवाज से ध्यान टूटा।

'मम्मा, सोनी दीदी की मम्मी ने उन्हें कल भी बहुत मारा था।'

' क्यूँ?'

'उनसे कांच का गिलास टूट गया था। वो बहुत रो रही थी।'

'ओह! बेचारी।

'ममा एक दिन पहले भी जब मैं उनके घर डॉल लेकर खेलने गयी थी तो उनकी मम्मा ने उन्हे बहुत डांटा था, फिर उनके पापा ने उनको कमरे मे बंद करके बहुत मारा था। मैं तो डर कर घर भाग आई थी।'

सुनकर मुझे बहुत गुस्सा आया। मुझे तो मन से उन दोनो पति-पत्नि से घृणा होने लगी थी। दिल कर रहा था कि पिताजी से कहकर इनसे घर खाली करवा दूं। ऐसे बुरे लोगों का घर में रहना किसे अच्छा लगेगा। रोज रोज का शोर और कलह और बच्ची का रूदन सहन की सीमा से पार हो रहा था।

'मम्मा, सोतैली माँ क्या होती है? सोनी दीदी की मम्मी सौतेली है क्या?'

'हम्म...'

''उनकी ममा कहां चली गई?'

'बहुत दूर।'

'पर क्यों?'

'पता नहीं।' मुझे सूझ नहीं रहा था कि इन बातों का क्या जवाब दूं?

'मम्मा आप मुझे छोडकर कभी मत जाना। मुझे सोतैली माँ से बहुत डर लगता है।'

ये कहते हुए स्वीटी मुझसे चिपक गई। उसकी आवाज में मुझे खोने का डर था।

'नहीं बेटा, मैं तुम्हे छोडकर कभी नहीं जाऊँगी।' मेरी बात से स्वीटी को कुछ आराम मिला ओर वो आश्वस्त हो कर सो गई।

शान्ता बहन फिर किसी कारण से सोनी को बुरा भला कह रही थी। सौतेली माँ कितनी निर्दयी, निष्ठुर और संवेदनहीन होती हैं। कैकयी जैसी स्त्री भी स्वयं परमेश्वर को अपने सौतेले पुत्र के रूप में पाकर स्नेह ना दे सकी, ना उनसे न्याय कर सकी और सौतेलेपन की पराकाष्ठा का पर्याय बन अमर हो गई। माता और विमाता का भेद कोई बदल नहीं पाया फिर चाहे वो स्वयं विधाता ही क्यूँ ना हों। प्रभु को भी मानव देह मे आकर दुमाता से प्रेम ना मिल पाया।

एक स्त्री, माँ होने पर जो ममतामयी हृदय अपनी संतान के लिये रखती है, अपना आराम और सुकून भूल कर अपना सर्वस्व उस पर सहर्ष लुटा देती है। वही प्रेम,वात्सल्य,त्याग और बलिदान की प्रतिमूर्त स्त्री, विमाता होने पर परायी संतान के लिये पाषाणहृदय हो जाती है। पराये जाये को दर्द मे देखकर उसका दिल नहीं पसीजता, उसे पीड़ा में देख उसका कलेजा नहीं फटता। शायद इसिलिए माँ निश्छल प्यार और अनुराग की ऐसी निर्झर नदी होती है जिसके स्नेह की अविरल धारा अपनी संतति के लिये कभी सूखती नहीं। ये प्रकृति का नियम है कि मनुष्य तो मनुष्य ब्रह्मांड के समस्त प्राणियों मे माँ ही अपनी संतान से अप्रतिम ममता

रखती है; टिटिरि की तरह वो ये समझती है कि वो अपने बच्चे पर आने वाले हर संकट, हर मुश्किल को दूर करने मे सक्षम है, फिर चाहे वो मुश्किलों का आसमान ही क्यों ना हो। पिता पर प्रकृति ने ऐसी कोई बाध्यता नहीं रखी है। वो प्राणदाता अवश्य होता हैं किंतु जन्म के बाद के उत्तरदायित्वों के लिये वह स्वतंत्र है। भावनात्मक बन्धन के सन्दर्भ में वो माँ की तरह बेबस और जकड़ हुआ नहीं होता।

'माँ है तो पिता भी है, माँ नहीं तो पिता, पिता नहीं रहता।' हाँ यही तो कहती थी मेरी स्कूल की सहेली रश्मि। विमाता के आने पर पिता भी बदल जाते हैं और वो उसी की आँखो देखने लगते हैं उसी के कानों सुनने लगते हैं, और तो और, उसी के दिमाग से सोचने भी लगते हैं। माँ के जाने से जीवन ही बदल जाता है। वो बताती थी कि कैसे सयुंक्त परिवार मे रसोई में ताई और चाचियां खाना बनाती ओर परोसती। परिवार के बाकी बच्चे जब मन करता हक़ से जाकर अपनी माओं से खाना मांग लेते, पसंद की सब्जी ना होने पर माँ से रूठ जाते और फिर कैसे उनकी माँ उनको मनाती और उन्हे दूध-मक्खन के साथ खाना देती थीं। वो सोचती थी अगर माँ जिन्दा होती तो उसे भी ज़िद्द करने और रूठने का हक़ होता। अगर कोई मनाने वाला ही ना हो तो कोई रुठे किस से? वो और उसका भाई सहमे हुए से खाने के लिये बुलाये जाने का इन्तजार करते और कितनी ही बार ऐसा होता की तब तक उसके भाई को नीन्द आ जाती। कैसे उसका छोटा भाई शाम सात बजे से कहता 'दीदी भूख लगी है, चाची से कह दो ना कि आज खाना जल्दी दे दे।' कैसे वो ताई-चाचियों को खुश करने के लिये कभी बरतन मांज देती कभी रसोई मे झाडू लगा देती। बदले मे सब कहते, 'देखो रश्मि कितनी समझदार है, कोई जिद नहीं करती, बिन कहे काम भी करने लगती है।' सब तारीफ के दो शब्द तो कह देते पर प्यार का एक निवाला कोई ना देता था। उसने बताया था कैसे एक बार उसका भाई उस से आकर कहता है, 'दीदी आज मोनू लड्डु खा रहा था। मुझे भी लाकर दो ना।' जब रश्मि ने कहा कि उसे मांगते डर लगता है तो फिर उसके भाई ने रोते हुए कहा था, 'दीदी चलो ना माँ के पास चलते हैं।' ये कहते-कहते वो कितना रोई थी उस दिन। आखिर एक दिन उसके पापा ने दूसरी शादी कर ली। पढ़ाई में अच्छी होने के बावजूद आठवीं क्लास मे उसका स्कूल ये कह कर छुड़वा दिया गया कि ज्यादा पढ़ने से लड़कियाँ बिगड़ जाती हैं। कुछ साल बाद उसकी शादी हो गई और फिर धीरे धीरे रश्मि मेरे ज़ेहन की तहों मे जाने कहां खो गयी। लेकिन आज इस तरह की जाने कितनी ही बातें मुझे याद आने लगी थी। विस्मृति की धूल जो अतीत की यादों पर पड़ी थी सोनी के आने से वो धूल उड़ने लगी थी। सालों के अन्तराल के बाद आज फिर से मुझे रश्मि नज़र आयी, सोनी में। इन्ही ख्यालों मे जाने कब नीन्द आ गई।

अगली सुबह मैने संजय से सोनी के साथ हो रहे बुरे बर्ताव का जिक्र किया। उन्होने कहा, 'ये उनका निजी मामला है। अगर हमने रहने की जगह दी है तो इसका ये मतलब नहीं कि उनकी पर्सनल लाइफ मे इंटरफेयर करें। वैसे भी बलवन्त भाई मुझे समझदार आदमी लगते हैं।'

'जो पुरूष अपनी पत्नी के कहने पर अपनी पहली पत्नी की नौ साल की मासूम सी बेटी को मारे वो क्या समझदार हुआ? सच कहा है किसी ने माँ के जाने से बाप भी पराया हो जाता है। बच्चों के प्रति पिता की निष्ठा और प्यार पत्नी के कारण होता है।'

'ऐसा नहीं है कि पिता को अपने बच्चों से प्यार नहीं होता।बस उसका प्यार माँ की तरह करुणामयी ओर ममतामयी नहीं होता। मेरा मानना है कि भगवान ने माता पिता दोनों को एक अलग और खास उदेश्य की पूर्ति के लिये बनाया है। दोनों एक दूसरे से बिल्कुल भिन्न हैं, इसलिये दोनों का प्रेम भी भिन्न है।। जहाँ माँ की गोद मे बैठकर बच्चा भावनात्मक संबल पाता है वहीं पिता का होना एक ऐसी छत्र-छाया है जिसके होने से बच्चे को अभयदान मिलता है। माता और पिता के प्यार को अगर एक ही कसौटी पर मापोगे तो पिता का मूक प्रेम कभी नज़र नहीं आयेगा। अच्छा एक बात बताओ क्या तुम्हारे पिता तुम्हे प्यार नहीं करते? क्या मैं स्वीटी को प्यार नहीं करता?' संजय ने बड़े शान्त स्वर मे कहा।

'करते हो।' कैसे कहती लेकिन मेरी माँ अभी जीवित है और मैं भी।

'छोडो इन बातों को सबकी लाईफ मे अपनी समस्याएं होती हैं । जाओ अच्छी सी चाय बना कर ले आओ साथ में पियेंगे। फिर मुझे कोर्ट जाना है।'

'लेकिन बलवन्त भाई....'

'लगता है बलवत भाई और दूसरी शादी करने के गुनहगार सभी पतियों की तरफ से जज साहिबा के सामने आज मेरी पेशी लगी है।'

हम दोनो खिलखिलाकर हंस पड़े।

रविवार का दिन था। आज पुराने घर की सफाई करवानी थी। सुबह के दस बजे थे। संजय ने बताया कि मज़दूर आने ही वाले हैं, इसलिये मैं जाकर रूम्स खोल आऊं। नीचे वाली मंजिल के ताले खोलकर मैं ऊपर वाले फ्लोर पर जाने लगी तो सोनी के रोने की आवाजें सुनाई दी। शान्ता बहन बाहर मिली मैने सोनी के रोने का कारण पूछा, तो उसने बताया की सोनी ने उसके नये सूट का कुर्ता प्रैस से जला दिया है जो उसे कल शादी मे पहनना था। उसके पापा को पता चला तो वो बहुत नाराज हो रहे हैं।'

'मगर तुम उन्हे रोकती क्यूँ नहीं, वो तो उसे बुरी तरह मार रहे हैं?' मैनें कुछ नाराज होकर कहा।

'ना बाबा ना ।उन्हे रोको तो वो बहुत नाराज होते हैं।अब उनकी बेटी हैं, मैं ठहरी सौतेली माँ। मेरा मुँह तो बंद ही रहता है।'

उसकी धूर्तता पर मुझे बहुत गुस्सा आया। इस बंद मुँह से निकलने वाले अपशब्द हम रोज़ ही सुनते थे।

मैने ठान लिया आज जो हो जाये मैं अपने घर मे एक मासूम बच्ची पर ये अत्यचार नहीं होने दूंगी।

मैं तेजी से उस कमरे की ओर बढ़ने लगी जहाँ से आवाजें आ रही थी।' कमरा अंदर से बंद है, पीछे से', शान्ता बहन की आवाज आई। मैं अब तेजी से कॉरिडोर की तरफ से होते हुए बाहर वाले रुम में गई क्योंकि उस कमरे का एक दरवाजा उसी कमरे में खुलता था जहाँ अभी बलवन्त सिंह सोनी को मार रहे थे और वो दरवाजा हम बंद रखते थे।' क्या हुआ भाभी?' शान्ता बहन पीछे से फिर बोली। मैने दरवाजे की सिटकनी खोल जैसे ही दरवाजा खोला अन्दर का दृश्य देख कर मैं आवाक रह गई। बलवन्त भाई ज़ोर-ज़ोर से अपनी हथेलियाँ ताली की तरह पीट रहा था और सोनी ज़ोर ज़ोर से रोने का शोर कर रही थी। साथ ही बच्ची को हंसी भी आ रही थी। मुझे कुछ समझ नहीं आया तभी दोनो की नज़र मुझ पर पड़ी। बलवन्त भाई ने हथेलियाँ पीटना बंद किया ओर सोनी ने रोना चिल्लाना। अचानक मुझे सामने देख बलवन्त भाई किंकर्तव्यविमूढ़ से बुरी तरह झेंप गये थे। उनकी दशा रणभूमि मे धराशायी कर्ण पर प्रेमश्रु बहाती कुन्ती जैसी थी जिसके सामने मानो पाण्डव आ खड़े हो गये हों, जैसे सत्य को छुपाने के दोष के लिये आज फिर युधिष्ठिर अपने शाप के साथ आ गये हों, ऐसा शाप जो न केवल उनके लिये हो वरन समस्त पुरूष प्रजाति के लिये हो या फिर समस्त पिताओं के लिये। बरसों से छुपाये उस सत्य को आज बाहर आने का द्वार जो मिल गया था।

तभी शान्ता बहन भी आ गई। बाप-बेटी के चहरे पर चिंता और संशय की रेखाओं के बादल और स्याह हो गये। जिस स्वांग को उन्होने बड़ी कुशलता से रचा तथा उतनी ही दक्षता के साथ चरित्र के मुखौटे पहन सबको भ्रमित किया था, उनकी वास्तविकता छुपाये वो मुखौटे आज उतर चुके थे और उन छद्म रूप के पीछे के असली चरित्र देख मैं निशब्द और चकित रह गई।

'बलवन्त भाई बस भी करो।' मैने ज़ोर से कहा।' छोटी बच्ची है, गलती हो जाती है। अब इतना भी मारता है क्या कोई?' मैने गुस्सा दिखाते हुए कहा। 'सोनी बेटा इधर आओ।' डरी सहमी सोनी मेरे पास आ गई। मैंने हाथों से उसके सर को सहलाया और पुचकार कर चुप कराने लगी।' शान्ता बहन तुम भी कुछ समझाओ बलवन्त भाई को।'

मुझे नहीं पता मैं क्या कह रही थी? क्या कर रही थी?

तभी स्वीटी आ गई, 'मम्मा आपको पापा बुला रहे हैं।'

'चलो!', ये कहते हुए मैं स्वीटी को लेकर सीढीयां उतर गई।

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समीक्षा:'मूक प्रेम'

देवेन्द्रा गुप्ता

"मूक प्रेम" सामाजिक यथार्थ पर रची गई एक मार्मिक कहानी है। युगों से लेकर आज तक हमारे समाज में असंख्य परिवारों के बच्चे सौतेलेपन का दंश झेलते आ रहे हैं। इस कहानी में परंपरागत सौतेलेपन की गंभीर समस्या को लेखिका ने अपने ढंग से यथार्थ के धरातल पर उतारा है। अपने आज के परिवेश में एक ऐसे ही परिवार को केंद्र में रखकर इस कहानी का ताना-बाना बुना गया है। भले ही यह विषय साधारण प्रतीत होता है, लेकिन यदि वह अपने अंतस को स्पर्श करने में सक्षम हो, तो उसको सार्थकता स्वत: सिद्ध हो जाती है।

अपने अनुभव के आधार पर कहानी रचियता ने निश्चित तौर पर एक नवीन दृष्टिकोण रखा है। कहानी का आरंभ एक सामान्य संवाद के साथ पात्रों का परिचय भी कर देता है। यह संवाद कहानी की सूत्रधारा मम्मी और नन्ही बिटिया स्वीटी का है। जैसे ही घटनाक्रम आगे बढ़ता है तो दूसरे परिवार का आगमन होता है जो इस कहानी के केंद्र में है, जिसमें कुल चार सदस्य हैं। बलवंत सिंह पिता, शांता बहन मां, सोनी व विक्रांत उनके बच्चे। मासूम सोनी सौतेलेपन के सताए हुए उन असंख्य बच्चों की तरह आहत है जिसका बचपन माँ के ना होने से समय से पहले ही नियती के हाथों छिन गया है। परिणामवश जिसकी स्थिति अत्यंत दयनीय है। कहानी के विशिष्ट पात्र हैं स्वीटी के दादा, जो अति विनम्र, दानशील, धर्मात्मा व दयालु प्रकृति के व्यक्ति हैं और सबके मददगार भी हैं। उनके पुराने घर में ही बलवंत सिंह का परिवार रहता है। संवाद और पात्र विषय वस्तु को जो जीवंतता प्रदान करने वाले तत्व हैं जो जिज्ञासा और उत्साह बनाए रखते हैं। "मूक प्रेम" कहानी में लेखिका ने पात्रों में सहज संवाद के अनेक अवसर तलाशे हैं जो कहानी को अन्तरंगता और निरंतरता की ओर बढ़ाते हैं।

कथावस्तु को आकार और विस्तार देने के लिए यथासंभव प्रसंगों की योजना लेखिका की कल्पनाशक्ति और तथ्यात्मक प्रवृत्ति को दर्शाती है। प्रसंगवश रामायण से विमाता के रूप में कैकयी का उदाहरण हो या महाभारत में से सत्य को छुपाने के लिए कर्ण और कुंती का चित्रण हो अथवा कल्पना के धरातल पर अतीत के झरोखों से सोनी के समकक्ष रश्मि की चलचित्र की भाँति स्मृतियां, सभी कहानी को संगठित कर सुदृढ़ आधार की ओर ले जाती हैं।

सूत्रधार के रूप में मम्मी कहानी में अध्ययन विद्यमान रहती है वह हर विषय पर अपनी प्रतिक्रिया देती है। उसका गुस्सा और झुंझलाहट उसे बेचैन करते रहते हैं। बलवंत और शांता के प्रति उनकी कटुता और सोनी के प्रति सहानुभूति पूरी कहानी में दिखाई देती है वह मां और सौतेली मां में अंतर देखती है कि जो मां अपनी संतान पर सर्वस्व लुटा देती है। वह जब सौतेली बनती है तो कैसे इतनी कठोर निष्ठुर और कुटिल हो जाती है -- ये प्रश्न बार-बार उसे

कचोटते रहते हैं। वह क्यों अत्याचार और दुर्व्यवहार की प्रतिमूर्ति बन जाती है? वह पिता के रूप को भिन्न-भिन्न दृष्टि से देखती है।

लेकिन जब उसका पति संजय पिता के प्रेम के पक्ष में तर्क देता है। तो कुछ हद तक सहमत हो जाती है। सोनी का पिता बलवंत पत्नी के सामने विवश व लाचार दिखता है। सोनी उसकी अपनी पुत्री है फिर भी उसका पक्ष नहीं ले पाता, कलह से बचने के लिए वह सोनी की बाल सुलभ इच्छाएं भी पूरी नहीं कर पाता। उसका बचपन छिन गया है। अन्य बच्चों की तरह उसे भी खेलने की लालसा है लेकिन उसकी इच्छा अपने घर में कोई मायने नहीं रखती। वह हर समय मां की मार फटकार व दुर्व्यवहार से डरी सहमी रहती है।

उधर बलवंत पुत्री को पत्नी के जुल्मों से बचाने के लिए एक अनोखी तरकीब ढूंढ लेता है। उसकी पत्नी आए दिन उससे सोनी की शिकायतें करती है वह उसे पिटवा ने के लिए पति को उकसाती है। बलवंत भी सोनी को पत्नी से अलग कमरे में ले जाकर पीटने का ढोंग करता है। सोनी भी चिल्लाने का स्वांग करती है।

दोनों दबी-दबी हंसी में अपना नाटक जारी रखते हैं। उधर शांता के मन में बड़ा संतोष होता है। पत्नी को भ्रम में डालकर पिता अपनी पुत्री पर मूक प्रेम की वर्षा ना जाने कितने वर्षों से करता आ रहा था। उसने बेटी का सानिध्य और स्नेह पाने का यह नया रास्ता ढूंढ लिया था, जिसका रहस्य केवल पिता पुत्री ही जानते हैं। हर रोज़ का यह रोना-चिल्लाना-पीटना सुन-सुनकर स्वीटी की मम्मी व्यथित हो जाती थी। इसका रहस्योद्घाटन फिर एक दिन ऐसी घटना के साथ होता है जब वह सोनी को पीटते हुए पाकर अचानक मुड़कर उनका दरवाजा खोल देती है। वह दृश्य देखकर आवाक सी खड़ी हो जाती है, क्योंकि घटना नें नाटकीय मोड़ जो ले लिया था। सोनी और उसके पिता अपनी चोरी पकड़े जाने पर हतप्रभ से खड़े थे। इतने में शांता बहन का प्रवेश सूत्रधार को भी तुरंत उस पिता पुत्री के नाटक में शामिल होने का संकेत देता है। यही भ्रम, संशय और रहस्य इस कहानी का चरमबिंदु है जो कदाचित इसे सामान्य विषय से विशिष्टता की ओर ले जाता है।

कहानी में पात्रों के अनुरूप भाषा का व्यवहार और वैचारिक पलों में भाषा की गरिमा का निर्वाह किया गया है। इसकी शैली में लेखिका के व्यक्तित्व व मौलिकता की झलक दिखाई देती है। भाषा में हिंदी के अतिरिक्त अंग्रेजी व उर्दू की शब्दावली का समावेश शब्दों, संवादों व वैचारिक परिवेश के अनुकूल किया गया है। 'मूक प्रेम' कहानी का शीर्षक प्रतिकार प्रतीकात्मक है। कहानी का विषय सामान्य होते हुए भी एक नवीन दृष्टिकोण प्रस्तुत करता है किंतु कुछ स्थानों पर ऐसा प्रतीत होता है कि बलवंत शायद अपराध बोध से ग्रसित है उसके पश्चाताप की परिणति पुत्री के मूक प्रेम में होती दिखायी देती है। पत्नी के सामने उसकी विवशता और पुत्री से विमुख रहना भी एक अन्य नाटक है जिसे वह मुखौटे बदलकर रोज़ खेलता है। कभी यह भी लगता है कि शांता को सोनी के प्रति ज़ालिम और क्रूर बनाने के लिए

बलवंत भी कुछ हद तक उत्तरदायी है। परिस्थितियों से समझौता करने व पत्नी के भय के कारण मासूम सोनी की भलाई व अधिकारों की बलि देने में उसका भी सहयोग रहा है।

समग्रत कहा जा सकता है कि विषय-वस्तु, पात्र संरचना, भाषा शैली तथा कथा प्रवाह की दृष्टि से यह कहानी सफल रही है। उद्देश्य पूर्ति, नाट्य विधान व नवीन दृष्टि का उद्घाटन कहानी को एक प्रथम स्थान पर प्रतिष्ठित करने में निश्चय ही एक सराहनीय कदम है।

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Audiences' Live Response

Abhyudita: What a story! And great narration.

Rajan Kaushal: Great Deepa ma'am...for a while we felt as if the story was being performed in front of us. Especially the dialogues were superb.

Anupama Pathak: Nice story Deepa.

Irene Rattan: Well-chosen words, Deepa.

Girija Sharma: A wonderful slice of life story, narrated very well indeed!

5

TWELVE YEARS LATER

DIPALI SHARMA BHANDARI

Just as Manjari picked the books to go to her class, Kamla came with a message from the office. Apparently, there was a call for her. Manjari walked the corridors, wondering… 'Who even calls on landline in this era of video calling…?'

'Hello', a voice she couldn't place said.

'Hello', she tentatively replied, trying to connect a face to the voice.

'Manjari here…'

'Anil…', there was a pause, 'How are you? I can't believe I finally located you!'

Manjari looked around and forced a smile. She spoke loud and clear into the phone, 'I am good. When did you come back?' Everyone went back to their files or phones.

'I came home this week. Wanted to talk but had no idea how to contact you.'

'OK' Manjari said, mindful of the inquisitive ears around her.

'I tried calling your old number…'

'We had it disconnected two years ago. Landlines have become totally obsolete…' she faked a smile, '... Wait; let me take your number. I have a class right now. Will call you later'. She fished a pen out of her bag and scribbled his number on the cover of her register. Thanking the Superintendent, she lied, 'Old friend… just back from Canada…. Planning a class reunion….' and ran for her class.

She breezed through 'The Road Not Taken', reminiscing of a time long before she could seemingly remember… when was the last time she had seen Anil? What had he said! Why had he called? And how did he get her number! Had twelve years passed already? She collected her books and went to the canteen. She asked Vicky for a cup of tea and slowly dragged herself to her favourite corner. She stared uneasily at the number scribbled on the register. She had said she would call. She picked her phone and dialed the number, pausing between each digit. Then she cleared the screen and went out into the garden. 'If I don't call him, he might call again on the office number'. She didn't want that. She dialed his number again. Anil picked the phone on the first ring.

'Hello…'

'Manju…' Anil said.

'Call me Manjari', she snubbed him.

'Sorry Manjari', Anil began, 'You don't know how long I've been trying to contact you. I asked everyone…. Not one gave me your number… you even disconnected the landline…. I searched you on facebook too… no luck. Then I went to your old home… your landlady told me your parents had shifted to Rampur and you were working in a government school in Himachal. You don't know how many schools I called and how many Manjaris I talked to….

'Why?'

'Please forgive me Manjari.' Anil broke.... 'I want to meet you….'

'I don't want to meet you, Anil. Please don't call me again…. ever…' and Manjari disconnected the call.

The phone pinged. Manjari went purple when she saw a message from Anil. Two days and twenty unseen messages later she called Anuj.

'Hi Manju! *Ki haal...?* Long time…'

'Yeah. Busy days… Exams ahead…'

'Hmm…. My elder one is in eleventh too…'

'You don't need to worry about Arshi…. She takes after Sarita, not you.'

'Ha Ha!', Anuj laughed.

Manju laughed too. Then she said quietly, 'Anuj, Anil had called. He is back.'

Anuj stopped laughing. 'What! When?'

'Two days ago.'

'How did he get your number?'

Manju filled him in on the details…

'How dare he? After all he has done!' said Anuj. 'Do you want me to talk some sense into him…'

'No Anuj. I just needed to talk to someone about it. He can't do any worse than what he already has.'

'Let me talk to Sarita if we can manage a weekend getaway. We'll try to come on Friday.'

'Thanks Anuj.'

Manjari disconnected the call and busied herself in her terrace garden. The bougainvilleas were in full bloom. She buried her fingers into the warm potting soil and started preparing the colourful terracotta pots for her succulents. She thought of the garden in her old home…roses, lilies, jasmines, the yellow *champak* and marigold. She played with her dolls in the garden with her friends. Once a *bulbul* had nested in the bush…. three nestlings… then overnight the nest was crawling with ants. She and her friends wrapped the half-eaten featherless young *bulbuls* in leaves and buried them under the rosebush…She had cried for days…. Anuj had laughed when she told him … he even told everyone else in class!

They were young then. Classmates. Best friends. Anil had transferred two years later. And then there were those awkward years when boys remained boys while girls grew into young ladies. One day Manjari was called to the staff room. Mrs. Vyas had a letter in her hand and started scolding her. Manjari was standing red-faced. Later Anil confessed that he had slipped the letter in her notebook during prayer. Manjari was upset but she did not tell anyone. Finally, she confided in Anuj. Anuj and the class held an intervention and warned Anil. This was the year of the board exams. Anil tried to talk to her many times but Manjari made sure she was never alone. Mrs. Vyas had called her parents and told them about the love-letter. After board exams, her parents shifted Manjari to a Girls' college. Anil stalked her and Manjari constantly ignored her. Anil finally joined an engineering college and left the city. Before leaving, he went to Manjari's home and declared his love for her. Manjari's father was so angry that he slapped him. Anil went away. The next week Manjari received a letter in the mail. Anil had threatened that she will not be able to show her face in the society once he was done with her.

Manjari had spent the next three years in fear. Gradually her fear gave way and she started laughing again. Two more years passed. She was returning home one evening when Anil approached her 'Manju, I love you. Please talk to me'

'I don't. Please let me go.' She said, her heart pounding. His threat at the back of her mind…

'Manju… I am going to Canada next week. I want to talk to your parents about us'

'No! I don't love you and I don't want to talk to you. There is no "us". You made life miserable for me all these years. What makes you think I love you?' She mustered all her courage and shouted.

'Manju…'

'Don't call me Manju!' She pushed him and ran home. Anil ran after her ….

……………………

Manju washed her hands. She poured a glass of water for herself and switched on the fan just as the phone rang. It was Anil. She picked the call and answered calmly, 'I don't want to talk to you or meet you. Do not call me again. I have registered a complaint against you. If you call me again you will be sent back to the prison where you belong.'

Manjari wiped the sweat off her face with her *dupatta*. A tide of pain came swirling around her and she winced. She felt a thousand ants crawling all over her. Anil had done what he had said. Even twelve years later people are not comfortable watching her leathery face…

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"Twelve Years Later": A Review

VIVEK NEGI

The story "Twelve Years Later" by the writer Dipali Bhandari is directed at the evil of eve teasing and stalking, the school going girls face, but cannot disclose it to their families. The fear of being misunderstood by their families, norms of honour and character assassination by the society forces the girls to keep this abuse to themselves so that the bigger objective of education does not come to a halt. It is Manjari who receives a scolding rather than Anil by the teacher on the letter landing in her hand. The family of Manjari does support her but a conditional one of making her continue with the studies at a girls' college. Anil keeps on stalking and dares to visit Manjari s house with a marriage proposal. He is slapped by her father. There follows a respite in Anil's activities, but his return leaves Manjari scarred.

Manjari reconstructs her life twig by twig showing her resilience and mettle inside. Anil is sentenced to twelve years imprisonment. Manjari completes her education amongst piercing stares of insensitive society every minute, every day, every glance and carves a corner for herself as a teacher where she finds her space. She rediscovers the beauty of life in hard bougainvillea and cactii.

The story is such that every teenager relates to, in the aspect of surge of love feelings one bathes in during adolescence. The initial plot construction gives an impression of a love affair entanglement of Anil and Manjari and the reader unaware of the beastly action of acid throwing on Manjari that Anil has done; does find a soft heart for Anil -- the lover who the reader thinks is a rejected lover due to the blows of destiny. The gradual revelation with the development of story immediately makes the reader despise Anil -- the criminal. One starts comprehending the connection between the three ants-gnawed innocent bulbuls in the nest in the garden and the innocent acid-attack victim Manjari.

The return of Anil after twelve years brings back the pestering but finds a strong defiant composed Manjari this time. Though Anil's mindset remains undeciphered as to whether his return is that of an obsessive lover or a repentant Anil yet the reader finds Manjari's decisiveness justifiable, as it is only she who has undergone the immense suffering -- mental, psychological, physical with the society beside as a mere spectator throughout her ordeal.

The story is striking indeed.

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6

TERROR

GURMIT BEDI

The two boarded the bus at Paraur, a place between Malan and Palampur. They were both turbaned Sikhs, robust and tall. Both were wrapped in blankets. They cast their eyes all around as they boarded the bus and sat down on the seat right behind the driver. There were about twenty people in the bus. Two or three women and the rest men. None of them Sikhs. Only these two were Sardars.

The interior of the bus was dimly lit but it was pitch dark outside. The passengers sat bent over with the cold. One of the Sardars turned his neck to look back, as if to inspect the passengers. Then he put his hand inside his blanket and began to move it back and forth. At this, the plump lady sitting behind him trembled with fear. She looked at her neighbours with terrified eyes. The horror on her face froze their blood.

'It was in the papers today that extremists have begun to infiltrate Himachal,' a passenger at the back whispered in another's ear, his voice quivering. The person sitting behind him saw him whisper. He prodded him with his finger and leaned his ear close to the man's lips. By the time the bus reached Arla, every traveller but the driver, conductor and the passenger sitting to the left of the Sardars, had come to know that two extremists were travelling in the bus. At any time now they would push the driver aside and take control of the steering wheel, or stop the bus someplace and kill all of them.

The alert eyes of the passengers were fixed on every movement of the Sardars. They saw one of them begin to cover his face with a white handkerchief, even as his eyes ranged this way and that. He covered his nose and mouth and knotted the kerchief. Only his two eyes were visible now.

'He is hiding his identity,' one old man surmised.

‘Now he will order the driver to stop,’ a well-built passenger warned another with his eyes.

Oblivious to all this, the driver was busy driving and the conductor, sitting on the front seat, had his eyes trained on the road ahead. Both were unaware of the potential danger. The person sitting next to the Sardars lolled to one side in his sleep. Then the Sardars began to speak with each other. Their voices were drowned by the noise of the bus. The masked Sardar nodded his head in agreement with the other. The bus was now passing through a stretch bordered by thick trees on either side.

‘They seem to be ready,’ a woman murmured clutching her child to her chest.

Just then the other Sardar bent down and began to remove something from his bag.

‘I’m sure there is an AK-47 in it. Extremists carry their AK-47s in bags slung on their shoulders, don’t they, and conceal them with their blankets. There’s no need to load them again and again. You just have to pull the trigger and turn the barrel around. He will also do this now,’ said a young man who had read about the workings of an AK-47 in the newspapers. He choked with the fear of impending death.

‘Why are they bent on killing us? How have we wronged them? Sardars were made to fight against injustice. What kind of Sardars are they who are preparing to kill innocent people,’ a woman passenger was about to break into sobs.

‘There’s not one man in this bus, who can face them boldly,’ another woman passenger glanced timorously at all the men. They were pale with fear; as if their blood had frozen in their veins.

Suddenly, a crackling sound was heard and the two Sardars drew close to each other. Then some chewing sounds were heard. One of them nudged the other and both exchanged smiles. Still smiling, they turned around and looked carefully at the passengers. The passengers could only see their eyes. Four intense imposing eyes. Then they both straightened their necks and seemed ready to get up from their seats.

Everyone, except the driver and conductor, was petrified.

‘Please stop here, brother. We have to get down here,’ one Sardar said politely. The driver applied the brakes instantly.

‘May God bless you,’ the second Sardar turned the door handle.

'When will you be available in the canteen?' the driver asked with some hesitation.

'Come whenever you want. We're always at your service.'

Laughing, the two Sardars stepped off the bus.

Taken aback, the passengers exchanged glances; each one trying to hide their embarrassment.

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Translation: Meenakshi F. Paul

"Terror": A Critical Review

KULBHUSHAN SHARMA

Aatank, a gripping story written by Gurmeet Singh Bedi, is translated into English as "Terror" by Meenakshi F. Paul. Translation, in itself, is a challenging task that demands expertise on linguistic skills, knowledge of culture, technicality, and skills to overcome the problems — lexical-semantic, grammatical, syntactical, rhetorical, pragmatic, and cultural issues. It is an onus on the translator to salvage the idioms and expressions used in a particular culturally-structured language representing the social milieu. The biggest challenge before a translator is to pick up a particular story for the translation.

The story under scrutinising lens provides a basis for 'Perspectivism'. According to German philosopher, Friedrich Nietzsche, Perspectivism, a philosophical view, is that all ideations take place from particular perspectives which means there is a strand of possible conceptual schemes, or perspectives on which judgment of truth and value can be made. It is a general tendency on the part of most people to pass a judgment or frame opinions about others too soon. The frailty of the human beings to turn into sharp critics and point out flaws or shortcomings in others, make them blind to their own faults, insecurities and mistakes. Instead of reacting, the need is to relate, to associate, to introspect and then reflect. In the modern world, where everyone is surrounded by machines and hi-tech gadgets, there is an urgent need to understand the three words − sympathy, empathy and apathy quite deeply. The world seems to move under the swooning impact of 'self syndrome' and the best option for carving a beautiful world of co-existence is a possible reality if humanity sheds the last word and imbibes the former two essential words by understanding the fellow human beings.

The story is well-crafted and written in a satirical tone. On a close reading of the story, it has all the basic elements of a plot. Right from exposition to denouement, it keeps the reader under a spell and arouses the curiosity to know what is to come. The plot and structure are like gravity. "Terror" has a well-knit structure, with the series of events assisting the overall design of the story. The setting is confined to a bus where the passengers react and form the opinions about the two Sikhs. Two turbaned Sikhs, tall and robust, wrapped in blankets, board the bus. The people seated in the bus get suspicious about them and start forming opinions, branding them as militants or extremists. The alert eyes of the twenty-odd people in the bus are fixed on every movement of the Sardars. The passengers think that they are about to kill them. Every passenger on the bus seems to choke with the fear of impending death. The fear lurking in the corner of their hearts make them think that why the Sikhs are bent on killing them. But the twist of the story occurs when the one of the two Sikhs pleads the driver to stop the bus because they want to get down. Towards the end of the story, the friendly exchange of words in between the driver and the two Sardars leaves every passenger taken aback. The realization dawns upon the passengers that the two Sardars, indeed, are good human beings ready to render a helping hand. Exchanging glances, the passengers try to hide their embarrassment. The language used is full of local colloquism and the translator has retained the certain nuances of local idiom. The dialogues of the story have been maintained as written in *Punjabi* – "Rab Tuhada bhalaa kare." As far as the character and characterization is concerned, the characters in the story are the people the reader associates quite easily. The translated version of the story has preserved the features of the characters portrayed as real. There is an apt portrayal of the physical description, action, inner thoughts, reactions and speech that makes the story compelling and gives the readers a strong sense of characters' personalities and complexities, making them vivid, alive and believable. The thematic core of the story underlines the basic human nature to form the opinions and pass judgment about others which leaves a scope for everyone to reflect and think deeply about the self. The writer wants to transmit the message through the story that the basic need of human beings to be loved and to be understood.

Finally, the story can be read conveying a message that people are not to be judged solely on the basis of their appearance, attire, race, nationality, religion, family or sexual orientation. Rather, they must be accepted the way they are.

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Audience's Live Response

Sangeeta Singh: How wonderfully the story weaves various perceptions of human psychology, which are all hinged to one incident – social conditioning …reminded me of Manto's short stories. Very powerful!

Jyoti Mishra: It's remarkable to realize that the co-passengers actually lived through the 'aatank' in their minds without it being there… perception matters.

Anupama Pathak: That's how human psychology works…amazingly narrated.

Namrata Pathania: How pre-conceived notions terrorise us!

Leena Vaidya: It talks about our pre-conceived notions and our prejudices.

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7

THE TEAR

Irene

His hand went right through the sudden tear at the elbow of the sleeve. There was a sound of a slight hiss as worn cloth gave way and his heart seemed to jump into his eyes -- how will he go out in torn clothes? Must he walk on a busy road in a worn-out shirt? What if he met his teacher and he saw it? What if he met one of the snooty girls of his class and she saw it?

"Haw! Don't know how some people can move around brazenly in broad daylight in tattered clothes! Simply standing first in maths doesn't mean someone is king of the jungle!"

The tear seemed not in his shirt so much as in his heart. He silently watched his mother wrap and re-wrap her flowing blue 'dupatta' around her face so he could see only her eyes flutter like two black birds in a rolling blue sky. It had now been almost seven months that his small world seemed to have heaved endlessly under his small feet so he could barely stand. Every morning his father's defeated shoulders spoke of a restive night and a worried dawn. No word was spoken but every one knew that Baba again wasn't going to work that day. Mummy would quietly make 'chapattis' on the griddle and serve with small helpings of vegetables or lentils and Baba would hang his head and eat in silence, before putting up his feet on the bed and going off into a trance, from which no one dared to wake him.

The room had now become crowded. None of them moved out of the well-ordered but cramped surroundings of the four walls because it wasn't allowed. It was difficult to live in the tiny room through the day or to move around while all of them occupied the same space. Toes stubbed against metal trunks and bedposts and many times the little boy had to roll over the bed to avoid stepping on his silent but affectionate father's feet. That occasional tumble made Ishu laugh, but never his Baba.

"Sleeping all together never made such a crowd but sitting around does," Ishu thought whenever he lay down on the one double bed that dominated the tiny quarters. On one side stood a couple of trunks which held the beckoning mysteries of his mother's treasures and on the other was a small plastic stool meant exclusively for Baba's steel mug of water and steaming cup of tea. On the trunks were piled up the rolls of a couple of 'razais' and blankets, covered with a clean bedsheet. Beyond that was the kitchen shelf which held an amazing variety of utensils, neatly stacked beside a rusting gas stove. Buckets of water occupied the pride of place all over the room, but drops of it were immediately wiped up by his cleanliness-conscious mother. Ishu lay around the whole day, his pent-up energy bursting out in sporadic explosions of childish tantrums and continuous rants of permission to play on the single mobile phone that Baba had managed to buy second hand from his past employer. After a couple of weeks of resigned confinement, the entire room seemed like the lanes of 'Dilli' during the cold winter months — dark, foggy and suffocating.

Ishu’s world hadn't always been so dismal. There was a time his father was ready before him and as he went off to school, Baba would trudge off too, whistling, in another direction, with his thick rope and paddings of cloth for his shoulders to carry loads and any other daily wages work. His loud, cheerful voice boomed beyond the curve of the road as he exchanged greetings with his colleagues.

Ishu's school was across the dispensary and was rife with laughter and calls of young voices. It had a few scraggly, half-wilted plants growing on its boundaries and about a dozen tall deodar trees in the distance. The rooms were small and held no furniture except for the teacher's table and chair and a blackboard; the students sat on small sack cloths provided by their mothers. Ishu knew that his mother had packed a few biscuits for him to munch on before he got his much-awaited meal in the school with the rest of the children. He knew that he had worked diligently at his arithmetic homework — not without invaluable help from his sharp mother — and was sure of getting a "Shabash!" from the teacher.

His mother, educated till the 10th class and quite bright in mathematics, had earlier worked as a maid at a number of houses, in order to supplement her husband's unreliable income. She would make food for them early in the morning, put it on the kitchen shelf for the two of them and move off quickly, throwing instructions at them for the day. They all would meet at home only in the evenings. In this mad rush, Ishu had started becoming careless and reckless towards his studies. Mummy had noticed that and became worried. One evening, Ishu had seen her talking quietly but seriously to Baba, casting glances at him as he pretended to study. From the next day onwards, she never went to work again.

Baba had managed to keep them afloat, working hard and refraining from wasting money. And Ishu's performance at school had soared.

He loved school. He loved his books. He loved to play with his friends on the small, uneven ground adjacent to the school building. The loud noise of children shouting, laughing, calling out to each other was like elixir to him and he made sure never to miss a single day of learning and playing. Last year he had received two certificates -- one for mathematics and another for highest attendance, which he proudly displayed from the small stage of the school. But those were the good days. Mummy had always warned that good days have a habit of flying off like a bird.

Ishu isn't happy any more. Father's job is off. Mother looks like a frightened, startled deer. Everybody's faces covered in masks; roads emptied neighbourhoods desolate. A panic, a constant apprehension, a sense of doom.... all around.

And school closed. Ishu could hardly believe it! His head constantly whirled with the ceaseless, confusing, worrying questions — Where would he play? How would he learn? What about exams? How would he get promoted to the next, ‘very very important’ Class 6 ? Where and how would he meet his best friends? How would he get his afternoon meal? His entire world as he knew it, crumbled before him, leaving him shell-shocked, tearful, uncomprehending and devastated. And today his mother, with a troubled look on her face, lowered her quivering voice and said, "Ishu, wear your full-sleeved shirt and the jeans your Mamaji gave you last year. We have to go to the sahebs' homes. Ration has finished."

That was all she said. That was all she needed to say....

And then Ishu's shirt sleeve ripped ....

He walked quietly beside his mother - both of them wrapped in thought, heading for the 'kothis'. They were oblivious to the changing landscape as they climbed up the slope: the well-designed gardens, the high, painted walls, the grand gates of the sprawling residences, the large, shiny window panes. Their feet seemed to pull them back, yet they dragged themselves towards the unknown.... Ishu burdened with the shame of an old, torn shirt and his mother with ... only God knew what.

As she stood before the women of the suddenly silent, apprehensive households, her eyes dark as hell's fires, her son stood beside her, his eyes lowered, shuffling his feet uncomfortably, trying to hold together the two sides of his torn sleeve with his other hand. They moved from one silently simmering house to the other, the small purse slowly filling up with money -- sometimes generously given, other times with tightly stitched mouths — the tear in his heart pulling apart the fabric of his being, his eyes filling up with unshed tears and his mouth with an unknown silence. By the time they returned home, Mummy seemed to be like an empty shell. Ishu wondered at the slight weight of her purse and the drooping, hollow blue suit that walked beside him and he felt a strange willingness for nothing envelop him.

When Ishu opened the door of his home, Baba's quiet presence filled him with some semblance of stability and assurance. He was standing at the gas stove, stirring a pot that contained a dismal comparison of the delicious — though small — dinners they used to have before the lockdown. The damp aroma of the broth filled the room with a tempered down hope for a life better lived in the future. He desperately tore away the offending, shaming shirt from his slight body and flew straight into his father's arms. Baba embraced him as Mummy settled on the side of the cot with an imperceptible sigh. Her tired eyes searched for that gleam of humour that she had always found in her husband's eyes whenever she returned home from work. Finding only a questioning gaze there, she dropped her eyes to slowly extricate her pouch from her handbag and counted out the cash from it.

“A month or so will pass easily," she said resignedly.

"And then...?" Baba’s expression silently asked, as some of the weight from his shoulders seemed to roll off and shift to hers. She straightened her aching back, reaching out for Ishu's shirt as she picked up the sewing box from the shelf above the bed.

"The madames looked tired," she replied as she threaded the needle.

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Review: The Tear

KAMAYANI VASHISHT

A lovely story; simple yet very sensitively written. The story is a narrative extension of a poem by the author which must find place alongside the tale because in many ways, the poem is its mother. One, because poetic utterance is the mother of emotional expression, two…because it was born before the story and then engendered it, three…because it sees Ishu's mother from another mother's (the author) perspective.

It was your eyes
The tide of uncertainty
Coming in and going out
Under the heavy sands of your scarf
It was your eyes
The raven depths
Like stinging black arrows
Piercing the secure bastions of my fort
It was your eyes
Vacuous with worry
There and yet not there
Quieting the laughter of my progeny
It was your eyes
That took a few pieces of paper
With the haughtiness paper deserves
Your pride intact, helping me to keep mine too
Your firm, rightful grip
My yielding one
That joined you and me
And made us complete women
With loving, giving eyes.
Thank you, o unknown woman.
May your family sustain
As mine does.

I read in this poem; a womanist note. Layli Maparyan (Phillips) says, womanism seeks to "restore the balance between people and the environment/nature and reconcile human life with the spiritual dimension”. It is through her motherhood that the author connects with another woman’s life that has been torn and disrupted by the pandemic. Both, the poem and the story when read together add a three-dimensional understanding to the life of the little boy and his family. His mother and Ishu, both come alive; at times more real than the images of thousands of homeless migrants who inhabited our television screens for months during the first Covid lockdown. Those were numbers and sometimes just anonymous faces on the screen. Often, one would try to construct histories and back-stories to their faces and give them a more detailed context than just a mention in the news. One would wonder what it meant to lose livelihood or home and what sense those children walking alongside their parents made of the whole experience. This story brought some relief in bringing the reader, personally to the little house of one such face in the crowd.

The Corona experience definitely altered each one of us, though in a million diverse ways. There has been an immense outpouring of people’s responses to the pandemic through numerous mediums. The creative mind, the sensitive heart and the spiritual, or for that matter even the non spiritual soul has had much to engage with. In fact, art and literature have been our best allies during this phase. Also, one of its best by-products.

The Tear tells us about the tear in the life of one such family that doesn’t have the luxury of calling the lockdown “a blessing in disguise”. The experience of being locked in a cramped space: a home that takes on a very different dimension when there’s nowhere else to seek respite from it is beautifully imagined and articulated. “Sleeping all together never made such a crowd but sitting around does,” strikes one as a fascinating observation! After a couple of weeks of resigned confinement, the entire room seemed like the lanes of 'Dilli' during the cold winter months - dark, foggy and suffocating. The experience of confinement flows is osmosis and reverse osmosis from the Covid restricted spaces into memory and back.

His memory of happier days, when Baba went about his job as a coolie and Maa, as a domestic help and himself going to the neighborhood school are all luxuries that become windows of escape from the non-ventilated existence now. “Ishu isn't happy any more”, a treacherously simple sentence about the little boy tells us all that’s not right in his life. From his perspective, that’s the whole problem!

Another striking element in the story is the shifting landscape of the realities of the neighborhoods. The mother and son are the connecting links between the two divergent realities; the homes languishing in ennui and a home restless because of the crisis of too little. There's both a physical and mental journey between the two spaces and this journey reveals the inevitable connectedness and interdependence of social layers. The play of the homonymous word, 'tear' lends itself also to the tear in insulated structures of families, social systems and personal memory. The tear in Ishu's heart pulling apart the fabric of his being, filling up his eyes with unshed tears and his mouth with an unknown silence is a beautiful semantic movement of the word that justifies its claim to becoming the title of the story.

The story is replete with visual imagery. There are some very vivid descriptions of the spaces that were forced to wear a new mantle during the lockdown. The same spaces we had inhabited became more real in unprecedented ways, so their details that went unnoticed became overwhelmingly oppressive. The torn shirt evolves into a metaphor for disrupted lives that were until now conducting themselves with a modicum of dignity because they were not out in the public space for all to see. Here, they're forced to bare themselves and their tragedies to others and a consequent sense of anguish and shame accompanies both Ishu and his parents. The image of the mother walking by Ishu's side; the hollow blue suit, speaks volumes of the deprivation that is pushing the family against the wall.

"The madams looked tired," she replied as she threaded the needle." The last line is powerful in all its simplicity. There's hope next month because the madams are looking tired. She might be called to resume work in the *kothis*. There's a ray of hope in the darkness of their little home and lives. Were it not for this line, it would be a sad narrative. This single sentence turns it around and leaves the reader with a faint taste of optimism behind a wry smile. The interdependence of all human life is both inevitable and a matter of relief.

One is reminded of Donne's lines:

> No man is an island entire of itself; every man
> is a piece of the continent, a part of the main;
> if a clod be washed away by the sea, Europe
> is the less, as well as if a promontory were, as
> well as any manner of thy friends or of thine
> own were; any man's death diminishes me,

because I am involved in mankind.
And therefore, never send to know for whom
the bell tolls; it tolls for thee.

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Audiences' Live Response

Meenakshi F. Paul: Irene's sensitive and compassionate spirit shines through in this pandemic story.

Anshu Kaushal: The struggle of these challenging times reflected through the eyes of the young boy… heart wrenching!

Namrata Pathania: A melancholic tale with vivid imagery and poetic expressions.

Sapna Pandit: Beautiful story Irene… Congratulations and thank you for such a sensitive and true to life portrayal of those who were forced to leave their homes and walk the streets for months together during the pandemic… loved the story.

Priyadarshini Sharma: Very sensitive delineation of helplessness and hope. Wonderfully narrated and analyzed.

8

APOCALYPSE

JAIDEV

Her expert eyes swept past the neighbours' tele antennas and settled on the saintly pine. It looked dimmed against the lavish cooper clouds. The pine was her old man. Still. The blazing passion of the vine around its trunk staggered her. The land below the tree had gone and houses, gutters and garbage had erupted like pimples. Still, the tree was there and it was her brooding old man. She had once written a poem to it, but no, the thought of the poem was a needle. She averted her face.

A cracker exploded in some distance and the House trembled. The House was a wounded animal. Papery rags of discoloured paint flew down from the kitchen wall. Another Diwali? Something flew up hissing before bursting into several-coloured globs. Then explosions like hurts. Clutching at the loose rusty railing, she sighed but was held by the golden sun half out of the cloud. It was melting fast above the Kasauli hill. The treetops were a long dazzling armchair for its mercury glory. Her eyes swam with the circling sun.

Once upon a time there was a sprightly girl and her dress, on that Diwali night, jingled and glittered with tinsels and beads. That night she was numbed by a strong tension as her eyes pursued the ghostly coil of silver grey, the vanishing trail of her *phooljhadi*. That translucent coil was a poem, and she had revived it again and again. The mother was watching her, from the kitchen window, and the silver plate with the goddess would shake in her delicate ivory hand when the *phooljhadi* rushed towards her. For a moment, the little girl stopped, for there were tears behind the incense curls. Tears again? She knew he was behind all tears. He had been wicked in the morning. He had cursed them both without opening his eyes. And he was lying on the great-grandpa's sofa on which one wasn't allowed even to sit. The mother had prodded her in the back and when she had touched his eyes with a "Happy Diwali, Pa!"

he had shouted them out. The tea was spilled on the soft, soft rug. The pink napkin on which she had drawn a greeting went into another puddle of black tea below the dusty armours of the family greats. He had cursed away the lunch and spent the whole day sitting between the black telephone and the life-size portraits. His golden pen had hung over his thick diary and he was gazing into the flames in the fireplace. Later, in the evening, the mother's *puja* had to be abandoned because he kept shouting abuses from his room. His ivory pencil was rushing upon paper and his grey hair shone in the lamp's orb. The room was blue with smoke and the carpet was a jungle of crumpled, rejected poems. And all the while the dying coughing Ramu Chacha kept standing in the cold, turning away the friends and their packets of sweets. In fact, late last night he had smashed decorations and lamps in the garden and also smacked at the pleading face of the old dying Chacha.

She had not noticed when her dark, six-fingered hands had risen and covered her wrinkled face. A sudden chill made her realise the sun was gone down the Kasauli hill. Her fingers opened and through the chink she saw the pine. It looked calm. Thank God, she said and pressed at her eyelids. Hard, so that red stars gleamed, in the pain of dark. The shimmery borders were the texture of the conch she had found, once upon a time, on the Konark beach. The pale withered mother had sighed with a smile when the girl held the conch before her dark rimmed eyes. The very next instant she had hugged her hard and kissed her sandy cheeks. He had chuckled at the conch. That night, he had hit his head on the bar table and when they pushed him into the room, he had demanded an immediate death. This was, she could feel now, his way of wishing them dead. The mother had died soon after.

Yes, the day is gone. Again, nothing has happened. Nothing except that Diwali seems to be nearby. One more idiot day, she thought, crushing flat a peel of paint. Someone in a new house gave a cheer and a chain of bulbs went on. Yes, the day is over and yet he isn't dead, hasn't died. They're just prolonging his stupid life, but... but what else, come on, what else should they be doing? Yes, yes of course, she repeated and felt rancid. Yes, sometimes he looks alive, he notices your entry sometimes; he even makes some splintery noise. Sometimes he is alive. And so what is one supposed to feel? Gratitude? Oh, no! Her nails were telling into her chapped palms. Please God, let him die!

A faint darkness throbbing with the crickets' calls was oozing up from the hills, encroaching upon the red in the sky. Tomorrow, I'll be up early; it's Sunday — another Sunday, another visiting morning, you see; she was now addressing the darkening top of her pine. May be, I'll gather a few roses, though he notices nothing. Not that he ever noticed anything. Except of course what he wanted; then he was a hawk. He was a hawk. He must have wanted the mother, so she loved him. Then he must have wanted her — so she was born. Then he must have wanted her to die — so she died, the poor reed of a mother. Then tie must have wanted poems, and when they did not come, well, then too.... She sobbed, and suddenly in her dim, tear-touched impression, her pine bent with pity, bent all the way, its trunk making a plastic crescent around the new block of flats, to touch her, console her. The tree was her true friend.

Of course, the tree did not bend, and she knew it. In her poem it had bent and stood between her breasts. Please, you mustn't grudge her this innocent fiction. She had little else. Besides, a bird lived on her pine, and the bird was important.

But now she wasn't looking at the tree. She was away, in that winter night, once upon a time, when she sat by a drafty window in that common coach. They were returning after the ceremony on the river bank. The evening had been horrible, what with the car breaking down and, afterwards, on the river bank, that fat, turmeric-coated priest spitting verses on her ashes. The priest had messed up everything, and the ashes mostly blew in their faces. He had kept scratching into the lines on his forehead, resenting everything the shiny bald priest asked him to do. And when the latter objected to his cigar, he murderously stamped his feet. In the coach, his collapse and his 'damn it!' had drawn attention and in misery she had dropped her bag and when she was feeling and gathering the things, her shoulder had touched his thigh and God! He had recoiled from the touch. Hurt and bewildered she had looked up, but he had hidden the face in his hands and groaned. She had wanted to pull out a cigar from his pocket and shove it into his mouth. But no, she just got back to the seat and watched the jostling reflections in the window pane. By the time, she could isolate his profiled face; the glass had begun to rattle. She was torn by the recognition that everything — his agitation, his roughness towards a fat woman, his recoil against her touch and now these ugly moans — everything was aimed at her. His "God, my God!" was a pair of pincers and these were trying to slash into her and tear some part open. The cigar was nothing, the dead mother was nothing. The girl had sat with a brimful of red-hot emotions and his "God, my God!" had grown more frequent.

A Shylock demanding his pound of flesh? Of her? What had she to give him? What do you want? She had breathed to the face in the glass. For no reason, his drunken mockery of the mother echoed in her ears. "My love is a junkyard, virtuous bitch," he would blare while locking the garage. With a fresh wave of sobs, she had recalled the jasmine face of the mother. Then, slowly it came, swooping like a terror. Her mother's face vanished and left her to her own, in the glass, her nose flattened against it. Something was churning inside her; it was enormous, hot, also ominous and dangerous. She was marooned on some pointed rock, its edge hurting her, revolving her like a weathercock, blood trickling down her thighs. Faintly she sensed all this drama was a preparation for something terrible; she would suffer, suffer and suffer; she would be a victim, get burnt or charred; something was at work on her heart; she would suffer like the mother. And her mouth would be sewn shut. He was groaning as in a rite, invoking some dark powers. He was demanding. But what? She convulsed in agony; something, some stone, was rolling up her throat, preventing her breathing. But... some part inside was expectant, too. Also excited, reckless. She pressed at her throat and whatever mountain it was that was looming up now from some dark waters made her breathe in exultation. Somewhere she was happy that the ashes were gone.

And thus had begun a lifetime of damnation and despair, she thought and nodded. Grimly, in the dark. A brood of images stirred up — bizarre, revolting, hideous. And she had lived through them all. The young girl dreaming of a huge fire-ball with its octopus' arms all in flames and she screaming "Mother" as it stood suspended over her and he smiling and asking her to catch it for him and she scorching and still her arms unfolding... and in the morning she bracing up for an abstract eternity of withdrawal and solitude — at school, in the club, before everybody; and then placing a rose in the tea tray—and he seeing and yet saying nothing. And one day she dedicating her poem on the mother to him, and he giving her a cold stare before locking himself up in the Study; and later "Mother" and three more poems he published in his own name and dedicated them to her! And she fixing tea for him and expecting only a word, one clumsy stupid word of explanation but he only complained of the overboiled egg and his hundred hells! And two years later he getting a collection of poems mined out from her diaries and earning some name as a sensitive feminist male poet! And he coming home drunk, reeking of garlic when she wouldn't lay poems and demanding an instant death. And her attempted escapes all aborting as jokes, leaving her exhausted but also convinced that she wasn't a person, wasn't anything except a shade, or a shell, and that was her life and her damnation. And all this, she knew, all this had been fixed that night in the coach. His "God, my God!" was a dark spell and she had said "Amen"!

Yes, everything was settled that night, though he had said nothing except "God, my God!" Somehow, the groans were the spell, she concluded in the dark. The pine was a phantom now. She thought of the time and rejected the idea of the meal. It is late again, she thought. And again, it is dark; this way I'll never see it. Still, her new poem was nearly done, and it was for the bird. May be, it's only right that I haven't seen it in clear light. In dark, it is mysterious but also special, my own, it's my dark bird, my bird on my pine. Tenderness lurked inside her and she wondered if someday it would perch on her shoulder.

Suppose he... This he was a queer poet-disciple of Him; he had seen some of his poems in a Melbourne magazine and had decided to call on Him while in India. His own poetry had been received well in his country but surely, he wrote, it was nothing beside the Indian Guru's, she had to hear his praising His sensitive entries into the feminine consciousness and once upon a time the irony of it all had made her collapse with a hysteric "Stop this!" But now she was only wondering, suppose he comes across her... no, His, His still, His, of course, His poem on the bird someday? Would he see the joke? And the irony? Would he remember her and how one evening he and she had stood in the balcony to watch the sunset? That was the first time the bird was noticed. It was hardly a bird, a strange fleeting lump, a ball rising in a trajectory towards the pine's top branches. It had almost touched her shoulder and she was jolted, and he had held her by his hairy arms. Whatever was it that got touched and conveyed? For he had gently bent his head and kissed her on her quivering lips. Suddenly the bird dived at them and after a confused circle rose again and vanished. The bird became a daily event, though next day the poet-disciple had been claimed by Him in some mysterious way for he did not speak even two sentences to her during his month-long stay. For hours together they were locked together in talk and laugh and their voices would trickle out of the closed room or window, and within a few days the poet-disciple had left his hotel and hung his sleeping bag in His inspired room. He was clever and had isolated hundreds of feminine nuances in His four volumes but even he had missed the meaning of the girl in the house or her hysteric tears, once upon a time in a party, at His bland "Meet my daughter, my muse, my secretary." His brief, remote "How do you feel now" was little better than a young art historian's "It mustn't be easy to be a celebrity's daughter" or a poetry teacher. "Why did you stop your studies like that?" Oh, he was clever all right, but how could he guess?

Then it came. A compacted ball slashing its way from behind her, it caused a sudden flutter in her bosom and her *dupatta* rose and fell with her heart. Her mouth opened and her eyes widened feeling the needles that were lights in the distance. Blood rushed and tingled at her finger ends. And then ... it was gone past the antennas. A branch fluttered above and it was quiet. She stood with dreams tantalized on her cheeks and her eyes drinking the dim vision that was. It was her bird, she told a mocking murmur within her; why else should it go swishing so close to her? And who knows one day, someday, may be for a second only, it will gently land on her pleading shoulder, its beak softly trickling, exploring down her neck. In her poem, it did that. Her eyes were lingering to where the pine top was, she let out a long, half-happy breath and her body shivered. It's cold, it really is.

The telephone rang in the dark house. She trembled: Is He dead? Or has He again done something horrible? Last time she was called at night was because He had hit the nurse with an open hot-water bottle, smashing her glasses. He was left unattended that night. God knows she had gone all the way and tried hard to be of use. She had changed the sheets, fighting against the foul stench and He had cursed her all the while. She had wanted to pound on his chest and blast him with some truths at last, but no, she had allowed the kingdom, and the power and the glory, to include her poems too. The new Lord's Prayer, chanted for him.

After a pause, the telephone rang again. She moved. Her steps were clockwork through the kitchen and the dining room. The hand that pressed the switches was stone. Is he dead? He cannot be, He is God, the Father, immortal, everlasting. His being in hospital is his show. He cannot die, she shouted to the stuffed tiger against the grandpa's gun, and the telephone grew silent. She heard the return of silence and breathed. Is this life? She asked facing herself in the oval mirror. And this, is this woman me? Once upon a time there used to be a girl, but since that night in the coach, everything had been his, no longer a me or my life. Thine is the kingdom.

The telephone again. The stone turned but did not move. Is this life? What has it meant? And He cannot die. First, she must be abolished. The bell persisted and the stone moved, sank on its knees and picked the receiver. Is this life please? With much accumulated rage she shouted, "Is He dead or not?" The speech exhausted her, and she listened, her eyes staring at the dusty chandelier. She dropped a pebble of yes into the instrument and pressed at her heart. He seemed to be dying at last, and a car was coming to take her at the gate. Is this life? And will it someday perch on my shoulder? And you who were so good at finding feminine nuances, did you never wonder, never suspect...? And is it fair? She asked the faded roses in the large vase while her hands gathered a green shawl. Is it fair, tell me please? She moaned at her sandals. And what do you make of one who steals her lover and her poems and her everything and spends an hour praising his iron will in giving up smoking? Is this life? She asked the gate. And is He dying?

"Let Him die," she says as the lift door is shut by a low-statured, bearded villager with an umbrella in his hand. The words shock him and he turns, arranges his glasses and says with confused anger, "What?" She is a stone and turns and faces the man's doll-like, field-smelling wife who in confusion folds her hands and presses them upon her bridal necklace. A child stands besides her holding her by a saree crease and a saliva thread is swinging from his lip. The lift crosses another level and the child frees the saree fold but tumbles. With a shock she realises the child is blind. Her mouth jerks open and she kneels down and holds his round milky face into her six-fingered hands. The hands are alien and the child recoils as she fills the face with kiss after kiss after kiss. There is something irregular in the act and the mother coughs and throws an anxious, urgent look at the man. He stops the lift and rather roughly herds away the child and the woman who looks very relieved at such a narrow escape from so obvious a witch. Alone now, she gazes at the numerous stub stains on the cage wall. Is this life? Is it fair? The lift crosses another landing and she wonders if he had noticed her pockmarks when the bird arrived and he kissed her. But what does it matter how I lost him? I lost him to Him. But is this life please? She is asking the lift and there is no one to watch how trapped this sagging, grey-haired woman looks as she crazily beats at the ascending metal frame.

She knows she is supposed to say something, perhaps thanks, to the anxious doctor who shifts a catheter to his left hand and leads her by the arm into the intensive care. She tries to form a word, her lips move, but nothing happens and she is a stone. Her nose twitches at the phenyl smell and she sets her eyes on the several bottle-stands around His head.

A syringe-carrying nurse holds a chart before the doctor. With a sigh, he waves his hand before His eyes. Nothing. But the eyes move when she reaches the bed. "A Corimin please, Sister," the doctor mumbles and the nurse selects a syringe. She brings her face right against His and at once something flickers across the web-veined eyeballs and the socket muscles expand shaking the dip tube in his nose. The eyes are purple but a red trickle is rising into them. And she is a stone.

Bubbles brew up at his suppurating lips. They are words, pus-stained and deformed, aborted kids, dying before they are allowed. She is a stone but bends. Right, wry, write, rite, what? Must be Write. Naturally. Write a poem on the death of a poet. Always an actor! She is a stone and bubbles are flying at her face and she smiles. Write! What else?

But write what? She suddenly panics and her hand takes the nurse's pad. His whole body is spasmic, a huge vibration and the noise from his lips is a beetle's hum. For a moment He seems to rise in the air and his eyes are two red embers. Pus bursts from the lower lip and the room seems to shake under his booming cough. With clots of blood flying out of his mouth, something emerges brokenly.

Once... up ... a time ... was ... a devil ... an he ... ate ... his ... delic... chil...

As his mouth shuts, she hits at her eyes. What is this? What did he say? She looks hard at the dead mouth, rushes at him, gets tangled in the catheter tube, and falls on his chest. Glass bottles crash and the doctor shouts for water. Her body is on fire as she shakes Him hard. "Did you say those words?" She screams into His dead eyes. "Of course, you did. I heard, I heard". Her fingers crawl like lizards all over the bloody face. For a second, she lowers her right ear at her own breast. Something is cracking, splitting in it. She is rocked by a massive sob and her tears fall on her face as her kiss forgives Him all. When the nurse and the doctor hoist her face, she realizes that the tension and purpose of her life is gone; she will never write a poem again; nor will the bird perch on her shoulder.

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Narrated by Praveen Kumar

Self-Blame and Self-Preservation in Jaidev's "Apocalypse"

MEENAKSHI F. PAUL

Apocalypse is usually understood to mean great or widespread destruction or devastation, particularly one brought about by violent means. It also means to reveal, unveil or disclose. In Jaidev's story both meanings are wrought with an acute sense of existential anxiety bordering on the absurd. Biblically, the apocalypse has special reference to the end of the days; it is believed that the horrific events of the apocalypse, drawn over ages, will be followed by the longed for Second Coming. The protagonist of "Apocalypse" is severely traumatised and left spent and empty after a lifetime of pitiless oppression; but the possibility of redemption is tenuously presaged at the end of the story.

"Apocalypse" is a dark, almost surreal, story about the brutalisation of a daughter by her father. Surreal elements in the story imbue it with inexhaustible meanings as its dark world articulates the unavowable in social discourse. The tone of the story is restrained to balance the horror of the narrative. "Apocalypse" is a scarring study of dehumanisation riding on the ego of an abusive failed poet and of his daughter overwhelmed with his violence and her father hunger. The story opens with the haunting image of a father substitute, the 'saintly pine' which the protagonist calls her 'old man' and her 'true friend' who even in her advanced age is still there as her 'brooding old man': "The pine was her old man. Still. The blazing passion of the vine around its trunk staggered her". The word 'still' holds one of the keys to unlock the complex plot of the story. It is strategically placed between the bedridden old father, the old man tree and the blazing passion of the vine. The tree is still there after so many years, holding out strength and security to the woman's dazed mind and body, which are tyrannised by her father. The tree is still and so is she; nothing has changed between them or in them, although all around the tree, change has "erupted like pimples". 'Still' the protagonist is staggered by the passion of the vine clinging to it. The passionate embrace of the vine is driven by self-preservation, for it would die if the tree died. She is the vine clinging to her tree, her life. The reader is driven into a vortex of events, thoughts and feelings in what appears like a meld of *The Bluest Eye* and *Metamorphosis.* The tree and the vine are the first of paired symbols in the story. There is a brief mention of a poem she wrote many years ago to the tree and now she averted her face to escape the 'needle' of the poem; however, as a Diwali cracker exploded 'in some distance,' 'the House trembled' and the woman is catapulted to the past as a witness to her childhood life. Instantly, we are introduced to an abusive, violent, temperamental, frustrated poet who reduces his fearful wife and daughter to tears and hits their old domestic worker — the 'pleading' 'dying coughing Ramu Chacha.' "The House was a wounded animal" suffering as much as the human beings from the violence perpetrated by the patriarch.

The father is on his death bed in the hospital and the woman wills him dead, yet the 'idiot' day is over and "he isn't dead, hasn't died". The death wish is juxtaposed to an earlier violent incident when "he had demanded an immediate death. This was, she could feel now, his way of wishing them dead. The mother had died soon after". 'Hasn't died' suggests that the woman believes her father is able to govern his and their lives according to his will. He exercises control on their life and death: "He must have wanted the mother, so she loved him. Then he must have wanted her—so she was born. Then he must have wanted her to die — so she died, the poor reed of a mother. Then he must have wanted poems, and when they did not come, well, then too...". The woman breaks down at this recollection and the tree reaches out to her compassionately. It is interesting to note that while the House has been capitalised, the pine tree is not, keeping the latter well within the precincts of the fantastical and thus, highlighting the pathetic aspect of the girl/woman's transferred subjectivity to the tree:

She sobbed, and suddenly in her dim, tear-touched impression, her pine bent with pity, bent all the way, its trunk making a plastic crescent around the new block of flats, to touch her, console her. The tree was her true friend.

Of course, the tree did not bend, and she knew it. In her poem it had bent and stood between her breasts. Please, you mustn't grudge her this innocent fiction. She had little else.

In the midst of this emotional turmoil, Jaidev introduces another pair of symbols: a hawk and a bird. "He was a hawk" when he wanted something. And, although the tree cannot hold her straying mind, it was vital because "a bird lived on her pine, and the bird was important". As the protagonist is identified with the bird, the juxtaposition of the hawk and bird directly leads to the life-altering pitiless incident that defines her life thereafter. She is now away on a train with her father. They are on their way back from performing the last rituals of her dead mother on the riverbank. What follows is a dark, turbulent and chaotic passage that suggests vicious incest and rigid control that the father exercises over her: "And thus had begun a lifetime of damnation and despair, she thought and nodded. Grimly in the dark. A brood of images stirred up—bizarre, revolting, hideous. And she had lived through them all". The 'victim' with her mouth sewn shut responds ambivalently from her "bracing up for an abstract eternity of withdrawal and solitude" to "placing a rose in the tea tray" for him. She dedicates her poem on the mother to him and he proceeds to publish that and other poems from her diary in his name. In a macabre irony, the poems earn him the title of a "sensitive feminist male poet". When she "wouldn't lay poems" he demanded "an instant death" convincing her "that she wasn't a person, wasn't anything except a shade, or a shell, and that was her life and her damnation". Damnation because the hawk has the bird firmly in its talons and there is no escape. Also, because she had said 'Amen' to his dark spell of "God, my God!" and thus willingly lost her soul to his profanation. This self-blame causes further anguish and trauma as she internalises the guilt, shame and anxiety of a victim who believes she has colluded in her own oppression. The vine clings fiercely to the tree for survival.

The spell is not broken even with the arrival of a queer poet-disciple of the father from Australia. The bird was first seen by her as she stood with the visitor in the balcony to watch the sunset and he kissed her. The young man is immediately commandeered away to his room by the father and the disciple, though clever, has no clue that the four volumes of sensitive poetry expressing 'feminine consciousness' are actually written by the "girl in the house" mentioned in the poems. The father deviously crafts her image with "her hysteric tears" and gaslighting which effectively turns away potential suitors—the poet, a young art historian, a poetry teacher. The bird of her subconscious self is also only intermittently and partially visible to her after that evening on the balcony: "May be, it's only right that I haven't seen it in clear light. In dark, it is mysterious but also special, my own, it's my dark bird, my bird on my pine. Tenderness lurked inside her and she wondered if someday it would perch on her shoulder". She is now writing a poem, '*His* poem,' for the bird and wonders if the poet-disciple would get to read it and grasp the irony of it. The woman feels momentary happiness as she thinks: "who knows one day, someday, may be for a second only, it will gently land on her pleading shoulder, its beak softly trickling, exploring down her neck. In her poem, it did that". Once again, the last line negates the hopefulness of the preceding line by removing the bird to the realm of imagination. It is noteworthy here that the bird does not ever sing. Not even in her mind. The disturbing voicelessness of the woman is starkly enunciated by the songless bird.

Mary Ann Mattoon writes in her introduction to Jungian psychology that both the male and the female child could have a father complex, and it could be either negative or positive. The protagonist of the story appears to have a paradoxical emotional and psychic relationship with her father. She steps up to change his fouled linen in the hospital when he hits a nurse with "an open hot-water bottle" but he still "cursed her all the while". At this juncture, the extent of her enslavement to him is brought out with piercing clarity by the author:

She had wanted to pound on his chest and blast him with some truths at last, but no, she had allowed the kingdom, and the power and the glory, to include her poems too. The new Lord's Prayer, chanted for him.

When the telephone rings, she is turned to stone, incapable of any emotion but disbelief: "Is he dead? He cannot be, He is God, the Father, immortal, everlasting. His being in hospital is his show". The phone falls silent as she stares at herself in the mirror; a woman looks back at her from where once there was a girl. The self-conscious moment acutely brings out the awareness of loss and the interminable time that has elapsed in the clutches of the hawk. The stare at the reflection, underline the absence and depersonalisation of self that for long have made her feel 'not real'. Distanced and disconnected both from herself and the world around her make it difficult for her to attach emotions to memories.

The telephone rings again and the hospital intimates her of his imminent death, 'finally.' She is overcome with the question "Is this life? And will it someday perch on my shoulder?" The question reverberates in her mind till "Is this life?" is juxtaposed with "Is he dying?" and eventually gives way to "Let Him die". The 'stone melts' for a fleeting second in the lift as she caresses the liftman's blind child with her six-fingered hands and plants kisses all over his face. Disconcerted, the man stops the lift and herds his bewildered wife and child out. Jaidev now describes her in an image hauntingly evocative of Pecola Breedlove of *The Bluest Eye* who asks each and all if her eyes were the bluest and who at the end of the novel: "[beat] the air, a winged but grounded bird, intent on the blue void it could not reach—could not even see—but which filled the valleys of the mind". The woman in "Apocalypse" asks each and all "But is this life please? She is asking the lift and there is no one to watch how trapped this sagging, grey-haired woman looks as she crazily beats at the ascending metal frame".

In the hospital she is a stone again and the full horror of 'is this life?' is encapsulated in the "Bubbles [that] brew up at his suppurating lips. They are words, pus-stained and deformed, aborted kids, dying before they are allowed". Habitually, she takes the nurse's pad and prepares to take orders on what to write, but the blood-clotted words that fly out of his mouth are completely unexpected:

Once … up … a time … was … a devil … an he … ate … his … delic … chil ….

She is shocked and incredulous: "'Did you say those words?' She screams into His dead eyes". The half-blurted confession of the dying man breaks the spell he had cast on her and something in her 'cracks and splits'—the stone splinters and a 'massive sob' escapes the woman: "her tears fall on her face as her kiss forgives Him all". This unprecedented and dramatic forgiveness scene at first appears parodic. The equivocal use of the words: 'deli'—was it delicate (like the vine) or delicious; and 'chil'—chilled or child, open the possibility of several parodic interpretations. In psychological terms, by his broken barely comprehensible words right before his death, he had robbed her of "the tension and purpose of her life". It can also be understood, however, in the light of Ezekiel 36: 26 in which God promises his people: "I will give you a new heart and put a new spirit in you; I will remove from you your heart of stone and give you a heart of flesh." The Bible posits this replacement as a rebirth to spiritual awakening and ultimate redemption; nevertheless, this forgiveness extracts a colossal price: "she will never write a poem again; nor will the bird perch on her shoulder". 'Is this life?' she had asked; it wasn't, and yet, tragically but redemptively, it is.

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Audience's Live Response

Kamayani Vashisht: Ma'am, the fact that you chose to use the word, "father hunger" in the very first few sentences, placed the story in its right context. Everything else falls in place...candidly and spontaneously. Thank you!

Anupama Singh: This story has once again taken me to the classes of M.A. where Jaidev Sir used to take us on the journey of the characters of stories/fiction and while listening to him many times we could not hold back our tears. Our whole focus used to be on every word that he used to utter and his deep expressions can never ever be forgotten.

Namrata Pathania: A harrowing tale of abuse.

Anu Jaidev: Ma'am, a privilege to hear you on this story.

Sangeeta Singh: Amazing analysis Ma'am. It was a rather complex story. Need to read it.

9

A SNOW TRAIL

JANESH

It was snow everywhere when I got up in the morning. So much snow was being witnessed after a long gap: eternity as it were..., nostalgia about such winter mornings of yore, when my grandmother would coax me out of the warm bed, chirping: "Get up and come out! Look the earth has been studded with silver..."[*"Uth Bhaua, uth! Dekh kedi chandi he chandi bikhri ri. Uth, uth…"*], suddenly possessed me. The eutony in her words invariably made me leap to the veranda, amazed not so much by the snow, but how it was projected by Ammaji, whose bed time stories full of gods and demons along with innocent human creatures braving the wrath of both, and of kings and queens, brave princes and lovely princesses, were catalysts enough to fantasize snowfall into something beyond mere ecstasy. Jacky, the dog next door, would start to pule and then growl impatiently, exhorting me to come often. I would quickly gulp down a cup of hot milk, allow myself to be wrapped in woollens without a frown, and run out wildly with Jacky, leaving our footprints on the pristine snow.

Later in the day, when I would team up with my cousins and other children of our Mohalla, I would cook up a story (not really a product of my imagination but a contextualization of childhood gossip) how snow was the messenger of Jacky's Mama, who would come to visit him shortly, making him so excited so early in the morning. Days would then be spent snowballing, sledging, creating a host of snow people and preparing *kulfis* by freezing a mixture of milk and sugar in layers of snow for our little private parties. But even in the midst of such happy times, the elderly boys in the group did not miss a chance to rag the young ones like us. It was January 30, and my cousin was persistently impressing upon me that we need to pay homage to our martyrs when the siren would be sounded. Overcome with patriotism, I could not have noticed him passing mischievous glances with others. I was in the midst of a sledge ride when the siren went off and I tried to stand up to offer a salute to the great ones, to find myself flying into a *nallah* along with my sledge orchestrated by an uproar from other boys. The bulk of soft snow graciously saved me from any injury though. My Big Brother, who first protected me from the bullies and would always keep my back in numerous gulley fights as I grew up. May the powers that be speed his soul… *Amen*!

The awe and amazement for snowy days instilled by Ammaji continued to grow on me over the years. Find it impossible to remain cuddled indoors even today when it snows. And today was very special with so much snow around. Quickly gulped down a spicy *parantha* with steaming coffee, zipped-up my jacket and stepped out on to the pristine snow, gently cherishing the first imprint of my feet on its softness. As I walked along, couldn't help imagining as if I were paving the way to some uncharted destination.

The heavy snow had taken its toll on everything around. Trees had snapped branches, power supply wires were dangling from helplessly bent poles and water lines were frozen cold, but what of these. Not the occasion to worry that there would be no power or water supply for days to come. The crystalline beauty of the icicles was gradually taking on shapes, sparrows having the fill of snow with their pecking beaks, monkeys dangling from the tree branches releasing a brief spurt of snow, people walking steady now to suddenly lie flat in obeisance to some unseen power, only to be stunned was a snow missile fired from somewhere as they got up. Ash and gravel were being laid to make a safe pathway to save restless souls from slipping into the dungeon of despair as others laughed hilariously at their fall, only to be chastised with a swift fall the next moment. It was a longish walk from Summer Hills through Chaura Maidan across The Mall and beyond The Ridge. As I moved from Lakkar Bazar towards the Snowdon, I halted at a tea stall. It used to be our favourite haunt during studentship in Sanjauli College. It used to be a boys' college back then. Boisterousness, hooliganism and other forms of physical jugglery were hallmarks of what appeared to the public about the life in college. The boys would form chains covering the entire road and force girls from other colleges to bow down in order to cross over. But the essential character of the college comprised a disciplined and deeply academic culture. Sanjauli Boys dominated when it came to joining professional courses or securing university positions in academics or the co-curriculars. And while the boys would tease and taunt girls along the Lakkar Bazar road, not a squeak could be heard if one of them happened to step inside the campus. And the famed rivalry between Sanjauli Boys and medical students from the IGMC which often erupted into violent clashes or the intra-college wars between different gangs of students. Studies would often be struck due to such incidents and most of us preferred to bask in the sun on The Ridge rather than braving the tundra that the way to the college was.

The tea stall, converted into a medical shop lately, was named *Anamika*. We would visit it several times a day for *chai* and sometimes *samosas*. As it was a relatively new eating joint then with few customers, we could sit there for as long as we desired. In fact, we were warmly welcomed as regular customers. The smoky cups of tea with ample time to spend over them would make us speculative – *fikro-fankaars* – as we loved to describe ourselves. There was not a problem in this universe which we could not resolve, empowered by the *amrita* infused in our beings by *chai* and *samosas*. There used to be so much excitement around the corner. India had hosted the Asiad and the NAM summit on a grand scale: a journalistic quip read – "Given India's ability to organize *tamashas* overnight, if God made the world in six days, India could have done it in five". The PSLV, our first satellite launch vehicle had been tested successfully. Rakesh Sharma had journeyed to space. It was some years since the Emergency and the atmosphere was generally euphoric. It was also the era of the cold war, of the disruption of the USSR, of gradual transition to a material and virtual world. There was more than a lot to keep us engaged.

The artistic strain in some of our comrades, be it writing, sketching, mimicry and the like was also sculpted here. Ayub (AK) would be busy singing melodious Mohammad Rafi Songs. Sodhi (SSS: Silly, Stupid Sodhi) would be busy sketching automobiles, aspiring to be an auto designer; or worrying about saving money to pay for his dog's bus fare when they went to Delhi in the winters as his father won't provide for it. Vineet (Vicky) was our Romeo, always working out ways to impress girls, always anxious about his appearance, constant subject of our taunts. Kartar (KT) would make every place pleasant with his ready smiles. Subhash (Bhashi) would turn aggressive over anything. I was the philosopher of the group; at least Pradeep (BB: Bhola Bhandari, the latter incidentally being his surname) always expected me to express some serious thought. BB was, still is simple and naïve, willing to believe everything. We would exploit his naivete to arrange food for us when we were broke: "*Oye BB! Kal hamare liye apne ghar se paranthe bana ke lana. Bhukhe rakha to bhoot banker tujhe tang karenge*" ["If you do not bring us *paranthas* to eat tomorrow, our hungry souls are going to torment you".] And BB actually brough something for us to eat. Surinder (Guru) was our reality instructor. He would taunt Vicky for his romantic dreams: *"Kake, tera yahan bhi barahwan number hai. Tujhse pehle gyarah line me lage hain. Chod yeh dhanda aur mast reh"* ["Don't waste your time here. Eleven other boys are already towing this line ahead of you"]; or incite Bhashi: "*Bhashi to badahi susheel balak hai, Bas isse panga mat lena, nahin to phod dega"* ["Bhashiis such a sweet angel as long as you don't bother him. He will just blast you, if you do"]. Again, when BB would ask me something: *"Usko chod, vah nahin bolega. Bas imagine karlega ki maine bol diya hai"* ["Who are you talking to? You will have to imagine his answer".]

We lost Bhashi to eternity...

KT had to bear the brunt of fate; lost his parents and young children to a mysterious illness. Gosh! How does he manage to smile still?

Vicky, we all knew, would not try to reconnect once he left.

SSS too disappeared from the scene

Guru came back to meet us twice and I went to his village near Udhampur in Jammu once. But the terrorism happened and all connectivity was snapped. I hear that he is doing well.

AK, BB and I being localites do bang into each other at times. A brief stroll with AK on The Mall or passionate chats with BB always makes our day. Last time I met BB, he drove past me in his car, caught my glimpse in the rear mirror and, without even thinking, reversed through the traffic to hijack me for a drive.

Memories…

One fine afternoon in *Anamika*, while we were guffawing and joking, waiting for tea to be served, I felt hot tea seep into my trouser, irritating the skin beneath. The service boy had spilled the tea while putting it on the table and on to my lap. I was about to swear and curse but supressed my anger as two deep innocent eyes stared at me without comprehension or fear: a little boy who had just been brought to the stall for work. *"Tumhara haath to nahi jala?"* ["Did you char your hand?"], I patted his hair as the owner scowled at the boy, pulled out my hanky to scrub my trouser and patiently waited for a fresh cup of tea.

It was after almost a decade that I found myself entering the old, familiar place. I nodded to the owner, who did not betray any sign of recognition. The young lad had grown and supported a beard and a long *'tilak'* on his forehead, was behind the stove. Trying to ring a memory of old days, I queried:

"Pehchana? Main apne doston ke saath yahan aata tha aur tum hamain chai pilate the? ["Do you recognize me? I used to come here often with my friends. You used to serve us tea".]

"Aate honge. Yahan to kai aate-jaatehain" [May be. So many people keep coming and going".] was his deadpan response.

I tried to gulp down his apathy and said, *"Chalo, ek garama-garam chai to pila do. Bahut thand hai"* ["Ok. Do serve me some hot tea. It's so cold".]

"*Ek chai in sir kobhi de dena"* ["Give a cup of tea to this gentleman also"], he passed on the order to another young boy who now did the serving and cleaning.

I waited for tea, feeling cold and fidgety over the lack of human warmth I had always experienced in this place. It was perhaps this unconcern which made me repeat my order loud, *"Jaldi do bhai!"* ["Give me tea, quick!"]. My old accomplice in so many games and little mischiefs, be it making faces obliquely at his employer or pinching the hips of our other friends while we sat for tea, let out a volley of abuses at his apprentice: "@#$&$#@&…. *Dikht anahin. Sahab ko kitne jaldi hai!"* ["You @#$&$#@&…. Can't you see, Sahab is in such a hurry?"].

Channnnn…….!!!!

A past memory was suddenly shattered like a glass, its unheard smash exploding my innards and the very spirit.

I instantly got up and walked out. Not only had the voice and physicality of my little friend of one time, with deep, innocent eyes hardened, a part of him was also perhaps stone dead. I couldn't imagine what my reaction would have been if he had spilled tea on my trouser today?

Sand and gravel had blotted the whiteness and purity of snow. Numerous feet had flattened its softness. Shopkeepers had littered it with scraps and leavings from inside. Someone had also peed on it.

I turned back homewards as the snow abruptly began to look grotesque….

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"The Snow Trail": A Trail of Inner Devastation

Praveen Kumar

Janesh Kapoor's disquieting story "The Snow Trail" begins with nature at its splendid best. It has snowed and there is whiteness all around. This whiteness has a special meaning for the protagonist because it flashes on his mind the fond memories of his past. He is reminded of his childhood when his grandmother would coax him out of a warm bed in harsh winters. She had an innate fondness for this beautiful natural phenomenon and wanted to instill the feeling of awe and wonder in the young boy. After an initial reluctance, quite normal for winters, the boy would spring to his feet and rush out to play amidst the silvery-white surroundings. Structurally, the story swings between the past and the present. The past is available in the story as nostalgia, while the present, devoid of childhood innocence and carefree attitude, is present through absence. It's also about the expectations we have from people around us, not necessarily from family and close friends, but even from those with whom our bond is shallow and tenuous. The story is a study in human relations, which though lend beauty and charm to our lives, yet can also be a big drain on our emotional energy.

Coming back to the presence of the grandmother, it is easy to pick up a number of linguistic and structural hints that point to the nature and depth of her influence on the young boy. Her words used to cast a spell on him, filling him with excitement and verve. Look at the words: "The eutony in her words invariably made me leap to the veranda, amazed not so much by the snow, but how it was projected by Amma ji, whose bed time stories full of gods and demons along with innocent human creatures braving the wrath of both, and of kings and queens, brave princes and lovely princesses, were catalysts enough to fantasize snowfall into something beyond mere ecstasy." In an attempt to show the impact of the grandmother's words on the child, the writer makes use of an uncommon, but a very evocative word "eutony". It comes from the Greek 'Eu' meaning good and from the Latin 'Tonus', meaning tension. Hence, 'eutony' conveys the idea of a harmoniously balanced tonicity, or in simple terms, the pleasantness that a word's sound produces in an individual. The grandmother's words and stories acted like a catalyst and fired his imagination. Her words had the power to transform snowfall into something mysterious. There have been numerous instances of great writers imbibing their grandparents' linguistic and imaginative capabilities; here too the protagonist is shown to have learnt from his grandmother the ability to convey ideas and feelings in beautiful and expressive phrases and images. Look at the following construction: "The crystalline beauty of the icicles was gradually taking on shapes, sparrows having the fill of snow with their pecking beaks, monkeys dangling from the tree branches releasing a brief spurt of snow, people walking steady now to suddenly lie flat in obeisance to some unseen power, only to be stunned was a snow missile fired from somewhere as they got up." The present spell of snow stirs his childhood memories, and he goes lyrical quite like his grandmother. Since the grandmother was a simple lady who, in all probability, would use uncomplicated words and expressions to convey her ideas and feelings, but her language still sparkled because it had a natural rhythm and authentic flavour. The child, now fully grown up, has picked up some of her grandma's linguistic felicities, and the moment it snows, he is overwhelmed with emotions, and his words, expressions, and imagery do full justice to his emotional state.

The situations, places and people in the story may look familiar to anyone who has either lived in Shimla or been a regular visitor to the town. The walk from the Summerhill to the Mall and onwards up to Sanjauli makes many of us nostalgic. Despite the bus service available between these places, a lot many people love to walk on foot, enjoying the easy pace and serenity of the surroundings. The charm of this route is enhanced by the presence of many tea stalls and general stores, selling daily necessities. The greatest appeal of these tea stalls lies in the personal touch and intimacy of interaction that is invariably seen between the shopkeeper and the customer. At times the quality of tea may not match the expectations, but the feel of the place and the ease with which one could sit there for an extended period of time makes the experience unique. One such stall found favour with the young protagonist and his friends, where they would routinely enjoy hot cups of chai and occasional samosas several times a day. One may not even realize, but the experiences of this kind gradually cement an unspoken bond between the place, the people who manage it, and the visitors. After the college days, friends stopped visiting the tea stall, but the current spell of snow ignites the protagonist's memories relating to their favourite haunt. The protagonist, driven by some inner compulsion, finds himself walking into Anamika, their *adda* for tea and stimulating discussions. This place had the resonances of a Bengali *adda* where intellectuals gather, sip cups of tea, and discuss a range of topics from politics to poetry. After a lapse of ten years, not only did the place but also its people looked a tad different. The protagonist's attempt to seek the past familiarity is almost rebuffed by the owner's heartless indifference. He tries to swallow his pride and overlook this apparent coldness, but the subsequent interaction too offers no comfort. Things deteriorate further when he displays impatience and wants to be served quickly. The same service boy, who was once forgiven by the narrator for his innocent immaturity despite spilling tea over his trousers, looks indifferent, almost contemptuous. This is too much for him to bear, and the narrator walks out stupefied at this unexpected and rather unpleasant turn of events. The past memories which felt so heartwarming and life-enhancing a moment ago suddenly lose their meaning: *"Channnnn…….!!!!* A past memory was suddenly shattered like a glass, its unheard smash exploding my innards and very spirit." A beautiful past ceases to exist that very moment. In a hugely embarrassing and painful realization, the narrator learns that a memory we cherish and nurture for years together may have no meaning for someone else. At times there is a wide chasm between our perceptions, and if we don't learn to negotiate this gap, it may have devastating consequences for us. In that moment of frustration, the snow begins to look grotesque, loses its whiteness, softness, and purity, and even looks ugly, littered with scraps and leavings: "Sand and gravel had blotted the whiteness and purity of snow. Numerous feet had flattened its softness. Shopkeepers had littered it with scraps and leavings from inside. Someone had also peed on it. I turned back homewards as the snow abruptly began to look grotesque…."

The writer succeeds in capturing the state of shock and disbelief in a masterly fashion by projecting the inner turmoil of the protagonist on to the world outside, more particularly on the pristine snow, which now loses its luminosity and begins to look ugly. A situation of this kind may arise when one person flows with time, changes, evolves, and adapts to the changed realities of life, while the other one remains stuck in the past, hopelessly wallowing in nostalgia. A shopkeeper, for instance, gets a chance to meet a significant number of customers routinely. Even students who visit them frequently depart once they pass out, only to be replaced by a new set of students. Shopkeepers in general don't nurture a special bond with any particular set of customers. For them it is business, a question of livelihood, but for a customer, it may be indulgence. So, their approaches are in sharp contrast to each other's. While a shopkeeper takes things in his stride and doesn't fuss about emotional issues, a sensitive person may experience emotional trauma in such situations. Since educated customers have language at their beck and call, they find it easy to intellectualise, blow it out of proportion, and feel things much more intensely than they should.

The story explores the fragility of an individual's inner world that doesn't have its own resources for nourishment, but merely feeds on the mercy of others, or on the possibilities of love or indifference they receive from a relative, friend, or acquaintance. The story is peopled with numerous characters having unique attributes, but they just provide a context and background, and have hardly any bearing on the direction or the thematic focus of the story. Except for the grandmother whose pervasive presence lends beauty and charm to the story, no other character leaves any lasting impression on the mind of the reader. One could question the writer why he chose to tantalise the reader with hints of so many different characters when he had no intention to delineate them in detail. Possibly the answer lies in the fact that their primary role is to contextualise the flow of the story, without having a bearing on the final outcome. That may be one of the possible reasons why the writer takes no pains to develop these characters beyond a certain point. Besides the narrator, it is only the grandmother who gets the writer's attention. With a few careful strokes, the writer brings her alive in a masterly fashion.

Some readers may have issues with the abrupt ending of the story, but I believe it is in line with the unanticipated and appalling shock the protagonist receives from the people who would otherwise evoke happy memories in him. It was totally unforeseen and so justifiably dealt a stunning blow to the narrator. By not offering a detailed peep into the protagonist's mind, the writer left it to the reader to feel the impact of the shock, depending on their understanding of the situation, or their level of sensitivity. The writer succeeds in his attempt to build a situation whose enormity can plausibly leave inerasable scars on the psyche of the individual who happens to undergo this kind of humiliating experience. The story offers a rewarding experience with its well laid out structure and a clear progression of thought.

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Audience's Live Response

Kamayani Vashisht: Thank you sir! Nostalgia is always beautiful…becomes better when it enters literature.

Girija Sharma: A slice of life…narrated in a manner most engaging with a lot of involvement…Congratulations Janesh.

Shikha Kapoor: Thoroughly enjoyed the story and the analysis. I guess most of us can identify with the emotions of the narrator.

Anjali Parmar: Thank you sir. A wonderful and lively portrayal of life.

Archana Sharma: Old memories well-knit into a story.

10

RADHE RADHE

Kamayani

Nirmalaji didn't take any of my calls that morning. It had become a ritual for me to call her every day on the way back from my walk. Not that I had been very close to her all my childhood and youth, but now on the wrong side of forty, I felt a newfound empathy for older people. I had, for most of my life, thought of her as the most crooked member of our extended family—clever, ambitious and selfish. Her smile had never convinced me of her love for the children of the family. It was never as exuberant as it was when she greeted her political colleagues.

I remember her husband busy himself with visits to hospitals, diligently filling prescriptions, bills and reimbursement forms. He would take out his files every day, reshuffle his papers, arrange them neatly and tuck them back safely under lock and key. And then, one day he died. Nirmalaji cried and cried. I wasn't moved by her crying. My mother told me; people have their own ways of mourning. She was sure Nirmalaji was sad; very sad.

This was about thirty-three years ago. Nirmalaji dedicated her life to the service of people in her district ever since. People sympathized with her childlessness even more. If only she had a son, she would feel more secure. Even a daughter would do. Her husband's demise freed her up of the domestic duties that had irked her sense of liberty. She had a lot of time on her hands now and she felt compelled to dedicate it to the service of society.

She had had to tone down the fire she was known for when she was in college. She was the first-ever girl to be elected President of the Students Union in the town's college. She dreamt of a bright political career. Senior leaders who visited the state always set aside time to get introduced to her. She radiated confidence and energy when she spoke on stage! Many, including her, imagined she'd become the country's

Prime Minister one day. Men around her were either scared of her or in love with her. She was called *Durga* by her political allies, and she believed it.

Somewhere along the way, marriage happened. She strived to keep *Durga* alive. But home and dreams never agreed with each other. She had never reckoned she would struggle to balance them out. She knew her priorities. Nation first! But priorities have a way of reshuffling themselves. She had imagined bedazzling the world with her large *bindi*; a symptom of the complete woman. After the first week of wearing red and pink silks, she realized her credibility lay in the *khadi sarees* and *Kolhapuri Chappals*. She started carrying a packet of smaller *bindis* in her *Jhola* and just before entering the party office, the size of the dot on her forehead would dramatically alter, and so would the look in her eyes.

Her colleagues, however, failed to reciprocate her enthusiasm. So, she worked harder than ever to prove her commitment to the nation and her capability to become…well, a worthy member of the party, if not the Prime Minister. Home would wait for her like a hungry, abandoned child. She began to despise all children. The very sight of them was disturbing. Marriage was expected to yield fruit. But she aspired to mother the entire nation!

And then, love happened. During one of the month-long training programmes of party workers, she met "J" who unlike the man at home, shared her idealism and participated in her dreams. They would sit in the lawns of the alien town every evening after the day's routine and discuss political thinkers. She knew there were limits she couldn't cross, and yet it was always a thrill to sneak under the barbed wire. The evening sessions gradually changed flavor without the political thinkers getting a whiff of it.

By the time she returned home, she was only half home. This man, the hungry home and the children waiting to be born were more repulsive than they had been a month ago. "J" and "N" wrote letters to each other with feverish frequency and then, they suddenly stopped. Nirmala would never know why. Routine took her in, and "J" became a thought that sneakily crept upon her once in a while, when her husband found the courage to demand her body. She had begun to forget what he looked like, but whenever he visited their bedroom, "J" was the sparkling Adonis they had jauntily read about the evening before they parted.

She managed to prove her commitment to her leaders; enough to be given a ticket in the state legislative elections. The pre-election campaign was exhilarating; just as she had dreamed. She felt like the *Shakti* that would save the world from doom. She wanted people to believe in her and they did! The man at home, his angry old mother and the children waiting to be born kept themselves busy with complaints and other chores. He contemplated *Sanyas* but that would affirm people's apprehension that the children waiting to be born, were not born because of him. So, he stayed and distracted himself with visits to doctors, prescriptions, Gold *bhasms* and *Shilajeet.*

She lost that election and subsequently, favour with party seniors and popularity amongst the townsfolk. But it all resurged when her husband died. She rose again— the Phoenix of a strong woman who never lost sight of her dream of serving the people but had to take a break from politics to tend to her ailing husband. Walking the streets of the town, talking to people in the villages gave her a new surge of life, the gasp of breath she needed to fill her aging lungs with. She fought another assembly election and lost. This time she felt broken, in spirit and in body as postmenopausal osteoporosis set in.

She sought refuge in *Bhakti*. After all, from nation to God is a noble trajectory to follow. As years went by, Nirmala increasingly found herself spending her days confined to her room. The hungry house needed her no more. She needed it. The life-sized calendar in her room was no longer a picture of her party's founding father. It had been replaced by one of Radha and Krishna. The calendar had expired ten years ago but who cared! Gods don't have limited shelf lives. All day she would drag herself around the house, managing the cooking, managing the cleaning, and managing her bones, all the while chanting "Radhe Radhe".

She gave me a call one morning, last year. I took the call unwillingly. She cried and I cried. I hated her for making me cry with her version of her story. I wondered why she had picked me to narrate her life to. I was two generations her junior. She said she always thought I understood her…even when I had been seven years old. She said she always wanted to explain herself to my angry eyes. I struggled to preserve my opinion of her, yet found myself calling her every day after that call. I waited for her to drop more details about herself and found myself sharing many of my own, ones I did not know existed.

Of late, she had started complaining of lack of sleep. Some days, we'd laugh. Some days, I'd struggle to keep the conversation going.

Some days, I wished she would completely fade into oblivion. But that day, when she did not take my calls, I got worried. I tried getting on with my day but when I couldn't, I mailed out an application for leave.

On reaching her place and getting in by the rear door, I found Nirmalaji lying on the floor, crying and unable to move. I helped her get up and reach her bed. She had obviously broken her leg. She was visibly distraught and shaken. Having called for an ambulance, I sat her down to prepare for the hospital. She wanted me to change her clothes.

"How did you fall, *Ammaji*?"

"I had climbed on the chair and the chair flipped"

"What were you doing trying to climb the chair so early in the morning?"

"I hate this Radha. I wanted to beat her up with my walking stick, but the chair flipped.

It dawned on me that she had dragged a chair to the wall and climbed on it to beat up the Radha in the calendar.

"Radha?" I asked, feeling perplexed. "Why would you want to do that?"

"She doesn't let me sleep. These two perform *Rasleela* all night long. It annoys me so."

"Hmm" I said, "We'll see what to do about them later. Let's get you to hospital first."

"Not them…only her. Make sure she's not there when we return. I want only Krishna on my wall."

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Review of "Radhe Radhe"

JANESH KAPOOR

We are now so much aware about the stereotyping of characters against the backdrop of social norms and how we are compelled, always or at least at times, to surrender to such stereotypes. It is the story of a failed life (which partakes of personal aspirations and interpersonal relationships) and the need to have at least one relationship or association with some other member of the human world through which we can find self-expression [or, maybe self-realization in a different dictum] – and age or gender need not be a bar to it, as is amply borne out by the initially unstable and then apparently a need-based association between the narrator and her protagonist, Nirmalaji, which is gradually transforming into a life-bond between the two. Of course, "Radhe Radhe" is, in a manner of looking at it, a story about human existence, and we do need a human environment to exist (I don't mind if such an expression sounds aphoristic, just as it might be viewed as a feministic (given the centrality of a woman protagonist and her life) as well as a post-modernist narrative as it interrogates and fails to arrive at any plausible answer or solution with regards to every situation and relationship, whether it be on the personal or familial front, something which is very much rooted in the immediate context of the story and therefore quite authentic.

The thematic context of the story offers an interesting contrast with the story by Rajan Kaushal, "Why Fear…" which was presented in the same session. While Rajan's story emphasizes the Divine schemata behind what happens in life, Kamayani's protagonist, Nirmalaji, again assumes existential dimensions in that she is not afraid to make choices of her own, which appear to be drastic from the social stand point, and she is left near alone to grapple with their consequences. Since we do not really know how to respond to a personage like Nirmalaji, we take refuge in self-acclaimed pity for her. For instance, after her husband has died, we pity her thus hoping that she would realize the 'blunders' she has been committing all along: "People sympathized with her childlessness even more. If only she had a son, she would feel more secure. Even a daughter would do". Maybe Divine Will matters or maybe it is a design to make people conform to the social structure….

In consonance with the thematic matrix of the story – a woman refusing to succumb to social stereotypes of a wife, daughter-in-law and a mother, or at least, attempting to carve out and establish her own identity despite her circumstances, the narrative tone is loaded with irony – pretty pungent at times. As a case in point, the sections which describe the relationship between Nirmalaji and her 'unnamed' husband may be taken up here. As the narrator states at the outset, "I remember her husband busy himself with visits to hospitals, diligently filling prescriptions, bills and reimbursement forms". The irony embedded in the description of her husband indicated that theirs' was a failed, insignificant relation from the very beginning, or that no attempt was made to build or resurrect it, except for occasional physical encounters [which too were doomed to fail – "he stayed and distracted himself with... prescriptions, Gold *bhasms* and *Shilajeet*" – and we later find our protagonist taking refuge in the romantic image of "J", who "was the sparkling Adonis" of her imagination]. And then Nirmalaji's husband dies 'unnamed'. In fact, no other character in the story is named except for the metonymic "J", whose brief association with her remains an unfulfilled, fleeting association for Nirmalaji's being. The reference to our protagonist as "Nirmala(ji)" keeps her a distant, sulking member of her social as also of her political tribe which she wished to use as launch pad for her ambition to 'mother' her nation, although her own children (that is if she desired them, we are not sure of that) remain "waiting to be born". Yes, the irony involved is pretty pungent as we can at times experience the stench emanating from Nirmalaji's existence, which might in turn be the stench of some un-owned, dumped aspect of our own being. She envisions herself as '*Durga'* and '*Shakti'* to enable herself as 'Mother of All' and this makes her political colleagues either love [respect] or fear her, but perhaps never approach and appreciate her as another human being. And just as the mythological 'Durga' reincarnates herself in different forms to battle the demons and protect her children, Nirmalaji is also constantly changing her 'avataars'. Her large red bindi and her silk sarees are thus substituted with *khadi, jhola* and small bindis to fulfil her political incarnation of the saviour of the nation. Such a transformation is deep-dyed in irony as, while it represents the desire to fulfil her own self, it also signifies her acceptance of political stereotypes to be able to do so in sharp contrast to the decrying of stereotypes – the key tone of her character.

Irony is also embedded in the narrative progression in the story, which is presented as a linear one on the surface but eventually characterized Nirmalaji's existential space paradigmatically. The linear progression in the narrative may be listed thus:

"Somewhere along the way, marriage happened"

….

"And then, love happened"

….

"[Then] [s]he sought refuge in *Bhakti*. After all, from nation to God is a noble trajectory to follow".

The events that 'happen' along the way are not a part of the schemata of things Nirmalaji has herself desired. These are merely contingencies of life that must be met with only to move on. However, when the apparently linear sequence of events is mapped out spatially to collate her life and being, the movement [hesitant to describe it as a transition] from husband to lover to God embodies the existential void in her existence which she fails to fill up because of the unbridgeable hiatus between the kind of life we desire and the one which is thrust upon us. [Maybe, some kind of balance, as proposed by Shivani in her story of the same title is required. I am reminded of lines by a senior poet from our state, Sh. Om Prakash Saraswat ji, in this context:

जब यह विश्व तुम्हारी आकांक्षाओं का विस्तार नहीं / तब तुम कैसे कामना कर सकते हो / सभी नागफनियों के कमल होने की ? (Writing from memory, might not be his exact words). As such, it may be observed that irony is the informing principle of the narrative structure of Kamayani's story.

The problematic construction and collapse of Kamalaji's world should be analysed from a woman' perspective for an authentic understanding thereof; however, I do not feel qualified to do so. The story and the protagonists' mental and emotional environ may be aptly approached and analysed from Erich Erikson's formulation of 'inner' and 'outer' space as characterising the basic orientations of women and men respectively towards life and its processes. According to Erikson, a clinical psychologist, who based his inference on the basis of play blocks given to a group of girls and boys and their subsequent 'constructs', women predominantly exist in the 'inner' sphere, which is well-ordered and peaceful, like the interior of a house [of course, such an assumption might be justifiably contested by feminist theorists who might ascribe the tendency exhibited by girls to social conditioning, but Erikson tends to view is as a distinguishing trait] as compared to the 'constructs' by boys which were mostly outdoor, automotive objects. While we may not dwell on the relative merits or demerits of Erikson's experiment, the potent idea being projected here is that the life-matrix of a woman is deeper and more precarious compared to that of a man. Kamalaji's life, her emotions and expectations fail to be internalised and integrated with her being as they are constantly thwarted by the existing social structures in familial, social and institutional domains, which lack an imaginative apparatus to allow scope and space for the individualisation of sorts that she seeks.

The use of language, words and expressions, used like a refrain, reinforce the formal progression in the story as well as its structurality. As a prominent example, let us look at the expression pertaining to children "waiting to be born" once again in this context. The expression is repeated twice and is subtly reincorporated into the narrative pictographically by positing 'home' as 'hungry, abandoned child'. The expression is specifically an epitome of the discontinuity between the desired and ascribed existence of Nirmalaji while ironically establishing the stereotypes one is expected to enact as a part and parcel of social existence and the apathy and patronizing sympathy that it entails from the social environs which keeps on changing in tone and tenor. The sentences are short and run-on syntactically which is suitable to establish the narrative contingency of the story. The use of such sentences keeps the reader alert and connected to the narrative tone and is also characteristic of the unwieldy gap between expectation and reality of individual and social existence.

However, the run-on or the run-through nature of the narrative also gives rise to some stylistic lapses. For instance, the reference to the angry mother-in-law should have been introduced early and integrated with the husband, hungry home and children waiting to be born in the course of the narrative. However, the reference is made casually, only once and remains a loose strand. Similarly, the 'Adonis' motif (or label it as you like it) with regards to "J" is a passing reference and fails to impress upon our perception of either the protagonist or of "J". The calendar bearing the couple-image of Radha-Krishan, which/who do/does not have a shelf-life, and is used both as a motif (text-centric) and an archetype of eternal or unfulfilled (?) love could also have been introduced in Nirmalaji's bedroom as the backdrop of her physical encounter with her husband while cohabiting with her ideal love on the imaginal plane. This would have enabled the emergence, disappearance and re-emergence of the motif/archetype with a symphonic ardour and force to encapsulate Nirmalaji's life saga in a blended, conglomerate manner towards the conclusion of the narrative.

Audience's Live Response

Namrata Pathania: A bold narrative of the dilemma of a woman who wanted to excel in her chosen field...who somehow keeps the illusion of perfect marriage going on... Great Kamayani.

Irene Rattan: That was a stupendous first attempt.

Sapna Pandit: That's what story tellers do. Restore order with imagination... Congratulations Kamayani... beautiful narration.

Girija Sharma: That was a beautiful story Kamayani... inimitably narrated in an idiom uniquely yours...with a great sense of theatre too...Congratulations!

Anita Sharma: Bold theme, beautiful portrait of a woman trying her best to live life at her own terms, simple clear short precise sentences, narrated with clarity and confidence! Congratulations Kamayani! Looking to have few more stories from you.

11

ममा

कुँवर दिनेश सिंह

अखिल अभी सात बरस का नन्हा, अबोध बालक था। दूसरी कक्षा में पढ़ रहा था। अक्षर-ज्ञान से कुछ आगे बढ़ रहा था। अपने आसपास की दुनिया के रंगों को पहचानने लगा था; रंगों की अपील को, रंगों के भेद को, बूझने लगा था। अभी कुछ दिन पहले उसकी माँ उसके लिए सुन्दर सी गुलाबी रंग की टी-शर्ट लाई तो उसने उसे पहनने में आपत्ति जताई। कारण पूछा तो कहने लगा, "ममा, पिंक कलर तो गर्ल्ज़ का है, मैं तो ब्लू या व्हाईट कल की टी-शर्ट लूँगा..." माँ देख रही थी कि बच्चा कुछ-कुछ बड़ा हो रहा था; अपनी पसन्द-नापसन्द रखने लगा था। न केवल रंगों की पसन्द करने लगा था, बल्कि टी.वी. सीरियल्ज़ में भी रुचि लेने लगा था। उसे अब पहचान थी कि किस चैनल पर कौन सा सीरिअल आता था, कौन किरदार थे और कुछ-कुछ सीरिअल में कथानक के प्रवाह को भी समझने लगा था। उसे लगभग सारा घटनाक्रम याद रहने लगा था। चरित्रों को, पात्रों को, समझने लगा था; उनमें तुलना करने लगा था। यही नहीं, अब वह अपने आस-पड़ोस के लोगों को भी विश्लेषण व तुलना की दृष्टि से निहारने लगा था। सभी को बड़े ध्यान से बाँचने लगा था; उनके आचार-व्यवहार को परखने लगा था। पड़ोस की आँटी को देखता तो उसको अपनी माँ की तुलना में देखता। स्कूल में मैडम को देखता, तो मन ही मन उससे अपनी माँ की तुलना करने लगता।

अखिल की माँ को उस दिन बहुत आश्चर्य हुआ और परेशानी भी हुई, जब वह स्कूल से लौटने पर उससे पूछने लगा — "ममा, आशीष की ममा कैसी अच्छी बनकर रहती है... आप वैसी क्यों नहीं रहती?"

"अच्छी का क्या मतलब? मैं अच्छी नहीं हूँ?" माँ ने बड़ी हैरानी से, थोड़े ऊँचे स्वर में पूछा।

"अच्छी हो... पर... वो... वो सुन्दर-सुन्दर माला पहनती है... बिंदी लगाती है... चूड़ियाँ पहनती है... लिपस्टिक लगाती है... आप क्यों नहीं लगातीं... ?"

"देख, बेटे... इन सब चीज़ों से कोई अच्छा या सुन्दर नहीं हो जाता... अच्छाई और सुन्दरता तो इन्सान में अपने अच्छे गुणों से आती है... पर तू क्यों सोचता है ये सब... बड़ों की बातों पर ज़्यादा ध्यान मत दिया कर, समझा... ?" माँ थोड़ा डाँट कर बोली।

माँ की डपट से अखिल थोड़ा सहमकर चुप रह गया। माँ ने उसे खाना खिलाया और फिर होमवर्क पूरा करने का आदेश दे दिया। वह ख़ुद अखिल के साथ बैठ गई और जब तक सभी विषयों का कार्य पूरा नहीं हो गया, उसी के साथ बनी रही। सारे कार्य में एक घण्टे से अधिक समय भी नहीं लगा। सरकार की नई नीति के अनुसार अब बच्चों को ज़्यादा होमवर्क नहीं दिया जा सकता था। ज़्यादा से ज़्यादा कार्य स्कूल में ही करवाया जाए, ऐसे निर्देश जारी हो चुके थे। इससे बच्चों को खेलने व अन्य पाठेतर गतिविधियों के लिए समय काफ़ी बच जाता था। अखिल की माँ ने उसे ताना मारते हुए कहा — "अब एक तो तुम लोगों को होमवर्क भी ज़्यादा नहीं मिलता, बस खेलने की तरफ़ ध्यान रहता है या फिर टी.वी. पर या फिर ऊल-जलूल बातों पर..."

अखिल ने माँ की बात पर ज़्यादा ध्यान नहीं दिया। और होमवर्क ख़त्म होते ही तुरन्त बाहर मोहल्ले के बच्चों के साथ खेलने के लिए जाने की इजाज़त माँगने लगा। इधर माँ ने इजाज़त में सिर हिलाया और उधर वह दौड़ पड़ा घर से बाहर...

पति की अचानक एक कार-एक्सीडेंट में मृत्यु हो जाने के बाद बच्चे के पालन-पोषण के साथ-साथ अन्य कई जिम्मेदारियों का बोझ अब अकेली निशिता के काँधों पर आ गिरा था। सास-ससुर गाँव में पैतृक घर में रह रहे थे। वे वहाँ की थोड़ी बहुत जो ज़मीन थी, उसकी देखरेख करते हुए बुढ़ापे का वक़्त निकाल रहे थे। महीने में एक-आध बार बहू व पोते से मिलने शहर भी आते। और बीच-बीच में, विशेषत: स्कूल से बच्चे की छुट्टियों के दिनों में और त्यौहारों के अवसर पर निशिता भी अखिल के साथ गाँव जाती रहती थी।

निशिता शहर में बच्चे की पढ़ाई व पालन-पोषण की जिम्मेदारी निभा रही थी। पति राज्य के लोक-निर्माण विभाग में अभियंता थे। अपने जीते-जी वे दो-मंज़िला एक मकान बनवा गए थे। ऊपर की मंज़िल अपने लिए रखी थी। नीचे की मंज़िल में दो-दो कमरों के तीन सैट थे, जो किराए पर दिए हुए थे। इनसे जो किराया मिलता, उसी से निशिता अब सभी ख़र्चे पूरे करती। एक तीसरी मंज़िल अभी बननी बाक़ी थी, मगर अभी यह निशिता के वश में नहीं था। उस की एकमात्र प्राथमिकता थी बेटे अखिल की अच्छी पढ़ाई और परवरिश।

अखिल की बातों पर निशिता ने थोड़ा ग़ौर किया, मगर फिर एक अबोध बच्चे की जिज्ञासा मात्र मानकर उसे गम्भीरता से नहीं लिया। उसे बिल्कुल समझ नहीं आ रहा था कि वह अखिल को किस तरह समझाए कि भारतीय समाज में एक विधवा को शृंगार करने की इजाज़त नहीं है। मगर उस छोटे-से बच्चे को वह समाज के क़ायदे-क़ानूनों व बंदिशों के बारे में क्या बता सकती थी। उसे ख़ुद समाज के नियमों को ठीक से समझना मुश्किल हो रहा था तो

उस नन्हे बच्चे को क्या समझा सकती थी। वह तो सिर्फ़ इतना जान पाई थी कि समाज के रीति-रिवाज़ों, मान्यताओं, विश्वासों व नियमों का शांत रहकर पालन करते रहो तो जीवन शान्तिमय ढंग से चल सकता है और यदि इनमें कहीं कोई टकराव पैदा हुआ तो अशान्ति तो होगी ही, जीना भी दुश्वार हो जाएगा। और समाज पुरुष-प्रधान होने से परम्पराओं व नियमों के पालन का सारा बोझ महिलाओं पर रहता है। पति के न रहने के बाद एक बेवा का तो मानो आधा अस्तित्व ही समाप्त हो जाता है। और जब आर्थिक स्वातन्त्र्य न हो तो आधा क्या, अपना अस्तित्व तो रह ही नहीं जाता; वह तो आश्रित हो जाती है दूसरों पर, परिवार पर, समाज पर। जहाँ कहीं वह लीक से हटी, समाज उससे (आत्म-)सम्मान के साथ सिर उठाकर जीने का अधिकार ही छीन लेगा।

वैसे आस-पड़ोस में बड़े समृद्ध घरों से कुछ नारी-मुक्ति संगठनों से जुड़ी महिलाएँ निशिता को उत्साहित करतीं एक सामान्य जीवन-यापन के लिए, किन्तु वह उनके बहकावे में न आती। उसे उनके वचनों में प्रचारबाज़ी ज़्यादा दिखती; सही चिन्ता व सहानुभूति कम दीख पड़ती थी। मन तो निशिता का भी करता कि वह भी रंग-बिरंगे, चटकीले कपड़े पहने, गहने पहने, शृँगार करे — एक समय था जब वह प्राय: जैसे कि करवा-चौथ के दिन, तीज-त्यौहार के मौक़े पर, नवरात्रि-दिवाली व बसन्त-पंचमी जैसे पर्वों पर और विवाहादि उत्सवों में ख़ूब शृँगार करती थी — सोने से लदी तो वह कैज़ुअली भी रहती थी। किन्तु कुछ वह समाज के भय से, अशान्ति के भय से, परम्परा में ढल गई और कुछ वह अपने पति के प्रति प्रेम व आदर के भाव के कारण भी समाज के चलन को स्वीकार किए हुए थी। लेकिन एक बच्चे को यह सब समझा पाना अभी सम्भव नहीं था। वह बेचारा तो अभी तक ठीक से पिता की मृत्यु को भी समझ नहीं पाया था।

निशिता सोच में डूबी हुई थी कि अचानक डोर-बेल बजी। उसने दरवाज़ा खोला, अखिल खेल कर थका-थका वापिस आया था। साँझ ढल गई थी। सूर्य रोज़ की तरह आँखें मूँदे प्रयाण कर चुका था। अखिल टी.वी. पर कार्टून नेटवर्क लगाकर बैठ गया। निशिता रसोईघर में रात के भोजन की तैयारी में लग गई। खाना पकाने में पौना-एक घंटा लग गया। इस दौरान अखिल कार्टून प्रोग्राम देखता रहा। खाना तैयार हो जाने पर निशिता घर के एक कोने में पूजा के लिए बनाए स्थान पर साँध्यकालीन पूजा-आरती के लिए गई। उसने अखिल को भी वहाँ बुलाया। दोनों पूजा के लिए बैठ गए। माँ ने जोत-अगरबत्ती जलाई और आरती गाई, बेटे ने भी ताली बजाकर साथ दिया। इस नित्य सन्ध्या के बाद दोनों ने भोजन किया। निशिता कुछ देर टी.वी. पर महिलाओं से सम्बन्धित सीरियल देखने में व्यस्त हो गई।

उधर अखिल अपनी छोटी-छोटी कार, जीप और अन्य खिलौनों के साथ खेलने में मशग़ूल हो गया। मुँह से गाड़ियों के चलाने की हुम्-हुम् की आवाज़ें भी निकाल रहा था। माँ

बीच-बीच में टोकती कि आवाज़ न करे। मगर अनजाने ही रुक-रुक कर वह फिर आवाज़ें निकालने लगता। माँ के व्यस्त रहने पर वह अक्सर इसी प्रकार ख़ुद ही में मस्त रहता।

टी.वी. पर सीरियल के ख़त्म होने पर निशिता ने अखिल से भी खेल बंद कर अब सो जाने को कहा। अखिल ने सोने से पहले एक कहानी सुनने की इच्छा व्यक्त की। उसके दादा-दादी भी उसे कहानी ज़रूर सुनाते थे। माँ ने भी पंचतन्त्र की एक छोटी-सी कहानी सुनानी शुरू कर दी। मगर अखिल ने यह कहानी पहले भी सुनी हुई थी। उसने कोई नई कहानी सुनाने का आग्रह किया। माँ का मन कुछ उखड़ा हुआ था। उसने अखिल को थोड़ा डपट कर वही कहानी सुनने को कहा और ज़ोर दिया कि वह जल्दी सो जाए। उसे सुबह भी जल्दी उठकर स्कूल जाना था। कहानी सुनते-सुनते उसे नींद आ गई और कुछ देर में निशिता की भी आँख लग गई।

अगली सुबह फिर से वही दिनचर्या प्रारम्भ। प्रात: जल्दी-जल्दी अखिल को तैयार कर आठ बजे निशिता उसे स्कूल तक छोड़ने के लिए निकल पड़ी। स्कूल क़रीब एक किलोमीटर की दूरी पर ही था।

रोज़ की तरह बेटे को स्कूल छोड़कर लौटने पर वह घर के सारे काम निपटाने में लग गई। दो बजे फिर से अखिल को स्कूल से लाना था। जल्दी-जल्दी काम निपटा कर निशिता पौने दो बजे ही स्कूल के प्राँगण में पहुँच चुकी थी। छुट्टी ठीक दो बजे होनी थी। और बहुत-से बच्चों की माताएँ भी वहाँ पहुँच रहीं थीं। छुट्टी के इंतज़ार में सभी एक-दूसरे के साथ चपड़-चपड़ गप्प-शप्प में लगीं थीं। हर रोज़ की तरह चर्चा के केन्द्रीय विषय थे — एक-दूसरे के सूट, साड़ी व गहने, या अपने-अपने बच्चों की बाल-लीलाएँ, या सास-ससुर की आलोचना, या पतियों के तौर-तरीक़े, या फिर बच्चों की क्लास-टीचर पर टिप्पणियाँ।

निशिता भी अखिल के घनिष्ठ मित्र आशीष की माँ के साथ बातचीत में व्यस्त हो गई थी। आशीष की माँ बहुत सुन्दर थी। माथे पर बड़ी-सी लाल बिन्दी उसकी आभा को चौगुना कर देती थी। दोनों बच्चों के बारे में ही बतिया रही थीं। एक दम घंटी की आवाज़ हुई। दो बज गए थे। छुट्टी की घोषणा थी। सभी अभिभावक बच्चों को लेने गेट पर इकट्ठा हो गए। अध्यापकों ने अपनी-अपनी कक्षा के बच्चों को लाईन में खड़ा कर दिया था और एक-एक करके बच्चों को गेट से बाहर भेज रहे थे। पाँच मिनट में अखिल भी बाहर आ गया। वह तुरन्त माँ की तरफ़ दौड़ा और अपना बस्ता उतार कर उसे थमा दिया। साथ ही आशीष भी आ चुका था और उसने भी अपना बस्ता अपनी माँ के पास दे दिया था। अब दोनों बच्चे हँसते-खेलते, बतियाते, हाथों में हाथ डाले, कभी एक-दूसरे के काँधों पे हाथ धरे जा रहे थे। दोनों की माँएँ भी बातें करतीं जा रहीं थीं। बातों बातों में आशीष का घर आ गया। उसकी माँ ने निशिता से कुछ देर रुकने को कहा। पहले तो निशिता ने कुछ आपत्ति जताई, मगर जब आशीष ने भी अखिल को कुछ देर रुकने के लिए ज़ोर दिया, तो निशिता मान गई।

इधर निशिता आशीष की माँ से बातों में लग गई। स्वेटर की बुनाई; किसी डिश की रेसिपी; बच्चों के विकास और कुछ पारिवारिक समस्याओं को लेकर चर्चा चल रही थी। उधर अखिल और आशीष अपने-अपने बस्ते उठाकर भीतर के कमरे में चले गए। दोनों अपनी बातचीत में व्यस्त हो गए। इस बीच आशीष की माँ ने चाय बना दी व बिस्किट के साथ निशिता को परोसी। बच्चों के लिए दो गिलास दूध कमरे में देकर आई। बातों-बातों में पता ही नहीं चला कैसे वक़्त बीत गया। दो घंटे बीत गए थे। घड़ी में चार बजते देख निशिता खड़ी हो गई और आशीष की माँ से चलने की इजाज़त माँगने लगी।

— "अब हमें चलना चाहिए... हमने आपका बहुत समय ले लिया..."

— "नहीं, नहीं, आप ऐसा क्यों कहतीं हैं? बहुत अच्छा लगा आप से बातचीत कर के... और... अखिल और आशीष भी कितने मस्त हो गए हैं... इन बच्चों की भी अपनी ही एक अलग दुनिया होती है..."

— "जी हाँ, यह तो सही है..."

— "कुछ देर और बैठो ना... बच्चों को खेल लेने दो... इन का जी अभी नहीं भरा..."

— "नहीं, नहीं, अब चलना है... अखिल को होमवर्क भी कराना है; थक कर यह सो जाता है, फिर काम का वक़्त नहीं रहता... घर के और काम भी हैं..." ऐसा कहकर निशिता अखिल को आवाज़ लगाती है, मगर बच्चे अपनी बातों में इतने खोए थे कि अखिल को माँ की आवाज़ें सुनाए नहीं दीं। आशीष की माँ और निशिता उन्हें बाहर लाने को कमरे में गईं। निशिता ने अखिल को चलने को कहा –

— "अखिल, घर नहीं जाना है क्या?"

— "ममा, हम वीडियो गेम खेल रहे हैं, कुछ देर रुक जाओ ना..."

आशीष भी कहने लगा, "आँटी, हमें गेम पूरी करने दो ना..."

— "नहीं बेटे, अब देर हो रही है; हमें चलना होगा... अखिल फिर किसी दिन आ जाएगा... (अखिल से) बेटे, अब चलो... अपना बैग उठाओ और चलो..."

अखिल ने उन्मना-सा होकर बैग उठाया और बाहर की ओर को चल पड़ा। दरवाज़े तक आशीष और उसकी माँ उन्हें बाय कहने को आए और उन्हें फिर आने को कहने लगे। निशिता ने भी उन्हें अपने घर आमन्त्रित किया और इसके साथ ही वहाँ से विदा ली।

घर पहुँचते ही, कपड़े बदलने के बाद, माँ-बेटा दोनों कुछ देर टी.वी. देखने बैठ गए। बेटा कोई कार्टून प्रोग्राम देखने की ज़िद्द कर रहा था, मगर माँ ने महिलाओं का कोई कार्यक्रम लगा

दिया था। ऐसे में बोर होकर दस-बारह मिनट में ही अखिल की आँख लग गई। टी.वी. से थोड़ा ध्यान हटने पर निशिता ने अखिल को सोते देख, उसे कम्बल ओढ़ा दिया। और स्वयं उसके होमवर्क और क्लास टीचर से मिलने वाले अन्य किसी निर्देश के लिए उसके स्कूल बैग से डायरी निकालने को बढ़ी।

बैग खोलने पर पहले बाहर की ज़ेब से उसने लंच-बॉक्स निकालकर धोने के लिए अलग रखा और फिर जब उसने डायरी निकालने के लिए बैग के अन्दर हाथ डाला, तो उसका हाथ कुछ ऐसी चीज़ों पर पड़ा, जिनके बारे में वह सोच भी नहीं सकती थी। लिपस्टिक, मस्कारा, बिन्दियाँ और कुछ चूड़ियाँ — ये सब चीज़ें देखकर उसका माथा ठनक गया। आख़िर अखिल ये सब कहाँ से लाया होगा? और क्यों लाया होगा? किसी के यहाँ से चुराए होंगे? पर किस लिए?

बच्चे की इस हरकत पर उसका मन उद्विग्न हो उठा। उसने तुरन्त सो रहे बच्चे को झकझोरना शुरू कर दिया।

— "अखिल... अखिल... उठ... अखिल... उठ ज़रा..."

अखिल की नींद टूट गई। आँखें मलता हुआ हैरानी में वह माँ की ओर देखने लगा। माँ को ग़ुस्से में चिल्लाते हुए देख वह सहमा-सा, अवाक् था। माँ फिर उसे झकझोरते हुए, उसकी लाई चीज़ों को हाथ में लेकर बोली —

"अखिल, ये सब कहाँ से लाया है तू?... बता कहाँ से लाया है?"

माँ को झल्लाते हुए देख अखिल डर के मारे कुछ नहीं बोला। उसे ख़ामोश देख माँ को उस पर और सन्देह होने लगा। अब वह उसके कान मरोड़ते हुए पूछ रही थी — "जल्दी बता, नहीं तो सज़ा मिलेगी... तेरी पिटाई करूँगी और वॉशरूम में बंद कर दूँगी..." उसके इस प्रकार बार-बार धमकाने पर अखिल कुछ रुक-रुक कर बोलने लगा –

"आ... आशीष..."

वह अभी कुछ बोल ही रहा था कि उसकी माँ ने प्जिर से चिल्ला कर झिड़का, "क्या? तूने आशीष के घर से चोरी किया ये सब?..." और साथ ही उसने अखिल के गाल पर एक तमाँचा भी जड़ दिया।

अखिल रोने लगा। माँ ने फिर उसके कान मरोड़ते हुए पूछा, "सच-सच बता तूने चोरी क्यों की?"

सिसकते हुए अखिल बोला, "ममा, मैंने चोरी नहीं की..."

— "फिर ये सब सामान कहाँ से आ गया तेरे पास?"

— "मैंने आशीष के साथ ऐक्स्चेंज किया है..."

— "क्या मतलब? क्या ऐक्स्चेंज किया है?"

— "मैंने उसे अपनी ट्वाय ट्रेन दी है... उसके बदले में ये लाया हूँ"

— "लेकिन ये सब किस लिए उठा लाया?"

— "आपके लिए..."

— "क्या? मेरे लिए?... लेकिन क्यों?"

— "वो आशीष की ममा भी तो ये सब लगाती है... तो आपके लिए भी ले आया..."

— "लेकिन तूने ऐसा क्यों किया?"

— "आप भी आशीष की ममा की तरह सुन्दर दिखोगी..."

अखिल के सिर पर हाथ फेरते हुए निशिता ने उसे गले से लगा लिया और कुछ पलों के लिए वह चुप्प सी हो गई।

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कहानी 'ममा' की समीक्षा

बलदेव सिंह ठाकुर

'ममा' कुँवर दिनेश विरचित कहानी है जो इनके प्रथम कहानी संग्रह **"जब तक ज़िंदा हैं"** में संगृहीत एक महत्त्वपूर्ण व मर्मस्पर्शी कहानी है। इससे पूर्व इनके अनेक काव्य-संग्रह, समीक्षात्मक व शोधात्मक ग्रंथ प्रकाशित हो चुके हैं। इन्हें कई साहित्यिक पुरस्कारों से नवाज़ा जा चुका है व इनके सृजन पर कई विश्वविद्यालयों में शोधकार्य भी हो रहा है।

'ममा' एकल (विधवा) नारी व उसके सात वर्षीय बच्चे की कहानी है। माँ का नाम निशिता व बेटे का अखिल है जो शहर में रह रहे हैं। पति की मृत्यु हो चुकी है जो मरने से पूर्व शहर में दो मंज़िला मकान बना देता है। निशिता ऊपर की मंज़िल में रहती है और नीचे की मंज़िल के दो कमरों के तीन सैट किराए पर दिए गए हैं और यही आमदनी उनके गुज़र-बसर का आधार है। सास-ससुर गाँव में थोड़ी-सी खेती भी करते हैं व कभी-कभार उनसे मिलने शहर आ जाते हैं। निशिता ने अखिल को एक किलोमीटर की दूरी पर स्थित विद्यालय में प्रवेश दिलाया है। वह नित्यप्रति उसे विद्यालय छोड़ने व लाने जाती है व इस बीच घर के सारे कार्य भी करती है। कुल मिलाकर 'ममा' का कथानक डेढ़ दिन का है। विद्यालय से लौटकर सायं माँ बेटे को होमवर्क करवा रही है। वहीं से कथानक शुरू होता है। होमवर्क करके बेटा खेलने चला जाता है और माँ इस बीच घर के कार्य निपटा देती है। अखिल खेलकर वापिस आता है तो टी.वी. देखता है। निशिता उसे पूजा-आरती में साथ बैठाती है। अखिल माँ से अपने सहपाठी- सखा आशीष की माँ की तरह सज-धजकर रहने का आग्रह करता है तो निशिता उसे कहती है कि सुन्दरता गुणों सी आती है। भोजनोपरान्त कोई कहानी सुनाकर उसे सुला दिया जाता है। दूसरे दिन निशिता बेटे को विद्यालय छोड़कर वापिस आकर घर के कार्य निपटाकर दोपहर पश्चात् पुन: लाने जाती है और वापिसी में आशीष की माँ के आग्रह पर चाय पीने को रुक जाती है। अखिल और आशीष वीडियो गेम खेलने में व्यस्त हो जाते हैं। चाय पीकर माँ- बेटा घर वापिस आते हैं। शाम को वही नित्यप्रति के कार्य और अखिल कार्टून लगाने की ज़िद्द के पूरा न होने पर उसी मुद्रा में सो जाता है। निशिता टी.वी. पर कोई महिला-कार्यक्रम देखने के बाद बेटे की स्कूल डायरी देखने के लिए बैग खोलती है तो हतप्रभ रह जाती है। यही कहानी का चरम-बिन्दु है। बैग में शृँगार की वो सभी वस्तुएँ थीं जिन्हें अखिल माँ को पहने देखना चाहता था। निशिता क्रोध में अखिल को उठाकर उसे चाँटा तक रसीद कर देती है तो अखिल उसे बताता है कि उसने अपनी ट्वाय-ट्रेन आशीष को देकर बदले में ये चूड़ियाँ, लिपस्टिक, बिंदियाँ आदि ली हैं। अन्त में माँ ने बेटे को गले लगा लिया और कहानी समाप्त हो जाती है।

कथानक का अन्त प्रेमचंद की 'ईदगाह' कहानी की तरह है। चिमटा और सौन्दर्य-प्रसाधन दो वर्गों व काल-खण्डों की आवश्यकताएँ और प्राथमिकताएँ हो सकती हैं। परन्तु उत्तरदायित्व और चाहत का भाव तद्युगीन परिवेश से ही आता है। अखिल अपनी माँ को किसी भी सूरत में अपने मित्र की माँ से कमतर नहीं देख सकता। यदि हामिद मेले में मिठाई न खाकर दादी-माँ को चिमटा लाता है तो अखिल भी अपनी ट्वाय-ट्रेन क़ुर्बान कर देता है। संवेदना का उत्स किस घड़ी और किस चित्तवृत्ति से होता है, इसकी मनोरम छवि प्रस्तुत कहानी में बहुत ही शिद्दत से उभारी गई है। माँ के सज न पाने की विडम्बना ही कहानी को मार्मान्तक बना देती है जिसके द्वारा लेखक ने रूढ़-समाज की वर्जनाएँ बख़ूबी उजागर की हैं। कथा की यह केन्द्रीय पात्र मानती है कि विद्रोह का मतलब अपने को अशान्त करना है— "समाज के रीति-रिवाजों, मान्यताओं, विश्वासों व नियमों का शान्त रहकर पालन करते रहो तो जीवन शान्तिमय ढंग से चल सकता है और यदि इनमें कहीं कोई टकराव पैदा हुआ, तो अशान्ति तो होगी ही, जीना भी दुश्वार हो जाएगा। और समाज पुरुष-प्रधान होने से परम्पराओं व नियमों के पालन का सारा बोझ महिलाओं पर रहता है। पति के न रहने के बाद एक बेवा का तो मानो आधा अस्तित्व ही समाप्त हो जाता है। . . . जहाँ कहीं वह लीक से हटी, समाज उससे आत्म-सम्मान के साथ सिर उठाकर जीने का अधिकार छीन लेगा।" निशिता का यह चिन्तन उसे समझौतावादी नारी बनाता है। वह रुढ़ियों को तोड़ती नहीं निबाहती है। परन्तु यह भी निश्चित है कि निबाहने के पश्चात् भी वह शान्त कहाँ है? क्या समाज के बाह्य घटनाक्रम की संरचना में ढलना शान्ति है? शान्ति तो अन्तश्चेतना से उभरती है जिस स्तर पर निशिता अशान्त है। सम्भवत: कहानीकार विधवा स्त्री की इसी दशा को उजागर करना चाहता है।

कहानी में भले ही पितृ-विहीन सन्तान की एषणा तुष्ट हो गई हो, परन्तु पति-विहीन पत्नी विडम्बनाओं में पल रही है; यही कहानी का मूल संवेद्य भी है। कहानीकार ने केवल दो पात्रों के आधार पर कथानक को गुंथा है। प्रासंगिक पात्रों के रूप में आशीष व उसकी माँ को सृजा है। बाकी सभी पात्रों का मात्र परोक्ष वर्णन है। पात्रों में परस्पर सामंजस्य की समझ है। किसी भी पात्र का दूसरे से विरोध नहीं है। माँ, बेटा, मित्र, मित्र की माँ या दादा-दादी; पात्रों में हर स्तर पर समझौता या संतुलित जीवन-दृष्टि है। शहरी परिवेश का जीवन काफ़ी कुछ औपचारिक भी होता है और यही सब कुछ 'ममा' कहानी में भी चित्रित हुआ है। इस कहानी के संवाद भी छोटे, वर्णन- प्रधान व पात्रानुकूल हैं। निशिता- केन्द्रित अन्यपुरुष शैली के स्व-संवाद कहानी का सबल पक्ष है। लेखक की भाषा पर पूरी पकड़ है। स्थित्यंकन के लिए प्रयुक्त सामान्य भाषा प्रभावशाली है और उर्दू, देशज शब्दों का प्रयोग भी सहजता से हुआ है। मनोवृत्ति को मुहावरों के प्रयोग द्वारा उजागर किया गया है। आप्तवाक्य सदृश अन्यतम कथनों का प्रयोग करके कहानीकार ने अपनी उत्कृष्ट कहानी कला का परिचय दिया है। यथा — "सूरज आँखें मूँदे प्रयाण कर चुका था।" समग्रत: कुँवर दिनेश को भाषा-शिल्पी कहा जा सकता है।

अतएव प्रस्तुत कहानी में परिस्थितियों की विडम्बनाओं में एक समझौतावादी नारी है जो सब कुछ समझते हुए भी विद्रोह नहीं करती है। सम्भवत: लेखक ने समाज की रूढ़-वर्जनाओं को दिखाने के लिए ही कथा की सृष्टि की हो। यद्यपि नारी-समाज का एक वर्ग, भले ही वह कम है, इन वर्जनाओं को तोड़ रहा है। यह भी हो सकता है कि अधिकाँश का चित्रण लेखक का ध्येय रहा हो। यदि बच्चे के चित्रण को केन्द्र में रखकर विश्लेषण किया जाए तो बाल- सुलभ चेष्टाओं व एषणाओं की माँ की ममता के साथ जुगलबन्दी (माँ के चाहने के स्तर पर) भी सुन्दर बन पड़ी है। अत: इस कहानी में जीवन्त परिवेश है जहाँ मानवीय भावों व संवेदनाओं के रेशों का विगलन कर समाजानुरूप रूपान्तरण हुआ है।

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Audience's Live Response

Irene Rattan: Dinesh sir, sensitive portrayal of the trials of a single mother. Commendable.

Leena Vaidya: Very touching stories Suneela ma'am and Dinesh sir.

Rajan Kaushal: Wonderful stories.... very touching... Congratulations Suneela Ma'am and Dinesh Sir.

Savita J.B. Singh: Loved the enunciation and pronunciation of Hindi by the anchor and the story teller... both sounded really good!

Irene Rattan: बलदेव सर, क्या बात कही है आपने। लेखक कहीं ना हो.... वाह!

12

THE HOLE IN THE WALL

MAHI YOGESH

"Why have I kept you? To doze off whenever you find a chance?" thundered Kishan Lal as Rahil woke up with peals of sweat on his forehead. Kishan Lal stood there pointing at the unwashed utensils and babbled, "Who's going to wash these, you or your Abba?" A nervous smile appeared on Rahil's sunburnt face when he realised that he was dreaming and that his secret was still well kept. A dealer had bought and brought fifteen boys, huddled in the back of a truck and Rahil was one of them. Four boys were from his village and eleven from the nearby villages. They were being taken to work in the factories and dhabas in the town. Kishan Lal had paid a handsome sum of ten thousand rupees for him, which his father and the dealer had shared equally. He was ten years old, with a slight frame but was tall for his age. He knew nothing about dreams or childhood. Kishan Lal, a stout man of over fifty, was Rahil's new master and his dhaba was his new home.

His job was to serve tea to the shopkeepers nearby, wash utensils, scrub the floor and obey Kishan Lal with a nod every time he was summoned. There was an old cinema hall in the locality and Rahil went there every day to serve tea to the Babu at the ticket window. He had seen young men loiter around the hall, waiting for their turn as one show ended and the other followed. He so much wanted to see a film, any film, but he knew Kishan Lal would never allow him to be absent from his duties for three long hours. The coloured film posters were all over the front wall of the cinema hall and they were alluring for the little boy. He had tried to catch a glimpse of the inside of the cinema hall by standing close to the entry door but was pushed away by the ticket checker.

He took tea for the shopkeepers at two in the afternoon and had to return with the empty glasses by 3 pm. This was the only time when he was on his own and away from Kishan Lal's sharp gaze and scathing tongue. One day, after he had served tea to the Babu at the ticket window, he went towards the back of the cinema hall to have his fun time there. He threw heaps of dry leaves in the air and urinated on the wall. There were two old film posters on the wall. On the first poster the colour of the heroine's saree had faded away and the other poster was half torn from the top with only two pairs of legs left in the lower half. He picked up a piece of charcoal from the ground and started drawing the upper body on the two pairs of legs. He drew a figure of a bald man with a big paunch on the first pair of legs; he looked at it and had a hearty laugh. Then he thought of making a thin man; so, on the other pair of legs he drew the figure of an extremely skinny person with a tube-like waist and a long face. This little act of creativity filled him with great excitement and he started tearing the poster with the heroine whose saree had faded; he thought, "If I cannot watch a film, I can at least keep an old poster." As he tore off the poster, he saw that there was a three-inch-wide hole in the portion of the wall that the poster had covered. There were loose stones and gravel in the hole which could be easily removed. He delicately removed the small stones and tried to clean the inner surface of this hole. As he emptied the entire contents of the hole, he began to hear some sounds, which were not familiar but exciting. To peep through this opening, he had to stand on his toes. A feeling of sudden exhilaration and joy took over his whole body when he saw that from this miniscule opening, he could see the screen on which a film was running. His eyes grew big. He thought, "Is this real, or am I day-dreaming?" This was what he had longed for and now without even buying a ticket he could see a film and that too at any time he wanted to. He was thinking fast, and then with a sudden feeling of fear he shuddered. He looked all around to see if there was anybody watching him but only found a crow flitting on the branch of a tree opposite the wall. He turned around and stood with his back towards the wall trying to hide the hole. Then he turned around again and peeped through the hole and realised he was standing on his toes. He looked for something that he could stand on comfortably, but not finding anything, decided to relish the movie that was going on. The time passed quicker than he had thought. He knew he had to go. His heart was racing. He thought aloud, "Nobody should know about this opening in the wall". He had to cover it up and make it look as it appeared earlier, before he had torn off the posters. He could not paste the torn posters back but he thought that later in the evening he could bring some rice from the shop to use as adhesive. He furtively started filling the hole with the loose gravel, mud and dry leaves lying on the ground.

Kishan Lal screamed as he saw him coming, "Hey you scoundrel, don't even think of running away. I have paid a lot of money for a scrawny fool like you," reminding Rahil his worth. All these dialogues got mixed up with the ones he had just heard while peeping through his little kaleidoscope. Hiding his joy, he returned to his work thinking of many more films that he would watch in the coming days. It was challenging for him to contain his excitement, but he tried as best as a child can try to hide his gratification.

Every day with a song in his heart, he made his way to the market to serve tea, and then rushed to the secret hideout to enjoy the panorama of the film world that it unfolded. He could never watch a complete film together. He had seen the same scenes from a film till it ran for a week and had later tried to connect different scenes from different films to make a movie of his own. His days were like dreams where he like a poor hero worked at a dhaba and whenever possible escaped into a film. His dreams were made up of the films that he had repeatedly watched. He got confused at how fast the life on the screen moved as compared to the real life he led. He wondered how the heroes in the films wore new clothes in different scenes whereas he had just one extra pair of shirt and pants to change. The worn-out sweater whose sleeves were short for his long arms was the only piece of warm clothing he owned. His mother had woven this sweater for him when he was eight and he still continued to wear it. She had made it with some leftover yarn which was noticeable in the haphazard pattern on his sweater. He could not read and write well. His father had taken a loan from the village landlord to marry off his two elder sisters and now he was bound to the landlord's fields and so was Rahil. He was sent to school at seven and removed after a year to help his father. When he turned ten, he was brought by a dealer to work in the town. He once cried his heart out at the death of the hero's mother and had longed to go home, but giving up this fun was too much for him. Through the films he had got close to a world which was more beautiful and vivid than his own pallid life at the dhaba. He had seen the mountains and the seas which he had only heard of. He felt intoxicated by the show of colour and dance and woke up mumbling the dialogues of a film.

One afternoon, while filling the glasses he spilled some tea on the floor. Kishan Lal noticed his exuberance. But today he said nothing, just stared hard. There were customers in the shop and Kishan Lal did not want to make a scene; so, he let him go. In the evening Rahil cleaned the tables and helped Kishan Lal to close the shop. As he turned to go to bed Kishan Lal gave him a forceful jerk and he almost fell to the ground. He pulled Rahil by his neck, brought him close and stared in his eyes. "Why are you in such a hurry to run to the market Hmm? Don't forget, my eyes are on you!" Rahil could not utter a word; Kishan Lal pushed him to the floor and left the shop. Rahil nearly choked and realised that at any cost he had to keep his secret safe. "I will lose my greatest amusement, if someone came to know about my wanderings, caught me peeping through the hole and reported it to Kishan Lal," he thought. So, he reduced his visits to the compound and decided to go there only thrice a week. Months passed and a whole year went by and his jubilance grew manifolds.

One morning when he was attending the customers, he heard a discussion about the Cinema Wala. He stood near the table to gather more, "So finally the Cinema Wala has hit the jackpot," one of the customers said. "The cinema hall is old but, the property has value," added the other. Rahil forgot to put the tray down on the table. "What are you listening to? Foolish boy!" screamed Kishan Lal and hit him on his head. The joy he had secretly found had taught him to ignore these fiery rebukes and blows with nonchalance. He had always felt jealous of the Cinema Wala whom he considered to be the happiest person in the world. "He can watch any film for free anytime and he makes money by showing it to others, what more can one ask for and now he has hit another fortune, God is extremely benevolent to him," he thought.

Rahil wanted to be a Cinema Wala when he grew up. By the afternoon, the conversation he had heard in the morning had lost its effect on him. He carried on with his work and as the favourite part of his day arrived, he joyously arranged the tea glasses in the metal carrier and made his way to the shops.

After serving the customers, he tip-toed to the back of the cinema hall. As he approached the place, he heard rustling of the leaves and saw three men inspecting the building and its compound. They had long measuring tapes and were noting down something. He stood there transfixed. All gaiety left him when he found company at his usually deserted hideout. "Have they found out about the hole and have come to mend the wall?" he thought. One of them saw him staring and shouted, "Hey you chokre run away from here." While he crossed the market place, he saw people hanging out in groups talking about the cinema hall.

A week went by and every day when he went to the compound, he found somebody there. The following week, when he went to the market, he heard that the film shows for the day had been cancelled. When he passed by the front gate of the Cinema Hall, he saw a lock hanging there. The bigger gate had a smaller entry to its left which had a broken latch. He sneaked in through that little gate and went towards the backside. Today there was nobody there. He slowly approached the hole and saw that nobody had even bothered to check it in all these days. He removed the loose gravel from the hole and peeped in but there were no colours or sounds coming from the other side. Today it was just a black hole staring at him. He stepped back and felt uncomfortable, the black hole disturbed him. He put the gravel back and hid it.

He could not sleep that night and woke up exhausted in the morning. While washing utensils he heard Kishan Lal say, "What kismet Cinema Hall wala has, inherited property is selling for crores." "Now our town will also have a touch of the big city," his customer added enthusiastically. Next morning while running for errands the people saw an enormous machine with jaws attached to an arm, move towards the market. People left their work and children left their play to follow the machine. Rahil also left his work and ran to see. A huge crowd had gathered outside the cinema hall. He shoved and pushed through the crowd to have a closer look. The sturdy gigantic arm of the machine hit the old building and the structure crumbled before his eyes. The crowd cheered at the demolition but Rahil's legs began to tremble. His face went pale.

He staggered to the dhaba where Kishan Lal was waiting for him, "Abbe, if you have had your fun get back to work or get lost!" The word 'fun' pricked him; he could not tell anyone what he had lost. He wanted to howl and cry but had to hold back. When he went to sleep, he was reminded of his village, the dirty lanes and one room home with no electricity. One day his father had handed him over to the dealer, who brought him in the back of a truck with fourteen young boys and they left their homes far behind to work and live amidst complete strangers at unknown places. He remembered his mother had stood at the door wailing and pleading to his father not to send him. His head began to ache. For a year these memories had not haunted him. He felt sick and wanted to go home to his mother. That night in his dream, he saw the scary machine with jaws approach him while he was peeping through the hole in the wall.

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Review: "The Hole in the Wall"

MEENAKSHI F. PAUL

"The Hole in the Wall" is an engaging story of Rahil, a ten-year-old boy, who was bought by Kishan Lal, a dhaba owner, for ten thousand rupees from a dealer who had 'bought and brought' fifteen young boys from the village. The story begins with Kishan Lal reprimanding Rahil for sleeping while there was much work to be done. This harsh tone of the beginning becomes a foil for the endearing tale that follows. Rahil worked a long day and slept in the dhaba itself. His work included taking tea to the nearby shops and cinema hall in the afternoon. He enjoyed doing this work because "This was the only time when he was on his own and away from Kishan Lal's sharp gaze and scathing tongue." Rahil was mesmerised by the larger-than-life posters pasted on the wall of the hall and longed to watch a movie. The glitz and aura of films fuelled his fantasy and dreams of fortune in the midst of his poverty and squalor. He was driven away, however, even if he peeped in the door of the hall.

Rahil has no friends and his family back in the village had sold him in desperation born of penury. He feels acutely alienated from the world and the magical world of films seems to offer him the only opportunity to escape his mundane existence. One day, he makes his way to the back of the hall for a few moments of solitary fun. He plays with the dry leaves and draws figures on the partly torn and faded posters. He decides to take home the poster of the heroine in a faded saree, for, "If he could not watch a film, he could at least keep an old poster.'" As he peals off the poster, a three-inch hole is revealed behind it. This hole in the wall becomes his passport to the land of fantasy in which he is the hero. The silver screen complements his daydreams and the friendless boy projects his feelings on to the cinematic characters. So, the individual self of Rahil seeks self-definition through fantasy, even as his relational self is marked by the absence of meaningful bonds of attachment. The hole in the wall dhaba in which he is condemned to child labour is displaced by the hole in the wall of the derelict cinema hall that offers him visions of love, adventure and heroism. Rahil's name means 'traveller' and 'to leave,' the dhaba and the hole in the wall map his journey from drudgery to daring.

Rahil watches sporadic scenes of films through the hole whenever he can and then strings what he has seen with what he imagines. The vivid scenes of far-off places, the dialogues and the emotions portrayed by the actors fuel his creativity. Rahil understands the significance of his find and when Kishan Lal becomes suspicious of his alacrity to go to the market, he rations his visits to the treasure hole to three times a week. After a year, Rahil discovers that the cinema hall is being sold. People gossip about it and men get busy measuring and evaluating the property. He peeps through the hole "but there were no colours or sounds coming from the other side. Today it was just a black hole staring at him. He stepped back and felt uncomfortable, the black hole disturbed him." From this point, the story moves towards its resolution fairly quickly. The unravelling of Rahil's dreams reaches its conclusion with the jaws of the pulveriser biting into the wall of the cinema hall. The end of the fantasy marks the return of reality and Rahil remembers his poverty-stricken home, his mother's wailing and his father's helpless deal with the agent which brought him to the town: "That night in his dream, he saw the scary machine with jaws approach him while he was peeping through the hole in the wall."

Mahi Yogesh has spun an appealing story of a child who faces the odds of life with a mix of gutsiness and innocence. Sadness and longing are kept at bay through daydreaming of an alternate life and the hole in the wall becomes a concrete medium to live that imagined life for Rahil. The tone of the story reminds one of Ruskin Bond's "The Thief" and Dicken's *Oliver Twist.* Yet, the story would benefit from scrupulous editing in order to smoothen the narrative and eliminate errors. Also, the characters in the story could be fleshed out some more to enhance the effect sought by the writer. The interaction between Rahil and Kishan Lal is quickly sketched out to keep the focus on the hole in the wall; nevertheless, the characterisation would gain from layering of their personalities, emotions, and experiences. While Rahil's character is envisaged from the child's perspective and can absorb the gaps, Kishan Lal's character remains fairly stereotypical. The impact of Rahil's alienation is thus weakened. The direct descriptions in the story hold out an opportunity for additions that may engage the senses more and help to fully conceive the world of the story. The exchange of dialogues, however, is sharp and ably maintains the momentum of the story.

The story makes us aware of the reality of child labour that is unabated in our society despite the constitutional directives and legislations prohibiting child labour. Rahil is one of the thousands of

'Chhotus' and 'Chhokras' who are robbed of their childhood and pushed into a life of toil and sweat. Rahil is also one of those who face hardships with ingenuity and grit. So, while the story ends on a note of pathos, it is hoped that the enduring spirit of Rahil, although bruised, may not be broken. This hope is ignited with the knowledge that Rahil had used the fantasy world of films to cope with the harshness of his existence but, at the same time, he had never lost sight of his reality.

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Audience's Live Response

Sangeeta Singh: Sad tragedy of lost childhood pleasures.

Anupama Singh: Mahi ma'am your story was full of mysteries, surprises, humour, satire and dejection. It was a gripping story indeed.

Anita Sharma: An introspective story! Congratulations Dr. Mahi!

Namrata Pathania: A brilliant analysis Meenakshi Ma'am.

Irene Rattan: I loved it that you took up a child as a protagonist. Despite the fact that we have various types of topics in our writings, but to give voice to the ignored and the voiceless is highly relevant and greatly required. Congratulations Mahi to depict a sensitive theme.

13

A GIRL AS OLD AS THIRTY

PRAKASH AVASTHI

I turned thirty last summer. My father is hopeful—rather he is almost certain that he would be able to fulfil his only remaining wish—the wish of being able to find a suitable match for me. Whenever father comes across a 'nice' young man or even a relatively older one, he loses his sleep and begins to make enquiries about him. Now, why on earth, that should not happen? Afterall, I am his only daughter and after mother's passing away, this responsibility weighs heavily on his mind. Isn't it bizarre that my younger brothers are married and living happily with their families but I, the eldest child of my father, am still unmarried? Ever since he retired, he sighs anxiously when he looks at me saying: "As old as thirty—and still unmarried!"

Father is not the only one worried about me. Even I want to get married now as almost all of my friends have been married long enough. Father keeps inviting home the prospective grooms—so many of them. So, what do I do? Granted, I am no princess, people would swoon over, but I am also not so wretched that I would agree to marry just about anyone. The 'boys' who come to 'see' me behave as though they have already 'accepted' me and the other way round too. Now, let's look at what happened yesterday. Father brought some spicy snacks from the market and whispered to me, "Sheela, get ready quickly and make some tea."

I asked him nervously, "Anything special Papa?"

"Yes, Mr. Ram Singh has been recently posted as tehsildar here. I have invited him over for tea."

"But Papa…", I wanted to say something but was interrupted by father.

"I have told you to make tea. If you want to ask me anything, ask later."

Meanwhile, come evening and the new tehsildar appeared on the scene. Seeing his blood shot eyes and an obese frame, I guessed at once that he takes less tea and more of other drinks. He must have been about thirty-five years of age but looked forty or more. Forgetting his age and that he himself was once an officer, father began to fawn on him brazenly while addressing him as 'sir' on more than one occasion. The tehsildar kept on boasting about his family, his well-placed brothers, his sister's forthcoming marriage to a high-ranking officer, his big house in Delhi and so on. When the discourse became too vulgar to bear, I quietly slipped out of the room on the pretext of preparing tea leaving him in father's company.

After a few days father invited a doctor who bragged about ministers being his patients: "With your qualification, I'll get you a job in no time. Making one phone call will fix everything. You know, there are as many as five ministers who are my patients."

Now what do I do with such people? After one such encounter, father asked: "What do you think about this boy?" I stood quietly with my head down not uttering a word. He shouted at the top of his voice, "What sin have I committed? You are thirty years old! I can barely sleep. Wherever I go, whomsoever I meet, I am asked only one question: 'Have you found a suitable match for your daughter? When are you marrying her off?', and so on".

For some time, father stayed quiet after this outburst and then said:

"Listen, what's your opinion about that engineer?"

"Papa…"

"Yes, say something".

"He is much older to me. I won't be comfortable marrying a person ten years my senior in age."

"You are too fussy. Remember…"

"Yes Papa, I have turned thirty."

"And that Civil Supplies Officer? He's not much older to you."

"I don't think I can communicate with him in any way."

"Oh, my child you're forgetting that you are not a young girl now…."

"Yes Papa, I'm as old as thirty."

Next day my brother came to visit us along with his family. My father was delighted to see the children. We were all sitting together. In the middle of a conversation, he suddenly stopped talking and then sighed. A heaviness seemed to fill the air. My brother and his wife sat in front of us, their gaze fixed on me as though I had committed some blunder. Father began his characteristic refrain, "How I wish Sheela too would settle down!" And now it was the turn of my brother to show sympathy: "Don't worry Papa! A suitable boy for her will be found soon." As everybody nodded in agreement, a wry smile appeared on my father's lips and he sighed again.

I pitied my father, who constantly faced the onslaught of the neighbours' and the relatives' inquisitiveness: "Not yet? Well, your daughter is highly educated, why would she marry just about anyone? It would have been so easy to marry her off had she been an ordinary and a less educated girl. Maybe we can help you." Father didn't know what to say. Though proud of me, he found it hard to counter these unsolicited opinions. It appeared people around me had nothing else to occupy their minds with but the thought of the long-delayed marriage of a girl as old as thirty.

Winter passed and the sleepy earth began to wake up slowly, ready for the birth of the spring. Soon the trees began to blossom turning red, purple, yellow and orange. The heaviness of the air gave way to a light fragrant breeze. The rustling of leaves and the songs of birds filled the air. The sun-kissed grass looked so inviting. As I stepped out of home for a stroll, I was overcome with a strange feeling of listlessness. Soon I found myself sitting on a bench, my eyes somewhat heavy. The gentle breeze soothed my spirits. I closed my eyes. After a while I heard someone humming a beautiful melody marked by a note of gay abandon rare to find in a systematically composed musical piece. "What a soothing tune", I said to myself and looked around. A young man was walking merrily across the road. He looked radiant as he smiled, gentle breeze lightly fanning his thick black hair. The moment he saw me, he stopped, looked at me, smiled intriguingly and occupied the bench opposite mine. He held a book in his hand and in no time was absorbed in it. After a while I realized I was feeling very light. Did I sleep? Was it a dream? No, no I was awake. The sound of the melody was still echoing in my ears. I looked around. The charming young man sitting right there had left. Stung by acute loneliness, I walked back home with heavy steps.

That day I just couldn't concentrate on anything. The haunting sound of the melody seemed to grip my mind as never before. I smiled when I remembered his smile. I hummed the melody he hummed. Seeing my excitement, father asked, "You look a bit nervous. Are you alright?"

"Yes Papa, why do you worry? I'm alright."

"This is not fair. Why wouldn't I worry? Thirty years old and no sign of marriage! You just don't seem to like anyone. At this age you cannot be a chooser. Sham Singh is a nice boy. Just today he was asking about your well-being. I fail to understand in what sense he is lacking."

"Papa, I have already told you what I feel about him."

"Tell me again."

"He behaves a bit awkwardly, always talking about money matters. Doesn't ever smile…"

Father nearly lost his temper, "Oh, my God! What do I do? Someone is old' someone immature, someone laughs loudly, someone does not smile…." I remained silent. He continued, "What kind of a boy are you looking for? Tell me what kind of a *gentleman* do you want to marry?"

I remained quiet for a while. Then suddenly I was astounded to find myself pouring out my feelings so passionately: "Papa, I would want to marry someone with a smile so charming that it would envelope me…who would hum a melody so sweet as to engulf me like the song of the Earth itself." Father froze up on the sofa—agape. "Oh my God! My fears have come true!" said he. After a pause, he muttered to himself, "This girl has gone crazy." I put my arms around his neck and said gleefully, "Papa, You're right."

I could not sleep that night. Every moment weighed heavily on my mind. The next morning as soon as father left home, I combed my hair, put on a light lipstick that matched with my lavender salwar-kameez and smiled in the mirror. "Will he come again?" I asked myself. That very moment, I heard the sound of humming that possessed me yet again. I found myself walking on the forest road to the spot where I had met him, until I reached the bench in the garden. The young man appeared again. I stood as though in a trance.

"Do you live here?" he asked me.

"Yes, I live here with my father."

He smiled again and waving a goodbye went his way. I waved my hand, wanting to say more, ask more but my lips froze and so did my feet. He walked some distance, turned and smiled yet again before disappearing. I spent the entire evening restlessly. A mixed feeling of anxiety and euphoria overpowered me again next morning. I could hear the rumbling sound of thunder in the distance. The dark sky looked ominous. Yet I stepped out. Suddenly, it started to rain. I ran for cover and reached the veranda. The young man stood in front of me near our front gate. Drenched in rain, he was still smiling. "Come inside. You'll catch cold", I said. Hesitant at first, he finally came indoors. We sipped tea together. I felt I was melting in his warmth as his voice softened. The rain had stopped. I looked around. He was here a moment ago. Where has he disappeared? A feeling of loneliness began to overcome me yet again.

My father who associated my state of mind to some delusion of sorts kept a dignified silence for a couple of days. His troubled look, however, revealed the disquiet within. Suppressing the commotion that raged inside his mind, he asked me coolly, "My child, what is the matter? Why do you look so dazed these days?" His fragile and ageing frame seemed to reel back under the weight of an undefined fear. I found it hard to look into his anxious eyes. I knew how desperately he wanted me to 'settle down'. The conflicting emotions of hope and fear gripped me.

"Haven't you taken a decision so far?" he asked once again almost pleadingly.

"Papa, I like a boy…"

"Is he one of those…?" he asked, his eyes beaming.

"No Papa"

"Then who is he? What's his name? What does he do?"

I realised I did not have answers for any of these questions. What I finally said sent father in a fit of anger.

"I don't know his name. I don't know what he does. But I know that I…I…love him."

"What nonsense are you talking?" he retorted.

Father stood dazed while I continued, "He is sensitive. He knows I am lonely, very lonely. I do not laugh. I do not sing. He understands me. Came to see me even as it was raining. I love him", I said, scalding tears rolling down my cheeks. Father could bear it no more. Quietly he walked out of the room looking very sad and remorseful.

The next morning, I began to wait again. But he did not come. Yet another day came. The sun shone brightly and the light breeze blew gently. But he did not come. I waited, waited endlessly. Birds chirped happily and flowers bloomed everywhere. Then came summer. But he did not come.

My father and family believe that I had concocted this story to console myself. The truth, however, is that he will certainly come. When flowers bloom in the spring and birds begin to sing in chorus, he will come—whistling and singing gaily, tugging at my heartstrings again with his warm smile.

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Translated from Hindi by Girija Sharma

Review: "A Girl as Old as Thirty"

JANESH KAPOOR

"I turned thirty this summer": The opening line of the story has a strong prophetic tone, almost ominous. Apparently, a statement of fact, it acquires an ominous character when read in the context particular to it. The person turning thirty is a girl who is still unmarried: a constant source of worry and consternation to her father, family and society because Sheela, the girl, is "As old as thirty – and still unmarried". The expression runs like a refrain with subtle variations throughout the narrative:

> "You are thirty years old! I [the father] can barely sleep."
>
> ……
>
> "Yes Papa, I have turned thirty."
>
> ……
>
> "When are you marrying her off?"
>
> ……
>
> "Thirty years old and no sign of marriage. . ."

Marriage is thus not a matter of individual choice. It involves the parents, the family and the society more than the individual concerned. Sheela's father is keen "to fulfill his only remaining wish – the wish of being able to find a suitable match for [her]". Father's wish is a natural one; she is his only daughter, the eldest of his offspring. It is but natural that she should be married and settled in life like her younger brothers. The responsibility "weighs heavily on his mind" ever since his wife passed away.

The emotional and filial responsibility of the father is permutated into anxiety by social pressures that he has to reckon with on a regular basis. The undercurrent of social mindset is explicit in the kind of observations that the neighbours and relatives make about Sheela having to remain single for so long: "Not yet? Well, your daughter is highly educated, why would she marry just about anyone? It would have been so easy to marry her off had she been an ordinary and less educated girl". He does not possibly find support from his sons either. The matter invariably comes up for deliberation during family meets only to end with mere comforting words for the father: "Don't worry Papa! A suitable boy for her would be found soon". The father can merely smile wryly and heave a sigh.

The antinomy in the views of the father and the daughter regarding matrimonial alliance is also quite apparent. While the father tends to think that the prospective groom should be well placed socially and economically, the daughter continues to cherish ideas of platonic love. As she once blurts out to her father: "Papa, I would want to marry someone with a smile so charming that it would envelop me. . . who would hum a melody so sweet as to engulf me like the song of the Earth itself". The poor father is sure the girl has turned crazy as he freezes up on the sofa: "At this age you cannot be a chooser".

So far, the atmosphere of the story is dull and dreary as the season is that of winter. We now have the advent of Spring and the mood of nature begins to turn colourful -- "red, purple, yellow and orange", which offset the "light fragrant breeze" to the tune of the "rustling of leaves". The filiations and passions of Sheela also appear to come of age. It appears like a dream to her. She sits on a bench, significantly in an outside space for the first time in the story and seeks to do away with the "heav[iness]" of her situation, with eyes closed. After a while she hears "a beautiful melody marked by a note of gay abandon". It came from a young man "walking merrily across the road". "He looked radiant as he smiled, gentle breeze lightly fanning his thick black hair. The moment he saw me [Sheela], he stopped, looked at me, smiled intriguingly and occupied the bench opposite mine". This is the moment when love happens; at least for Sheela. She is unable to concentrate on anything that day or sleep at night, her mind gripped with the "haunting sound of the melody".

The next morning Sheela finds herself getting ready and moving out to the same space in expectation of another encounter with the young man: "I combed my hair, put on a light lipstick that matched with my lavender salwar-kameez and smiled in the mirror. Will he come again? I asked myself". Her expectation is duly fulfilled as the man appears again, humming melodiously. The two also exchange a few words in a bid to get acquainted. Sheela is left "euphoric" again with a mix of anxiety about the coming day as the young man disappears out of view. Next, they meet at Sheela's place, who invites him inside as it is raining and the man is drenched. While they sip tea together, she feels like melting in his presence. Then the rain stops and the young man disappears yet again and is never seen thereafter.

However, to Sheela the young man represents love and the dream of the life she yearns for. As she vehemently declares her resolve to her father to wait for the young man as she loved him, the latter is remorsefully sure that Sheela had concocted the story to console herself.

Seasons begin to pass but Sheela continues to wait full of hope. It is on this note of hope that the story concludes: "The truth, however, is that he will certainly come. When flowers bloom in the spring and birds begin to sing in chorus, he will come – whistling and singing gaily, tugging at my heartstrings again with his warm smile".

"A Girl as Old as Thirty" has been translated from a relatively long Hindi story by the translator's late father – indeed an eloquent tribute to his cherished memory. On the linguistic level, the translation does total justice to the original. There is no jugglery of words, which unfortunately is the case with some translations when the translator is unable to find and organize words and expressions which are close to the original context. In fact, the language used by the translator is fluid enough to carry the readers with it, even if they do not have access to the original story. In fact, the translation reads like an original story in English. The colour and the sound images used by the translator to establish the two parts of the story – the first characterized by anxiety and the second by hope – have also been fine tuned to the original. The trajectory of the story is marked by a leap of hope and faith, which accords meaning and purpose to Sheela and which would perhaps also sustain the father. Indeed, no matter how cliched it sounds, it is our hopes and dreams which keep us going. . .

While the translated version is a shortened re-writing of the original, a couple of passages pertaining to interaction between the characters could have been retained for an authentic contextualization of the translated version. The most pertinent one is the brief conversation between Sheela and the young man as the two sit together as it rains and thunders outside. At this juncture in the original, the young man observes that he would certainly come to meet Sheela again if it does not rain – a strong indicator that he might not continue to be there for Sheela with the passage of time despite her dream of building a platonic relationship with him. As another instance of what could have been included is the brief banter between Sheela and her younger brother over the apparent change in her demeanor, who can now be seen happy and chirpy. This would have infused the translation with a sense of sibling teasing so common in families and also served as a qualifier for the element of hope which infuses the story.

However, a story is not something which is always available to us in black and white and may be translated word-for-word. It is what can be abstracted from a narrative, recounted and retold in a different idiom. A successful translation is one which captures the essence of the original story and does not move very far away from the source. Girija Sharma's translation seeks to retell the original story which is better suited to the new readership most likely to read it in English only. It is a successful translation which accords a new life to the original story.

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Audience's Live Response

Priyanka Vaidya: I was lost somewhere in emotional swings of a woman turning thirty!

Anupama Singh: Such a poignant truth has been narrated in such a touching manner by you ma'am. Bitter truth of our society.

Neelima Kanwar: Beautiful story, beautifully narrated...thank you, ma'am.

Sangeeta Singh: In the context of Girija Madam's story, I would also like to applaud the decision of the cabinet for having increased the marriage age of girls from 18 to 21.

Kamayani Vashisht: Thanks Ma'am Girija, for that engaging story. Enjoyed the conclusion.

14

AND THEN THE RIVER FLOWED

PRAVEEN KUMAR

Words fascinate me and I love to listen to the cadence their arrangements produce, but the moment I attempt to dress up a character or weave a story, words fail to fall into place and grow viscous. Restless tales lurk faintly in some corner of my mind, but they refuse to take a body and a shape. An overwhelming amorphous cloud muddies the river of thought, hangs over it like an ill omen, and spews in fits its chaotic elements, leaving me dazed. Don't know from where and why this cloud floats in, adding to my misery, leaving me fuming, shredding my faith in my creativity that has been drilled foolishly into my mind by those who claim to know me. Not that I am a lost case, incapable of writing sensible sentences, but this cloud comes with such frequency that I begin to lose faith in the unfounded belief that I'll bloom into a writer someday, whose words will cast a spell in the literary world.

I had never been inclined to write stories or novels. Deficient in empathy, inquisitiveness, and other writerly traits, I hardly fit the bill. But those whose orbital paths coincide with mine feel persuaded that a writer is waiting to be awakened within me, something I have failed to notice myself. Their faith and insistence weigh heavy on me. Should I yield to their misplaced faith or stick to my unartistic self? An attempt was being made to carve a writer out of an unimaginative bore, who hardly took interest in people's lives – probably the key ingredient in a writer's repertoire. The impasse into which our conflicting views had crystallized implored a quick resolution for my sanity. A part of me was willing to oblige and do the bidding, but a lot many other faculties refused to cooperate. The pen would hardly move across a page; it felt an insurmountable friction and dragged on producing uninspiring phrases and poorly conceived metaphors. Despite the unease I felt, I was too arrogant to admit to myself that I was incapable of looking into human minds besides being helplessly pathetic in capturing the externalities.

The nature of my professional work and personal aspirations of a different nature, obstructed my journey to literary stardom, which anyway was hardly a possibility or priority. But new possibilities can knock at the door of life most unexpectedly; they can peep through unexpected dark alleys, which often hide from view the light at the farthest end. My possibilities came wrapped in shock when I had to move unwillingly to a remote location; I was left bewildered and distraught, incapable of processing the new challenge with sense and sensibility. A seven-hour drive through the seeming wilderness landed me into a non-descript village that had nothing to boast of except its menacing peaks, lovely people, and a captivating place of worship. The journey to this land, which had existed in my mind only as a name and not as a living, habitable place, was an endless encounter with fear. Incessant rain had added to the magic, mystery, beauty, fear, and other undefined emotions. The arrival was no less dramatic as I bumped into a group of important-looking people, who extended a warm welcome, which failed to evoke a positive feel in the tired, frustrated, and unhappy soul. Someone offered to comfort, "You are fortunate because the presiding deity of the temple herself decided to requisition your services for the people of this area." This aggravated my irritation, but it did not take long before the import of the words dawned on me. The place had an ineffable magnetism and charm that soon gripped my soul and made me bow before it in total submission.

It was getting dark and the towering hills surrounding the place filled me with intense sadness. Cold added to the misery; the presence of a known face was the only source of comfort. It was a revelation that even insignificant things had the potential to alter the rhythm of life; they could trigger joy or sadness instantaneously. And when would joy turn sour, filling one with sadness, none could predict. Days rolled by and I found that a new world of innocence and natural grandeur was opening up before my eyes. While my workplace was always teeming with people whose painful stories of myriad ailments weighed heavy on my conscience, the sprawling complex, where I was offered a place to live, was always abuzz with activities, sounds, and an endless flow of people. There was never a single dull moment here. People from all strata of society could be seen paying obeisance to the goddess. Politicians, high officials, and common people came from far and wide, giving it a unique charm. But for me the greatest charm lay in the simple folk who served in the temple with utmost devotion and love. An instant bond was forged with the people and the place, the physicality of which lasted till my departure, but on an astral plane, it'll remain with me for life. Their existence is eternally etched on my mind.

The place had a mesmerizing beauty; tall deodars and rocky cliffs had an overpowering presence. Long walks in the wilderness, peppered with an animated discourse on spiritual matters by a junior colleague Prabodh added ineffable joy to the whole experience. I behaved like a curious seeker in his presence. He was more than willing to indulge me. In response to a question like, "Why does isolation and absence of human company leave us sad and insecure?" he would dwell on the intricacies of human emotions and of his encounters with the spiritual masters. Metaphysical questions fascinated him, and he loved to expatiate upon these topics. He had read a great deal of literature of this kind, and had spent a few months deep into the uninhabited mountains, where none except the seekers of the divine truth lived and prayed. "Sir, listen to this piece of music, and forget about your doubts; the melody, rhythm, and divinity of this rendition will dissolve your doubts in no time," he would assure me. He always spoke with conviction and passion about spiritual matters. His incredible stories often left me both mesmerized and puzzled. During one such sessions, he said, "I met a very old yogi, who looked resplendent, and with no sign of physical or mental decline that comes with age. I sat in his company enraptured for days together and listened to his discourses, which had multiple strands of spirituality, science, and metaphysics." Stories of this kind filled me with a deep sense of wonder and awe. On other days, we would sit together on a mound, deep into the wilderness, and enjoy music in total silence. An inexplicable joy flowed unhindered though no words were exchanged.

While Prabodh took me on an inner journey, Balu, an apparently arrogant yet caring man, made my life joyful in his own humble way. He was a dynamic and talkative worker whose presence could be felt in every part of the temple. I was drawn towards him, and he too showed his fondness for me. I couldn't understand why I liked him because I had never been good at forging friendship with simple village folk. I realized for the first time the power of innocence and love; generally, only in books do we speak about such emotions in glowing terms, but these come alive when we see them manifested in a living being. Balu interacted with me like a friend, would even tease me freely, without inhibitions. He always spoke to me with a feigned annoyance whenever I took leave. "Sirji! I don't like your frequent absences. I am very angry with you. Why do you go to your hometown so frequently?" he would act as if he were really displeased. I would then explain my position, try to convince him that the number of days I had spent away from the place were too few to give him reason to feel indignant. Jeevan, the cook, was another friend whose love had its own fragrance. Without being demonstrative, he would try to feed me with love. He had noticed my lack of excitement about food and made it a point that I ate well. He would invariably insist on having an extra chapati, and would always keep my share of the sweet dish despite it being in short supply. Once he got to know that I was seeking transfer from the place, he was dismayed. "Sirji! I don't like the idea at all. Why don't you serve here till retirement?" he expressed his disapproval of my plans. I knew it was nothing but attachment. He expressed similar sentiments on a number of other occasions. He tried to pressurize me by telling, "Devi ma would be unhappy with you if you decide to go." Expression of such pure love from someone with whom my association was relatively new put me in a big dilemma. Should I listen to my family, which needed me and was desperate to have me back, or should I respect Jeevan's loving insistence on prolonging my stay here? It was a difficult call. Moreover, the place which in the first few days of my arrival had looked so dreary and deserted now began to look like a home – vibrant, joyful, loving, and very charming. A heady mix of the people and the place had captivated me; it was beyond me to break the spell.

Long periods of isolation on Sundays or holidays were blissful. My soul, which looked tattered and joyless after years of constant inner chatter and restlessness, physical movement and emotional upheavals began to heal. Equanimity, an easy calm, and love for nature, fellow humans and simple things of life began to take roots. On certain occasions I spent whole days confined to my cramped but beautiful room. I couldn't understand why I never felt lonely, unhappy or disoriented despite not meeting a single soul for long hours. Peace pervaded my whole being, and no physical deprivation or discomfort could destabilize me. Even power cuts during the snowfalls were not much of an inconvenience. A new understanding of life was taking hold of me. Each had their contribution – loving friends, which included colleagues, temple *sevaks* and also the nearby shopkeepers with whom I had little in common. I felt it for the first time that we constituted a large human family in which each member was equally important; education, affluence, poverty, caste, class, or position hardly mattered. We were all notes of a symphony, and unless all notes played in tandem, the only possible music was a jarring noise. I was beginning to take interest in the multiple strands of the web that made society. Children returning from school on foot, who respectfully bowed and touched our feet gave me as much joy as our occasional encounters with high-profile visitors to the temple, who often interacted with us as we were regarded the intellectual elites in this rural setting.

A great many local patients thronged the hospital with their sufferings and ailments. Some came with routine problems, but in some cases, long years of neglect had taken their diseases beyond a point of no return. They struggled to articulate their symptoms, but their bodies said it all. Nikita and Vinti, my two nursing associates, came to their rescue. With utmost patience and empathy, they dealt with them, mediated and bridged the linguistic gap that lay between these simple folks and a medical specialist, who revelled in medical jargon. To my innocent inquiry how they could ever remain calm and helpful, they would respond with innocent smiles and disarming candidness. "They are almost family, directly or indirectly. In villages, the boundary of our love is not so well-defined. It spills beyond a mother, father, brother or a sister; a neighbor or a distant acquaintance has an equal right to sit in the comforts of the canopy of love we erect," said Nikita with conviction, leaving me speechless. Are they a different human species or is my understanding of human nature so pathetic and limited? The smell of festering wounds, medicine, and the huge associated paraphernalia that attempted to steal life from death, couldn't dampen the spirit of my team; the challenging working conditions couldn't kill their free-flowing humaneness and abundant charm. The whole atmosphere palpitated with a vibrancy and exuberance that their collective energies produced. Their tender approach to human connections, their quiet and unobtrusive presence released a torrent of ideas that had been simmering, waiting to spill over into images and phrases.

Life has its own logic and pace. We can hardly visualize what lies in store for us. What seems profitable to us may prove disastrous in the long run, while a seemingly unwelcome event may bring joy. My life too took a decisive turn when, one fine day, I picked up the pen and found that the company of these beautiful characters had mobilized my writerly instincts that had lain dormant all these years. I had met these artless souls only for a few months, but their ruggedness, commitment, unbounded love, and passion for life ignited in mean urge to look deep into human soul, to experience the rhythm of life intimately. I had never known that our lives had so much richness and depth, love and beauty. Their lives were an unfathomable quarry where multi-hued gems lay embedded. The seeds that had lain unsprouted all these years had finally found the soil they had been yearning for; tender offshoots were joyfully playing with the wind. The frozen mass of ideas and emotions was beginning to thaw. The river carried in its life-giving waters a huge mass of the fertile earth. The pen, which had got stuck in the rough furrows and looked rudderless, was finally moving freely, happily, energized with the human spirit and warmth. It was feeling uncaged and natural, almost like a river swollen with water, soil and uprooted trees during a calamitous flood. The barrier was finally crossed and a new, beautiful, resplendent view was emerging from behind the fog that had blinded my gaze.

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Review: "And Then the River Flowed"

JANESH KAPOOR

"And Then the River Flowed" by Praveen Kumar appears to be based upon the age-old adage that the seeds of creativity are inherent in all 'thinking' human beings, particularly if they enjoy a certain degree of felicity with regards to linguistic expression as well. This is particularly true of the present writer-narrator, who has always been "fascinate[d]" by words: "I love to listen to the cadence their arrangements produce. . ." and can feel "Restless tales lurk faintly in some corner" of his mind. Numerous friends and well-wishers constantly endeavour to provide a fillip to the dormant creative zeal of the writer, who feels both encouraged and constrained by such efforts as the moment he attempts "to dress up a character or weave a story, words fail to fall into place and grow viscous", thereby "shredding" his "faith in [his] creativity".

However, as Carl Gustav Jung has averred, our creativity (yes, we are innately creative, lest the opening sentence of the review be taken as an ironic indictment of sorts) comes to the fore when we feel ourselves to be the captives of some unwarranted situation or perceive an existential challenge to our being (often the result of the finitude of existence). It is when the narrator gets posted to a remote hilly area in the course of professional life and is subject to certain experiences, which are a proscription as well as a transcription of the hitherto routine life, offset with the basic inertia to change and the gradual interiorizing of the inapposite involvement in the new ethos apart from the professional-scape that the river of creative expression begins to flow.

Initially the writer-narrator is "bewildered and distraught" and is "incapable of processing the new challenge with sense and sensibility" The very idea of working and living in a "non-descript" village that has "nothing to boast of except its menacing peaks, lovely people and a captivating place of worship" after having apparently lived in cities for long. The very journey to the place "was an endless encounter with fear". It is to be noted that the narrator's description of the place is characterized by antithetical expressions as they combine anxiety and awe. The sentence which precedes his arrival at the new place is a case in point: "Incessant rain had added to the magic, mystery, beauty, fear and other undefined emotions".

However, it is the mystery and majesty of the place with its "ineffable magnetism" which soon charms the narrator who begins to view his transition to the place as a revelation of sorts: "it was a revelation that even insignificant things had the potential to alter the rhythm of life; they could trigger joy and sadness instantaneously. And when would joy turn sour, filling one with sadness, none could predict". It is the pervasive human environment of the temple and the unflinching faith of the people from every quarter of life in the benevolence of the deity, along with specific people who form a close proximity to the narrator that his experiential matrix begins to transform into an all-embracing and inclusive spectrum of being. He forges "an instant bond" with "the simple folk who served in the temple with utmost devotion and love" and observes that the "physicality" of his encounter with "the people and the place" "lasted till my departure, but on an astral plane, it'll remain with me for life. Their existence is eternally etched on my mind". He never finds a "single dull moment" in life now.

Some personages from this new environ find a special mention in the story. One is Prabodh, a young colleague of the narrator, who ignites a spiritual spark of sorts in the narrator. Prabodh's "animated discourses on spiritual matters" during long walks "in the wilderness" added 'ineffable joy to the whole experience". While Prabodh never accords any concrete answers to the queries by the writer-narrator, he apparently deepens the mystery and seductiveness of matters immaterial through suggestive stories and musical recourses, leaving the latter "both mesmerized and puzzled", vicariously perpetuating his "deep sense of wonder and awe" associated with the place.

Another such personage is Balu, "an apparently arrogant yet a caring man', who made the narrator's life "joyful in his own humble way". Balu would interact with him "like a friend" and tease him "freely, without inhibitions" and pose to be angry when he would proceed on leave on account of familial or social obligations. Jeevan, the cook at the temple, also finds a loving place in the narrator's memory. Jeevan made it a point that he ate well, was aware of his lack of appetite, perhaps due to his different eating habits, and would reserve some sweet dish for him to boost his appetite.

The writer-narrator can now consciously experience the transformation in his personality and outlook. Long periods of aloneness no longer bothered him. "Equanimity, an easy calm, and love for nature, fellow humans and simple things of life began to take roots. . .. Peace pervaded my whole being, and no physical deprivation or discomfort could destabilize me". He is able to meditate upon and realize a rare harmony with every aspect of being alive: "I felt for the first time that we constituted a large human family in which each member was equally important; education, affluence, poverty, class, or position hardly mattered. We were all notes of a symphony, and unless all notes played in tandem, the only possible music was a jarring noise".

Eventually, it is Nikita and Vinti, the two nursing associates of the narrator, who "mediated and bridged the linguistic gap" that often proved to be a barrier between him and his patients", and who remain ever calm and helpful to the simple folks who come to him for medical intervention in their diseases which "long years of neglect" had in some cases move "beyond a point of no return". For Nikita and Vinti, these patients "are almost family, directly or indirectly" both by virtue of their profession as well as the sense of belongingness and association with the place and its people. While the narrator's association with Prabodh, Balu or Jeevan is more of a personal alchemy, It is the care and concern which these two women exude towards humanity at large, which is perhaps an epiphany-like event for him and ushers a total transformation of his being variously orchestrated by his sojourn in this peculiarly humane place with a harsh geography. He is able to shed the persona of an intellectual struggling with words and ideas and respond intimately to the human condition around him, like a *sahaj sahridaya*. And the river of creativity begins to flow. . . .

Structurally and linguistically "And Then the River Flowed" is a well-knit story. There are no loose strands in the narrative and the syntagma fall into place perfectly. The choice of words is rigorous and orchestrates the state of mind of the writer-narrator characterized first by doubt and anxiety before moving to the mountainous incluse and later by acceptance and equanimity. Thus, antonymic words and expressions like "incapable", "unfounded", "unartistic" [*sic*], "insurmountable", "uninspiring", "unexpected" and the like abound in the first half of the story, but rarely occur in the later part. However, the language used remains too elite and does not capture the linguistic peculiarity of the rural, mountain people, whose feelings and expressions appear to be merely approximated even when assigned within quotation marks. While the "cadence" of strictly English language may be well maintained here, the words really "fail to fall into place and grow viscous" at least in this instance, as the language used by the rural folk tends to be idiomatic and inflectional. The use of "Sirji!' as an endearing form of address is the only exception one can pin point in this context.

Moreover, despite the delimited canvas of a short story with little scope for growth and development of context and characters, one feels as a reader that the depiction of the mountainous environment and the characters drawn there from lack due cross-matching. Various characters could have been assigned some traits drawn from their environment. For instance, Balu's apparent arrogance could have been likened to the towering hills and his caring attitude to the melting snow.

"And Then the River Flowed" is eventually a self-reflexive story wherein the writer tries to explore and justify one's creative process vis-à-vis the changing circumstances and environment of his personal and professional life. There is hardly any event executed on the external plane: the substance of the story comprises the thought process of the narrator which borders on the philosophical. However, all said and done, there lies embedded within the narrative, a larger story which is about life itself and its *joie de verve*.

Audience's Live Response

Priyanka Vaidya: Beautiful expressions and metaphors! This story teaches the art of living. Mesmerizing!

Sangeeta Singh: "Candid confessions" of the evolution of a writer/ artist/human being. Autobiographical inward journey of a mystic-seeker. Love the sheer flow of the river of words....

Kamayani Vashisht: The story of a story! Praveen sir, let this be the beginning of a new journey.

Shivani Chaudhary: An unconventional pastoral narrative... inner chatter, peace, harmony, charming human touch... how well contours of life have been painted. It's a story of finding extraordinary happiness in the ordinary.

Swaraj Raj: A tellingly effective memoir of a story. Sir, we are the people we meet, the stories we read, the stories that we tell ourselves about ourselves… we grow, we evolve in response to all that we encounter… very well narrated.

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छतरी

प्रियंका वैद्य

ज़िंदगी के चालीस बसंत वो अकेले बिता चुका था, एक अजीब सी कश्मकश थी जो उसे रोकती थी। अकेले रहने की आदत हो गई थी।

अजीब आदत थी, होटल में मिली लड़कियों के बारे में जानने की कोशिश करता। उनके अतीत की तहें खोलता और फिर ज़िंदगी और भावनाओं की कृत्रिमता को देख ठहर जाता। किसी को उस जीवन में धकेला गया था और किसी ने ख़ुद उस बाज़ार में क़दम रखा था।

व्यापार दोनों परिस्थियों में था। उसे ज़िंदगी एक बाज़ार और भावनाएँ वस्तुएँ लगती थी। गहरा उतरने की आदत थी और वो सारी कहानियाँ उसके अंतर्मन को घेरे रखती, उनसे निकलना आलोक के लिए सम्भव न था। कभी कहीं बाहर जाता तो भी बड़े शहरों की गलियों में बिखरी कहानियों को समेटने पहुँच जाता। कभी किसी को नज़र भर नहीं देखता, न किसी को स्पर्श ही करता। रात भर आराम कुर्सी पर बैठ मद्धम रोशनी में उन लड़कियों की अनकही और अनसुनी दास्तानों को सुनने की कोशिश करता।

आलोक एक महीने की छुट्टी लेकर मुंबई चला गया। किताबें पढ़ने का शौक़ था और किताबघर में जाते ही उसकी मुलाक़ात एक ख़ामोश पहेली से हुई, जो शब्दों का समंदर थी, फिर भी कोई हलचल नहीं थी।

अजीब पागलपन था। कुछ पाने की होड़ नहीं थी, कुछ जानने की होड़ थी और वो समंदर अथाह था और गहराई का कोई अंत नहीं। वो डूबता चला गया।

आलोक इस बार किसी ऐसी लड़की से मिला जिसके जीवन की गहराई तक पहुँचना सम्भव न था।

वो लड़की साँवली थी। गहरी काली बड़ी-बड़ी आँखें और घुंगराले बाल।

“तुम्हारा नाम क्या है?”

जो नाम आप दे दो। जितने लोग आते हैं उतने नाम दे जाते हैं और मैं अपना नाम भूल चुकी हूँ, याद करना भी नहीं चाहती। उन लोगों को भी कहाँ मेरा नाम याद होगा, मेरा चेहरा भी भूल गए होंगे।

बत्ती भी जलाना नहीं चाहते, ख़ुद का चेहरा जो छुपाते हैं।

'मैंने नाम पूछा और तुमने सब बता दिया, नाम नहीं बताया।"

"नाम पहचान से जुड़ा है, पहचान तो गुम है, इन बदनाम गलियों में।"

वो हंस दी। हँसी उसके चेहरे से अलग ही नहीं हो रही थी और ख़ुशी उसकी ज़िंदगी से बहुत दूर थी।

"आप पहले हैं जो बात कर रहे हैं। ... बात कौन करता है?"

फिर ख़ामोशी ने दस्तक दी।

दोनों चुप हो गए। बाहर बारिश हो रही थी। आवाज़ थी तो बस बूँदों के टपकने की।

"परदा खोल दो। हाँ! खिड़की भी खोल दो।"

"अरे! यहाँ कोई कभी भी खिड़की खुलवाने वाला नहीं आया। तुम अजीब हो।"

"क्या पीना पसंद करोगे?"

"काली चाय अदरक वाली। बस!"

'अच्छा! तुम तो अपना नाम बताओगी नहीं, नैना सही रहेगा!"

'नैना! नैना! नैना!' वो नाम दोहराने लगी। कितना अच्छा है सुनने में!

नैना ने कभी कुछ नहीं देखा था, रोशनी उसे डराती थी और रात अंधेरी होती थी। नैना की बड़ी-बड़ी आँखों में काजल ऐसे लग रहा था मानो आसमान को काले बादलों ने बाँधने की कोशिश की हो।

"तुम्हारे बाल तुम्हारे चेहरे पर गिर रहे हैं, तुम्हारी आँखों को देखना है। इन्हें बाँध लो।"

हाँ! चाय अपने लिए भी बनाना।"

वो मैथिली में कोई गीत गुनगुना रही थी। उसकी आवाज़ बहुत सुरीली थी और बारिश की रिमझिम में संगीत घुल गया था।

"गाओ, चुप क्यों हो गई?"

"चलो खिड़की के पास दो कुर्सियाँ लगा दो, वहीं बैठेंगे।"

ज़ोर से हंस पड़ी, एक तो चाय और ऊपर से सर्दी की बारिश।

"बैठ जाओ, गाना अच्छा गाती हो। कुछ और सुनाओगी, अभी तो बारह ही बजे हैं, पूरी रात बाक़ी है। चाय अच्छी है, थोड़ी और बनाओगी?"

वो चाय बनाते हुए गाना गुनगुनाती रही और आलोक उसके मैथिली भाषा में रचे हुए गीत में उसका अतीत खोजता रहा। मानो शब्दों के पीछे का सच, धड़कन में छिपा संगीत सुनना चाहता हो।

"मैथिली कैसे आती है?"

"ये गीत कहीं रुह में बसा है, बाक़ी कुछ ठीक से याद नहीं।"

अतीत याद नहीं, पहचान पता नहीं, नाम का अस्तित्व नहीं, जानने की चाह नहीं। आलोक खिड़की से दिख रही बूँदों में खोया हुआ था। कुछ ढूँढ रहा था। मैं कभी खिड़की नहीं खोलती! ख़ासकर जब बारिश हो रही हो तो! आलोक नैना की मरुस्थल हुई आँखों को देख रहा था।

पूरी रात बिना कुछ कहे और सुने बीत गई। ख़ामोशी चीख़ रही थी। कुछ था जो अभिव्यक्त हो रहा था, कुछबिना प्रयासों के मुकम्मल हो रहा था।बिना कोशिश के घटित हो रहा था। आलोक बिस्तर पर सो गया, नैना ने बरसों बाद बारिश को देखा था और वो सोफ़े पर बैठ एकटक खिड़की से बाहर देखती रही और कब उसकी आँख लग गयी उसे पता न चला।

अगले दिन सुबह फिर वही चाय।

"साहिब! ऐसे आप यहाँ नहीं रुक सकते।"

"मैं तुम्हें क़ीमत दे दूँगा।"

"किसकी क़ीमत, अदरक वाली चाय की?"

"हाँ! चाय और गाने, दोनों की!"

वो मुस्कुरा दी। पहली दफ़ा उस ठहरी सी लहर में उफ़ान आया था। रेडियो पर प्रधानमंत्री जी का संबोधन आ रहा था, उन्होंने कहा, "जो जहाँ है वहीं रहे!" लॉकडाउन लग गया था।

"क्या तुम मुझे लॉकडाउन ख़त्म होने तक यहाँ रहने दोगी?"

ठीक है। वो गीत गुनगुनाते हुए रसोई में चली गई।

"हाँ! कल का एक सवाल अधूरा है। तुम्हें बारिश क्यों पसंद नहीं?"

“इस सवाल का जवाब ज़रूर दूँगी। “बोलते-बोलते वो चुप हो गई। दोनों अपनी ख़ामोशियों में खो गए। आलोक दिन भर कम्प्यूटर पर अपने काम करता रहा। नैना कहानियाँ लिखती थी। उसकी डायरी में सबकी कहानियाँ दफ़्न थी। आलोक उस डायरी को पढ़ना नहीं चाहता था। वो नैना के मुँह से उसकी दास्ताँ सुनना चाहता था।

"क्या कभी किसी से प्यार हुआ है? किस बारे में लिखती हो, खोने के बारे में या पाने के बारे में?”

“पाया होता तो यहाँ नहीं होती।”

प्यार की तलाश भी अजीब है। प्यार की तलाश में खोया हुआ आदमी सब पर विश्वास करता है। सभी को दिल में पनाह देता है और सब की ज़िंदगी में आग के बाद बची राख की तरह बिखर जाता है। राख फिर किसी उन्मुक्त हवा के झोंके के इंतज़ार में ठहरी रहती है।

पुनः जलना चाहती है, जो सम्भव नहीं। राख से खेलते-खेलते अनेकों चिराग़ बुझ गए। बुझने से जलने तक का सफ़र और जलने से पुनः बुझने तक का सफ़र भी अजीब होता है। प्यार नाम का लफ़्ज़ मेरी कहानियों में कभी था ही नहीं। अगर ग़लती से उस शब्द को लिखती तो आँखों में पड़े रेत की तरह वो शब्द चुभता। पुनः मिटाती और कर देती अलविदा उस एक शब्द को जिसकी सदा से तलाश थी। पीछे भागती। अब लफ़्ज़ों से काम नहीं चल रहा था।

वक़्त की कँटीली धरातल पर पुष्प के बीज बिखेरने की सोचती और कोई उसे प्रताड़ित कर रौंद कर चला जाता। जिजीविषा के लिए फिर नए रास्ते पर चल पड़ती।

वृक्ष के सब फूल तोड़ दिए गए थे।

जो भावनाएँ थी वो घुटने लगी थी।

प्यार तो कभी था ही नहीं। प्यार ठहरता ही नहीं। प्यार तो स्वच्छंद हवा सा गुज़र जाता है। प्यार की तलाश ख़त्म हो चुकी थी। हृदय अनेकों आघात सहकर पाषाण होने लगा था। जब कभी कोई उसकी तरफ़ मुस्कुराकर देखता तो उसके बढ़ते हाथों को देख वो सहर जाती और विलीन हो जाती। स्पर्श उसे ग्लानि से भर देता। वो स्पर्श से भागती।

जिस स्त्री के जीवन में कोई पुरुष नहीं होता, वो सब के लिए आसान लक्ष्य होती है। सब उसकी भावनाओं के साथ खेलने को तैयार होते हैं। प्यार क्या इंसानियत ही मिल जाए, काफ़ी है।

सूरज ढल रहा था। हवा में ठंडक थी और नैना को एक रबड़ देते हुए आलोक ने कहा,

"तुम्हारे बाल बँधे हुए अच्छे लगते हैं!”

मानो उसकी काजल लगी गहरी आँखें पूछ रही हों, “आख़िर तुम चाहते क्या हो?”

नैना ने अलमारी से एक छतरी निकाली और ज़ोरज़ोर से रोने लगी।

"आप मुझसे बात करना चाहते हो ना? कोई बात ही तो नहीं की किसी ने, किसी ने कुछ नहीं पूछा कभी। ये छतरी बाबा ने दी थी, दो दिन पहले ही मैं ग्यारह की हुई थी। वो रंगबिरंगी छतरी। मैं रात भर उस छतरी को अपने साथ लेकर सोती, डर था कि मेरा छोटा भाई ले लेगा। आज मैं ख़ुद उसे देना चाहती हूँ, वो नहीं है। मैं ये छतरी लेकर नदी पार गाँव से दूर अपने रिश्तेदार के घर गई थी। बस यही छतरी थी मेरे पास। नदी में भीषण बाढ़ आ गई, घर और घर का कोई भी सदस्य नहीं बचा। सब डूब गया। सब तरफ़ पानी था, ज़िंदगी का कोई निशाँ नहीं था। सबकुछ डूब गया था। लहरें भी मेरे घर का पता नहीं बता रही थी। सब सवाल निरुत्तर रह गए। मेरी ज़िंदगी किस दिशा में जाएगी,मुझे नहीं पता था।मेरी नौका और मेरा सफ़र कुछ भी मेरा अपना नहीं था।"

"मेरे पास बस एक छतरी बची थी।"

"मैं दोबारा कभी अपने माँ-बाबा को नहीं देख पाई। उनकी तस्वीरें भी डूब गई थी। कुछ भी शेष नहीं था, उनके चेहरे मेरे दिल में अभी भी कहीं दफ़न है।"

"बाबा का हाथ मैं अभी भी महसूस करती हूँ, माँ की गोद मुझे बहुत याद आती है।"

"छोटा भाई अभी भी मेरी चोटी खींचकर मुझे सताता है।"

"वो गिरने के बाद बिना कपड़े झाड़े फिर से भागना, धूल से नहाना और फिर भी खिलखिलाते रहना, मिट्टी से सने चेहरे में जो हँसी फूट पड़ती थी वो बहुत याद आती है।आइने से ज़्यादा माँ की आँखें हाल बताती थी और ख़ामोशी भी बहुत शोर करती थी। भोलापन ऐसा कि प्यार जीतने के लिए रिश्वत नहीं देनी पड़ती थी और थोड़ी सी चोट लगने पर कई हाथ सहलाने के लिए उठते थे। पिता की उँगलियों से मेरी उँगलियाँ छोटी होती थी और पकड़ कर लगता था कि दुनिया के सबसे ताक़तवर आदमी ने थाम लिया और अनंत साहस भर जाता था। अब किसी भी दौलत से वो मिट्टी में सनी मुस्कान, साहस, छोटी उँगलियाँ और लम्बी दौड़ नहीं ख़रीदी जा सकती।"

"ग्यारह साल की उम्र में कोठे पर उन चीख़ों को सुनने की आदत हो गई थी और मेरी ही उम्र की छोटी-छोटी लड़कियों को गुड़िया तोहफ़े में दी जाती थी।"

"पंद्रह साल की उम्र में मैं वहाँ से भाग गई। एक किताबों की दुकान में काम किया, पढ़ने का शौक़ था।

उस दुकान की मालकिन ने एक कमरा दे दिया। पढ़ाई फिर से शुरु करने में मदद की। अंधेरे कमरे में माँ-बाबा से बातें करती, वो नहीं थे, उनका एहसास बाक़ी था:

“बाबा! मैं पढ़ूँगी, अब मास्टर तो नहीं बन सकती। ज़िंदगी ने ऐसे पाठ पढ़ा दिए हैं, परंतु मैं पढ़ूँगी।”

“हिंदी के सब लेखकों को दुकान में बैठ पढ़ लिया, जो ख़रीददार किताब लेते थे उन्हें लेखक और किताब के बारे में बता देती। कहानियाँ सुनाना आदत हो गई थी।”

वक़्त बीतता गया ।”

“वो बैसाखियाँ काटने लगीं। फिर से क़ीमत चुकानी पड़ी। कमरा महँगा हो गया और ज़िंदगी सस्ती। कुछ महीने ही हुए थे कि फिर से वही सिलसिला शुरु हुआ। अब की दफ़ा पढ़े-लिखे लोग थे, कोठा भी नहीं था। मैं थी और मेरा छोटा सा घर, बिना एहसास का जिस्म और बिना धड़कन का दिल।”

“सुना है आपने? ये दिल धड़कता ही नहीं।”

“अभी भी कालांतर में किताबघर जाती हूँ .... तभी आप मिले! ज़िंदगी कहानी बन गई है।”

एक ज़माने से किसी ने कुछ पूछा नहीं था। सब सिर्फ़ बताते है, कोई बीवी से परेशान, कोई घर से, कोई नौकरी से, कोई ज़िंदगी से। कोई नहीं पूछता कि मेरी क्या कहानी है।

वो छतरी को गले सा लगाकर रोने लगी, काजल बह गया था।

बाल अभी भी बँधे थे।

आलोक उठा, “तुम अदरक वाली चाय लोगी?”

“मुझे कहानियाँ पसंद है, क्या रोज़ शाम को एक कहानी सुनाओगी?”

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प्रियंका वैद्य की कहानी "छतरी" की समीक्षा

हेम राज बंसल

साहित्य बहुत सी ऐसी कहानियों से भरा हैं जो वेश्याओं के जीवन और उनकी वास्तविक पहचान से जूझता रहा है। कमला दास (1934-2009) की कहानी "ए डॉल फॉर दी चाइल्ड प्रॉस्टिट्यूट" (1977) तेरह वर्ष की लड़की रुक्मणि की जीवन व्यथा पर प्रकाश डालती है। बाबूराव बागुल (1930-2008) की "वुमन ऑफ दी स्ट्रीट" और निरंजन की "दी लास्ट कस्टमर" भी हृदय विदार कर देने वाली कहानियां है। इस्मत चुगताई (1915-1991) और कुर्अतुल ऐन हैदर (1926-2007) की कहानियां भी समाज के इस कड़वे सच को उजागर करती हैं। लेकिन प्रियंका वैद्य की कहानी एक ऐसी ही मजबूर लड़की के जिस्म से न शुरू होकर समझ से शुरू होती है। “छतरी” कहानी एक उस लड़की के बारे में बताती है जो एक ऐसी दुनिया में खुद को खोया हुआ पाती है जहां उसकी अपनी कोई व्यक्तिगत पहचान नहीं रह पाती। कहानी के मुख्य पात्र ओजस के जीवन को देखें तो वह उन लड़कियों के बारे में जानना चाहता है जो उसे होटल में मिलती हैं। उसे इस बात से पीड़ा पहुंचती है कि कैसे जीवन मात्र एक बाजार की तरह है और भावनाएं बनावटी। वह लड़कियों की जिंदगी में उतर जाना चाहता है, उन्हें समझना चाहता है, उन्हें क्यों और कैसे खुद को बेचना पड़ रहा है, क्या वो ये सब किसी पारिवारिक समस्या के कारण कर रही हैं या किसी आर्थिक समस्या के कारण या फिर स्वेच्छा से।

कहानीकार आलोक को एक ऐसे पात्र के रूप में दर्शाती हैं जो बिलकुल अलग है जिसका मकसद किसी के जिस्म तक पहुंचना नहीं बल्कि उनकी आत्मा, मन और परिस्थितियों को समझना है। यही कारण है कि जब वो मुंबई जाता है तो एक क़िताबघर में एक ऐसी लड़की को पाता है जिसको आज तक किसी ने समझने की कोशिश ही नहीं की थी। जब वह उससे उसका नाम पूछता है तो वह बताना ही नहीं चाहती। उसे यह जान कर आश्चर्य होता है कि किसी ने आज तक उससे उसका नाम जानने की कोशिश ही नहीं की थी। उसके पास आज तक जितने भी ग्राहक आये थे वो केवल शारीरिक भूख मिटाने आये थे, कभी बत्ती तक जलाने की कोशिश नहीं की थी उन्होंने क्योंकि वो किसी को भी अपनी पहचान नहीं बताना चाहते थे। इसलिए सबको अँधेरा पसंद था और अँधेरे में ही लौट जाना ताकि उनकी सामाजिक पहचान पर कोई नकारात्मक प्रभाव न पड़े।

प्रियंका वैद्य ने काफी मार्मिक ढंग से इस लड़की का चित्रण किया है। भिन्न-भिन्न लोग उसे अलग-अलग नाम से पुकारते थे और इस कारण वह अपना नाम तक भूल चुकी थी। आलोक का उसे नैना नाम देना उस लड़की की वास्तविकता को दर्शाता है। ये नाम उसे अँधेरे से उजाले की तरफ लाने की कोशिश करता है, और समाज को भी एक आइना दिखाता है कि हमें कैसे अपने नैना, अपनी आँखे एक वेश्या की जिंदगी को एक अलग दृष्टिकोण से देखने के लिए खोल देनी चाहिए। नैना का खिड़की कभी न खोलना भी उसके अंदर के अँधेरे को दर्शाता है। खिड़कियां जीवन में नयी संभावनाओं, अवसरों की प्रतीक होती हैं और इसी के साथ-साथ बाहरी दुनिया की झलक भी दिखाती हैं लेकिन नैना के सन्दर्भ में ये सारी सम्भावनायें मात्र एक कभी न पूरे होने वाले सपने के समान थीं। इसके अलावा किसी ने भी उसके दिल की खिड़की में गहराई से नहीं देखा था और न ही किसी ने उसकी पीड़ा को भांपने का प्रयास किया था। यही कारण था कि वह कभी भी कमरे की खिड़की नहीं खोलती थी और दूसरा यह भी कि लोग अपनी पहचान छुपाना चाहते थे।

लेखिका नैना की तुलना उस वृक्ष से करती हैं जिसके सारे पत्ते और फूल तोड़ दिए गए हो। नैना के जीवन से भी सारी खुशियों रूपी फूल तोड़ दिए गए थे। कुछ वक़्त के तमाचों ने तो कुछ उसका जिस्म नोचने वाले विलासिता में डूबे लोगों ने। उसका बारिश को पसंद न करना उसके जीवन की एक असहनीय घटना से जुड़ा था। उसकी रंग बिरंगी छतरी के साथ नैना की बहुत सी यादें जुड़ी थी क्योंकि वह उसे उसके बाबा ने दी थी। यही वो छतरी थी जो उसके बाबा के प्यार का प्रतीक थी और इसी छतरी ने उसे उस भयानक रात को बचाया था जब गाँव में आई बाढ़ में सब कुछ डूब गया था। बची थी तो सिर्फ वही क्योंकि उस रात वह उस गाँव में नहीं थी। इसी विवशता के परिणामस्वरूप एवं अनाथ होने के कारण ग्यारह साल की अवस्था में ही उसे वेश्यावृति में धकेल दिया गया था। प्रियंका कोठे की वास्तविकता को सबके सामने लाकर नैना के माध्यम से उस पर व्यंग्य करती हैं जो नैना की उम्र की बच्चियों के चीखने की आवाज़ से पता चलता है। नैना साहस जुटा कर जिस तरह से भाग निकलती है और एक पुस्तक की दुकान पर काम करती है वह उसके होंसले एवं विश्वास को दर्शाता है। लेकिन अब फिर से बढ़ती मंहगाई के कारण उसे मजबूरन वही करना पड़ता है -- जिस्म बेच कर गुजारा। और अगर प्रियंका के शब्दों में कहूं "बिना एहसास का जिस्म और बिना धड़कन का दिल", कुछ इस तरह का जीवन हो जाता है नैना का।

आलोक नैना के दुःख भरे जीवन वृतांत से आश्चर्यचकित हो जाता है। वह उसे जीवन जीने की एक नई आस देना चाहता है। लेखिका कहानी को एक औपचारिक अंत नहीं देती, वह यह पाठकों पर छोड़ देती हैं और यह कहानी के अंतिम प्रश्नवाचक वाक्य "क्या रोज़ शाम को एक कहानी सुनाओगी?" से प्रतीत होता है। हालाँकि आलोक वहां लॉकडाउन के बीच फंसा था लेकिन उनकी यह मुलाकात हमें उनके जीवन के एक सुहावने भविष्य की ओर इंगित करता है क्योंकि आलोक बहुत ही सहानुभूति पूर्वक नैना को समझने की कोशिश करता है।

नैना की रंग-बिरंगी छतरी जीवन के उन रंगों अर्थात खुशियों का प्रतीक है जो उसके बाबा उसकी ज़िन्दगी में भर देना चाहते थे। नैना का छतरी से हमेशा चिपके रहना उसके बाबा की उसके ज़िन्दगी में परोक्ष रूप से निरंतर साथ रहने का एवं सहारा देने को दर्शाता है। जिस प्रकार से छतरी बारिश और धूप से हमारी रक्षा करती है वैसे ही नैना के बाबा की भेंट की हुई छतरी उसे विकट स्थितियों से निपटने में ढाल का काम करती है।

"छतरी" 2020 में प्रकाशित कहानी-संग्रह *लॉकडाउन डायरी: कही-अनकही कहानियां* में सम्मिलित है। यह कहानी आर्थिक विवशता में जूझ रही औरतों के मजबूरन जिस्म-परोसने की पीड़ा को दिखाती है। लेखिका की इस तरह की औरतों के प्रति संवेदना का पता इस बात से चलता है की उन्होंने पूरी कहानी में कहीं पर भी 'वेश्या' शब्द का प्रयोग नहीं किया है। यह कहानी आर्थिक विवशता में जूझ रही औरतों के मजबूरन जिस्म-परोसने की पीड़ा को दिखाती है। आलोक जैसे समझदार पात्र का होना भी इस बात का परिचायक है कि ज़िन्दगी में वेश्याओं को भी प्रेम की जरूरत होती है, उन्हें केवल शारीरिक भोग की वस्तु मात्र नहीं समझा जाना चाहिए। वे भी भावनात्मक रूप से भूखी होती हैं जिसे शारीरिक विलासिता कभी पूरा नहीं कर सकती। कहानी की सुंदरता इस बात से भी है कि कहानी में मुख्य रूप से दो ही पात्र हैं और वो सही मायने में एक-दूसरे से 'बात' करते हैं और एक-दूसरे के मनोभावों को समझते हैं। यह कहानी हमें *जॉर्ज बर्नार्ड शॉ* (1856-1950) के मशहूर नाटक *मिसिज़ वारेन'स प्रोफेशन* (1902) की भी याद दिलाता है जिसमें उन्होंने कहा था कि वेश्यावृति एक नैतिक चूक के कारण नहीं बल्कि आर्थिक जरूरत की वजह से उत्पन्न होती है। इसी के साथ-साथ यह कहानी बाल तस्करी, मुख्यत: मासूम लड़कियों की तस्करी तथा उन्हें जिस्म बेचने के धंधे में धकेलने वाले लोगों पर कड़ा कटाक्ष है।

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Audience's Live Response

Jyoti Mishra: Very passionate rendering Priyanka...

Seema Bawa: Mesmerizing!

Shikha Kapur: Amazing! Lovely narration!

Shelley Narang: Loved your rendition …so passionate, yet delicate.

Arun Guleria: Very heart touching story Priyanka... exposed hidden layers of society… great.

16

WHY FEAR WHEN I AM HERE

RAJAN KAUSHAL

Mohan was literally dragging his feet while returning home. Earlier, he would always want to reach home as early as possible. He had never experienced this kind of eerie feeling where he would just keep walking and should never be home. What troubled him the most was that he was fired from his job despite his honesty, loyalty and dedication towards the company. This thought crossed his mind relentlessly despite giving his heart and soul to the company, the owner didn't think at all before his dismissal. He was the oldest employee of the company and had been there from the day when the owner Mr. Chopra inaugurated it by breaking the coconut. He had seen the firm meandering through the tumultuous path during recession to achieving the culmination of success, eventually. When he arrived near his home, the kids playing in the street thronged him for grabbing their toffies as he would bring toffies for them everyday. "Today I have nothing, kids," said he and showed his empty hands.

He had a surge of extreme sadness on his face. The moment he pressed the doorbell, he didn't know how he would face his wife and kids. His wife opened the door but without making an eye contact with her or uttering a word, he went inside. The wife could sense the change in his behaviour. She gave him a glass of water and sat near him. Then after a moment she asked, "What happened, are you Ok?"

He said in a very low volume while gazing at the floor, "Rekha, I have lost my job. They have fired me."

The wife was shocked but remained quiet for a while. But after a couple of minutes she asked, "But why did they do it?"

Mohan sighed and explained, "You know our employees' union has been on strike as we haven't been paid our bonus and dearness allowance this year. The owner gave the reason that company had incurred a huge loss this year. Being there in the accounts department, I knew it was true. So, I appealed the union leader to give our owner some time and apprised him of the losses this year. But he got infuriated and alleged, "You are a minion of the owner and a traitor of our union." It had happened a week back. That very day, our employees in the union shouted the slogans against me, calling me a traitor. Today, union made a compromise with the owner and agreed on half bonus and half DA now and half of it to be paid the next year."

"So good, union has compromised," she interrupted.

Mohan continued, "Yes, but along with that they kept a demand in front of the owner."

"What demand," asked Rekha exasperatingly.

He replied, "The demand to fire me. When the owner resisted a bit then they threatened to go on the strike again. The owner called me and handed over the letter of dismissal and asked me to leave," his voice cracked while saying this.

His wife said, "But why did you advise the union leader when there was no need?"

When Chopra Sir, the founder of the company was on his death bed, one day he had called me. He asked me to help his son in handling the company being the oldest employee. He reposed trust in me. He valued my loyalty towards the company. So, there was nothing wrong in telling the union leader about the loss the company had incurred. It was true," he answered.

"But what did you get after telling this truth", asked Rekha in frustration.

"Don't worry I'll find another, job soon," replied Mohan.

"I had told you so many times to make a job shift to any other company. You could get better salary in any other company as per your long experience and capability. But, you never listened to my advice. You have not faced any interview in the last 20 years. It is not going to be that easy," responded Rekha.

Mohan didn't answer and just got up and went to the balcony. He sat there and kept ruminating over the happenings in his life. His wife brought him a cup of tea and the newspaper. "Why newspaper," he asked.

"So, that you may look for a new job," replied she.

Being a practical lady, she knew that he would have to try hard; only then he could secure a job. She never wanted the family to suffer for want of money. They required money for the education of their kids. The son was pursuing his engineering whereas; the daughter was preparing hard to get admission in a medical college. Both the kids were in dire need of money for their fees and other expenses. The next day Mohan appeared in an interview. They were looking for a fresher and not a very experienced guy for the post as they wanted to shell out a miniscule amount as emolument. There were very few jobs available, as most of the business houses were firing employees to cut their costs and fight with the recession in the market. A couple of companies could give him a job but he couldn't justify why he left the previous one. So, he was considered as a candidate with dubious reputation. He had tried everything but still couldn't obtain a job even after two months. Their savings had gone and there was an urgent need to get a job. His wife tried to contact a few relatives to help him getting a job, but to no avail. Mohan prayed every day in the evening but he was unable to concentrate on prayer. The picture of his *Guru* and his words, "Why fear, when I am here" flashed through his mind. He remembered what he had said in his discourse when he had gone to his *ashram* long ago, "Keep your intention correct, I'll take care of the rest."

But this question kept disturbing him that despite his right intentions, honesty and diligence he was suffering so much. A couple of tears fell down his cheeks while gazing at the picture of his *Guru.*

Though, his wife would come up with one idea or the other every day to secure a job but even after acting upon her advice myriad times, he could not obtain a job. Today, when he finished his evening prayers, she had been waiting eagerly with one more idea to divulge.

She said, "I was thinking that at times, especially in troubled times we ought to take the help of our relatives."

Mohan stared at Rekha angrily but again she started giving her logics,

"Why not, relatives should help each other"

"Who are you talking about?" asked Mohan.

"Krishna, Bhai Sahab, whom you had met, at the demise of his father," she answered.

"What are you saying? I hope you haven't gone crazy. He is my grandmother's sister's grandson, such a distant relative. I had met only once during the last rites of his father, 18 years back. He doesn't even live in India. He has his business in South Africa," he blurted out.

"So, what, but you told me that he had asked you to tell him if you ever need any help, he would," she responded.

He said that as everybody says, during exchanging pleasantries. He might not be even remembering my name. It has been 18 years, nobody remembers such a small meeting of just a couple of minutes," replied Mohan instantly.

"You please try, if he doesn't recognise you then don't take it to heart," she pleaded.

His son Lokesh was listening to this conversation and he started searching Krishna in South Africa on the Facebook. Facebook showed 37 results. He quickly went to his parents who were chatting in their balcony.

"Baba, Facebook is showing all these profiles when I searched Krishna in South Africa. Can you identify his face Baba?" asked Lokesh.

Mohan looked at the profile pictures on his son's phone one by one. But he couldn't find him, so he said, "Look for Krishna Awasthi."

"Ok, Baba," replied Lokesh.

As he searched Krishna Awasthi, there came a profile showing the picture of a gentleman. He was dressed in a charcoal black suit and had radiance on his face. There was an aerial view of the Cape Town as the background of his profile picture.

"Baba, look at this picture, is he Awasthi uncle?" asked his son with excitement.

"A little smile came on Mohan's face and he said, "Yes, he is Krishna Awasthi."

"Lokesh, can you get his phone number from the Facebook?" mother asked Lokesh instantly.

"Let me see, Mummy, if he had put his phone number on the Facebook," saying this, Lokesh started searching again.

"No, he hasn't Mummy, but we can send him the friend request from Baba's Facebook account. Let's see whether he accepts it or not?" replied Lokesh.

"Ok, send it, immediately," responded mother.

Though, Mohan was reluctant to send the request but he remained mum.

His son immediately sent the request and said, "Baba, if he accepts your friend request, it means he has recognised you. We will proceed further, only then, don't you worry Baba," Lokesh assured him as he knew the nature of his father, that he never wanted to lose his self - respect.

Now everyone got busy in their works. Lokesh checked the Facebook in the evening. He literally ran to meet his father who was on the terrace and was lost in his thoughts. Lokesh danced there like a kid but his father snubbed him, "What are you doing, Lokesh, have you gone mad?"

"Baba, Awasthi Uncle has accepted your friend request and he has sent you a message on the messenger, saying, 'Got connected to you after a long time, and how are you, Mohan?" informed Lokesh.

Mohan didn't believe his ears and said, "Are you sure, Lokesh?"

"Yes, Baba, yes, very sure," saying this he danced again.

"Stop this Lokesh, he has just accepted my friend request. I have not got the job yet, so, please stop this," said Mohan irritatingly.

"Come on, be optimistic, Baba, I am very hopeful, you will get the job soon, if he gives his reference," responded Lokesh.

Mohan came down; he himself typed a message on the messenger and described the entire scenario in short. He requested him to give a reference so that he could get a job and he would be grateful. After half an hour, Mr. Awasthi sent a message asking for Mohan's phone number. As Lokesh sent his phone number just after a couple of minutes Mohan received a call from Mr. Awasthi's office. His assistant connected him to Mr. Awasthi.

After exchanging the pleasantries, he explained the entire story in short though Mr. Awasthi listened the entire narrative sympathetically but he said, "You do one thing send me your CV across on my email address which I am sending you on messenger, let me see what I can do.

Ok, dear I will get back to you in a couple of days," and he kept the phone down.

After this, Mr. Awasthi straightaway called up his personal Assistant and told her, "See, Suzane, I'll send you a CV in a shortwhile, immediately forward that to the HR department of our office in India. I want to know every detail about this guy, his work, conduct, reputation where he was working in his previous company. I want to know the reason why he was fired from there."

"Right sir," Suzane answered.

After a couple of days Suzane came and handed over the report about Mohan which they had received from their Office in India.

The report said the same version what was narrated by Mohan in his telephonic conversation. Mr. Awasthi asked Suzane to connect him to Mohan.

He said "Hello Mohan, see instead of giving you a reference for the job, I offer you a job in my company in South Africa, you will get good salary and other perks, a nice place to stay as well. What do you think about the offer?"

"Well, I will have to think, Bhai Sahab," replied Mohan.

"What is there to think Mohan, have you got your passport?"

"No, I haven't," Mohan replied.

"Anyways that can be made fast these days. You make up your mind, get your passport ready and call me back, I'll send you the Air ticket, take care, bye." As he talked about the offer to his wife and kids, they couldn't believe it to be true.

"What are you thinking now, please go and get your passport made," said Rekha.

"I will help you Baba, I know the process, as I have got it made," said Lokesh

"What happened why you are so sad Baba," asked his daughter Meghna.

"Nothing, Meghna, I had never thought that a day would come when I would be forced to leave my company first and then my country. Now I have no other choice but to accept the offer," replied Mohan in a very sad tone.

As Mohan called Mr. Awasthi after a week and informed him that he had got his passport ready. He immediately told him that he would be sending him the tickets soon on his mail.

Next week, Mohan flew to South Africa, and there the official car was ready to receive him. He was taken to the guest house where he got fresh and changed his clothes. Then he met Mr. Awasthi and, in the office, he met him like a professional and spoke to him formally. He handed him over his appointment letter. He was given the designation of the senior Accounts Officer in the central office of Awasthi Group of industries. Mr. Awasthi would also sit in that office itself. From central office all the offices located in various locations are supervised by the owner and the staff. He was given a fully furnished apartment from the office as promised. Mr. Martin Astle was asked to brief him about the various companies of their work and his work in the accounts department. Every day Mr. Awasthi would keep a strict vigil on him through the CCTV, and he found him very punctual and sincere. When Mohan's senior Mr. Astle was called to report about the performance then he simply said, "He is excellent, Sir. He has got a thorough understanding of accounts."

Mr. Awasthi was elated to know that. He again called Suzane and asked her to check the credibility and character of the new employee by tempting him by wealth, wine and women. He was offered a huge bribe to change some entries in the accounts but he refused straightaway and acquainted Mr.Awasthi of the happening. Even other temptations of wine and women were not paid any heed by him. So, he had got through even in this test. His domestic help was asked about his lifestyle at home and he said, "This guy leads a very boring life, sleeps at 10 o'clock and gets up at 5 O' Clock in the morning. Starts his day by praying in his little temple and ends the day again by praying. He eats whatever is given to him, no likings at all, no drinking; no partying God knows what he lives for."

Just after a month, he was promoted as the head of the accounts section. Now Mr. Awasthi started discussing various problems his companies were facing. Every time he posed a problem in front of Mohan, he gave him an ideal solution. This clearly showed that Mohan not only was exceptionally well in accounts but even in other matters related to the company as well. Mr. Awasthi could sense that though Mohan had worked in a smaller company for so long but he was quite aware of the fact where the international market was moving as he was in the habit of reading the Economic Times everyday. Mr. Awasthi also realised that he had a vision for taking the company ahead but till that time he had neither got the opportunity nor any exposure to prove his mettle. One day when Mr. Awasthi called him and asked him, "Mohan, are you happy here?"

"Yes, Sir", Mohan replied.

"Then, why don't you look happy?" he asked further.

"I don't know, Sir, but I had not thought about coming here, living here as I was very happy in that small company. The owner Mr. Chopra had asked me before leaving for his heavenly abode to be there and look after the company. But destiny had something else in store for me," replied Mohan.

"I understand that, anyways it has been three months now, go home, spend a couple of weeks with your family, then bring them here, if possible," suggested Mr. Awasthi.

"Yes sir," Mohan replied.

"You pack your stuff you will get the ticket soon"

The moment Mohan leaves Mr. Awasthi spoke to his manager in the Indian office and directs him to initiate the deal of taking over KGX Pharmaceuticals. As the company was incurring losses and the price quoted was more than the expected so the owner readily agreed. Within a week KGX Pharmaceuticals had become Awasthi Pharmaceuticals. Mr. Awasthi called up Mohan and asked him about his plan of coming back to South Africa. He said, "Sir, I would be back after a week."

"Ok, you may stay there for three months now," said Mr. Awasthi.

"But I have come here just for a couple of weeks, Sir," responded Mohan.

"But I want you to be there, as I have recently taken over a new company there which was incurring losses. Now it is your job to transform that company into a profitable one as I have appointed you the CEO of that company."

"CEO of a company…., which company is that, Sir?" asked Mohan in surprise.

"KGX Pharmaceuticals," responded the boss.

After hearing that, Mohan felt choked, and couldn't speak a word.

There was silence for a few seconds and then Mr. Awasthi said, "Mohan, check your mail, your appointment letter has been sent along with all the details of the company. You may take any assistance required from our office in India. They will be contacting you soon. Once you join me in South Africa, after three months then you have to give me a presentation on what changes you made in the company. Wish you Good Luck! Take care." Mr. Awasthi kept the phone down after saying this.

Mohan sat down and it was still difficult for him to believe that he would be the head of the company from where he was fired six months back. Such a turnaround in such a short span seemed like a dream. When this news was divulged to the members of his family they were overjoyed and celebrated the occasion. But that night Mohan went to his *Pooja* room locked himself. He looked at the picture of his *Guru,* read again those words written at the top of the picture, 'Why Fear When I Am Here' and tears trickled down his eyes, he remembered the day when he was fired and he couldn't understand why he was fired. Now he realized that everything happened because of the divine plan as God wanted to give him something better.

He wiped his tears, got a pen and paper and wrote down about his priorities in the company. Then he wrote down those things he thought were necessary to do in the company but he couldn't do due to lack of authority. Next day, he went to the company, hung the picture of his Guru in his room first, with the title, 'WHY FEAR WHEN I AM HERE.' Some employees were shocked to see him as their CEO. Whereas, some were apprehensive as to what would happen to their jobs, others were happy too as they knew that he was a very good human being. He called up a meeting of all the employees, which had never happened in the history of that company. Earlier, only union leaders would speak to the management. Then they used to pass on that message to the employees. They would communicate the matters as per their petty motives. Mohan in this meeting delivered a brief speech, "Dear, fellow workers, first of all accept my greetings. I had joined this company as a junior accountant 20 years back. I vividly remember the day when this company was inaugurated by our worthy owner Late Chopraji. I still see myself as an employee who has been here from its inauguration till now, except for the six months' time. I love this company as it has given food, shelter and everything, I and my family needed and even to yours as well. I never ever felt that I was working just for salary. I expect the same love from you for this company. If you look after this company nicely, in return it will look after you and your family in same proportion. Many people may have some apprehensions that I may be revengeful and I may take their jobs away. But believe me I am not going to do anything like that. I know what happens when you lose your job, I won't let any of our employee go through that tragic experience at all. I know, you all are capable and have done a great job here. One thing I want you to keep in your mind that you all will grow if the company grows. Love this firm and this firm will love you in return. Each one of you may come and meet me anytime if you have any problem. No other channel is required. Two employees don't need any other channel to talk to each other. If we can bring this company into profit, it will be beneficial for each and every employee. Now every month we will meet and discuss our progress, problems and plans. Let's take it as a challenge that in this quarterly report we will bring this company into profit. Wish you all good luck."

This speech had infused a great confidence among the workers and now an environment of positivity had started prevailing there. In the same way he had a meeting with his sales executives and motivated them. He established a better communication with his wholesalers and offered them incentives too.

He did achieve what he had planned to achieve. When the next quarterly report came his company had attained profit. Share value of the company had gone up. When he went to South Africa, he delivered a presentation on how he converted a sick company into a profitable one. Mr. Awasthi was simply overjoyed. Though he was thoroughly professional but he couldn't stop himself rising from his chair and hugging him. Afterwards, he asked him to manage the Awasthi Pharmaceuticals from South Africa and he was given the task of CEO of two of his South African companies and was asked to bring the required changes in them in the next couple of months. He studied their entire system. With great precision he understood the problems and tried to find their solutions. He was doing a wonderful job. A few days later Mr. Awasthi called him up and asked him to come to his home in the evening. He went there and it was a huge mansion. As he sat in the drawing room waiting for Mr. Awasthi but then a servant came and told him, "Sir, is calling you in his bed room."

He went there and saw that Mr. Awasthi was in his bed and was looking sick.

"Sir, what happened, are you fine," asked Mohan.

"Come, Mohan, sit here," replied the boss.

Mohan sat on a chair near his bed. Mr. Awasthi held Mohan's hand and a stream of tears rolled down his eyes. He said, "Mohan, my brother I am dying."

"What are you saying Sir?, asked Mohan, absolutely shocked.

"Mohan just listen to me, about 10 months back, I got to know that I had blood cancer. I went to USA for my treatment but even there the doctors told me that I had maximum one year. They told me that for about 10 months I would be able to work but towards the end I would feel as if there was no energy in my body. I was not afraid of the death at all. But I was extremely worried about my son Shankul. He was a drug addict and when after remaining in rehabilitation centre for one year though he gave up drugs but suffered from acute depression. Now he doesn't speak much. He remains in his own world. I had already lost my wife a few years back. Now there would be nobody to look after him that was my chief concern. Another worry was my 18000 employees who work in my companies. What will happen to their future and their families? When that day, I sat for meditation and these words written there 'WHY FEAR WHEN I AM HERE' started flashing my eyes, again and again. Then picture of my Guru flashed. That was a kind of sign that don't worry. That very evening, I got a Facebook request from you. I put you to various tests but you passed all the tests. Now I don't think there could be a better successor to my business empire. Please, take care of my son Shankul. Treat him as your own son. Whatever belongs to me is yours from now onwards."

Mohan sat there shocked and speechless. He was gazing at the picture of his *Guru* hanging there in front of him with the words 'WHY FEAR WHEN I AM HERE'. He had now understood the divine plan completely why he lost his job and why he came there. It was all God's plan. The lawyer was called and all the formalities were completed. That day onwards Mohan remained there with him and every evening he would spend with him. He would sit near him till late in the night talking and assuring him that he would take care of his son and his business very well. After a couple of months Mr. Awasthi passed away very peacefully. No need saying that Mohan loved and looked after Shankul just like his kids.

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Review: "Why Fear When I am Here"

SHIVANI CHAUDHARY

Dr Rajan Kaushal's "Why Fear When I am Here" is a story about faith, telepathy and the tendency of our belief system to govern our lives. The author has done a commendable job in driving home the point that every end marks a new beginning. So, when something goes wrong, God wants us to get something better. It is thus, a story about trusting the divine plan.

Everyone has their own belief system but we all feel, at certain points of time, our lives moving according to a masterplan. We are like blocks of a jigsaw puzzle; we are arranged to complete an array which is then broken to create a fresh one. Our conscious choices may result in small decisions like choosing a profession or a place of work but we do not consciously choose our looks, parents, place of birth, etc. We can't negate the fact that life happens to us. There is a superpower that brings us sufferings, joy, gain or loss.

The story has a well-knit structure. There are two subplots in the story which merge through the punch line 'why fear when I am here' thus justifying the title of the story. The protagonist of the story Mohan connects both the subplots.

The first subplot unravels the crises that Mohan is undergoing. Despite being sincere, loyal and one of the founder employees of Mr. Chopra's company KG Pharmaceuticals, Mohan loses his job. His wife Rekha emerges as a strong and practical lady who does not want her family to suffer a financial crunch. She suggests Mohan to ask for help by contacting people who could help them come out of the situation. Their son Lokesh is pursuing engineering and their daughter Meghna is preparing hard to get admission in a medical college. So they need funds. Mohan sits in meditation and visualizes his guru's words 'why fear when I am here'.

In the second subplot we come across Krishna Awasthi, his business empire, his son Shankul, his Secretary Suzane and Mr Astle, Mohan's senior in South African setting. Mr Awasthi is Mohan's distant relative – his grandmother's sister's grandson.

The two subplots meet when Mr Awasthi offers Mohan a job in his company but puts him through ordeal by testing him in every possible way. The reader is shocked when the revelation of Mr Awathi's Blood cancer is made. His wife is already dead; his son Shankul who used to be a drug-addict and now faces anxiety issues needs care and supervision. It is when Mr Awasthi is reminded of his guru's words, 'why fear when I am here' that he gets a call from Mohan asking for help. Mr Awasthi wants Mohan to take charge of his business empire and his son Shankul. Mohan becomes CEO of the company he was fired from and Mr Awasthi passes away peacefully. Mohan takes good care of Shankul and the conflict of both the subplots gets resolved. The ending gives relief to the reader and also suggests the reader to have faith when life gives troubles.

The story begins with a metaphor of returning home empty handed, without toffees for kids. It is ironical that the one who used to bring hope and toffees to the kids who thronged around him is hopeless and helpless today. The opening of the story describes how layoff shakes a person completely, more so, when the person has been more than dedicated and sincere in his services. Here I'm reminded of Dr APJ Abdul Kalam's quote: "Love your job but don't love your company, because you may not know when your company stops loving you."

The story holds significance during the COVID crisis when many employees are being laid off. Crises like losing a job, a shelter or a partner paralyze the world of the sufferer and nothing seems to make sense.

In such cases end of something seems like end of everything. But there is always a reason why things happen the way they do. This is what Dr Rajan Kaushal's story is all about.

Mohan loses his job, his credibility and his reputation. He is literally dragging his feet while returning home. His anguish, despair and guilt weigh heavy on him so much so that he hesitates to face his wife and children. He is devastated. Unlike Saul Bellow's Tommy Wilhelm (who loses his job as he fails as a salesman and an actor) and also unlike J.M. Coetzee's David Lurie who is forced to resign due to his misconduct, Mohan has lost his job despite his right intentions and efforts. Mohan tells a truth about the company's financial position which evokes the anger of the employee union which ends up calling him a traitor and the owner's minion. How his own fellow workers blame him and turn against him without knowing the truth reminds me of Gurmeet

Bedi's "The Terror" which Prof. Meenakshi F. Paul has translated. In both the cases people become polarized in their opinion and blame the innocent, without knowing the truth. They suffer due to what psychologists label as mob behavior.

Mohan is suffering despite his right intentions, honesty and diligence. Whenever someone gets sufferings in life, a train of unanswered whys keep disturbing the sufferer. Here in the story even the reader feels Mohan's pain and fails to understand why bad things happen to good people. It's been two months since he has lost his job. Now he is running out of funds and has responsibilities to shoulder. He is unable to find a new job. He feels lost.

It is rightly said that when all doors get shut a small window opens up. For Mohan this window opens up in the form of his guru's words, why fear when I'm here. Then begins the search for Mr Krishna Awasthi, Mohan's distant relative, who could help him find a job.

We see technology and social media playing a great role in the story. Krishna Awasthi is searched through Facebook. E-mail, messenger, CCTV cameras-all provide the necessary communication set up. The new technology and social media that we all fear is estranging people from one another is seen as a medium to connect. Their speed and economy help Mohan and his family sail through their troubled phase.

Mohan's hard work, sincerity and efficiency are appreciated by Mr Awasthi and his team. His karmas ultimately bring him the desired *prarabdha* and then his entrepreneurial journey begins. In this way the author establishes the reader's faith in Mohan's virtues and skills. This is how the story becomes inspirational.

Success does not come to him easy and overnight. He possesses the entrepreneurial skill set which enables him to succeed. Besides financial skills, knowledge of international market, crisis management skills, he possesses good communication and leadership skills. He also has good customer service skills, analytical and problem-solving skills, strategic thinking and planning skills. He communicates effectively with his employees, sales executives and wholesalers.

Speeches of leaders carry huge influence. His speech as a new leader infuses confidence amongst his team. He introduces an open-door policy so that anyone could meet him. He gives his employees a conducive environment to work in and a work culture that encourages them to work to their full potential. As a result of which productivity increases. It all happens because Mohan has the capacity to translate his vision into reality. His leadership steers the company into a profitable position.

Mr Awasthi is impressed with the results that Mohan has brought forth. When we meet Mr Awasthi for the first time in the story, we visualize him as a mighty, established businessman but when the story unfolds, we get to know how he is suffering and has limited time left. He is worried about his son who is struggling too. He fell a prey to drugs and then to depression. After his stay at a rehabilitation centre, he needs to be taken care of.

Here the author successfully drives home the point that all of us have our own struggles. The grass looks greener on the other side but we fail to realize that life has a different question paper for all of us. We all have our phases.

Mohan's tough phase is over now. His virtues are rewarded. He becomes the CEO of the company he was fired from.

Here I am reminded of Steve Jobs, the co-founder of Apple who was fired from his own company. In 2005, Jobs was reported to say: getting fired from Apple was the best thing that could have ever happened to me. Then he founded his second company NEXT which was ultimately acquired by Apple and Jobs became its CEO.

The story becomes didactic when Mohan is tempted with money, wine and women but the integrity of his character wins him good fortune. There is also a moral statement in the story that the whole story rests upon, "Keep your intention correct I will take care of the rest."

In Dr. Rajan Kaushal's story we find that Mohan and Mr Awasthi get connected through magical words of their guru 'Why fear when I'm here'. These words resonate with Aamir Khan's famous dictum from the movie 'Three Idiots': Aaaal ij well/ All is well. Faith is all about trusting something you cannot prove. It may include fooling oneself into believing that everything's going to be fine. This is why, I believe, carrying lucky charms or wearing lucky dresses work because they make you believe that everything is going to be all right.

It's true that faith does not solve all the problems of life, but neither does logic nor science. Mohan's prayers and Mr Awasthi's meditation re-inforce the Power of Prayer elaborated by Dr APJ Abdul Kalam.

So, here we have a powerful story that gives hope to one and all, especially during the testing Corona crisis. I congratulate Dr Rajan Kaushal for writing such an inspirational story and I am thankful to ETF for giving me the opportunity to review this beautiful story.

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Audience's Live Response

Vandna Kumari: Rajan sir, amazing story which revolves around few words but with a great vision.

Girija Sharma: An absorbing story Rajan… was glued to the screen throughout... congratulations!

Anupama Singh: Very interesting story Rajan Sir which is replete with a positive note of faith and uprightness.

Abhyudita: Rajan sir, the story envisions today's scenario. Congratulations!

Shikha Kapur: Rajan sir, you are an amazing storyteller!

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बिल्लियां

रेखा

ड्राइंगरूम में आते हुए बाबूजी आदतन नज़रें झुका लेते हैं। ऐसा नहीं कि वे खाने से पहले ''या देवी सर्वभूतेषु क्षुधारूपेण...'' जप रहे होते हैं। वे तो बस दरवाज़े के सामने वाली दीवार से आंखें बचा रहे होते हैं। जैसे ही वह दीवार पीठ पीछे चली जाती है, वे आश्वस्त होकर अपनी तयशुदा कुर्सी पर बैठ जाते हैं। अब जो दीवार सामने है उससे आमना–सामना होने में कोई भय नहीं, कोई खतरा भी नहीं। वह तो केवल दीवार है। लेकिन जिस दीवार से वह नज़रें बचा रहे होते हैं एक वही दीवार ऐसी है जो केवल पत्थर–गारे की चिनाई नहीं है। वह धीरे–धीरे सांसें लेती हैं। दम साधे खड़ी रहती है। बाबूजी से आंखें मिलते ही अंधेरे में बिल्ली की आंखों की तरह कौंधती है और किसी अपशकुन की तरह रास्ता काट लेती है। फिर खाना तो क्या दिन भर का चैन चौपट हो जाता है।

कभी–कभी बाबूजी अपने इस बेपेंदे डर से परेशान भी हो जाते हैं। आखिर ऐसा भी क्या है? क्यों नज़रों में नज़रें डालकर नहीं देख लेते एक बार? अपने बूते, अपने हाथों एक–एक ईंट चिनवाई है इस घर की। सब दीवारें उन्हें अभयदान देती हुई दृढ़ता से खड़ी हैं। सिर्फ यही एक दीवार नज़रें पड़ते ही काली बिल्ली बनकर गुर्र–गुर्र करने लगती है। बाबूजी अपने भय के समर्थन में कई तर्क जुटाने लगते हैं, जैसे कि नैपोलियन जैसा व्यक्ति भी बिल्ली से बहुत डरता था। या कि जैसे कई लोग चूहे को देखते ही संतुलन खो बैठते हैं या कौकरोच देखकर छलांगें लगाने लगते हैं... आदि–आदि।

कल जब किटी छोटा–सा बलूंगड़ा (बिल्ली का बच्चा) उठाए रसोई के दरवाज़े पर खड़ी होकर सासु मां से कटोरी भर दूध मांग रही थी तो बाबूजी झट से अपनी लाइब्रेरी से बाहर आ गए थे। किटी बिल्ली की नरम पीठ पर हाथ फेरते हुए कह रही थी... ''हाऊ क्यूट–हाऊ क्यूट'' और तभी बाबूजी किसी अनिष्ट की आशंका से अवश होकर किटी के हाथ पर झपट पड़े थे। बलूंगड़ा किटी के हाथ से नीचे लुढ़क गया और म्याऊं–म्याऊं करता हुआ कहीं सरक लिया था। किटी कह रही थी... ''मैं इसे पालूंगी'' और बाबूजी उसे इस तरह देख रहे थे जैसे वह धर्म बदल लेने का सार्वजनिक एलान कर रही हो।

वे धीरे–धीरे अपने कमरे में लौट आए थे और कुछ हैरान से थे कि उनका हाथ ऐसा बेकाबू कैसे हो गया। बहू के सामने मर्यादा का उल्लंघन कैसे कर बैठा उनका अपना अनुशासित हाथ।

किटी पांव कुछ अधिक ज़ोर से पटकती हुई अपने बैडरूम में घुस गई। खीझकर गहरी सांसें लेती हुई कि कैसे पूर्वाग्रह ग्रसित लोगों के बीच आ फंसी है उसकी ज़िन्दगी। ससुर के सामने क्यों चीखकर नहीं कहा उसने– ''मुझे अपने शौक पूरा करने का पूरा हक है।'' कैसे सह लिया उसने वह मूक प्रहार।

ब्याह के पहले बरस पति के साथ रहते हुए उस बीहड़ प्रान्त में किटी के पास लम्बा खाली समय रहता। पति सुबह ही काम पर निकल जाते और लौटने तक शाम ढल जाती। कॉलेज और साथी–संगियों का साथ छूटे अभी अधिक समय नहीं बीता था। गृहस्थी के दायित्वों की जकड़न भी कसी नहीं थी। बहुत अलसाये हुए से दिन थे। ऊंघते–जागते। धीरे धीरे अपने नए किरदार को समझने और उसमें उतरने की छोटी–छोटी कोशिशें कर रही थी किटी। ऐसी ही एक कोशिश के तहत उसने जूट के एक टुकड़े पर कुछ कसीदा करने की बात सोची। हो सकता है अब भी कहीं मन में मासी के वे शब्द कुन–मुन करते हों जो उसने अपने घर की बैठक में अपनी होने वाली सास के सामने सिर झुकाए हुए सुने थे – ''इतना पढ़ने–लिखने के बावजूद घर के कामों में बहुत रुचि रखती है किटी। सिलाई–कढ़ाई–बुनाई सबमें निपुण है।''

गर्मियों की सूनी, लम्बी दुपहरी में घड़ी टिक–टिकाती रहती और किटी नहाई–धोई, डियोडोरेंट की सुवास में डूबी–डूबी जूट के रेशों में काले डोरे घुमाती रहती। कुछ ही दिनों में जूट के उस टुकड़े पर से दो काली बिल्लियां अपनी नीली–हरी–आंखों से टुकुर–टुकुर झांकने लगीं। ऐसे सीधे बेबाक देखती हुई कि मानों आंखें मिलते ही गुर्र–गुर्र करने लगेंगी। गर्मियों की छुट्टियों की उपलब्धि स्वरूप बिल्लियां लौटीं। किट्टी ने लकड़ी के सुन्दर चौखटे में मढ़वाकर उन्हें सजा दिया। न जाने क्या सोचकर उसने डाइनिंग रूम की इस दीवार को ही सर्वाधिक उपयुक्त पाया।

बाबूजी 'या देवी सर्वभूतेषु...' कहते हुए जैसे ही डाइनिंग रूम में घुसे, उनकी पहली नज़र जैसे बिल्लियों ने बांध ली। काली डोरी से उकेरी गई वे बिल्लियां कुछ ऐसी जीवंत लगीं कि बाबूजी को आभास हुआ मानों वे अभी कह देंगी... 'म्यांऊ'! उस दिन वे खाना नहीं खा पाए। लगा जैसे बिल्लियां उन्हें नज़र से बांधे उनके हर निवाले को भूखी आंखों से ताक रही हैं।

उन्हें पता था यह किटी की कलाकृति है। उसे आहत किए बिना इन्हें इस कमरे से नहीं हटाया जा सकता। ''ये मनहूस बिल्लियां यहां से हटाओ–'' पत्नी से यह कहना जितना आसान था बहू से यही कहना उतना ही कठिन। बहू पढ़ी–लिखी आधुनिक लड़की थी। कुलीन थी। कला अभिरुचियों से बखूबी परिचित। उसके आने से घर की साज–सज्जा, रख–रखाव में कई परिवर्तन आए थे। बाबूजी देखते थे कि उन्हें कुछ अनुकूल, कुछ प्रतिकूल भले ही लगे, बाकी

सभी सगे–सम्बंधी, मित्र–बंधु बहू की अकूत प्रशंसा किया करते। सौंदर्य बोध का एक नया आयाम उसने इस मध्यवर्गीय पारिवारिक परिवेश में जोड़ा था।

बाबूजी जब भी कभी पत्नी से अपनी पसंद या नापसंद की बात करते वह हर बार यही कहकर बात टाल देती– ''कर लेने दो जी। जो जी में आए करे। हमारे दिन तो अब लद गए। इनका ज़माना है। इनका घर–बार है। कहते हैं ना – जेड़ा ध्याड़ा, तेड़ा बरतार ा– यानि समय के अनुकूल बर्ताव।''

बाबूजी को पत्नी की इस प्रवृत्ति पर कभी तो आश्चर्य होता है, कभी क्रोध आता है। वही पत्नी जो उनकी पसंद के साथ कभी आसानी से सहमत नहीं हो पाती थी, जो अपनी बात से कभी टस से मस नहीं हुई, वही बहू और बेटे की अनुकूलता के साथ कितनी आसानी से ढल जाती है। कहां से आता है यह लचीलापन? सहनशीलता, उदारता, स्वीकार या फिर स्त्रीसुलभ व्यावहारिकता? कभी–कभी तो वे भीतर ही भीतर तमतमा कर रह जाते। उन्हें लगता यह एक तरह की चालाकी है। सत्तापक्ष से समझौता कर लेना... पर फिर वे रुक जाते। तो क्या वे भी यह मान रहे हैं कि घर में भी सत्ता का हस्तांतरण हो चुका है? रिटायरमेंट के साथ ही उनके सिंहासन पर बेटे–बहू का राज्याभिषेक कब और कैसे हुआ, उन्हें तो पता ही नहीं चला। हां, केन्द्र में बदली हुई सरकार का आभास उन्हें घर की बदल रही नीतियों और कार्यक्रमों में जब–तब होने लगा था। कभी हनुमानजी के कैलेंडर की जगह पवन वेग से दौड़ते हुए घोड़ों ने ले ली। कभी तानपूरे पर झुकी मग्न मीरा के स्थान पर कोई मांसल सांथाली आ विराजी। गेंदे और चम्पा की क्यारियों में कैकटस विलसने लगे। गौरी गैय्या का गऊग्रास पालतू कुत्तों में बंटने लगा। कुछ फर्नीचर नौकरों के कमरों को शोभायमान करने लगा और कुछ बरसों की जानी–पहचानी अपनी सी चीज़ों ने अपने अस्तित्व की रक्षा के लिए बैकयार्ड में शरण ली। बाबूजी ने हर परिवर्तन की फाइल पर अपनी अच्छी या बुरी टिप्पणी दर्ज की और हर फाइल मुंह पर फीता बांधे चुप लगा गई।''

बाबूजी देखते और कभी कभी बनियान उतारते हुए या कमीज के बटन बंद करते हुए बुदबुदा देते– ''वसक वतकमत बींदहमजी...''। वे उन दिनों पहुंच जाते जब शैली की 'वैस्टविंड' पढ़ते हुए उन्हें भी अपने भीतर सर्वध्वंसकारिणी एक प्रचण्ड आंधी की पदचाप सुनाई देती थी। फिर वे स्वयं ही उसके आवेग से थककर शिथिल और त्रस्त होकर बैठ जाते और एक लाचार सा अर्थानुभव उनके हाथों में कांतिहीन पत्थर की तरह चुभता रहता – परिवर्तन कर पाना और परिवर्तित हो जाना – इन दो स्थितियों के बीच का अंतर ही यौवन और जरा का अंतर है। तर्क को कुछ और आगे ले जाते तो लगता पत्नी ने तो सर्वथा हार मान ली है क्योंकि औरतें जल्दी बूढ़ी हो जाती हैं। लेकिन वे स्वयं? वे अभी हार नहीं मानेंगे। सामना करेंगे आखिरी दम तक।

दूसरे दिन किटी ने देखा बिल्लियों को डाइनिंग रूम की दीवार से पदच्युत करके बाहर गैलरी में स्थान दिया गया है। जूतों वाले रैक के कुछ ऊपर। यह भी कोई जगह हुई? दरवाज़ा खुले और स्वागत में जुड़े हाथों की जगह बिल्लियों

के पंजे नज़र आए। आव देखा न ताव उसने उन्हें वहां से भी उतारा और प्रतिकार स्वरूप बाबूजी के बैडरूम के दरवाज़े से कुछ हटकर टांग दिया।

बाबूजी वहां से गुज़रे तो 'बाबूजी पाय लागूं' की तर्ज पर कोई जैसे आते–जाते कहने लगा, ''बाबूजी, म्यांऊ–म्याऊं!''

बाबूजी उसी दिन बाज़ार से एक लोकप्रिय, लोक मनभावन पेंटिंग का प्रिंट लेकर आए और डाइनिंग रूम की खाली पड़ी दीवार पर टांग दिया। हल्की क्रीम रंग की दीवार पर फूलों का गुलदस्ता महक उठा। बाबूजी को कुछ सार्थक सा कर पाने का संतोष हुआ। डिनर के समय किटी की नज़र पड़ी। स्वतः नाक सिकुड़ गई... ''हाऊ कॉमन।'' दूसरे दिन वह प्रिंट बाथरूम में पहुंच गया।

बैडरूम की तरफ आते हुए बाबूजी अभ्यासवश जपते हुए आते... ''या देवी सर्वभूतेषु निद्रा रूपेण संस्थिता...'' और दोनों काली बिल्लियां उनकी नींद पर झपट्टा मार कर गुर्र–गुर्र करने लगती।

बिल्लियां उन्हें सदा से संस्कारवश अशुभ और अमंगलकारी लगती हैं। मक्कार, धूर्त, चालाक, लिंबड़ी और लालची। बचपन में मां कभी–कभी छोटी बहिन को 'लिंबड़ी बिल्ली' कहकर डांट लगाती थी क्योंकि वह खाने की चीजों के इर्द–गिर्द डोलती रहती थी। किसी भी निरीह प्राणी पर कैसे तो झपटती है बिल्ली। चोरटी कहीं की। चोरों की तरह दबे पांव चलना। घात लगाकर हमला करना। उनके ताऊजी से सुना था उन्होंने... ''बिल्ली आंखों वाली औरत का भरोसा नहीं होता। चंचल, लुभावनी परन्तु दगाबाज़।'' पता नहीं क्यों सभी उनकी मां की चाल की तुलना बिल्ली की चाल से करते हैं। इतनी धीमी, सुकुमार कि चले तो आहट तक न हो– बिल्ली की तरह। वे चाहते हैं सारे उपमान बदलकर रख दें। अपनी दिवंगता मां की किसी भी स्मृति के साथ बिल्ली जैसी कुलच्छणी को न जोड़ें।

जब इंटर में पढ़ रहे थे, उन्हीं दिनों एक फिल्म देखी थी। उसमें काला जादू करने वाले जादूगर की मेज़ पर बैठी हरी आंखों वाली बिल्ली आज भी जब याद आती है तो पल भर के लिए सांस रुक जाती है। कहां तो कहा जाता था कि खाना परदे में खाना चाहिए। भली–बुरी नज़रों से बचाकर और कहां अब डाइनिंग रूम में सम्मानित पुरखों की जगह डटी हुई हैं ये कलूटी बिल्लियां।

बाबूजी ने हठात निश्चय किया कि बिल्लियां इस जगह से हटेंगी नहीं तो वे सबके साथ मेज़ पर बैठकर खाना छोड़ देंगे। पत्नी के सामने यह घोषणा की तो वह उल्टे उन्हीं पर बरसने लगी – ''इसे ही कहते हैं सठिया जाना। बिल्कुल बच्चों जैसा स्वभाव हो गया है आपका। बालहठ करने लगे हैं इस उम्र में आकर। छोटी–सी तस्वीर इतने प्यार से बहू ने बनाई है। तारीफ करना तो दूर, घर में कुहराम मचा रखा है इसे लेकर। खबरदार जो और कोई बावेला मचाया। जहां है, वहीं रहने दें इसे... क्या फर्क पड़ता है... बड़ा वही जो मन मारे।'' कोई कुछ भी कहे, अपनी नींद और भूख में ऐसी घुसपैठ वह कैसे सहें? घर में इतनी अवहेलना हो तो व्यक्ति की अस्मिता बचेगी कैसे? बाबूजी ने दूसरे दिन बिल्लियां

हटाईं और बच्चों के कमरे में टांग आए। इस बार तो पत्नी सरेआम बहू के खेमे में शामिल हो गई।

''बच्चों के कमरे में काली बिल्लियां? मति मारी गई है क्या आपकी? कहीं रात को डर जाएं तो?''

एक बारगी मन में आया, पत्नी के हाथ पकड़कर गिड़गिड़ाएं ''अरे तुम भी नहीं समझोगी? मैं भी बिल्लियों से डर जाता हूं... अंधेरे में ही नहीं, उजाले में भी।'' यह भी मन में आया कि बिना किसी से कुछ भी कहे चुपचाप इसे उतारें और लकड़ियों वाले कमरे में छुपा कर रख दें। आखिर यह रामायण, महाभारत, गीता, कुरान या जपुजी साहब जैसी कोई पवित्र कृति तो है नहीं जिसे महाप्रलय से बचाना ज़रूरी हो या राजा रवि वर्मा का कोई पुश्तैनी चित्र जिसे धरोहर की तरह सुरक्षित रखा जाना ज़रूरी है। फिर एक दूसरा भाव आया कि कला को नष्ट करना अपराध ही नहीं वहशीपन और कायरता भी है।

बाबूजी कुछ शर्तों पर अपनी बात रखना चाहते हैं। मान लें कि डाइनिंग रूम में फूलों के गुलदस्ते का चित्र फूहड़ अभिरुचि दर्शाता है और यदि वहां जानवर ही सजाए जाने हैं तो गाय और हिरण जैसे भोले और वात्सल्यमय पशु क्यों नहीं? बाबूजी धीरे–धीरे समझने लगे हैं कि ये सब अब फैशन में नहीं है। विश्वास और भोलेपन की जगह संशय और चालाकी ने ले ली है। शायद इसीलिए फैशन में हैं बिल्लियां।

बाबूजी बहुत कुछ कहना–सुनना चाहते हैं परन्तु गृहस्वामी की मर्यादा आड़े आती है और वे चुप लगा लेते हैं।

फिलहाल बिल्लियां बच्चों के कमरे से भी हटा ली गई हैं और अब लाइब्रेरी की शोभा बढ़ा रही हैं। अब बाबूजी डाइनिंग रूम के लिए फलों का चित्र ले आए हैं ताकि बहू का घायल मन कुछ तो शांत हो। बहू ने देखा तो फिर टिप्पणी की – ''यह घर है या कुंजड़े की दुकान।'' बाबूजी चुप रहे। बहु–बेटियों से खुल्लम–खुल्ला विवाद करना शोभा नहीं देता।

वाकयुद्ध की जगह शीतयुद्ध चल रहा है। बिल्लियां सचमुच कपट चाल चल रही हैं। बिल्ली का वैभव हमारी संस्कृति का पराभव है। यह हमारी अभिरूचियों में पाश्चात्य हस्तक्षेप का द्योतक है। बाबूजी इस धर्मयुद्ध में अर्जुन की तरह विषाद मग्न खड़े हैं। किस पर शस्त्र उठाएं? शत्रुपक्ष में उनके अपने ही बाल–बच्चे हैं। लाइब्रेरी जो उनका अभयारण्य थी आज जब सुबह 'सर्वमंगल मांगल्ये...' जपते हुए वहां घुसे तो सन्न रह गए। नारायणी भगवती सरस्वती की जगह वही कुलटा बिल्लियां मूंछें निपोर रही थीं। उनका चेहरा निष्प्रभ हो गया। घायल सन्नाटे से घिरा हुआ वह चेहरा, घुटी हुई चीखों से विकृत वह चेहरा... नहीं, अब और नहीं... बाबू जी के हाथों से गाण्डीव छूट कर गिर पड़ा।

कौन जीता, कौन हारा? यह बताने वाला कोई शेष नहीं। बिल्लियां अपनी जगह लौट आई हैं शाश्वत सत्य की तरह घूरती हुई – भयहीन। बाबूजी उनकी ओर पीठ किए चुपचाप खाना खा रहे हैं।

Kitty Alias Baloongda: The Many Lives of Cats in Rekha Vashisht's "Billiaan"

ANU JAIDEV

This review will begin with addressing aspects of formal composition in the short story "Billiaan" to illustrate how they relate to the complex trooping of cats in the story. It will offer an analysis of Babuji, the beleaguered father figure in terms his mental monologues and conversations (or the lack thereof). It will also address the positioning of women in the narrative space, the unnamed wife, and the daughter-in-law, Kitty. It will deliberate upon the central crisis on which the story hinges: Kitty's act of defiance of weaving cats into a screen, and the brilliant intertextual referencing of another set of felines in a different screen. The review will then focus on the reconfiguration of domestic space in terms of the almost postmodern assemblage and re-assemblage of artifacts, pictures and other household objects. The consequent discussion will focus on the joys of patriarchy and the rather elusive discourse of power. It will examine the articulations regarding gender, authority, intergenerational power struggles, fears and the whole build-up around the inarticulable: all in the space where potentially anything/everything may be said.

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'Billiaan' by Rekha Vashisht is a neat narrative triumph. In terms of composition, the story is clean and elegant. It is beautifully structured: almost Austenian in its brevity, unforced and unpretentious, inviting the reader into a simple immersion in the textual space. For a story so endearingly simple its achievements are rather tremendous. It reveals its treasures one bit at a time, in terms of carefully ordered details and observations. For instance, Babuji's reluctant admission of his fear of cats drops into the text, one innocuous detail at a time. From a psychoanalytical perspective, his fear, bordering on phobia would not be about cats per se, it would be about what the cats represent to him. The build-up is as compulsive as it is reluctant. The narrative utilizes a judicious mix of aggregated memories, assessment of the self, reasons and rationalizations through interplay of flashbacks and flash forwards. What emerges is a deliciously detailed composition hinging on the concretized image of unexpected/ unsuspecting stray kitten, the 'baloongda'. Babuji's unlikely nemesis actually gets him to act out of character when he literally pushes the little kitten out of Kitty's arms: and he is painfully aware of it. The stray kitten (that later morphs into the screen woven with cats) is rejected in this initial moment as at once unworthy of note and also unreasonably terrifying (clearly capable of inducing a completely knee-jerk phobic response). The articulation of Babuji's turmoil comes through the closely aligned narratorial voice: the only plausible strategic machination for articulating the inarticulable.

Cats are troped in the story as culturally loaded metaphors with a complex self-referentiality as they erupt into the textual frame and then linger about. The cats are positioned within the two intergenerational, different cultural orientations that collide and culminate into the central crisis. Cats, or rather the different ideas of cats, have an organic, contextual reference point within the story. Cats inspire very different kinds of emotions in the two contenders. The stray is a furry pet for Kitty-- "how cute, cute, cute'. For Babuji, it is a tad repellent, inane, inferior and beneath contempt. Miniature feline, at once domesticable, and potentially wild, vile and ominous. To be fair, cats are rather belated entrants in the domestic space in this provincial part of the subcontinent. Naturally for him, Kitty's adoration of the kitten as "how cute" smacks of a much more serious breach. In the narrator's description, it is akin to almost the cardinal transgression of religious conversion.

Babuji catalogues several negative stereotypes associated with cats: vile strays, tricksters, clever dissemblers, surreptitious predators…and cat-eyed-unfaithful- women. And bit by bit he divulges his own fear and disgust of them. The memory of a green-eyed black cat in a film he saw as a young boy that still sends shivers down his spine. His mother's reprimanding of his kid sister as a greedy, gluttonous kitten, forever loitering about eatables. And finally, his uncle's comparison of his mother's walk with the walk of a cat; perhaps the most insidious and problematic of all his memories, fueling his need to be rid of cats. He needs to cleanse his mother's memory from any stain of anything cat-like, but he can't. At best he can repress it, only to have it come alive in walls breathing cats. Incidentally, all the cat traits he catalogues are also subliminally suggestive of his unsayable assessment of the daughter-in-law monikered Kitty.

In an adept illustration of inter-generational, gender dynamic the story presents Babuji, the supposed formidable father figure, at a complete loss when he is dealing with his daughter-in-law. It is by definition an uneasy relationship, based entirely on discretion, consideration and the ability to hold one's tongue. His almost visceral revulsion towards the kitten is his communication. Kitty's belated response is in terms of the end product of her talent and aesthetic judgement: her woven cat-screen, which she gifts to the household. She installs her cats right where she wants them without saying anything to him. Whether her choice of subject for this artistic maneuver is incidental or deliberate, is beside the point. Essentially, she is right on target. Given that she is the daughter-in-law, the domestic space and its arrangement is supposed to fall in her purview and it is his intrusions vis-à-vis the positioning of the cat-screen that are regarded as completely unwarranted.

Kitty weaving cats into her screen is reminiscent of another weaving literary predecessor. The intertextual reference to Aunt Jennifer weaving tigers into hers is quite unmissable (See Adrienne Rich's "Aunt Jennifer's Tigers"). But Kitty isn't Aunt Jennifer yet, and the kittens have a little growing up to do. Moreover, cats seem relatively harmless, or do they? A tiger burning bright on a screen would be a declaration of fierce assertion, but it probably wouldn't be as creepy as a cat waiting to pounce, and of course the metaphor would be lost. Or maybe, Kitty is aware that she can accomplish so much more with cats than her predecessor could with tigers. With this single stroke, she over-rides Babuji's authority without confronting him. A fine instruction in stealth-strike for Aunt Jennifer!

The story skillfully engages with this almost postmodern assemblage and re-assemblage of artifacts, pictures and other household objects: moving/removing; placement/replacement/substitution of the screen from the dining room to the entrance wall, to the library, to the kids' room, and back to the dining room. The screen, pictures, artifacts are tokens through which power is negotiated here. The constant changes in domestic art illustrate the unquiet currents underlying them. Most of the changes are innocuous: too ordinary to be taken up and yet the chipping away at authority is unmistakable. The calendars, flowers, paintings, even routine practices of everyday piety are in a kind of transition. The '*gau gras*' for instance, is replaced by scraps for cats and dogs. Kitty's aesthetic decisions render the established-order-of-things in the household completely redundant. But then, the order of things is about so much more than just how things are ordered. The modification of space and images after all has significance beyond what is said and acknowledged. These are gestures and maneuvers which happen in lieu of the conversation that can't happen. There's a new lesson for Babuji in the new, altered order of things in his universe. Power is not necessarily vested in any person who seemingly dispenses it: it is dialectical and discursive, and its accents change over time. Old order changeth. Indeed. In his traumatized isolation, Babuji alone sees what the deliberate fixtures of the screen are capable of doing. Alas, he can't say anything: not even to his wife.

The story thus, makes a fine illustration of the joys of patriarchy. Patriarchy, which privileges a certain class of men for a major part of their lives, ultimately renders them incapable of adequate personal or even domestic articulations. The *noblesse oblige* code does demand that one doesn't complain and doesn't explain. After all, how does the master of the house say that he is so terrified of the cats, he can barely breathe? How does the lion ever acknowledge feeling completely upstaged by a cat? The story, thus, brilliantly illustrates the plight of paternalist inflexibility versus feminine adaptability and inventiveness. His wife adapts, siding with the children (in an almost cat-like betrayal?) Kitty works around his disapproval of her intended pet. The cats find a way. He alone stands disempowered.

In his interview with Derek Attridge titled "This Strange Institution Called Literature", Derrida talks of the literary space as the space where potentially anything/everything may be said. The narrator breaches the silence around the unsayable dilemma here, hence the story. The true triumph of the story however, is in its effective verbalisation of the nature of the unsayable and the impossibility of articulation: authority trapped in its own sense of consequence cannot articulate its vulnerability. The most important conversation, which goes round and round in circles in Babuji's head, can never actually be had: even if it means that he may barely breathe while he swallows his food, potentially for the rest of his life. That is his fait accompli and his tragedy.

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Audience's Live Response

Vivek Negi: Narration with a difference. Experience is definitely matchless. *Bahut aadar Ma'am apko.*

Sangeeta Singh: Amazing intertextuality.

Pankaj Singh: What a beautiful narration of complexity of human emotions and sensitive and often difficult family relations!

Kulbhushan Sharma: Beautiful narration of the story Rekha ma'am. Human relations are complex and this assertive complexity brings in originality and multiplicity of it. Your story portrays life and its diverse facets. In-depth analysis Anu Ma'am.

Girija Sharma: A masterly analysis of a deeply symbolic story. Not only the story… you too articulated the inarticulable so lucidly…

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NIRVANA

SANGEETA SINGH

Jay could hear the voice say "Go north." He couldn't help it. He knew if he didn't obey the voice all his sanity would be ripped off of him. He packed a bag for himself, stuffing all kinds of essentials he could find. After having an early dinner, he went to bed. Waking up early, he took a quick hot shower and caught the bus that went north. He waited anxiously for guidance from the voice.

Jay was among the very few people of the world who could hear this voice. It was called the white voice. He had found out about this voice a few months ago. He had help from another man, who, like him could hear this voice. It would've been overwhelming for any ordinary person but he was unfazed by it. He had thought that this was just a dream. However, after hearing his guide's story he started trusting the voice. He believed it was real.

The concept of the white voice was hard to explain. It didn't really have an exact meaning however, one thing was clear, it could only be heard by someone who was about to attain enlightenment. Buddha saw his previous lives flash in front of him because he had summoned the white voice. It was this very voice that guided him to nirvana. Jay was about to get enlightened.

Unlike Buddha, he hadn't summoned the white voice. It just appeared to him. He didn't meditate, didn't practice any rituals and yet he achieved what millions only dream of. It was simply because he had continued the cycle of life and he had suffered for all his sins. All his actions had had reactions and his life had come to a still place. He wasn't moving forward or backward; he simply existed. It was time to relieve him from this endless cycle. It sounded impossible.

He could still feel human emotions. The main difference was, most of his emotions were stirred neither by him nor the world outside but by the white voice. He was going to be part of the bigger power soon. He obeyed the white voice because it was true.

Jay had been waiting for another order from the voice. He had to keep going North till then. Maybe he would end up in the mountains. He didn't care much. The bus stopped at a station for some time. He got out for some fresh air. He stretched his hands and entered the convenience store nearby. He sipped the canned coffee he had bought from a vending machine. He stuffed some other snacks he had bought in his bag. The journey may just be long.

It was late at night. Everything was dark outside. There were only three people in the bus, including Jay. The bus moved on an empty road. It was very silent. He was about to fall asleep when he heard the voice again. "Stop!"

All sleep vanished from his eyes. He sat upright on the seat and patiently waited for the bus to stop. When it finally reached the station, Jay got out and waited for further directions. The voice whispered in his mind "Find Sita Mane, daughter of the carpenter."

Jay looked around. He wanted someone's help to navigate through this unfamiliar little town. An old drunk man was sitting on a bench at the station. "Sir?" Jay asked.

"What do you want?" The man replied.

"Sita mane, the daughter of the carpenter." Jay said, mimicking the monotonous white voice.

"Yeah. She lives nearby." The old man took a swig of liquor and gave jay the directions. Without wasting any time, he went to the carpenter's place. It was a warm and cozy looking house. Jay knocked at the door. A young beautiful woman stood in front of him.

"I am here for Sita Mane." He said. The woman was hesitant to entertain this stranger who arrived at such ungodly hour.

"Yes. That would be me." She managed to say.

"I wanted to talk to you about the dreams you are having these days. They are true.'' As soon as he finished the sentence Sita froze. Her eyes were wide open, she wasn't blinking. She was completely stunned. She tried to say something but Jay beat her to it.

"Let me explain this. The voice that tells you that it is approaching is truly approaching. It tells you of everything you did in this life and you cannot resist it. Does it scare you?"

"Yes. It doesn't feel normal." Sita mumbled. "Please come in." She added as he followed her inside. They sat in the well-lit living room to discuss things further.

"The voice doesn't feel normal because it isn't any ordinary voice in your head. It is very supreme. We call it the white voice. I hear it too. It speaks to me even when I am awake. It has given me a task. I have to tell you about this white voice. If I don't do this you will resist it. It's human nature. We are just not ready to face something that powerful. I resisted it too. My guide helped me embrace this voice. It is actually very gentle. I am aware of everything that has happened to me in the past. I also know of things that will happen in the future. It is very simple."

Sita nodded. She was not sure if all this was true but something about Jay made her feel that he was the most honest man in the universe. Jay's simple and calm aura soothed her and she knew she could trust him.

"I will tell you everything. Then you can be with your thoughts, the white voice will guide you further. You will attain peace." Jay stated. It was already late. He decided to stop talking. Sita sensed his wish and agreed to let him stay at her house that night. He slept in the living room.

They both were awake early next day. Sita made breakfast which they ate in silence.

"Let me continue from where we left. The white voice will completely take control of your thoughts gradually. It will tell you about your current life first. Everything you did, every decision you made, everyone you met, every emotion you felt will be displayed to you with perfect details. You will start to understand how your mind works."

Sita listened to him in awe. She was completely focused on every word that comes out of his mouth. This flash of memories would be scary, he told her but she felt that she could get through this step.

"Not only your current life, the voice will tell you about all your previous lives too. You will know who you were before you became Sita Mane. We all have an energy inside us, which keeps us alive. It is our truest self. The voice will show you the journey of this energy. You will get to know all the places this energy has been. You will be fully aware of everything. The voice will then take control."

Sita kept nodding. She couldn't believe she was going to experience all this. After her father's death she was left alone. Her mother had passed away when she was very young. Sita started practicing meditation to get peace. This was what she wanted: to get rid of the cycle of birth and death. Whenever she meditated, she was in a peaceful place. She wanted to be there forever. It was almost time to get fruit for her labor.

"You have to complete one task before you finally get enlightened." Jay spoke, pulling her back into reality.

"What task?" she asked.

"You, like me and many others, have to guide the next person who is about to get enlightened. Like I explained, many people resist the voice. It hinders the process. It is sad that in the modern age people are so engrossed in all the illusions, bound by possessions and relations that they fail to see the truth even if it approaches them. That has created a need for guides like us. We have to carry out this task of teaching others otherwise; everyone will be stuck in this world forever."

"Why not teach everyone about this? Everyone wants to get rid of this suffering after all."

"Many people have tried. I am afraid I won't be able to teach anyone about summoning the voice because I never did that. The voice itself approached me. Even if I told everyone about how the voice works, would they believe me? In the end, it is about faith. Someday all your sins and deeds would be balanced. You will reach this point that day. That's all I can say. Not everyone needs to be taught. Some things should be experienced at the right time." Jay answered.

She seemed to be satisfied with the answer and stayed silent. There was nothing more to it. He had completed his task. He was waiting for the white voice to stop all his thoughts. It was better if he left. He soon announced his wish of going back home. Sita did not protest.

Jay attained enlightenment. The voice emptied his thoughts and replaced them with the final truth. He was at peace. Soon his being became connected to the supreme power. His physical body glowed before its energy left it. He was free from everything.

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"Nirvana": A Review

SUBHASH VERMA

The great Indian Yogi, spiritualist, and teacher Paramahansa Yogananda has said, "Before embarking on important undertakings, sit quietly, calm your senses and thoughts and meditate deeply. You will then be guided by the great creative power of spirit". 'Nirvana' focuses on the universal themes of enlightenment, self realization, spiritualism and intuitive experiences of common person that lead people to be directed or guided by, in Paramahansa's sense, 'the creative power of spirit'. The author brings the Buddhist philosophy back to life in her own way in this gripping story about Jay, the central character, who is on the threshold of getting enlightenment. As Jay is about to get enlightened, he can see the state of the world and life with clarity but to complete the cycle he has to explain the nature of the white voice to someone else, who is trying to resist it and fails to comprehend its stance. Having awakened, he realizes his duty to help other people find freedom from suffering. He himself is guided by someone to embrace this voice and now he has surrendered himself to it. It is the same white voice which was summoned by Lord Buddha and it 'guided him to Nirvana'. It can 'only be heard by someone who was about to attain enlightenment'.

Jay's journey to attain enlightenment commences with the instructions of the white voice, initially to 'Go North' and later to 'find Sita Mane, daughter of the carpenter'. Like Jay, Sita is also a pure soul that's why the voice itself approached them. Jay finally succeeds in completing the enlightenment cycle after explaining the true nature of the voice to Sita. Now the inner tumult of Sita is pacified but before getting enlightenment she has to 'guide the next person who is about to get enlightened'. At the end of the story Jay attains 'Nirvana' where his mind is at complete peace and his soul is merged with the universe.

The title 'Nirvana' fits the story it is meant for. This is because the concept of Nirvana is discussed throughout the story and at the end the protagonist is in a state of consciousness beyond emotions, positive or negative. His desires and sufferings all go away and he is 'free from everything'.

The story explores the roles of metamorphosis that impact human lives at various places. Metamorphosis as a personified presence in the story seems to be regulating and governing all life, dissolving the line between the past and the present. In 'Nirvana' metamorphosis is a pivotal concept and the author has used it as the unifying theme to bring out the reality of life. Perhaps, it would be relevant to call metamorphosis, in the modern times, an alternative perspective of reality, a subtle governing principle of life. Jay and Sita's evolution from human beings to enlightened individuals is caused by metamorphosis which leads to growth. Metamorphosis as a doer in the story plays two roles in Jay's life: as perspectives, bringing psychological and emotional change and as transcendence, bringing spiritual change. After being physically and emotionally or intellectually transformed, the final step of life is self-realisation or 'Nirvana', a philosophical term for spiritual transformation and at the end of the story both the characters are completely metamorphosed.

The theme of intuition leads the readers to meditate or cogitate on the total immersion of the soul into the universality of life where everything is connected. It appears that when someone acts positively by heart and soul, she/he reaches closest to the universal force. This thematic conception of the story appears reflecting the fact that our life stories, world's history of the present and future are written by the same metaphysical hand. Reference of Lord Buddha regarding the 'Nirvana' or 'Enlightenment' is given but Jay doesn't go through any sort of meditation on the basis of that we can declare him enlightened. But Jay's attitude can be analysed in context to the Buddhist principle expecting that Jay would not have any unfulfilled desire of material things and he has no attachments with the external or physical things around him and he has reached the neutrality about the martial world. So, he might have had that level of developing intuitions in him that turns to be the 'white voice' and draws him towards other similar but unfamiliar fellow Sita in the North. There is another probability that can be observed through modern psychological or neurological tools where such persons are usually kept under treatment declaring them mentally ill. But this probability itself gets faded when another character Sita is convinced with Jay's journey to her home accepting the fact that she was passing through the same dilemma or thought processing which has been clearly discussed by Jay with her.

The story is beautifully structured where the plot and setting take place in an unknown place where journey of the protagonist is guided by the white voice, 'Go North'. Jay takes bus from unknown place and the reference of the 'empty road' and 'only three people in the bus' provides the hint to anticipate the setting of place as rustic or hilly. The reference of the journey to the end of the 'mountain' again offers hint that the protagonist is journeying from plain or semi-plain to mountains. However, the thematic weight of the story overlaps the events of the plot as well as the setting. Characterization is woven with unique human beings guided by the mysterious metaphysical force. The story carries the readers to the spiritualistic journey of Jay whom we come to know what the 'voice' is really about. No extra information about the main character is given and the readers are compelled to believe in the events happen to occur with the intuitive experiences of Jay and the other female character Sita. The characterization reflects the spiritual magnetism among the characters affirming the universal idea that each character in this world serves a particular purpose determined by the supreme power. The point of view is third person omniscient where narrator castes light on the analogy of the flow of self realization in two chief characters. The narration moves forward directly as per the direction of the metaphysical 'voice'. The soul of the chief character Jay is about to be liberated from the sin of the world and the character appears more ripen in context of 'self realization' while the character of Sita is walking on the similar track. Jay hands over the spiritual flame of 'self realization' to Sita who can feel its eternal power and does not ask even a single question from the unknown person, Jay, about his uncalled and uninvited arrival at her home. Through its cardinal characters the story delineates two-fold moral message that one must make the knowledge comprehensible for dissemination and goodness is redeemable. The narrative indicates the intuitive mind and self realization of a person who can pursue, listen or feel the voice of God.

The story can also be analysed through the post-modernistic lenses. Many people believe in the 'voice' of supreme power and intuition and take them as guidelines to instruct their lives. They even think that dreams in form of intuitions are the "Language of the World" that liberates people from the cycle of birth and death. Jay's journey to 'Go North' as per the direction of 'white voice' is postmodern stance of "hyper-reality" that proves truthful when another character Sita unquestionably accepts what Jay articulates to her.

The story can be judged by the sharpness of its preception, which stirs our feelings and thoughts into motion. Carrying deep philosophical meaning, the story is multi-layered and enigmatic. It is written in simple but powerful words and hence it is precise and meaningful. The story opens up the research and dialogues regarding the theme, conception and dimensions of spiritual principles of human life. Is Nirvana the only way to liberate oneself from the unnecessary suffering that most of us inflict upon ourselves by resisting various attributes of reality or calling them wrong? Is there something like a transcendent metaphysical space one in some sense occupies once fully liberated? These questions are intentionally left unanswered by the author and the readers are impelled to ponder over them.

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19

BALANCE

SHIVANI CHAUDHARY

Shanaya used to be a feminist but now she is not.

Born to the parents who believe educating girls is as important as educating boys, she grows up into a confident working woman. Her parents and Sangharsh's parents decide to have a meeting. She meets Sangharsh, the horoscopes match and the families slate the date for their wedding. Shanaya and Sangharsh get married. She is glad no one has a problem with her working as a teacher after marriage. That's the only concern. She just wants to puruse her career in teaching and she believes everything else can be adjusted to. She believes love begets love and respect begets respect. So, she is not worried. She knows if she cares enough and manages things well, it's not difficult to make a place for herself in the new family and have a life of her choice. All the literary discourses that she reads and teaches to her students turn her into a feminist. Her dad's words keep echoing in her mind, "You are not only going to make rotis and do the laundry, you'll have a life too." She is a woman of opinion and wants to raise her voice for equality but little does she realize that her perception is not all that would govern her life.

Sangharsh is more than what Shanaya could ever ask for. A thorough gentleman -- caring, understanding, adaptive and well-behaved. His professional prowess coupled with benevolence could win any heart. He endears everyone with his enchanting smile. Shanaya is so fortunate to have him and so is he to have her as all he wants from her is adjustment and he has understood that she is capable of doing it. He had lost his mother in his early childhood and has been brought up by a step-mother and father to whom he owes everything. He is indebted to them and wants to keep them happy in every case because they have brought up a motherless boy, they have educated him. They are everything for him.

Shanaya understands his concerns and leaves no stone unturned to please them. Gradually she learns to place her in-laws' preferences and choices before hers. She is so inspired by Sangharsh's personality that she feels blessed to bring happiness to this guy's life who has struggled all his life through after losing his mom. His aura is magnificent and he spreads positive vibes wherever he goes. He gels with everyone and always helps the needy. He treats all human beings equally irrespective of status, gender, caste or religion. He could easily be friends with rickshaw-walas, drivers, his subordinates, seniors and higher authorities, MLAs, entrepreneurs or anyone. Shanaya learns life lessons through his conduct. His life journey had only polished and refined him. He does not have bitterness for anyone.

Shanaya is amazed at the possibility of existence of such a man. "I must thank my lucky stars. He is so broad minded…does not believe in gender defined roles…does not try to dominate me…feels happy if I take decisions and he wants me to celebrate life. He never tries to change me. He adores me the way I am. Such men only exist in movies or in fairy tales. I must have done some good karma. All my life through I have not come across someone like him… someone with so much of basic goodness, compassion and generosity." He definitely deserves the best life and Shanaya is all set to give him one.

Pretty soon Shanaya realizes the dictum that she had forgotten *'Har kisi ko mukammal Jahan nahi milta, kisi ko zameen toh kisiko asmaan nahin milta.'* (It's not possible for everyone to get everything.) She has got a perfect husband and a perfect job but her frequencies don't match with her in-laws. They have different value and belief systems. They view 'the new bahu' (the newly wed daughter-in-law) as someone who would rob their son of his money. They believe she will take their son away and will break their home. They have set norms for bahus of the house. They say the elder bahu has taken away their son as they moved out of town for their jobs. Now they just have the younger son. He must obey them and so must his wife. They have all terms and conditions ready and Shanaya is just expected to adhere to them. They tell her how much they have done for her husband and it's now his and her duty to obey them and look after them. They make it clear to Shanaya that whatever they have belongs to their daughter (who happens to be step-mother's daughter) and whatever she wants, she should get from her parents. They have no love or respect for the new bahu; they only have conditions and expectations.

Every now and then they tell Shanaya how short and ugly she is as compared to their tall and handsome son. They often pass comments stating that her parents didn't give enough in marriage though there were other alliances from richer parties for their son. They make fun of her parents too which becomes unbearable for Shanaya. Her parents are valuable for her. They have raised her without selfish motives. Through her thick and thin, they have been with her. For her, all elders deserve respect but her in-laws believe that boys' parents are superior and they can take liberty to comment on her and her parents. She sees all feminism that she has read collapsing in front of her eyes. She is helpless and speechless.

Shanaya should only listen to. She should never raise her voice otherwise they would tell Sangharsh that all that they have done for him has gone waste because she does not respect them. They want respect from the bahu whom they despise and ill treat all the time. They have a license to mock at her, to be angry at her for nothing and to complain about her but she being a bahu is not expected to react. They support Sangharsh in whatever he does but oppose Shanaya for everything she does. Shanaya understands the ploy; it is a strategy that they use to please him and to gain financial benefits from him. They boast of being rich but it is only Sangharsh and Shanaya who plan and finance everything. In-laws have hoarded the liquid assets for their daughter and want Sangharsh to construct a house. He does not mind because they have already done enough for him. They have financed his education and wedding. They have done it for a motherless boy.

Shanaya could easily draw a parallel between Sangharsh and the protagonist of the movie 'Beta'. In her favourite TV series, 'Ashoka' Sangharsh is akin to Bindusar who is so blinded by the honey-coated words of step-mother Helena that he respected her as his own mother whereas she wanted him to die so that her son becomes the king. Sangharsh's father had lost his first wife to suicide so he never opposed his second.

In-laws keep a vigil over Shanaya and they ensure that she does not have any time left to enjoy. They keep her engaged in chores and other activities. They convey to her that watching TV and indulging in hobbies suit unmarried women. They get furious at the very idea of her wanting to go for a movie with her friends. 'There is enough work at home and you want to go for a movie. Going to theatres is nothing but wasting time and money.' They would say with a grave tone. Shanaya's conduct is under constant scrutiny so much so that whether she does something or she does not, she is always held guilty. Her action or inaction, her words or her quietness, her involvement or detachment – everything offends her in-laws. She feels choked and unwanted. It becomes almost impossible for her to bear the brunt all the time. Shanaya keeps adjusting. Women around her have strict in-laws and hubbies who are not supportive. At least, he doesn't say anything to her or to them. He is a great manager and he knows how to strike a balance.

She has only two major faults -- one she comes from another home and second, she is their 'bahu'. Had she been a neighbour or a family member, they would have sung praises of her management skills. Being an early bird, she goes to a nearby temple daily to seek the blessings which give her strength to carry out her hectic routine. Then cooks breakfast and packs lunch, gets her son ready for school and drops him to school. Meanwhile she does the laundry, arranges things, cleans the house and quickly glances at the grocery requirements she should get while on the way back home from her college. She makes sure that all meals, snacks and tea are prepared and served well in time. She takes care of her son Dhruv's requirements and homework. She takes her in-laws out for buying essentials that they need.

Since she is not a perfect driver, she doesn't mind the comments she regularly gets from her in-laws for her driving. Even her parents would have probably felt the same way but they wouldn't demean her every now and then. "It's not fair to compare your parents with your in-laws. Your parents love you for who you are. Their love doesn't lessen with your mistakes. In-laws can't love you. They are only trying to accommodate you." Shanaya consoles herself. Fulfilling her responsibilities towards the family motivates her and makes her happy but there are untangling snafus since her in-laws are not happy with her and they are constantly poisoning Sangharsh against her. "Girls break families. Their parents' interference can break homes too. Keep them at one arm's distance. She is now our bahu. She must only think about us." Shanaya could clearly perceive how boys' parents' interference could break homes too but saying it would hurt Sangharsh and she wants him to be happy at any cost. "Let them say and do whatever pleases them. Let them complain", she tells herself.

They hate it when he takes her out for shopping. They don't have any problem when he goes for parties or short trips with his friends but they cannot stand it if he goes out with her. At home they never like it when he gives her time or even takes her name. They don't want him to take her to their relatives either. Encompassed by patriarchal and orthodox set up, Shanaya feels suffocated. She has no choice but to adjust without complaining. She feels like spending time with her parents but she is not allowed to visit them often. It is only Sangharsh who manages to take her to her parents' place on a few weekends when his parents are at their village. Had he not been so supportive, Shanaya would have given up. She doesn't have the grit to face it all alone. She is not quiet because she is weak but she values Sangharsh more than being right so she just keeps adjusting.

Sangharsh is aware of her plight but he is feigning ignorance because he does not want to hurt his parents by speaking in her favour. Shanaya tells herself, "Had it been a female-dominating society, Sangharsh would have come to stay with her parents. What if they found faults with him? What if they despised him? What if they kept passing on comments to him? Would she have spoken up? Probably, she would have been in a similar fix. She too wouldn't have courage to oppose her parents. She too would have tried to make a balance. So, it's not a question of being a boy or a girl. It's a question of balancing the two opposite poles, two types of people who will never like each other." So, she can't blame Sangharsh, it's the lop-sided system that has to be blamed. His parents' insecurity is to be blamed. The system does not provide security to aged people. So, like most of the parents, Sangharsh's parents want him to be their support in old age, and after all, that is why they brought him up! Why should they waste their time and energy to think about Shanaya's happiness?

She has her solid support system. Besides her husband and son, her parents and siblings adore her. She grows in teaching and research at a professional college. Her students and their queries keep her engaged. She is blessed to have wonderful colleagues, mentors and friends. She has passion for two things – English and Teaching. She is living a life of her choice. At her workplace she loves to read and write. She is fond of writing poetry and listening to talk shows and interviews. What she enjoys the most is engaging her students in activities like group discussions, mock interviews and extempore. She feels victorious when she witnesses their stage fear vanishing. She gets rewarded when her students improve their pronunciation after attending her accent learning modules. She feels immense proud when her students send her messages thanking her after being placed or selected for a job. Grooming her students and seeing them evolve add to her self-esteem. When she counts on the number of blessings, she feels highly grateful.

At home, everything is fine but her in-laws keep complaining. Sangharsh does not pay a heed. He is a busy professional and a socialite. He just wants his space. He is doing great community service and social welfare activities. Such petty issues do not matter to him. He is capable of earning name and fame. He has told Shanaya his heart. He has told her "I am not someone who is satiated after a 9 to 4 job. I like socializing. I have my goals. I have my circle. I want to enjoy my life." Shanaya does not mind giving him space because he has noble thoughts. He adores her. He is someone who has taught her the meaning of celebrating every moment. He has given her wings. He supports her in living her dreams. He plans weekend parties and outings for Shanaya whenever his parents go to their village on weekends. So, there is no one to say things. He wants peace and he does not want to annoy his parents. At the same time, he wants his wife to enjoy her life too. He is a good son and a good son never prioritizes his wife and children over his parents, although he had tried to intervene once when he saw Shanaya getting choked because of his step-mother's remark. What his dad said in response to Sangharsh's initiative shocked Shanaya even more*, "Tere saahi na sadka ch rulde. Iha tu chakki ne kursiya par bathalya."* (Motherless boys like you stray on the roads. She picked you up and helped you become a professional.")

Shanaya is all praises for Sangharsh. He has been through a lot but never gives way to negativity. He is a devoted son, a loving father, an excellent professional, a helpful friend and an adorable husband. He has struggled a lot after losing his mom to suicide. As a sensitive child, he diverts his attention to others' pain and develops a notion that it is his duty to resolve people's problems to whatever extent he can. He grows up into an empathetic intellectual whom everybody adores. He is an expert in his field. All his seniors, juniors and friends are in awe of his wonderful behavior and management skills. He undertakes counselling of sad, depressed and lonely people at his personal level. He has rejuvenated many lives. He is adept at doing chores, extremely fond of cooking. He wants to help Shanaya whenever he gets time out of his busy schedule but is shamed by his parents that he being a boy can't do women's jobs. Ironical! He had learnt the chores since childhood as he used to help his step-mother.

It has been a decade since Sangharsh and Shanaya have been saving money to build a house of their dreams. Rented accommodations have their limitations. After a great search they could find a suitable flat under construction. Sangharsh pays the advance and books it in Shanaya's name. "Why my name?" she enquires disbelievingly. "You are the backbone, you deserve it" says he. "But won't your parents be annoyed?" "No," assured he. To their surprise, his father also helps them in paying the advance amount for the house. His father's paternal property is disputed and he is getting old. So, he wants to live with his son. He wants Sangharsh to be his support and Sangharsh is honoured to be so.

Everything seems to be fine but for the attitude of his parents towards his wife. His parents' constant poisoning and interference has now started having an adverse effect on Shanaya and Sangharsh's relationship. He wants to be the best son and she wants to be the best wife but the balance falters.

He succumbs to the increasing pressure, increases his intake of drinks and starts smoking. She opposes all this. He dislikes her for opposing him. She could see his deteriorating health so she urges him to reduce his drinking and smoking but he stops listening to her. He starts avoiding her. She is unable to make sense of this stupor but she is trying and praying for things to improve.

Nothing works. One day she returns from her college to find him dead.

Unbelievable! What a shock! A shock for everyone! What a mysterious demise!

"Was he hypertensive? He probably got a heart-attack."

"Was something wrong with him? Did he succumb to some pressure?

"Was he depressed?"

No clear-cut closure but one thing is clear that he is not going to come back. Shanaya wants to die too. She would never know what he took in mind. She is in utter doldrums. She has lost all faith. How could her God let him go? Was no one in the universe powerful enough to stop the disaster from happening – the sun, the moon and the gods she worshipped? She realizes, "Life's mathematics is a little different. If one goes out of two, nothing remains." Life has lost meaning for her but she can't quit. She needs to live for her son, her fatherless son who needs to be taken care of. A life full of challenges awaits them.

His parents are shattered too. They probably failed to realize that things are to be used and people are to be loved. After his demise they buy the house which Sangharsh wanted to. Now it belongs to the step-mother and her daughter. Now in-laws don't have a problem to send Shanaya to her parents' place. She moves out with her son.

She often says to her son Dhruv, "I wish he could have stayed and I would have gone." One day Dhruv comes to her and goes like, "Never say that again mom. Maybe the world needed him more than you but I need you more than anybody else. I'll stay strong, if you stay fit. Mom, please don't lose your balance… look at me… I am just nine. I want to live my life happily. See what all I have gone through. After losing dad I can't afford to lose you. Look at me and you will have several reasons to live. Look at nana nani (maternal grandparents) and all those people who love you. They can't see you depressed. Unfortunately, dad is gone but we are still here. We are here to live not to ruin our lives. Dress up the way you used to. Do what makes you happy. Dad is in our hearts and one day, everyone will go. Let's live till then." The same wisdom that Sangharsh possessed has suddenly dawned upon naïve Dhruv. Losing a loved one is the most painful experience but who can challenge the cosmos or God, if there is any? Who can challenge the laws of universe? Who has got an answer for why someone dies? But we have to pick up the pieces and stand up again till our time comes. Though it is true that a comeback needs to be much stronger than the setback.

"Did he take pressure for something? …He probably got a heart attack or maybe it was some kind of a mistake. Issues were there in his life since his childhood but what could have broken such a strong man? He never complained, never misbehaved. He got enough recognition and love. Could he give up? No! He was extremely strong. Who does not have issues in life? Could anything break such a fighter? No! Impossible! But… Remember 'The Titanic!' The unsinkable sank! It did. All of us are vulnerable. Who knows what's coming up next? … He can't ruin our lives. He knew he meant the world to us. He loved Dhruv too…but who knows when death would knock at the door?"

Men are mentally less strong than women. They express less. They believe in escape strategy to overcome their stressful situations. Just like women, men have their pressures too. They have a pressure to earn and a pressure to pay debts to their parents. All their lives they can't do enough for their parents even if they do everything possible for them. They have to maintain a balance between professional and personal lives. They have a constant pressure to maintain a balance between their parents and wife. Some men face false allegations by women. Some are made victims. They are doubted more; they can be easily conspired against. In many homes they are not taught basic life skills believing that their mothers, wives and sisters would be doing everything for them. Men are so dependent on women. They have competition from women in almost all domains. Men have to deal with property issues. They compete and fight with their own siblings for property and other issues. They face peer pressure too. Parents, friends and relatives are constantly comparing them with more successful people. As a result, they have to handle professional jealousy too. Men talk about peace treaties and world affairs but they don't know where their own lives are heading. They have a pressure to know the latest happenings. They have pressures of various kinds. Shanaya has seen the constant pressure that men face, even the very strong ones. So, her notions of feminism collapse. "No, it's not about a man or a woman; it's about striking a balance."

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Narrated by Praveen Kumar

Review of "Balance"

HEM RAJ BANSAL

The story "Balance" by Shivani Chaudhary is very touching as it dwells on the sensitive topic of suicide/death and sensitizes us about the importance of viewing each individual as a unique human being, having her/his personal space and limitations and it is something which is above the purview of gender. It revolves around the life of the protagonist Shanaya who deems herself lucky for having been born in a family that gives equal space to both male and female children. Shanaya learns a lot from her mother who does not only manage her professional life efficiently but also performs household chores really well. The accommodative and liberal ambience of the home can be seen in the fact that Shanaya's father is a vocal critic of gender dictated roles. He wants his daughter to excel in life-enhancing skills and competitive ambience rather than just push her for the drudging life of a family cook. Be it the playing of the games that boys do or anything else, she feels proud of her father for being so affectionate and liberal towards her.

Though the stories of discrimination against females in families where daughters are considered a burden, a liability to be raised up, educated and married off, makes Shanaya simmer with indignation, her father's showering of profuse affection on her encourages her to face challenges in life boldly. As an avid reader, she knows about women oppression through the feminist discourse too. It is one of the reasons that like the second wave feminists, the protagonist believes in financial independence of women which does not only make them self-dependent but also self-confident. So, the narrator grows up to be a feminist and by offering lessons to her students, she develops feminist sensibility in the early years of her career.

The author shows that marriage is not at all an institution of oppression (as is often said) if the partner gels well with a professional woman. Shanaya's eventual marriage with Sangharsh is such a tale of happiness in the early years of marriage where she does not only find him adorable, caring, and sensitive but also respectful of her teaching career. The author projects the husband figure in a highly idealized way. From being a philanthropist to a dutiful son, he transcends all the barriers of class/caste/religion which can be seen in his befriending the people from various classes of life. As for Shanaya, she feels really blessed to have Sangharsh as a husband whose heart brims with love for all and he makes her believe that life is something to be celebrated than lamented.

However, her in-laws had different expectations from her. She sees things go awry for her. For her parents-in-laws, a daughter-in-law implies one who does not only rob the husband of his money but also causes separation/conflicts between the son and his parents. For them, a *bahu* is synonymous with someone who like a witch exercises a bad influence on her husband. The author speaks against such stereotypes about a newly-wed woman wherein she is viewed just as a beast of burden and at the same time not as a maker but a breaker of home. Moreover, the bahu, (Shanaya) is not seen as a daughter but simply as someone who can never be a loyal member of the house despite her utmost devotion to the family. The author also exposes the problem of dowry in the story as Shanaya and her parents are constantly subjected to humiliation and jeering remarks for having given her little dowry. It is this emotional abuse which proves too much for the innocent and sacrificing Shanaya.

The author shows how Sangharsh feels indebted to his step-mother and father who raised them when he lost his biological mother at a tender age. She equates him to the hero of the movie *Beta* and Bindusar of the TV Series *Ashoka* who love their mothers, not realizing the fact that the same mothers in the stories wish their son dead. Though the step-mother does not wish Sangharsh to be dead in the story, he is misused for his sacrificial nature.

A woman who has enjoyed independence at her parents' home and has nourished dreams of a better life with a bright professional career with her husband, too many restrictions from her in-laws stifle her very womanhood and individuality. Despite serving her parents-in-laws with full dedication, Shanaya does not receive any love from them. As a teacher she does perform her duties well and as a bahu she takes care of the needs of all. Apart from it, she takes care of her son and his studies. Yet her conservative parents-in-laws object to her every move of finding out time and space for her or going for movies or looking for other means of steaming out her pent-up emotions. It is what makes her life, existence unbearable. However, she goes on bearing everything silently as she believes that at least her husband does not ever speak a word against her and manages things really well. She rues the fact that her in-laws believe in their son's contentment at the cost of their daughter-in-law's happiness. It pains her to learn that her parents resent their son's giving time/space to her or taking her for shopping or so. Though Sangharsh tries to strike a balance between the two extremes, his spirits ultimately begin to dampen.

Sangarsh, indeed, does a lot of *sangharsh*, that is, struggle. His struggle begins since the death of his mother and continues unabated. Working for the poor and the destitute, managing the parents who are demanding, showering love on his wife, who feels otherwise reduced to a zilth in her house, shows how he tries to balance things in life. However, at the same time, he is governed more by a sense of gratitude to his parents as he does not want to offend them. He is also conscious of the ever-worsening situation of his wife, yet with the passage of time, things prove too disastrous for him to handle. Besides, a decade long blessed marital life, the too much parental interference begins to weigh heavily on the psyche of Sangharsh and eventually leads in his taking refuge in alcohol and ultimately suicide, an act of self-destruction and at the same time of all those who are left behind, especially the wife and the son. It does not only shatter Shanaya but also makes her question the existence of gods whom she worshipped every morning and evening.

Interestingly, the author does not bear any grudges against the social set-up, patriarchy, or gender roles. Despite her story's central character being a victim of gender stereotypes, she does not like the label of being a feminist. What she puts forth through her story is empathy. An empathetic eye and a sympathetic heart to view the pressure that men cope with when confronted with two diametrically divergent situations. Through the sad plight of Shanaya and the tragic suicide of Sangharsh, Shivani Chaudhary also makes a case for men who have constant pressures from many ends to bear with. She gives more importance to individuals than to men or women. The author pleads for a genderless society and a genderless society does not mean a sexless society. The moment we try to view everything from male or female gendered perspective or identities, the problem rises from there. The author makes a sort of plea to strike a balance between the gendered spaces as she also makes us think of the insurmountable pressures that men undergo. Dorothy Sayers (1893-1957) in her essay "Are Women Human?" also highlights the same concerns of treating an individual as a human being first than a man or woman.

Audience's Live Response

Deepti Vaidya: Beautiful narration of complex human relations!

Jyoti Mishra: Absolutely spellbound by Shivani's heart-rending story & more so by Praveen Sir's lucid presentation...

Prajya Mishra: Sensitizes us about the position of men in matrimony...nine-year old son seems to have a grey head on young shoulders. Shivani thanks for giving us a slice of life.

Irene Rattan: Gripping narrative

Anjali Parmar: Very emotional, wonderfully narrated sir.

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THE MAD MYSTIC IN HEAVEN

SOM P. RANCHAN

Gods and Goddesses were sitting in a rectangular hall with gold and silver pillars, hundreds of them. The hall was silvery white inside, but the outer walls were opalescent. There were many gates to the hall. One gate was for small time deities to whom the lesser mortals prayed for the health of their cattle and children, and for giving them reprieve from distress and mundane pressures. The middle gate of the Cornelia was for the middle echelon deities for praying to for radical improvement of fortune and for knowledge and expertise in a given field laced with a bit of superior, ineffectuating insight. The high arched gate was for the entry of gods and goddesses who were a notch or two below the principal gods, The Big Three with their be-decked spouses. Thy Big Three did not enter through any gate. Being transcendent they parachuted down from an occult aperture in the roof of the hall. No other deity had seen the *lapis lazuli* roof, though one could see the high ramparts of ruby.

It was a custom with this divine club to hold an annual meeting in mid heaven, wherein they invited a mortal from the earth-plane, together with an entourage, to check the rise and fall of their numinosity in the world down below.

But this time a lone mystic was invited. It was not clear if he was to be quizzed about his identity. Chitragupta did not have his card in the reference file. An undercurrent of excitement ran through the august assemblage. Even the Big Three with their consorts wore expressions of puzzlement, though they did their best to cloak it because their reputation among the thousands of gods was that they were know-all. They were omniscient.

In came a non-decrepit looking frail mystic in his *dhoti*, with no upper garments. He had a trance-idiot look though the cut of his face was Germanic. He broke into a dance, clapping his hands like he was elongating tortillas. Suddenly the dance stopped and he stood stock still with no sway to his body, as if he were in a psychotic state.

The august audience was stunned. They had never seen anything like this before. In the earlier conferences, the earthlings had come and read their reports on virtues and vices prevailing and fighting it out on the earth-plane. So many secretaries and secretaries of the high echelon deities stenoed the gist of the reports for the lesser gods and the gist of the gists for the big Three, who stashed away the files. It was more of a charade than for real because the earth-plane continues in its eonic stasis.

Suddenly the stock-still figure came into awareness. In the twinkling of an eye, he grew in size, his chest enlarged preternaturally and heaved: "O Gods and Goddesses, small and large and the Sovereign Three! It is time you left mid heaven and deigned to come down and mingle with the earthlings. The membrane between the earthlings and you is now translucent if not transparent. Mingle with them in work and play."

"Hey Tasuki! son of Vasuki, who is around Shiva's neck, who rolls in the creeks like water, and as a prince in festivities and parties, seducing old maids and bonny lasses, just for the heck, because you have a fluent libido. But it's time you loved. I don't mean fidelity or loyalty. No, no, no Tasuki, you are free. But love from the feeling centers."

This was a bombshell. Tasuki sitting close to Shiva felt hit by the dart of truth. His fickleness was lanced. All the deities squirmed as the mystic spoke with authority. Putting an inch or two more to his size and widening his affectionate grin, he swept Vishnu and Lakshmi with his gaze and spoke with a boom. His voice had become baritone: "Forget the Avtar routine. The paradigm is an old hat. No more trips and no more laying through vaunted scriptures. Humans now are gods, some active, majority in latency, many in varying stages of fluency, a few dormant. But the new age has unfolded an existential practicality and its numinosity. Vishnu, there is no need for your *dues ex machine* interventions. You are cashiered. You may as well go to *Vaikuntha*. If Lakshmi wants to go with you, fine! Though being *rajasik*, she would like to be down there, giving beauty, elegance, exquisiteness on the run to the high born and the lowly, though the categories pertain to the range, scope and quality of consciousness, not to wealth, status, privilege and prestige."

The impulsive Lakshmi got up, materialized into an owl and started flying from pillar to pillar, raining down the buff brown feathers. The gods and the goddesses stood up and broke into a stampede to collect the Golden Fleece.

By now the mystic took on the miens of a Lord and resumed: "Now you have Lakshmi's talisman, a feathery piece, I bid you to dance."

Began the dance. In the ensuing, they forgot their rank. For a few moments their hierarchy was gone. Democracy ruled. Strange, they sat down wherever they were. Now they were comingled. They felt a new ease that they had never experienced for ages and eons.

The sage stopped suddenly. Went into a trance. From its depth he spoke with a whispery resonance: "Saraswati! Come down with me and emanate. Brahma too can follow with a *syn pados* smidgeon. Down below we have to give to the world and the teeming humans and unfold the lotus of the psyche winged with executive imagination. Good bye gods! It was a privilege being with you. Prior to it I had experienced you in meditation and muse and in *Bhava Samadhi*."

After he disappeared, the gods and goddesses continued sitting for a while. A thought flashed through the bee hive – they had been like sleeping for ages to the world, and for eons, being high and happy, indifferent to the world below, merely intervening spasmodically to keep the keel from sinking into the oroborus of the sea.

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Narrated by Janesh Kapoor

Disrupting the Divine, 'Opalescent,' and Mid-Heavenly Realm: A Review of Som P. Ranchan's "The Mad Mystic in Heaven"

ROSHAN LAL SHARMA

The story, "The Mad Mystic in Heaven" by Som P. Ranchan, has been compiled in the book titled *All My Boys and Other Stories* published in 2005. Viewed simply, the story is about a "mad", "lone", and "non-decrepit looking frail" mystic with "a trance idiot look" and a face with a "Germanic" cut. With a dhoti and unclad upper body, the mystic bears unusual demeanor in contrast to the august gathering of Gods, Goddesses and middle- and lower-rung deities to ascertain the degrees of their "numinosity"/divine influence in the human world. The mad mystic, in fact, represents the mortals in the meeting, but proves to be far wiser, sharper and wittier not only compared to earthlings but also the divine, complacent beings. He ends up offering them correctives of diverse sorts. Interestingly, Ranchan does not view Gods, Goddesses, deities and wisdom figures as abstract, unapproachable entities; he rather engages them as tangible/ veritable presences in human life as their importance lies in the profound impact that they have on the human heart and mind through (verbal/non-verbal) interaction, psycho-spiritual and alchemical transformations.

The modernist curve of Ranchan's creative idiom (both in poetry and fiction) is well-known and so is his uncanny ability to lend contemporary flavor to mythological stories, tales, themes and motifs. "The Mad Mystic in Heaven" evidences narrative experimentation not only at the level of narration but treatment of content as well. The mad mystic is a disruptive as well as a levelling narrative device that Ranchan employs to first disrupt the hierarchical splendor of mid-heaven "august assemblage" and then to ensure non-hierarchal, equal and democratic commingling of the Gods, Goddesses and lesser-/lower-level deities. Being wont to hierarchies symbolized by different entry gates to the rectangular and spectacular meeting hall for Gods, Goddesses and "small time deities" hailing from different categories, the august gathering could never think in terms of breaking the hierarchic norm. "The big three", "principal gods" have been delineated in the story in a reverse order as Shiva, Vishnu and Brahma. Intriguingly, Shiva's spouse has not even been referred to even in passing whereas the "bedecked spouses" of Vishnu and Brahma have been duly acknowledged but assigned special roles concerning weal and welfare of humanity. The big three Gods are privileged enough just to parachute down at will — a power that the gods and goddesses at the second rung did not enjoy alongside "small time deities".

Ranchan introduces the "lone mystic" rather dramatically. Chitragupta's reference file missing the mystic's card containing his personal information, mystifies the latter's identity even further. It causes bewilderment even to the principal gods who try their best to camouflage it behind their so-called "omniscience". Upon entering the hall (a sacrosanct "rectangular" space not meant for the ordinary human folks), the mystic (an official invitee to the mid-heaven meeting) suddenly starts dancing in a weird, psychotic fashion clapping his hands as if "elongating tortillas", and then abruptly stands "stock still" in front of the astounded divine audience. The mystic, besides defying the past protocol for earthlings, also baffles the "big three" who find it hard to take the entire spectacle for real, simply because it appears more of a charade than actual.

As the mad mystic regains consciousness and becomes aware of his surroundings, he grows into an enormous being. With his preternatural physical proportions, the mystic exhorts "Gods and Goddesses, small and large and the Sovereign Three" to mingle with the denizens of the earth. He has a piece of advice for Tasuki, son of Vasuki who adorns Shiva's neck that despite his "fluent libido", he should love from the "feeling centres", which implies that libido, or eros loses sense if there is *bhavashunyata*, i. e. lack of genuine 'feeling-register' (an expression that Ranchan would use so frequently in day-to-day interaction) that emanates from relational principle in life. Owing to such authority and confidence in an idiom hitherto unheard in mid-heavenly realm, each one of the participants in the conference becomes nervous and thus squirms as the mystic comments on fundamental *Vritti* signifying an integral, body-mind-soul behavioural mode that regulates/governs us.

It is worth mentioning that the writer has focussed on Tasuki rather than Shiva and his spouse, Parvati, who is not mentioned even once in the entire story, compared to the bedecked Lakshi and Saraswati, the spouses of Vishnu and Brahma respectively. The mystic then increases his size and grins a little more, and tells Vishnu and Lakshmi that they should shed the old and dated "Avatar" paradigm. He denounces the "vaunted" scriptures indulging in pointless adulations of Gods and Goddesses and thereby declares that nowadays "[h]umans . . . *are* gods"—some active, fluent and "majority in latency" (italics mine). Almost like Jungian Wise Old Man, the mystic sounds rather prescriptive as well as emphatic as he spells out the new age norm wherein "existential practicality" is viewed in numinous terms. He suggests it to Vishnu that he may go to Vaikuntha along with Lakshmi in case she so wishes; else, according to the mystic, she could go down to the human world wherein she can bless folks with "beauty, elegance and exquisiteness" based on "range, scope and quality of their consciousness" rather than their status or birth. The mystic's language thus is unconventional, deconstructive as well as challenging as it impels the reader to think afresh about the human world as well as the divine beings who need to be humanized so that their relevance to the common folks could be ascertained without of course compromising the core symbology associated with them.

As if taking a cue from the mystic, Lakshmi gets up, materializes an owl (her vehicle which mythologically signifies intelligence, wisdom, and prosperity), flies from one end to the other and rains down "buff-brown feathers". The whole gathering becomes frenzied to catch hold of the "golden fleece", that is the auspicious feathers of the owl which the mystic terms as Lakshmi's "talisman" strategically. The talisman is undoubtedly magical as the mystic's *leela*/ play persists all the more overwhelmingly. Taking on the "mien of a Lord", he invites the gathering "to dance", which, in fact, is a ploy to dissolve hierarchies, break boundaries and commingle all the divine beings non-hierarchically and non-discriminately with a view to make democracy prevail. As if under a spell, they do his bidding, and experience an unprecedented lightness and "a new ease" in their celestial selves after "eons".

Slipping into a trance even at this juncture, the mystic-sage tells Saraswati with "whispery resonance" to come down as "an emanate" along with Brahama who is allowed just a miniscule space. In fact, Saraswati has a major role to play down there through "unfolding the lotus of the psyche winged with executive imagination" in a world with "teeming humans". It implies connecting human beings with their souls and creative imagination, which has been lying in latency.

On that note, the mystic bids them adieu and thereby reveals before them how before this meeting, he had experienced them in "meditation and muse and in Bhava Samadhi". The mystic, at this point gets rooted to the ground where he meditates, engages with his creativity and also experiences Bhava Samadhi—a state of ecstatic consciousness upon realization of one's *Ishta* (god/deity/object of worship) through constant devotional practices. It may outwardly appear *sahaja* (effortless) but requires intense devoutness and devotion that can trigger inner transformation through attaining oneness with the *Ishta*. To put it simply, it is a merger of one's *bhava* with his *Ishta* which endows him with the depth as well as the expansion of consciousness. He disappears thereafter leaving them dumbfounded with a thought that flashed through the "beehive" (signifying the co-operative, divine society) that all the divine beings have been lying in deep slumber in the sense of being indifferent to the world down there for a long while, and would intervene only at the time of crisis and to "keep the keel not sinking into the ouroboros of the sea". The need of the hour thus is that their numinosity/divinity, rather than "spasmodically" remaining available to all the mortals, is available incessantly irrespective of their hierarchal position/ status.

The mystic's lesson to the divine beings has profound political implications as well. In fact, this story may also be viewed as political allegory. A highly stratified and hierarchized world unfolds before us right from the very beginning. We have divine beings representing various classes/ categories such as the "Sovereign Three" Gods, Goddesses, lower-and middle-rung deities along with the lesser mortals. The heaven is a space which is deeply complacent but ironically conscious enough to ascertain annually whether their numinosity is still acceptable/ influential in the human realm or not. It implies that Gods and Goddesses want to ensure that the power equation between the divine and human kingdoms should remain as it has been all through without ever bothering to do anything extra for the lesser mortals. The same situation has been prevailing year after year, and the divine gathering would quite curiously listen to reports from the human world with all its problems.

This time around, however, something dramatic happens as the notes taken down by the secretaries are stashed away by the "big three" due to the arrival of a puzzling presence called the mad mystic, who emerges as an unconventional forced mythifying and demystifying the mid-heaven divine hypocrisy that gets exhibited/showcased every year. Ranchan has employed him as a disruptive narrative device to set aside the "big three" via disallowing them relatively lesser narrative space and preferring instead to engage with the Lakshmi and Saraswati instead because of their meaningful presence and role in the human world that has gone awry and totally off-center. Thus, the mystic as a demythologizer enables the divine figures to engage with/ awaken to the human world and its problems. In his poetic universe of dialogue epics, Ranchan engages with Wisdom Figures (such as Christ, Mother Sharda, Swami Vivekananda, Sri Aurobindo, Lord Krishna, Manjushri and Baha'u'llah) symbolising finer essences that imbue man's being with an immortal, changeless aspect. That is precisely the reason why man in his mortal aspect keeps looking up to them and thus seeking guidance to grapple with problems in day-to-day existence.

Structurally, the story has a well-thought-out narrative frame which in no way seems congealed/coagulated. We, in fact, come across narrative fluidity in the form of frames of diverse sorts. Firstly, there is the basic frame in the form of a deeply hierarchized divine assembly taking place in an opalescent hall; secondly, there is the mystic's frame which counteracts on the basic frame and disrupts it by creating other semantic frames such as Tasuki-Vasuki frame, Lakshmi-Vishnu frame and Saraswati-Brahma frame, which have been deconstructed in a way that they could be disentangled from their loaded, conventional and mythological significations. The mystic offers a counter-discourse, a counter narrative not only to neutralize numinosity/divinity around the "Sovereign Three" but also to point toward its finitude/limitation via embedding them within existential, human world/ context. Tasuki lacks *bhava* register, which is a must; Lakshmi's "beauty, elegance and exquisiteness" would be more useful and meaningful for earthlings irrespective of their status or class; Saraswati too needs a non-incestuous space [devoid of Brahma] to be herself and thus ennoble and enlighten the human world with a wisdom that puts all classes/hierarchies in a perspective. The postmodernist curve of the story thus becomes evident as the grand narrative concerning the "big three" overarching gods (Brahma-Vishnu-Mahesh) has been layered with several mini narratives that have been carefully lent prominence. Abrupt disappearance of the mystic towards the end is both dramatic as well as unconventional. A hint has been dropped through a potent thought that flashes through the "beehive", a metaphor for 'rank-less', 'status-less', classless and non-hierarchal, divine democratic presences, that they need to awaken to the fact called human existence, and thus 'repurpose' their role in making it better.

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Audience's Live Response

Anu Jaidev: A very nuanced and sensitive reading. Thank you, Dr. Roshan.

Anupama Singh: Marvellously deliberated upon the story, Roshan Sir.

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BIMLA "PAAGAL"

SUNEELA SHARMA

'In-mate of Nari Niketan passes away due to multiple organ failure.'

The news, along with the photograph of the deceased, silently screamed out my failure in my face from the tiny column of the page five of the newspaper where it was inconspicuously squeezed in. I perched my magnifying glass over the column to read it again.

This was Bimla Paagal – my Bimla, whose death they had reported. The news said that Bimla had died, due to multiple organ failure within a week of being sent to the Nari Niketan at Shimla. As I sat still in the glazed balcony of my flat, the symphony of a busy early morning routine, composed of cooing of the birds, calls of garbage boys, newspaper boys, milk walas, morning bhajans, coupled with the cacophony of monkeys romping from one rooftop to the other played around me in its usual melody. In the middle of all this, I suddenly felt a numb inside, the first symptom of a shock perhaps, I thought, even as I managed to pretend to go through the rest of the newspaper with my morning cup of tea. Screams and a commotion from outside withdrew me out from my trance. As I stood up, I saw a petrified little school boy standing on the tiny winding road below the four-storied building where I lived. He was wiping away tears of terror from his face with one hand, a school bag on his back as he clutched a lunch box tightly in his other trembling hand. Two huge monkeys, their eyes glued onto the lunchbox, were moving slowly towards him. Out of nowhere, shouting and running from the opposite direction, friends of this boy pounced upon the monkeys. Moments later, the tables had turned, with the boys now chasing the monkeys away instead. I stood there, admiring the brave boys, and their unity that gave them the strength to chase the dangerous monkeys away. A gust of cool breeze drew my attention to the dark, towering, thick grove of giant pine trees dotting the famous Jakhu hill of

Shimla behind our building. One couldn't help but admire the majesty and strength that resulted from their collective number, due to which perhaps, they had survived the axe of modern multi-storied concrete jungle of Shimla. Single trees are slaughtered everyday…. like the lonely Bimla. I lamented aloud to myself, realizing that *we*, Bimla and myself had finally lost our struggle within and outside.

"Oh, why couldn't we all be strong? Why couldn't we fight it all together?"

As I sat there looking at the blue canvassed sky with patterns and shapes of clouds chasing each other, I remembered ***Bimla "Paagal"***. She too was a lonely tree, who got slaughtered.

I still remembered the day I met Bimla for the first time, as if it was only yesterday. She and I, both were about the same age then… I was thirty-one years old and it was 27^{th} April, 1998, my first day as a teacher in school and my second one in Rampur, the quaint little hometown of my husband. Located at about a hundred and twenty kilometers from Shimla, with a resident population of only a few thousands and an equal number of floating populations, this town is the biggest supply base for civilians as well as the armed forces, beyond the capital city of Himachal Pradesh up to the borders of China and Tibet. After my father in law's death, we had decided to return home from Mumbai.

This particular morning was an important one in my life; I had to make it to my new job at the school before the eight o'clock prayer time, as was stipulated in my appointment letter. A family servant well versed with the labyrinth alleys of the old town market was deputed to guide me to my destination. Engrossed in my own thoughts as I turned around a sharp corner, I collided into her–and she fell in a heap, literally knocked down by my momentum. The next moment I was pulling her up but to my surprise she was a heavy and lifeless bundle. As I struggled to pull her upright and pleaded to her to get up, my attention was arrested by two huge black eyes that stared into mine from a dark, mud-stained pudgy face, while ringlets of short curly hair spilled all over and dark swollen lips said to me in a dead monotone -- "I'm hungry, give me food, they tore all my clothes, give me your clothes". Dazed and unable to unlock my eyes from those captivating ones, I groped for my lunchbox inside my bag, and shoved the two paranthas that I had cooked for myself towards her, got back on my feet and rushed to join my new job. Memory of a young naked body, barely covered with a tattered blanket, kept haunting me even as I went through my day. Those big dark eyes kept challenging me. My new colleagues interacted with me during the lunch break and provided me the opportunity of narrating my curious encounter of the morning to them. I was told that she was -- Bimla Paagal, and one must change one's path if one came across her as she was an inauspicious jinx. On further inquiry a colleague informed me that Bimla belonged to a village nearby. A few years ago, she fell in love with a truck driver and eloped with him from her home. He turned out to be an agent supplying girls to transporters for their needs on the road. Ever since then, she became the play doll of these highway transporters, often used and thrown by them and therefore abandoned and disowned by her own family, when she turned towards them for protection. With this knowledge about her, my eyes searched for Bimla as I walked back from the school that day but she was nowhere to be found. That evening, over a cup of tea, I mentioned her name to my mother in law. Peering at me from behind her favorite magazine, she gave me a quizzical look through her spectacles and asked how I knew about her. After my recount of the incident, a stern -- "she is the shame of our town and should be avoided" made it obvious that no further discussion on the topic was going to be entertained in the house and my curiosity was left to itself.

Life thereafter moved at an automated pace in the months that followed as all of our family got busy settling down. Bimla was by now a part of my life in Rampur. I would often run into her, in the alleys of the town market, at bus stand, on the road or at the Chaudhary adda, where goods were loaded and unloaded from trucks on the road. I would quickly turn my face away as soon as her big black accusing and my guilty, helpless eyes would meet. Sometimes *She* would be in tatters and at other times her torn clothes were replaced by new ones. By now, it was easy to guess that her new clothes would remain new only till the time she was 'picked up', and the tatters on her body meant the inevitable had happened again on the road in the dark of night -- a grim and sad reminder of the façade of a safe, secure and cultured society that we all pretended to live in. Over the years, to me, Bimla became a silent symbol of this naked truth of our civilized society. And my muffled protest against her suffering was limited to my own limited status of a modest bahu amidst a social system where traditions worship a woman as a Devi but persecute her as a stigma, a scum as soon as she tries to put a step beyond the boundaries of the stereotyped role set for her -- her Lakshman Rekha. Braving it out I tried sharing my concerns for Bimla amongst my peers at times but this drew a helpless submission to the inevitable, given the fact that Bimla had been abandoned by her family who had no doors to take her back.

In the following year I began teaching at the government college of the town which was a couple of kilometers away. We often heard that Bimla had lost her mental equilibrium, which did not surprise anyone as she had been noticed eating garbage and even the hospital waste. Then we heard stories of Bimla delivering and killing her own new born babies twice, even throwing stones at anyone who ventured near her. One day I found her near the gate of our college, amidst a crowd of students, her naked body tangled with strips of torn clothes around it. We had to procure clothes and food from the college hostel for her. This time she was different. She lay there, refusing to respond and move. We were forced to leave her but next morning she was gone. A few days later she was located in another place again in *torn clothes*….

Years rolled by and Bimla Paagal's life and identity merged with the highway transport life of Rampur. The small town too underwent the changes of digital world, the wonders of electronic media and people including me flowed with its pace. She was often seen lurking in the town's alleys, scavenging for food in the gutters, garbage bins, wedding parties and as for her tale of new and torn clothes, it lost the race with the new life that moved at a pace faster than the story of Bimla. Her story was told to the young girls of the town as a moral lesson of the dire consequences if a girl decided to leave her home, and for their mothers … a metaphor of our own muffled identities.

Many years passed and now I was posted at a college in Shimla. One Saturday evening, an old friend from Rampur, called me. Our friendship dated back to the times of our own struggle for identity. Nowadays she was running an NGO for differently abled children at Rampur. Her mention of Bimla's name brought so many memories flooding back. She announced on the phone that she had finally filed a petition to the court for Bimla's protection and healthcare. As a result, the court had ordered her treatment at the state hospital at Shimla. She requested me to ensure that Bimla should not be declared mentally fit at the hospital and sent back by the medical authorities, as was done with patients in police custody under legal directions.

As I read Bimla's medical report that was mailed by my friend to me, to my dismay I discovered that Bimla had lost her health completely due to prolonged sexual abuse and neglect of physical hygiene. She was heading on for multiple organ failure and could be HIV positive. I felt a pang of guilt and shame for having forgotten her. Her black eyes haunted me in my sleep that night, pleading for justice and by the morning I had decided, I had to do something. I called another friend who runs a nursing college in the city and together, we went to welcome Bimla on her arrival at the city hospital. The moment our eyes met, I looked straight into them. A catheter was attached to her for the ablutions she couldn't render herself. It was clear at the first look that she was now too weak and needed an attendant but the doctor on duty almost threw me out demanding me to prove that I was related to Bimla. I was able to click a few pictures of Bimla though, and shared them on facebook to seek help. We received no less than two hundred likes but no hand came to hold hers'. I spent a sleepless night as the stipulated period of police custody was to lapse the next day and she would be sent back to Rampur. Now was the time that an honest medical report stating her pathetic mental and physical state would be required for her salvation and directions from the court for sending her to a mental re-habilitation centre.

Next morning, I met the head of the department where Bimla was under treatment. The doctor seemed more concerned with his position in question rather than the situation of the poor victim. When I told him Bimla's saga of loss of innocence, physical, mental and emotional torture, her social boycott, my association with her, I sensed a shadow of sympathy cross the stoic face of the doctor. He requested me to put my signatures at the base of two blank sheets, promising to do all he could to help save Bimla's life. Confident that Bimla would be shifted to the mental rehabilitation centre for further treatment the next day, I thought of spending some time with her before leaving the hospital. As I went near her bed, she looked up at me like a fresh lily flower. The nursing students from my friend's college had washed and scrubbed her. She looked so pretty wearing a bottle green kurta and black slacks with her hair neatly combed into a plait.... The girls were really pampering her and she seemed to have won the hearts of all the patients in her ward. Someone had even kept a small bunch of roses near her bed. I asked her what she wished to eat and with a strange glint in those dark eyes, she raised her arms spreading them wide like a little child and demanded lots of oranges and chocolates which we procured for her. Had I known that that night when I left Bimla, it was to be forever, I would have tried to fulfill all the rest of her wishes too. The next day Bimla was sent to a home for Destitute women with a medical report that declared her mentally, physically and medically fit and a witness, Suneela Sharma, had certified that she was never sexually assaulted nor was she found indulging in any activities indicative of mental illness .Bimla, declared medically fit, was sent by the court to a Nari Niketan where she passed away after a week, coincidentally on the International Day for women. Bimla was released from the body prison of a woman while the whole world celebrated freedom for women.

The frantic ringing of the doorbell brought me back to the present and I realized that I was surely late for my 10:00 am lecture. I rushed to the door to let in Renu, an orphan girl who had lost her father during the construction of our building fifteen years ago, and since then she had been adopted by the families of our neighbourhood. She was just fifteen then when she started helping us all in our house chores, now a mother of two daughters herself. As Renu stared at me with a perplexed look in her eyes, I gave her a tight hug, made a silent promise to Renu and took her inside.

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Review of "Bimla 'Paagal'"

ABHYUDITA GAUTAM

Bimla Paagal is a tragic and pathetic tale of a woman, who had fallen in love and eloped with her partner only to be betrayed, abused and discarded like a scrap. The sanctity of her marriage was ruptured, her chastity torn down, her dignity shattered and mutilitated just like the clothes on her body that barely managed to cover up her wounds. In fact, it would not be wrong to say that this tale is about a woman who fell victim to circumstances and is unable to overcome the emotional trauma that led to repeated physical assaults on her body ultimately resulting in her mental retardation and imbalance.

The story is about the failure of the society to protect her and provide Bimla the basic necessities of life like food, clothing and shelter that every individual has a right to. Her succumbing to her wounds and illness was a tragedy that the writer and her fellow workers regret in the story as they tried to rehabilitate her but could not succeed in their noble mission.

Suneela Sharma has narrated the story beautifully in an autobiographical manner with a lot of sensitivity, adopting the stream of consciousness technique by chronologically depicting the events in her own life as well as that in Bimla's life giving it a structure of a parallel narrative. There is pathos in the story as one feels pity and sympathetic for the protagonist.

The present day is when she hears about Bimla's death as the morning routine sounds of the milkman, newspaper boy, morning hymns, garbage boys and the monkeys jumping on the rooftops reverberated in the background. The four stages of the writer's life which are the significant turning points in her life also narrate the four different phases of Bimla's life.

The first instance is her first day to her school where she had joined as a teacher when on the way she bumps into a hungry and nude Bimla on the road, second one was when she was settling down her family business in her hometown when she observes Bimla wandering around like a lunatic who had evidently been sexually exploited. By the time the writer joined her services of college cadre, Bimla's condition had deteriorated to such an extent that she had transformed into a scavenger feeding on garbage and leftover food and had even started pelting stones at people who came near her, as a natural defense instinct against the men who had assaulted her. Suneela Sharma shifted to Shimla when she was told that a petition for Bimla's protection and welfare was filed in the court.

The writer's own struggle to find an identity for her been narrated simultaneously with Bimla's story who was the same age as the writer. Both the women's lives run parallel in the narrative.

Bimla, the name, means pious and pure, but ironically, she was subjected to very heinous atrocities in her life that devastated her chastity.

The writer has symbolically represented the retaliation in her story when she narrates the incidence of the struggle of the school boys with the aggressive monkeys trying to snatch away their lunch boxes. The need to be united and gather ourselves to fight against the challenges of life is portrayed, which the writer and her fellow social workers failed to do.

There is also a comparison between the cutting and felling of the trees with the sexual exploitation of Bimla. This theory of Eco Criticism is evident through the reference of trees in the story and we are reminded of *The Flowering Tree* by AK Ramanujam where a woman's body is disfigured by the human hands that pluck the twigs of the tree violently.

Labeling of Bimla as 'Paagal' or a jinx is the issue of witch-hunting that the authoress has brought in. We can make a reference to *The Madwoman in the Attic* by Gilbert and Gubar as Bimla has been isolated and alienated in the society that thought of her as an outcaste or a monster.

Bimla's roaming around without clothes, becoming a subject of ridicule and loathing is a stark contrast to the worshipping of the Goddesses in our country. Sustaining on garbage and wasted food by Bimla is an indication of malnutrition that the women of the Asian countries suffer, which has been elaborated upon by Amartya Sen in *A Hundred Million Women are Missing.*

The issue of personal hygiene of women is also another aspect to ponder over, which is the basic right that every woman is entitled to, and that Bimla could not access as she was not provided the personal and secure space even during her menstrual cycle.

Bimla's toothless smile shows her hope to overcome the adversity of life even though she was unarmed and unguarded to face this cruel world. Teeth may symbolically be taken as natural defense tools that she had shed making herself vulnerable to the wolves of the society.

The story ends tragically when Bimla, a probable case of HIV positive dies due to multiple organ failure, much to the utter dismay of the writer and her fellow social workers.

However, the story ends with another ray of hope for the writer who opens up her door to a new domestic helper called Renu, in whom she sees an opportunity to compensate for what she could not accomplish for Bimla.

The story of Bimla "Paagal" seems to have come a full circle.

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Audience's Live Response

Irene Rattan: Suneela ma'am, your words carry the pain of your heart.

Priyanka Koundal: Such a beautiful and heart touching story.... Ma'am, I want to read it again, so can you tell me please from where I get this? Any soft copy?

Leena Vaidya: Suneela Ma'am we had a somewhat similar case over here in Bhunter, Kullu. I just informed the authorities and now feel sorry that I never followed it after she was shifted to Shimla.

Namrata Pathania: There is a Bimla Paagal in every nook and corner of our society. What is lacking is empathy and action.

Savita J.B. Singh: Suneela Ma'am...you have touched on a very sensitive topic... Heart rending... the plight of women!

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नंगा आदमी

योगेश्वर शर्मा

साँझ होते ही अँधेरा उतर आता है पटरियों पर। फिर आग–धुएँ से सिसकते हुए, इंजन के पैर छुक–छुक, छुक–छुक सरकने लगते हैं – धीरे–धीरे, तेज, फिर बहुत तेज।

सन्नाटे के पिंजरे से शब्दबेधी एक लम्बी सीटी टकराती है चीत्कार की तरह। कुछ पंछी मर जाते हैं। कुछ उड़ जाते हैं। कुछ बेदम हो जाते हैं।

जो मर जाते हैं वे 'आप मरे, जग परलय' निजात पा जाते हैं रोज–रोज के 'परपंच' से, डर–भय से। जो उड़ जाते हैं, वे अंधेरे में पंख फड़फड़ाते रहते हैं। जो बेदम हो जाते हैं, वे दूसरी चीत्कार का 'धड़ाका' स के लिए खुद को तैयार से करने लगते हैं।

सीटू को बेला–कुबेला यह रेल की सीटी बहुत बुरी लगती है – डायन, मनहूस। हर वक्त जान साँसत में रहे। भरम लगा रहता है कि यह कहीं 'टुंडी राक्षस' की तरह फड़फड़ाकर उसके टपरे को न लपक ले...पटरियों से उतरकर।

निगोड़ी नींद उचट जाती है अधराते।

फिर आज तो नींद यूँ भी उखड़कर आ रही थी।

सुखनी – उसकी बीवी, बड़ी–बड़ी आँखों में पुतलियाँ–सी नचाती फिरती है, न जाने किस करमजले के पल्ले बाँध दिया उमर भर को। न कभी प्यार की बात करे...। उनींदी रात को भड़भड़ाकर उठा देगा, 'सुखनी री, देख तो घड़े में आटा है बचा हुआ? कल का जुगाड़ हो जाये तो एक दिन छुट्टी करूँ। जिस्म दुःख रहा है सारा। ठंड लग गयी है शायद।

सीटू को अपने नाम से चिढ़ है। सुखनी को अपने नाम से।

और यह गाड़ी भी ऐन गला खंखार–खंखारकर गुजरती है यहाँ से – आधी रात को। न कोई सुर, न कोई ताल। इसके बेढंगे डरावने रिरियाने पर कौन ताली पीटने बैठ जाये, कौन गाने नाचने लग जाये।

सीटू जब पैदा हुआ था तो इसी धड़धड़ाती गाड़ी की लम्बी सीटी बजी थी, उसके बापू कहते थे। अन्दर माँ को जोरों के दरद थे – हाल–बेहाल। उधर

गाड़ी की सीटी बजी और इधर लालटेन की काँपती–झाँपती लौ में बूढ़ी दाई ने चट खुशखबरी फेंक दी ठीक आँगन में, 'राजा बेटा हुआ है रे तेरे, मंगतराम।"

अँजुलि भर बतासे बचाकर रखे थे आड़े दिनों के लिए। मंगतू ने सारे के सारे लुटा दिये।

फिर वह मन ही मन गुनने लगा – बेटा हुआ है, अच्छी बात है, लेकिन राजा बेटा! है न बेढंगी बात! वह घंटों हँसता रहा और उदास होता रहा।

लेकिन एक काम कर दिया उसने फटाफट। बेटे का नाम रख दिया सीटू।

सीटू को आज अपने दिवंगत बाप की याद आ गयी है बहुत बुरी तरह से। रात के साढ़े बारह बजे हैं। पीला–पीला चांद चमकता है – ऊपर, बहुत ऊपर आकाश में। सामने के ऊँचे–ऊंचे पहाड़ ज्यों के त्यों खड़े हैं चौकन्ने। गाड़ी का चीखना सुनकर उनींदे से सकपकाकर उठ बैठे हैं जैसे। अजाने बोल की परछाइयाँ–सी फेंक रहे हैं रस्सी की मानिंद–ठीक सीटू के सामने ...

'कुछ बोल सीटू, रात कट जाये।'

अब सीटू क्या बोले? माघ महीने की कड़कड़ाती ठंड! दाँत बज रहे हैं ठकाठक। गूदड़ को फेंक–वेंककर आँगन में उकडूँ बैठ गया है सीटू। बीड़ी का सुट्टा लगा रहा है भकाभक।

ससुरी नींद उचट गयी है। उचटी कहाँ? नींद आयी कब है आज? सोचता है – जो हुआ बुरा हुआ।

बापू ने मरती बार कहा था –

'सुन बेटे, तेते पाँव पसारियो, जेती लांबी सौर। हमारे 'बंस' में यह मुहावरा बहुत पुराना है। सम्पत्ति, जायदाद, घर, जमीन... यह सब बसीयत करने की 'परथा' हमारे खानदान में कभी नहीं रही। हम अपने सपूत बेटों को अपनी पिलपिली पसलियाँ, खुरदुरे हाथ, बिवाई पड़ी एड़ियाँ, बोझ से बौने हुए सिर, छिले हुए कन्धे भर दिखा देते हैं। हमारे बेटे बड़े सयाने हैं। जिस्म पर टाँकी हुई इस पथरीली भाषा को खुद समझ लेते हैं। कोई शिकायत नहीं करते। ...देख बेटे, तेते पाँव पसारियो जेती लांबी सौर। चादर छोटी पड जाये तो उकडूँ हो जाना। फिर भी कम पड़े तो पेट, टाँगों को छील–तराशकर छोटा–छोटा कर लेना। कपड़ा बित्ता भर भी रह जाये, कोशिश करना वह अंगुलि भर कपड़ा तुम्हें पूरा ढक ले। नंगा मत हो जाना! हम अनपढ़, गँवार, गरीब, मजदूर लोग हैं। नंगा होना शोभा नहीं देता हमें।

'अब समझा सीटू कि उसके दिल में 'होल' सी क्यों मची है इतनी? ठीक–ठीक नींद क्यों नहीं आ रही है? सीटू खुद हैरान कि बापू क्यों याद आ रहे हैं आज इतने जोरों से?

शायद आज वह नंगा हो गया है पूरा का पूरा। किसी ने देखा या नहीं, उसने देख लिया है अपने आपको।

वह दबे पाँव सुखनी के सिरहाने बैठ गया। 'हौले' से उसके माथे को छूकर देखा – 'सुखनी री, आटा.....?' लेकिन रुक गया बोलता–बोलता। बात कुछ और करनी थी। मुँह से निकल गयी कुछ और।

सुखनी ने 'ऊँह' करके करवट बदली।

अजीब औरत है! न ढंग का पहने। न ढंग का खाये। न ढंग की बात करे। न ढंग से सोये।

लेकिन यह दोष किसका है? क्या सुख दिया है सुखनी को उसने अब तक?

सीटू सोचते–सोचते रुक गया। पहले ही काफी बोझ है सिर पर। अब और क्या–क्या सोचे सीटू?

पिछली गर्मी में सुखनी बोलती थी, 'अबकी जाड़ों में हाड़ खड़–खड़ करेंगे, देख लेना। गुदड़ सारे फट गये हैं। देखते नहीं कपड़े सारे उधाड़ी–उधाड़ी हो रहे हैं। तू तो मर्द है, नंगा रह लेगा। मैं औरत जात हूँ। सुई से जिस्म में कहाँ–कहाँ बखिये लगाती फिरूँगी।

सीटू को बस गुस्सा ही आ गया, 'कैसे कहती हैं तू कि मैं नंगा रह लूँगा। बापू ने कहा था... खैर, रहने दे। आगे से ऐसी बात मत करना। कहे देता हूँ।'

बुड़बुड़ाता रहा सीटू यूं ही। बीड़ी का सुट्टा खींचता रहा गहरे, बहुत गहरे। धुएँ के घने काले कोहरे में उसने अपनी सारी जिन्दगी की परिक्रमा कर ली। गोल–गोल छोटा–सा दायरा है। कितनी देर लगती है घूम–घुमाकर वापस अपनी जगह आने में। लेकिन उस दायरे में 'घुमाव' बहुत है। सारा रास्ता काँटों और नुकीले पत्थरों से अटा हुआ है। पीछे देखता है तो ठोकर लगती है। आगे देखता है तो अँधेरा दिखायी देता है, घटाटोप।

बड़े–बड़े सपने उसने कभी नहीं देखें।... अब तो बस एक साध रह गयी है। उसकी झोंपड़ी के किवाड़ लग जाएँ, बस। बाकी जो है, ठीक है। दो तीन–साल से सोच रहा है। पैसों का ' बेसगी ' जुगाड़ नहीं बैठता। साध ज्यों की त्यों धरी–धराई पड़ी है।

बिन ब्याहा था तो ठीक था। लेकिन जब से सुखनी को ब्याह कर लाया है, किवाड़ की चिन्ता खाये जा रही है रात–दिन। दिन भी खैर निकल जाता है। रात का डर लगा रहता है। चोर–डाकू, गुंडे–उचक्के... शहर की आबोहवा गाँव को डस रही है धीरे–धीरे। खटका लगा रहता है हमेशा–हमेशा कि कोई कोठरी के भीतर झाँक रहा है। खैर, चुराने लायक चीज तो कोई नहीं है घर में। जो है वह सुखनी है। जवान–जहान उसकी बीवी। गऊ जैसी सीधी। लेकिन दुनिया की नजरें तो सीधी नहीं हैं। कभी–कभी घर लौटने में देर हो जाती है तो सुखनी के साथ–साथ किवाड़ का ध्यान आ जाता है झट्ट से। 'धड़ाका'–सा उठता है कलेजे में। घर में पक्के किवाड़ नहीं हैं तो घर पूरा नंगा–नंगा लगता है। सुखनी बैठी है सामने। सुखनी नहा–धो रही है। सुखनी बतिया रही है।

सुखनी खिड़–खिड़ हँस रही है। सुखनी सो रही है।... सुखनी सीटू की ओर देखती है और खुद को ढीला छोड़ देती है। 'सरम' लगती है कोई और देख ले तो! ...

जब से जंगल का काम लगा है, लकड़ी का गोदाम यहाँ बना है। आरे की मशीन लगी है, शहर से लोग यहाँ आये हैं। देहात के कच्चे–पक्के घरों में किवाड़ों और तालों की जरूरत पड़ गयी है।

गुंडे–उचक्के पक्की सड़क छोड़कर पगडंडी पर इधर–उधर मँडराने लगे हैं। कुछेक पगडंडियाँ तो खुद बना लेते हैं ये। ताकते रहते हैं अगल–बगल जीभ चिपचिपाते हुए। लकड़ी के गोदाम के कारिन्दे, जंगल के ठेकेदार, सरकारी आदमी उसके टपरे के सामने से इकले–दुकले टहलते, खंखारते, भद्दी हरकतें करते रहते हैं। किसी दिन उन्होंने देख लिया कि घर में किवाड़ नहीं हैं तो! सीटू का दिल बैठ जाता है। कभी–कभी दिहाड़ी का काम छोड़कर वह पहरा देने बैठ गया है घर पर खुद दहलीज पर बैठ जायेगा और सुखनी को ओट में कर लेगा। लेकिन जब घड़े में आटा चुक जाता है तो सुखनी जबरदस्ती धकेल देती है उसे काम पर।

कभी–कभी ऐसा भी हुआ है कि इधर गाड़ी की सीटी बजी, उधर सीटू झटपट उठकर अंधेरे में अपनी बीवी को टटोलने लग जाता है। बगल में सोयी है चैन से। ठीक है। सुखनी कभी–कभार जागी होती है गुपचुप। सोचती है सीटू रोम–रोम में रसी–बसी अँगड़ाई ले लेगा अभी–अभी और सुखनी को जगा देगा झकझोरकर। लेकिन मन की जब ऐसी हालत होती है तो सीटू उसे जगाता नहीं। उल्टे खुद जागकर उसे थपथपी देकर सुला देता है।... थोड़ी–थोड़ी कोशिश करती है सुखनी सीटू की बात समझने की।...

सीटू बस सोचता जा रहा है। एक नजर बाहर, एक नजर भीतर। एक ख्याल यहाँ, एक ख्याल वहाँ।

मिसिरजी आये थे इलाके में। कैसा भाषण दिया था। दो कोस दूर तक आवाज बुलन्द थी। खादी के कपड़े झकाझक। गले में 'डोलरे' के फूलों के हार। सारा इलाका टूट पड़ा था भाषण सुनने के लिए। मिसिरजी कभी हँस जायें। कभी चुप हो जायें। बस देखते–सुनते बनता था। एक–दो बार तो भाषण देते–देते रो पड़े। खूब याद है। अभी की बात है – हुए होंगे सात–आठ महीने। जंगल के ठेकेदार, कारिन्दे सब अगल–बगल में चहक रहे थे। भाई–भतीजे थे उनके या अपने खास आदमी। दौड़–भागकर पूरा इन्तजाम किया था उन्होंने। शहतीरों का ऊँचा चबूतरा बनाया था, जहाँ से मिसिरजी भाषण दे रहे थे। मालिक लोगों ने कहा था, 'सीटू, नेताजी का भाषण खत्म हो जाये तो शहतीर उठाकर वापस 'टाल' में रख देना। भीखू, भाद्रू, मोहन, लच्छू सबको कह दिया है। समझे! हर शहतीर का ध्यान रखना।'

सीटू इसलिए अगली लाइन में बैठ गया था, ठीक चबूतरे के पास। कभी नेताजी को देखता, कभी शहतीरों को। भाषण की कोई–कोई बात समझ में आ

जाती, बाकी राम जाने। लोग ताली बजाते थे। वह बीड़ी मुंह में खोंसे पहले इर्द–गिर्द लोगों को देखता। फिर चट्ट से दोनों हाथों को एक–दूसरे से मिला देता।

परताप है नेताजी का। बोलो, नेताजी जी जय!

मिसिरजी की आँखों में आँसू थे। जाने क्यों? ठीक सामने बैठे सीटू की ओर उन्होंने देखा। सीटू को समझ नहीं आया कि वह ताली बजाये या रोये। फटाफट बीड़ी बुझाकर कान में खोंस ली।

मिसिरजी चबूतरे से उतरे। बड़े प्यार से सीटू के कन्धे पर हाथ रखा। बाँह से पकड़कर उसे चबूतरे पर ले चले। खड़ा कर दिया सबके सामने उसे पलक झपकते सब कुछ हो गया। सीटू बुत बना सब देखता रहा। भीड़ में से दो–चार आवाजें भी आयीं, 'मंगतू का छोरा है, सीटू।' कुछ लोग हँसे भी। लेकिन मिजिरजी की आँखें गीली थीं–

'देखो, इस गरीब मजदूर की हालत देखो। हद है। आजादी के बाद क्या मिला है इसको? यह भी इन्सान है। भाई है यह हमारा। गांधीजी ने कहा था 'अंत्योदय'...। मैं कसम खाकर कहता हूँ कि यदि मैं चुनाव जीत गया तो सबसे पहले इस भाई की हालत सुधारूँगा।'

तालियाँ! ...

सीटू को कुछ वक्त लगा इस 'सदमे' से उबरने में। लेकिन वह पूरी तरह गद्‌गद् हो गया। 'साक्षात्' नारायण है मिसिरजी। इतने आदमियों के बीच तड़ाक से बोल गये, 'यह भाई है मेरा'। धन्न हो महाराज! कहाँ राजा भोज, कहाँ गंगू तेली।

साथी मजदूरों ने टहोका दिया, 'अब तेरी चांदी है, सीटू।'

बड़ी रात तक, उस दिन सीटू शहतीर उठा–उठाकर 'टाल' तक पहुँचाता रहा। लकड़ी के ठेकेदार, बाबू सब उसकी पीठ ठोंक रहे थे, 'मिसिरजी खुश होकर गये हैं, सीटू।'

थोड़ी–सी दारू, विलायती, पिला दी उन्होंने। पी गया गटागट सीटू। मिसिरजी की जूठन थी। परसाद समझकर पी गया।

घर आकर सुखनी पर रौब झाड़ने लगा, 'तू क्या समझती है मैं दो टके का आदमी हूँ? जानती है, मिसिरजी ने आज सबके सामने सीटूराम के कैसे–कैसे बखान किये?'

सुखनी ने आँखें मटकायीं। दारू का सारा नशा गायब?

'सुखनी री, अब काम हो गया समझ। चबूतरे पर हाथ पकड़कर ले गये मुझे। मेरे कन्धे पर हाथ रख दिया यूँ। बोले, यह भाई है मेरा यूँ रखा हाथ।' सुखनी को गुदगुदी हो आयी।... एक बीड़ी और सुलगा ली सीटू ने।... मिसिरजी जीत गये। 'मन्तरी' बन गये।

सीटू ने दारू पी ली 'खुशी' में। अबकी विलायती नहीं थी। पल्ले से खरीदी–सस्ती देशी। 'पव्वा' भर चढ़ा लिया। बस सिर को चढ़ गयी। गिरता, पड़ता, लड़खड़ाता, हँसता, गाता, नाचता रहा सीटू। घर आकर बीवी से झगड़ा कर दिया।

'दारू पीकर अपनी घरवाली से झगड़ता है तू। लड़नी है तो कोई बड़ी लड़ाई लड़।'

यह बड़ी लड़ाई क्या होती है? भूषण बाबू यूनियन के जलसे में बोलते थे, 'अपने हक की लड़ाई लड़ो।'

अब यह 'हक' क्या होता है? एक टंटिया हो तो समझे सीटू! ...

मिसिरजी 'मन्तरी' बनने के बाद इलाके के दौरे पर आये तो रंगत ही दूसरी थी। जीप, कार सब अगल–बगल में। जमघटा लोगों का। सब एक सुर में बोले, 'मन्तरीजी जिन्दावाद!' रंग–बिरंगे फूलों के हार। मन्तरीजी दोनों हाथों से 'परनाम' करें सबको।...यह अगला जन्म, पिछला जन्म कुछ नहीं है। जो है, बस इसी जन्म का खेल है। प्रताप है महाराज का। जिसको है, उसको है। जिसको नहीं है, उसको नहीं है।... सीटू सोच रहा है।

'धन्न हो मन्तरीजी। माटी 'पवित्तर' हो गयी।'

'धार परोली' के मेले में भी ऐसी चहल–पहल नहीं देखी।

भाषण के वक्त इस बार फिर सीटू चबूतरे के ऐन बगल में बैठा था। परन्तु मन्तरीजी अबकी आसमान को देख रहे थे और धरती के जीवों को 'उपदेश' दे रहे थे।

डाक बँगले में भी बड़ी धक्कमपेल थी। बाहर कुर्सी लगाकर बैठे थे मिसिरजी। फैसले पर फैसला सुना रहे थे। जिस तरंग में सीटू उनके करीब दौड़ा, उसी धक्के में सन्तरी ने पीछे धकेल दिया, 'लाइन में आओ।'

'क्यों आयें जी लाइन में? मिसिरजी भाई हैं हमारे।'

मिसिरजी ने हल्ला सुन लिया। उधर देखा। सीटू ने धड़ल्ले से दोनों हाथ जोड़कर 'परनाम' कर दी। 'सायत देखा नहीं मिसिरजी ने।' इधर उधर घूम–फिरकर जब भी मिसिरजी की नजर सीटू पड़ती, वह तड़ से परनाम कर देता–पूरा दुहरा होकर। एक बार, दो बार... पाँच–सात बार ऐसा हो गया। 'पहचाना नहीं सायद।'

अगल–बगल जंगल के ठेकेदार, मन्तरीजी के अपने खास आदमी, अफसर, साहब लोग, पंचायत के परधान, सरपंच... बड़े–बड़े लोग बैठे थे।

सीटू को कसक रह गयी कि परनाम का जवाब नहीं मिला ठीक से। मैनाराम सरपंच की नजर उठी कोई दो घंटे बाद।

'क्या चाहता है, सीटू तू? मन्तरीजी से कोई फरियाद करनी है। क्या?'

अब देखा मिसिरजी ने साफ–साफ।

'कौन है यह आदमी?'

सीटू को लगा जैसे जोगनी के 'ढाँख' से किसी ने उसे धक्का दे दिया। लेकिन सँभल गया एकदम। हो सकता है उसके सफेद पाजामे और लट्ठे की कमीज से न पहचान पा रहे हों। 'सादी' के टैम का सूट है। खास–खास मौके पर पहनता है सीटू।

'मैं सीटू हूँ, जी। पिछली बार...।' जंगल के ठेकेदार लोगों ने घूरती हुई नजर से देखा उसे। वह डर गया। उन्हें डर अपना था कि कहीं वह दिहाड़ी, मजदूरी... जुल्म, जबरदस्ती का बखेड़ा न छेड़ दे यहाँ।

सरपंच ने कहा–'इसका नाम सीटू है जी, जनाब।'

मन्तरीजी हँसे। पान की पीक थोड़ी–थोड़ी होंठों से बाहर निकल आयी।

सरपंच ने सीटू को टहोका दिया–'जल्दी बोल सीटू, क्या तकलीफ है तेरी? मन्तरीजी को टैम नहीं है। बहुत से काम है करने को।'

'जी...।'

सूझा नहीं ऐन वक्त पर कि क्या बोले? भाई वाली बात तो गयी।अब नये सिरे से बात शुरू करनी थी। लेकिन फँस गया। अब बोलना जरूरी था कुछ। उसके मुँह से निकल गया –

'जी, मेरे घर में बिजली नहीं आयी आज तक।'

सभी हँस पड़े जोर लगाकर। मन्तरीजी ने मुंह टेढ़ा किया, 'तेरे घर में बिजली नहीं आयी आज तक? बाहर तो सारे झकाझक हैं...घर में बिजली लगवाने का ठेका सरकार का नहीं है।'

'हैं! कहाँ गलती हुई सीटू से?' उसने दूसरी बात पकड़ ली 'झट्ट' से –

'जी, जी मेरे घर का दरवाजा करीब चार साल पहले टूट गया था। दीमक ने चट्ट कर दिया था पूरा का पूरा। बाप–दादा के टैम की लकड़ी थी।... अब किवाड़ नहीं है।... सरम लगती है।'

मन्तरीजी ठहाका लगाकर हँसे। उनके साथ सभी हँसे जोर–जोर से। दूर खड़ा सन्तरी भी सारी बातें साफ सुन रहा था। कन्धे पर बन्दूक रखे सीधा–सपाट खड़ा था। लेकिन हँसी से कमर में लचक आ गयी।

हँसने वाली बात करता है सीटू।

पता नहीं यह क्या हो गया। अब कुशल इसी में थी कि सीटू भी जोर लगाकर हंस दे, बस।

मन्तरीजी अभी पिछली बात पर गौर कर रहे थे, 'सरम काहे की, रे? तू नंगा रहता है अपने घर में क्या?'

सीटू हँसता–हँसता एकदम चुप हो गया। कैसी बात कर दी मिसिरजी ने? वह सब कुछ सुन सकता है, लेकिन मजाल कि ऐसी बात कोई बोल जाये। सुखनी नहीं बोल सकती उसके सामने।

बस, भाग आया वहाँ से बुड़बुड़ाता हुआ।...पहचाना नहीं। कोई बात नहीं... लेकिन नंगा क्यों बोला मुझे?

दिमाग छोटा है उसका वरना सबका हिसाब बता देता साफ–साफ।

औकात नहीं है वैसी वरना सारी बात साफ–साफ कह देता।

थोड़ी–बहुत दारू जब खोपड़ी में चढ़ती है तो पैर काबू में रहें या न रहें, दिमाग में चीजें जरूर साफ दिखायी देती हैं। पूरी तसवीर खिंच जाती है लकीर लकीर। डर भी नहीं व्यापता।

बड़ी–बड़ी गालियाँ हलक से बाहर उमड़ने को आती हैं। खून पीते हैं आदमखोर।...सतवादी हरिचन्दर बने हैं। नंगे हैं खुद पूरे। सीटू को गाली देते हैं।... सबकी साँठ–गाँठ है। मिलीभगत है। हिस्सा–पत्ती है बंधी हुई सबकी। जंगल का ठेकेदार, खास–खास कारिन्दे, सरकार के आदमी...मन्तरी, सन्तरी... सब। बापू कहता था–सारा देस नंगा हो गया। तू मत नंगा हो जाना, सीटू।... बुड्ढा मरती बखत दो टके की सीख दे गया मुझे! बिलकुल हिजड़ा बना गया मुझे!

किसी ने आवाज दी। शायद भीखू था।

'इतनी मत पी, सीटू। जा, तेरी घरवाली देख रही होगी तुझे।' सुखनी की याद बड़ी देर से नहीं आयी थी। गुस्सा उतर गया। लेकिन 'फिकिर' चढ़ गया एकदम।

'किवाड़ नहीं है टपरे में।... ये मुए लुच्चे–लफंगे, गुण्डे दुनिया भर के। इनका क्या दीन–ईमान! माँ–बहन, कोई नहीं है सायद इनकी! इन्होंने धरती की सरम नंगी कर दी है। पेड़ को तने से टटोल–टटोलकर जमीन पर बिछा देते हैं लम्बा काट–मसलकर जंगल के जंगल साफ कर दिये हैं। बेसरम! बेहया! जंगल के सारे कपड़े चिथ–चिथकर उघाड़ी कर दिये हैं इन्होंने।'...

धक्क से सोचता है सीटू। डाक बँगले में जो औरत फरियाद लेकर खड़ी थी, शुक्र है वह सुखनी नहीं थी। लेकिन कोई तो थी बेचारी!

भीखू, उसका दोस्त, दूसरे दिन डाक बँगले से विलायती दारू की खाली बोतलें ले आया बोरी भरकर! उस सुबह इलाके में बहुत कम मुर्गों ने बांग दी।

'चोर हैं सारे एक नम्बर के सारे देस को खा गये। पेट नहीं भरा भुक्खड़ों का।'

क्या हुआ अगर सीटू भी छोटी–सी चोरी कर ले तो। ...

लकड़ी के बड़े–बड़े गोदाम हैं। शहतीर के बड़े–बड़े 'टाल' हैं। सारी उम्र उसका बाप शहतीर को कन्धा लगाता रहा है। सीटू भी वही काम कर रहा है लड़कपन से। दिहाड़ी मजदूरी जो भी है, वही है। 'पुगाव' कैसे हो? इधर वे एक रात दारू, दावत... जाने किस–किस में हजारों खर्च कर दें बैठे–ठाले। कोई हिसाब माँगे इनसे!

बहुत बार सीटू को ऐसा लगा है कि वह अकेला है। दिल के भीतर जो है, उसे सहारे की जरूरत है। इकट्ठे हों सब। मिलकर कुछ ठीक–ठीक बात की जाये। ठीक–ठीक बात सोची जाये।... दो–चार कभी इस लाइन पर सोचते हैं तो मालिक लोग चट्ट से दाना फेंक देते हैं उनके आगे और ये भाई चाटने लगते हैं थूथन से।

दाना उसे भी फेंका था।

'तेरी तरक्की कर देंगे हम, सीटू। तेरी घरवाली...।'

सीटू का मुँह एकदम अंगारा। मुट्ठियाँ भिच गयीं।

'तुमने देख कैसे लिया मेरी घरवाली को?'

थर–थर काँपता है सीटू।... शहतीर कन्धे पर लादे हुए एकदम दिमाग सुखनी की ओर मुड़ जाता है। सीधी–सादी औरत है। बैठी होगी इस समय चूल्हा फूँकने। प्याज का छौंक लगाकर आलू की सब्जी बना रही होगी और अनजाने में बाहर भी देख लेती होगी इधर–उधर। कहाँ तक आँखें मूंदे! दरवाजे के 'पल्ले' नहीं हैं इसलिए खुँखार भेड़िये दिन को भी चबेनी ढूँढते रहते हैं...खौल रहा है सीटू। हड़बड़ा गया है।

'सीटू, वजन सम्भाल। शहतीर डोल रही है।' दूसरी ओर से शहतीर को कन्धा दिये हुए भीखू ने चेतावनी दी।

सीटू मन ही मन सोचने लगा, 'अच्छा! सम्भालता हूँ वजन। करता हूँ कुछ जुगत।'

चोरों के घर में सेंध लगाने की बात उसने बहुत दिनों से सोची थी। लेकिन हिम्मत नहीं पड़ रही थी। 'धड़ाका' लगता है बड़ा! मालूम नहीं ये इतनी बड़ी–बड़ी चोरियाँ कैसे कर लेते हैं और खुशी–खुशी झेल लेते हैं। डोलते रहते हैं खूब, रौबदाब से। इनकी औकात भी ठीक, जात भी ठीक। सिरताज, धरमराज बने रहते हैं। सीटू को रास्ते में पड़ा हुआ दस पैसे का सिक्का उठाने में भी अपनी पूरी रूह से लड़ना पड़ रहा था इतने जोरों से!

शहतीर के 'टाल' और आरे की मशीन में चिरे हुए कुछ 'फट्टे' उसकी नजर में थे बहुत सारे। उसने केवल दो 'फट्टों' पर नजर गड़ा ली। मौका पाकर 'टाल' के खूब नीचे छिपा दिया उन 'पल्लों' को सारे हालात को,समझना,परखना जरूरी था पहले! आठ–दस दिन तक 'पल्लों' की किसी को फिकर नहीं हुई। सीटू ने दुबारा मौका सम्भाला और 'पल्ले' उठाकर थोड़ी दूर ले गया। झाड़ी में छिपा दिये एक जगह...।

थोड़ा–थोड़ा सफर तय किया नाप–तोलकर। महीना भर के बाद दो 'पल्ले' घर तक पहुँचे। उसने घर के पिछवाड़े छिपा दिये करील की झाड़ी में। बिना किवाड़ के एक कमरे में चोरी का बित्ता–भर सामान छिपाने की गुंजाइश नहीं थी।

सुखनी उस 'टैम' पानी का घड़ा भरने गयी थी।... यूँही सवाल–जवाब करेगी। उसके पेट में बात पचती कहाँ है? भक्क से बोल देगी किसी से।

दूसरे दिन दिहाड़ी पर नहीं गया सीटू। सारा दिन सुखनी को ओट में लिए बैठा रहा दहलीज पर। सोचता रहा गुपचुप। बस, सोचता रहा। सुखनी ने पूछा। कुछ नहीं बोला। सुखनी डरी। शायद 'भूत–परेत' की छाया पड़ गयी हो। पूछना जरूरी था। सीटू ने झिड़क दिया बुरी तरह। खाना भी नहीं खाया। सांझ हो गयी। सुखनी रोते–धोते सो गयी। सीटू जागता रहा।

बस, यही बात है कि सीटू आज रात माघ की ठंड खा रहा है–आधी रात को। वैसे रोज गाड़ी की सीटी उसका इन्तजार करती थी। आज उसने पहल की है इन्तजार करने की।

बापू याद आ रहे हैं इतने कि पूछो मत। दोनों ओर संकट है। किवाड़ खुले रहते हैं तो सुखनी और किवाड़ बन्द हो जायें तो सीटू। नंगा होना दोनों में से एक की किस्मत में लिखा जरूर है!

बीड़ी के धुएँ में वह जिन्दगी की पटरी पर बेतहाशा दौड़ रहा है–हाँफता–काँपता। छोटी–सी पटरी है। मुड़–मुड़कर वहीं से चक्कर काटती है बार–बार। सीटी की कर्कश आवाज आती है कानों में।

लौट जाये सीटू क्या?

क्या जवाब देगा लोगों को... वही लकड़ी है। मोहर लगी है। देख।

क्या कहेगा लोगों को। 'एकाएक किवाड़ कैसे लग गये? तेरी औकात क्या है?'

एक दरवाजे के 'पल्ले' भर लगाने की औकात न हो आदमी पास तो लानत है ऐसी जिन्दगी पर!

'टाल' के कारिन्दे, जंगल के ठेकेदार वहशी, खूँखार आँखों से घुड़कियाँ देते फिरेंगे उसे।उसके टपरे को देखकर मुँह से अजीब–अजीब सी लार टपकाते फिरेंगे – 'वाह बच्चू, हमारी ही लकड़ी और हमसे ही परदा!' तोड़कर रख देंगे वे किवाड़ को। सुखनी नजर में गड़ जायेगी ठीक सामने। ऐसी हालत में बोल भी क्या पायेगा वह? सिर नीचा किये अपनी चोरी माननी पड़ेगी उसे। लाख दलीलें दे ले वह झूठ बोला नहीं जायेगा। अक्ल कम है। दांव–पेच नहीं जानता। फिर दुनिया बड़ी समझदार है! जानती है किसकी बात का विश्वास करना है और किसका नहीं।

जिनके पास वसीला है, वे सौ–सौ खटके सम्भाल लेते हैं बिना खरोंच खाये। यहाँ छोटी–सी बात नहीं सम्भल पा रही है। लगता है जिस्म में साही के काँटे चुभ गये हों। सिर के सारे बाल जैसे मुंडिया गये हों एक रात में। ठंड है बहुत, लेकिन आज की ठंड ज्यादा भारी पड़ रही है सीटू को।

एक तरकीब सूझी है उसे कि इन 'पल्लों' को वापिस पहुँचा दे अभी इसी 'टैम' मशीन और 'टाल' के पास। साफ कह देगा कि गलती से ले गया था। वापस पहुंचा रहा हूँ... लेकिन गोरखा चौकीदार सौ–सौ सवाल करेगा। शिकायत करेगा मालिकों तक। फिर अपनी नेकनियती को साबित करने के लिए उसे जाने क्या–क्या करना पड़े?... ऐसे मौके होते हैं जब उसे अपने साथी, उसकी बात को वजन देने वाले बहुत कम दिखायी देते हैं। वह अकेला पड़ जाता है सबका अपमान सहने के लिए।...

रात के लगभग साढ़े तीन बज गये इसी सोच में। हट–फिरकर बात वहीं पहुँचे बार–बार...।

फिर उसने सब कुछ पक्का सोच लिया। वह पिछवाड़े गया। करील की झाड़ी से 'पल्ले' निकाले। लालटेन का तेल छिड़ककर माचिस लगा दी 'भड़ाक' से।

आधी रात में आग की लपटें उठीं पिछवाड़े से। सीटू अपने दोनों हाथ–पैर, दिल–दिमाग... जुदा–जुदा... फिर इकट्ठा तापने लगा उकड़ूँ होकर। पूरी चैन से।जी में तो आया था कि सुखनी को भी उठा दे, 'उठ भागवान, आग ताप ले। देवदार की है।' फिर कुछ सोचकर उसने सुखनी को नहीं उठाया। वह आग अकेले ही तापी उसने।

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Narrated by Praveen Kumar

योगेश्वर/नंगा आदमी : जितना बड़ा व्यक्तित्व : उतना ही श्रेष्ठ रचनाकर्म!

दिनेश धर्मपाल

यह बात मैं बड़ी तसल्ली के साथ योगेश्वर को लेकर कह रहा हूँ। उनकी रचनाएं अपने समय का वह दस्तावेज हैं जिसकी स्याही कभी फीकी नहीं पड़ सकती। वैसे ही नहीं कह रहा हूँ कि उनकी रचनाएं कालजयी रचनाएं है, खानापूर्ति के लिए नहीं हैं। हंगामा खड़ा करना उनका काम नहीं है। वे समाज में चल रहे सरोकारों से पूरी तरह से परिचित हैं। किसको क्या मिलता है? किसको क्या नहीं मिलता? इस सबसे परिचित हैं। यह गरीबी, यह दासता का स्वर पराधीनता में जकड़ी आजाद देश की आधी से ज्यादा आबादी का रूदन—सब लेखक की आखों के सामने है। कौन है जो आमजन को शोषण की आग में आगे धकेल रहा है और स्वयं पीछे बैठा तमाशा देख रहा है? योगेश्वर का मानना है कि काम होगा तो विकास तो होगा ही। चकाचौंध करने वाला विकास भी होगा। कुछ के लिए देखने का और कुछ के लिए चांदी बनाने का। मंगतू का बेटा सीटू और सीटू की घरवाली सुखनी विकास की इसी चकाचौंध के शिकार हैं। तभी उनकी जिंदगी में अंधेरा है। शब्दवेधी सन्नाटे के पिंजरे से टकराती चीत्कार पैदा करती सीटी के आवाज से एक दूसरी आवाज अस्तित्व में आती है और वह है सीटू! मंगतू के यहां बेटा आया है। लोगबाग बधाई देते हैं : ''राजा बेटा आया है।'' बेटे तक तो मंगतू को ठीक लगता है पर राजा बेटा अखरता है। गरीब का बेटा राजा कैसे हो सकता है? बचे हुए कुछ पताशों को ही तो वह खिला पा रहा है जो बधाई देते हैं। उसके पास देने को दूसरा है भी कुछ नहीं। जिंदगी की आखिरी रीत निभाते—निभाते तक मंगतू कुछ भी तो नहीं दे पाता सीटू को, न उपहार रूप में और न सीख रूप में, बस यही एक कथन : 'देख सीटू, हमारी बसीयत, हमारी जमीन, हमारी जायदाद, हमारे सपने हटफिर कर पिलमिली पसलियों, खुरदरे हाथ, बिबाई पड़ी एडियां, बोझ से बौने हुए सिर, छिले हुए कंधे और पथरीली भाषा के सिवा और कुछ नहीं – भले ही हम अनपढ़, गंवार, मजदूर, हमें बस एक ही बात याद रखनी है : कि हम कभी नंगा न हो! नंगा होना हमें शोभा नहीं देता!''

भले ही माघ महीने की कड़क ठंड। टापर में बजते दांत। गुदड़ को फेंक फेंक कर उकडू बनते सीटू और सुखनी, छोटी पड़ती चादर – फिर भी नंगा नहीं होने की बात कर मंगतू नहीं रहा।

सांझ के भयावह दृश्य से शुरू होता नंगा आदमी, पटरियों पर उतरता अंधेरा, सीटी के सन्नाटे से पिंजरे से टकराते चीत्कार पैदा करते, खौफनाक मंजर, परिणाम में कुछ पंछी मर जाते, कुछ उड़ जाते, कुछ बेदम हो जाते, मानो

नियति के तीन मंजर! इसी त्रासदी से गुजर रही मानवता! मरने वालों को तो रोज के प्रपंच से मुक्ति मिल जाती है, बाकियों को यह भी नसीब नहीं। बेदम हुए वे फड़फड़ाते रह जाते हैं। सीटू को यह रेल की पटड़ी और उसके चारों ओर मंडराने वाली विषैली सुरलहरियां अच्छी नहीं लगतीं। उसे लगता है हर समय जान सांसत में है। उसे डर है कि कहीं यह डरावनी आवाज टुडी राक्षस की तरह फड़फड़ाकर उसके टापरे को न लपक ले!

अनास्था का यह स्वर सीटू के चारों ओर अहर्निश मंडराता है। और पिता मंगतू का यह बोल भी कि गरीब भले ही क्षण प्रतिक्षण निंगोड़ी रात और उखड़ती नींद के आगोश में मृत्यु का उपहार क्यों न पा ले, पर उसे कदापि नंगा नहीं होना है। नंगा होना गरीब को शोभा नहीं देता!

सीटू को बार बार लगता है : घर में किवाड़ का न होना एक तरह से नंगा होना है। उसके यहां बाबा आदम के जमाने का बना किवाड़ दीमक ने चट कर दिया और तब से लेकर आज तक वहां न तो नया किवाड़ लगा और न ही बचे खुचे किवाड़ की मुरम्मत ही हो पाई। उसकी मनः स्थिति इसे लेकर हर पल दुविधा में रहती है। उसे कई बार लगता है जैसे घर नंगा हो गया है। कई बार लगता है कोई है जो देख रहा है, झांक रहा है। पर किसे क्युं देखना चाहता है कोई? ऐसा क्या धरा है यहां? फिर सहसा स्मरण हो आता है कि सुखनी तो है। उसे अपनी चिंता नहीं, सुखनी की चिंता है, जो अजीब औरत है, जो न तो ढंग का पहन पाती है, न ढंग का खाती है और न ही ढंग की बातें हो पाती है उससे और न ऐसा है कि इतमिनान से वह ढंग से सो पाती हो!

वह सोचता है : इसके सिवा जिंदगी में और है भी क्या? दाल, लून और रोटी और फिर चैन के दो पल और बेफिक्र नींद! बार बार एक उलाहना वह स्वयं ही स्वयं को पारित करता दिखता है : आखिर उसने उसे ऐसा दिया ही क्या है?

फिर जब वह सुखनी की ओर लौटता है तो वह एक प्रायश्चित की मुद्रा में दिखाई देता है। ऐसे जैसे सबके लिए जिम्मेवार वह एक। सुखनी में किसी भी तरह का कोई दोष दिखाई नहीं देता। उल्ट वह तो एक मार्गदर्शक की भूमिका में आ खड़ी दिखाई पड़ती है जो मानों उसे सचेत कर रही हो : ''अब के जाड़ों में हाड़ खड़ खड़ करेंगे, देख लेना!'' वह उसे स्मरण कराती दिखती है : ''गुदड़ सारे फट गए हैं, सारे उघाड़ी उघाड़ी हो गए हैं।''

यहां तक तो ठीक है, पर फिर एक उलाहना भर स्वर : ''तू तो मर्द है, नंगा रह लेगा। पर मैं औरत जात हूँ, सूई से जिस्म में कहाँ कहाँ बखिये लगाती फिरूंगी?''

सीटू से न कहते बनता है और न सुनते बनता है। बड़बड़ाता है कि मैं नंगा रह लूंगा। बाप का असर उस पर भी है। बाप भी नहीं चाहता होता कि उसका बेटा और उसकी बहु नंगी रहे। नंगा होना गरीब के लिए अपमानजनक है।

गहन अवसाद में चले जाने के सिवा सीटू के पास बाकि कुछ है नहीं। बीड़ी का गहरा सूटा लगाने के सिवाए शेष कुछ दिखता नहीं। उसे लगता है यही बीड़ी गरीब की अमानत है। वह बीड़ी पीता है, जोर का कस लेता है, गहरा सूटा लगाता है तो उसे यह बात सामने आती दिखाई देती है कि धुंए के घने काले कोहरे में मानो उसने अपनी पूरी जिंदगी की परिक्रमा कर ली हो। इस धुंए में उसे गोल गोल छोटा छोटा दायरा बनता दिखता है। उसे प्रतीत होता है कि उसकी जिंदगी कुछ इस दायरे की भांति ही है– गोल गोल छोटे छोटे दायरे में बंटी हुई। पर उसे सहसा लगता है कि उसके इस दायरे में घुमाव बहुत हैं। सारा रास्ता कांटो नुकीले, पत्थरों से अटा हुआ मानों। आगे कुआ पीछे खाई। ठोकर लगने के पूरे अवसर मोजूद। इससे बचाव नहीं। अंधेरे–घनाटोप अंधेरे में बच कर निकलना उसे असह्य जान पड़ता है। सब रामभरोसे छोड़ देता है।

एक पति होने के नाते सुखनी को लेकर उसकी चिंता जाइज है। सुखनी गऊ जैसी सीधी। न कोई छल और न कोई कपट! एक बात उसे भीतर ही भीतर कचोटती है कि जब से वह उसे ब्याह कर लाया है उसने उसे कुछ नहीं दिया। यहां तक कि झोपड़े का किवाड़ तक। उसे जब भी सुखनी का किसी भी बात को लेकर स्मरण आता है तो साथ में किवाड़ का भी स्मरण अनायास आ जाता है। उसे लगता हैं बिना किवाड़ के घर नंगा है। किवाड़ नहीं तो घर नहीं। दिन तो जैसे तैसे निकल जाता है, सवाल रात का है। यह सवाल और भी भंयकर शक्ल ले लेता है क्योंकि यहां लकड़ी का गोदाम जो खुल गया है और फिर लकड़ी के गोदाम के कारिंदे, जंगल के ठेकेदार, सरकारी आदमी उसके टपर के सामने इकले उकले, टहलते खंखारते, भद्दी हरकतें करते...

फिर विकास की इस धारा में पंगडंडियों का बनना। नई पगडंडियों का बनना। योगेश्वर पगडंडी को गंतव्य तक पहुंचाने का एक जरिया मानते रहे हैं। दो धाराओं को जोड़ने वाला एक जरिया। लेकिन यहां पगडंडी एक विरोधाभास लेकर उपस्थित होती दिखती है। छोटी छोटी पगडंडियो तो होती ही हैं, पर नई पगडंडियों का बनना कुछ और उकेरता है। ऐसे में सीटू को लगता है कि देहात के कच्चे पक्के घरों में किवाड़ों और तालों की जरूरत पड़ गई है। गुंडो उच्चकों द्वारा अपने हिसाब से नई पगडंडियो को बनाना संकट का सबब है। ऐसे में सीटू ध्याड़ी का काम छोड़ पहरा न दे तो क्या करे? खुद दहलीज पर और सुखनी को ओट में पड़े रहने को बाध्य न करे तो क्या करे? सीटू सोचता है। बस यही डर खाता और डराता है सीटू को कि कोई देख न ले कि घर में किवाड़ नहीं। जब भी सीटू ऐसा सोचता है तो उसका दिल बैठ जाता है। उसके भीतर एक डर वहम के रूप में बैठ जाता है। गाड़ी की सीटी बजते ही सीटू अंधेरे में टटोलने लग जाता है जबकि वह एक कोने में पसरी पड़ी है।

सुखनी को लेकर उसका सोचना अनायास नहीं। वह एक एक दृश्य को अपनी आंखों के सामने देखता रहता है जैसे कोई कहानी चल रही हो। वह देखता है : सुखनी बैठी है सामने। वह देखता है सुखनी नहा धो रही है। वह देखता है सुखनी बतिया रही है। उसे लगता है सुखनी खिड़ खिड़ हंस रही है।

उसे संतोष होता है कि सुखनी सो रही है और जब सुखनी उसे देखती है तो बस एक ही संवाद : ''तुम होते हो तो खुद को ढीला छोड़ देती हूँ। कोई देख न ले : सरम लगती है।''

क्या करे सीटू जबसे विकास की बयार ने यहां दस्तक दी है, लकड़ी का गोदाम, आरे की मशीन, शहर के लोगों का प्रवेश हुआ है, उसने महसूस किया है जैसे सबकुछ गड़बड़ा गया है, ऐसे जैसे खूंखार भेड़िए दिन को भी चबेनी ढूंढते फिरते हैं। थकाहारा सीटू अपने लिए विश्राग के कुछ क्षण चाहता है। उसे लगता है जैसे उम्र हो गई हो विश्राम किए हुए। इसीलिए वह सुखनी से घड़े में आटे होने की बात पूछता है। आटा बचा हो तो ध्याड़ी न करे, विश्राम कर ले! जब समय लगता है वह दबे पांव सुखनी के सिरहाने जा बैठता है। उसके माथे को छूता है और ऐसे में आटे का प्रश्न फिर जिंदा होने लगता है। सीटू की जिंदगी आटे, किवाड़, ध्याड़ी इन सबके बीच फंस कर रह जाती है।

सीटू सोचता है कि वैसे तो इस झोपड़े में ऐसा कुछ नहीं चुराने को। कोई अमानत नहीं अपने पास जिसे कोई चुराना चाहे। पर फिर सहसा यह विचार प्रवेश पाते देर नहीं लगती कि सुखनी तो है उसकी अमानत, उसकी जिंदगी, उसकी जिंदगी का मकसद! जवान जहान जो कुछ भी है सुखनी ही तो है। वह सोचता है कि दुनिया सीधी नहीं है, दुनिया की नजरें सीधी नहीं हैं। वह सोचता है कि घर में किवाड़ नहीं तो घर नंगा नंगा लगता है, खाली खाली! उसे लगता है कि जैसे शहरी मानसिकता ने गांव को डस लिया है। पगडंडी के रहते हुए भी नई पगडंडी का बनाना : आखिर किसलिए? गाड़ी की सीटी का बजना, सीटू का सुखनी को तलाशना। सुखनी को लेकर बैठा यह कैसा डर है जो उसे खाए जा रहा है। वह भीतर ही भीतर सोचता है। क्या यह सब उसकी सलामती के लिए नहीं? वह जोर देकर सोचता है। एक नज़र भीतर, एक नज़र बाहर, एक ख्याल यहां और एक ख्याल वहां! कैसी विचित्र स्थिति? कई बार वह महसूस करने लगता है कि हो न हो पगडंडियां उसके भीतर भी बनने लगी हों। जो मुक्ति को नहीं नग्नता को उकेरती हों।

मिसिरजी का प्रकरण कहानी को नई धार देने के लिए तो है ही, शब्द के अर्थ को गहराई से समझने के लिए एक औजार का काम भी करता है। विकास की बयार बही है। चुनाव इसी विकास को प्रतिबिंबित करता है। मिसिरजी आए हैं। पयादों को पता है कि वे जीतने वाले हैं। उनके लिए शहतीरों का एक चबूतरा बनाया जाता है ताकि वे वहां से एक दमदार भाषण दे सकें। उनका दमदमाता भाषण और उनका दमदमाता व्यक्तित्व यही दो उस दिन की चर्चा के विषय है। सीटू, भीखू, भादरू मोहन, लच्छु, सबको ठेकेदार का आदेश चबूतरा सजाने को और समाप्ति में टाल में रख आने का। डोलरे के फूलों से सजता महकता टमकता गिसिरजी का चेहरा सभी के लिए आकर्षण। दमदमाते भाषण सबके लिए कौतुहल से कम का विषय नहीं, इसके कई कई रूप हैं। इसमें गुस्सा है, इसमें शांति है, इसमें अश्रुओं का नीचे टपकना है। भाषणोपरांत मिसिरजी का चबूतरे से नीचे उतरना और सीटू को गले लगाना और सबके

समक्ष यह घोषण करना कसम खाते हुए : ''देखो भाई लोगो, इस आदमी को आजादी से क्या मिला? लाचारी बेबसी यह सब! लेकिन आगे ऐसा नहीं होगा, मैं इस बार जीतूंगा तो ऐसा कुछ नहीं होगा, मैं सबसे पहले इस ध्याड़ीदार मजदूर की गरीबी दूर करूंगा। इसके जिंदगी से अंधेरा हटा दूंगा।''

सीटू मिसिरजी के शब्दों के आगे परास्त। आंखो मे आंसू लिए वह सोचता है कि मिसिरजी इन्सान नहीं सीधे नारायण का अवतार हैं! जब सब बधाई देते कहते हैं सीटू तेरी चांदी हो गई है तो वह फूला नहीं समाता। वह मिसिरजी की जूठन की बची शराब पीता है, जब पूरा असर नहीं होता तो खुद के पैसों की देशी पीता है। घर आकर सीधे सुखनी पर धोंस जमाता है। हटफिर कर उसके पास सुखनी ही तो है। सुखनी से सामना हुआ नहीं कि नशा छुमंतर। सुखनी नसीहत रूप में कहती है : ''मेरे से क्यु लड़ता है? लड़नी है तो कोई बड़ी लड़ाई लड़।'' उसे जलसे में भूषणबाबू जी की तकरीर का एक वाक्यात स्मरण हो आता है : ''बड़ी लड़ाई : अपने हक की लड़ाई।''

इसी प्रकरण का दूसरा दृश्य बड़ा भयावह है। मिसिरजी जीते ही नहीं, मंत्री भी बन गए हैं। अपने पहले दौर पे हैं। सीटू की खुशी के ठिकाने नहीं। वह उनके समीप पहुंचना चाहता है और उन्हें यह स्मरण दिलाना चाहता है कि तब ऐसा कहा था। लेकिन यह क्या मंत्री के अंगरक्षक तो उसे धक्का मार कर बाहर का रास्ता दिखा रहे हैं। सीटू फरियाद रखता है : ''मेरी झोपड़ी में बिजली नहीं।'' मंत्री का उत्तर : ''बिजली लगाना सरकार का काम नहीं है।'' सीटू फिर से फरियाद लगाता है : ''मेरे घर में किवाड़ नहीं है।'' मंत्री का उत्तर : ''अपने घर में काहे का भय?''

सीटू को लगता है जैसे जोगनी के ढांख से किसी ने धक्का दे दिया हो। अब न हंसने को और न रोने को। गुस्सा आ रहा है मंत्री पर : सतवादी हरिश्चंद्र बनते फिरते हैं। उसे फिर से बापू की बात का स्मरण हो आता है : सारा देश नंगा हो गया है। तू मत नंगा होना सीटू। उसे साफ दिखने लगा है : ''ये मुए लुच्चे, लफंगे, गुंडे दुनिया भर के, इन्होंने धरती की सरम नंगी कर दी है।'' राजनेताओं और उनके अर्दलियों की विवेचना करते करते वह उन पर बरसना शुरू होता है : ''जंगल के जंगल साफ कर दिए हरामियों ने, बेसरम बेहया, सब चिथ चिथ उघाडी कर दिया इन्होंने।'' फिर वह उल्ट कर देखता है और विचार करता है : ''डाक बंगले से जो औरत फरियाद लेकर खड़ी थी, शुक्र है वह सुखनी नहीं थी। फिर भी कोई तो थी बेचारी, सारे देश को खा गए, फिर भी पेट नहीं भरा भुक्खड़ों का!''

उसमें भी हल्का सा ऐसा कुछ करने को मन करता है। वह भी चोर के घर में सेंध लगाने बारे सोचता है। लकड़ी का गोदाम, शहतीर के टाल आरे की मशीन में देखे थे उसने, चीरे हुए कुछ फट्टे चार पांच ही तो चाहिए किवाड़ का मुंह ढापने को! पहले वह वहीं छिपाता है, फिर समय देख वहां से कुछ दूर करील की झाड़ी में छिपाता है। बिना किवाड़ के कमरे में तो तिल भर कुछ रखने की जगह नहीं।

वैचारिक द्वंद्व की शुरूआत यहीं से। गरीब चोरी भी नहीं कर सकता। वह कई तरह के सवालों से घिर जाता है : क्या क्या जवाब देता फिरेगा लोगों को, एकाएक किवाड़ कैसे लग गए? मोहर लगी लकड़ी यहां कैसे पहुंच गई? क्या टाल के कादिंदे जंगल के ठेकेदार, बहशी खुंखार आंखों से घुड़कियां देते फिरेंगे। उसे बार बार यही बात घेर लेती है कि वह कदाचित झूठ नहीं बोल पाएगा। बार बार बापू की कही यह बात दस्तक देती : नंगा मत होना सीटू, हम गरीब गवार मजदूर, हमें नंगा होना शोभा नहीं देता! फिर भी वह अपने भीतर महसूस करता है कि वह पूरी तरह से नंगा हो गया है। यह बात अलग है कि उसने बड़े सपने नहीं देखे, बस एक ही साध : झोपड़ी के किवाड़ लग जाए, बस। यही डर उसके भीतर समाया है, तभी तो वह भीतर होने पर भी वह उसे बाहर तलाशने लगता है। जवान जहान उसकी बीबी सुखनी! गाड़ी की सीटी का बजना और सीटू का बाहर जा सुखनी का तलाशना।

बहुत सी बातें हैं जो सीटू का परेशान करती हैं–

मंत्री का यह कहना, ''सरम काहें की रे, कोई नंगा रहता है अपने घर में कोई क्या?''

फिर अनायास बहते मिसिरजी की आंखों से आंसू।

खादी के कपड़ों, डोलरे के फूलों से सजे गिसिरजी।

माघ महीने की कड़क ठंड, कंपकंपाती ठंड से शुरू हुई कहानी 'नंगा आदमी'– जब शीर्ष पर पहुंचती है तो भी वही कंपकंपाती ठंड पर आरे की मशीन से चुरा कर लाए चार पांच फटों पर मिट्टी का तेल छिड़क अकेली आग सेंकता सीटू, सारी आग...

शायद ही उसकी इससे जिंदगी भर की कंपन शांत हो।

❖❖❖

Audience's Live Response

Ursem Lata: हल्कू का कम्बल, होरी की गाय, जोखू का पानी, झोंपड़ी में किवाड़...

Janesh Kapoor: An iconic story wonderfully narrated.... We are happy and satisfied that we have been able to associate with senior रचनाधर्मी of our state through the medium of Monday Musings'. Kudos to Praveen Sir for executing the idea of short story fest so successfully!

Namrata Pathania: The metaphor of nakedness is beautifully presented in this story of human struggle for existence.

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Painting by Neeraj Kapoor

लोक संस्कृति के विविध पक्ष

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LOK SANSKRITI KE VIVIDH PAKSH
BY
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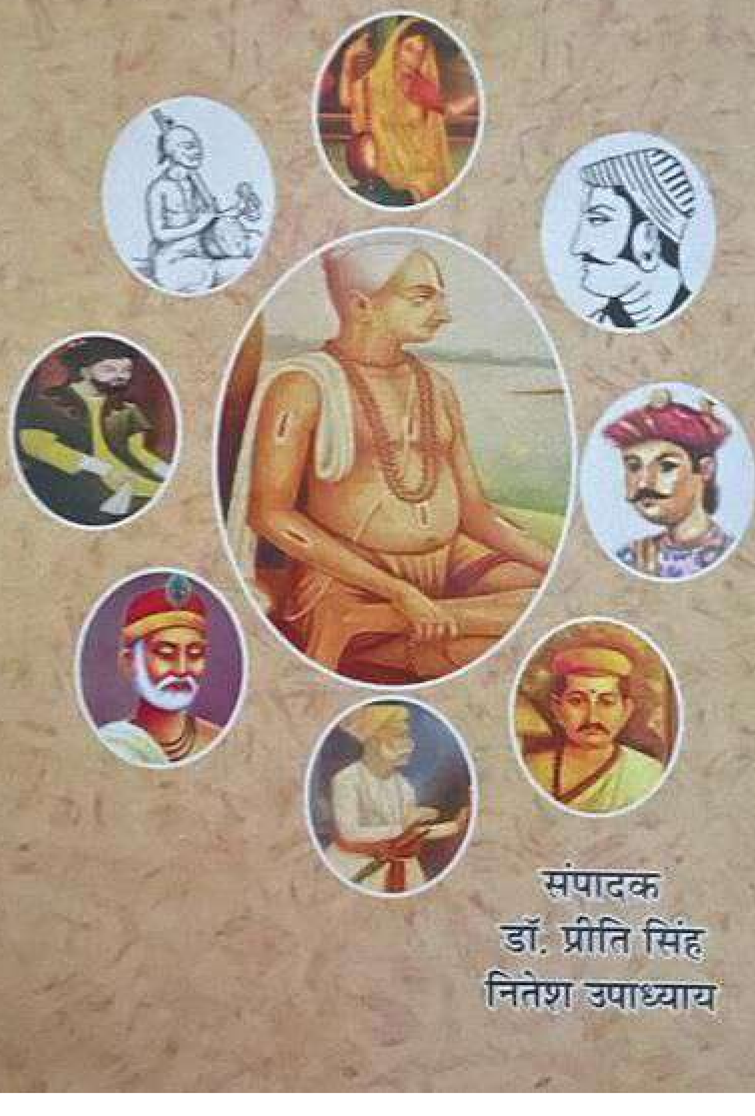
लोक में नानक

– डॉ. प्रिया शर्मा,

'लोक' शब्द अत्यंत व्यापक और सम है, यह ब्रह्म की ही तरह अनंत अक्षर और असीम है, जीवन का प्रतीक और जन का पर्याय है। 'लोक' की सीमा केवल ग्राम या साधारण जनता तक ही सीमित नहीं है, बल्कि समस्त चराचर मात्र में 'लोक' की समीचीन अलंकृति ही परम उपादेय और मांगलिक है। 'लोक' मनुष्य के हज़ारों विश्वासों, रीतियों, रिवाज़ों, रूढ़ियों, व्यवहारों, परंपराओं और संकल्पों से बनता है। लोक अनंत है, असीम है। लोक धरती से आसमान तक फैला है। जहां–जहां तक मनुष्य की बुद्धि और कल्पना पहुंचती है, वहां–वहां तक लोक की सीमा मानी जा सकती है। सृष्टि की उत्पत्ति से लगाकर प्रलय तक लोक रहेगा। जब तक मनुष्य रहेगा, तब तक लोक रहेगा। 'लोक' की व्याख्या बहुत विशद है, वह सदैव चिरंतन है। 'लोक' काल का अनुगामी है। काल निरंतर है, नित्य है, लोक भी निरंतर और नित्य है। आदमी की मृत्यु के पूर्व और पश्चात, काल की तरह लोक भी विद्यमान है। काल की गणना संभव है, लोक की गणना नहीं हो सकती लेकिन वह सर्वत्र होता है। काल को खंडों में विभक्त किया जा सकता है, लेकिन लोक को विभाजित करना संभव नहीं। लोक की शक्ति अपार है।[1] लोक शक्ति साहित्य में वेदों से लगाकर उपनिषद्, आरण्यक, पुराण, तांत्रिकी से होते हुए संस्कृत, अपभ्रंश वाङ्गमय में पूरी ताकत के साथ अभिव्यक्त हुई। गुप्त, राजपूत और भक्तिकाल के भारतीय समाज की कला साहित्य संस्कृति में लोक शक्ति संपूर्ण ओजस्विता के साथ प्रकट हुई। मुगलकाल में लोक शक्ति शिवाजी,

मध्यकालीन कविता

अवधारणा और स्वरूप



संपादक
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10.

गुरुनानक के काव्य में धर्म और आचरण की प्रासंगिकता

डॉ. प्रिया शर्मा

गुरुनानक देव सिक्ख धर्म के जन्मदाता थे। उन्होंने आदर्श जीवन के जो सिद्धांत मनुष्य को दिए, उन्हें हम सिक्ख धर्म के नाम से जानते हैं। यह धर्म सबसे अधिक बल ईश्वर की भक्ति अर्थात् नाम–स्मरण पर देता है और मनुष्य को निरर्थक विश्वासों तथा कर्मकाण्डों का त्याग कर उच्च, निर्मल, धार्मिक तथा सदाचारिक गुण धारण करके आचरण को पवित्र रखने की शिक्षा देता है। इसका कार्य–क्षेत्र यहीं तक सीमित नहीं बल्कि इसका मनोरथ तो मनुष्य को जीवन के सभी क्षेत्रों धार्मिक, सामाजिक, आर्थिक तथा राजनीतिक आदि में उचित मार्गदर्शन करवाना है। अपने इस मौलिक तथा स्वतंत्र धर्म को प्रचारित करने के लिए गुरुनानक जी को उस समय के प्रचलित धर्मों का विरोध भी सहना पड़ा, क्योंकि उन्होंने उनके अनुचित कार्यों का बड़े साहस तथा निर्भयता से खंडन किया था। प्रत्येक मनुष्य चाहे वह किसी भी धर्म को मानने वाला क्यों न हो, उसे उस धर्म की असलियत का भी अहसास कराया। ऐसे गुरुनानक देव का धर्म मज़हबों, सम्प्रदायों से ऊपर उठकर मानवतावादी भावना तक आ पहुँचता है।

वास्तव में धर्म की कसौटी मनुष्य ही है। मनुष्यों की विशिष्टता दिखाने वाली यदि कोई वस्तु है तो वह धर्म है। 'धर्म वह है, जो व्यक्ति और समाज को धारण करे, उसका पोषण और संवर्धन करे। गर्नारी महतो ने धर्म को कर्म का पर्याय माना है– 'वह कर्म जिसका संपादन, किसी संबंध या गुणविशेष के विचार से उचित और आवश्यक हो, धर्म है।' औचित्य की दृष्टि से धर्म सार्वकालिक, सार्वदेशिक और सार्वभौमिक है। समाज में रहते हुए व्यक्ति जिस धर्म का पालन करता है, वही मानव धर्म है। धर्म व्यक्ति को दूसरों पर श्रद्धा एवं विश्वास करना सिखाता है। हजारी प्रसाद द्विवेदी 'धर्म को अन्य व्यक्ति में श्रद्धा उद्रिक्त करने वाला' मानते हैं। धर्म मानव का उदार कर्तव्य है। 'यह कर्तव्य मानवीय संबंधों और परिस्थितियों के विविध रूपों में चरितार्थ होता है। धर्म

DAE-BRNS High Energy Physics Symposium
High Energy Physics Symposium
DAEBRNS 2018, HEPS 2018: **XXIII DAE High Energy Physics Symposium** pp 995–1000

Majorana Unitarity Triangle in Two-Texture Zero Neutrino Mass Model and Associated Phenomenology

Surender Verma, Shankita Bhardwaj & Monal Kashav

Conference paper | First Online: 19 May 2021

Part of the Springer Proceedings in Physics book series (SPPHY,volume 261)

Abstract

Non-zero value of θ_{13} has, now, shifted the focus of the neutrino oscillation experiments to measure leptonic *CP* violation. If neutrinos are Majorana particles, the mixing matrix contains two Majorana-type *CP*-violating phases along with Dirac phase. In the present work, we have established a possible relation between Majorana *CP* phases and geometric parameters of Majorana unitarity triangle (MUT) in two-texture zero neutrino mass model. Similar relations can, also, be derived for other theoretical and phenomenological neutrino mass models. For two-texture zero models, we find that Majorana *CP* phases depend on one of the interior angles of MUT. The non-trivial orientation of MUT in complex plane and its non-vanishing area suggests that *CP* violation is inherent in two-texture models.

Keywords

[spsdollar2dollarsps] Violation **Unitarity** **Texture zeros**

References

Roshan Lal Sharma teaches and in the Department of English, Central University of Himachal Pradesh, Dharamshala. He has authored *Walt Whitman* (2000), *Shorter Fiction of Raja Rao* (2008), *Envisioning Walt Whitman from Sufic Perspective* (in press), co-authored *Som P. Ranchan: Dialogue Epic in Indian English Poetry* (2012); edited with critical introduction *Rustic Tales* (2014); co-edited, text books namely *The Curtain Raised* (2013), *New Vistas* (2004), *Under the Spotlight* (2002), *Communication in Contemporary Scenario: Its Multiple Dimensions* (in press); and translated from English to Hindi a modern folk ballad, *Shirgul Parmar* (2008). As a bi-lingual poet, he has published two collections of poems namely *Mount Karol and Other Poems* (2008) and *Lahar Lahar Bahav Aur Anya Kavitayen* (2008). He also has more than fifty research papers to his credit. Having taught for more than twenty five years, he has supervised 11 M.Phil and 12 PhD scholars. Sharma was awarded Fulbright Senior Research Fellowship during 2007-08 for his book project on "Sufi Interpretations of Walt Whitman" at the University of Wisconsin-Madison in the United States. He was also awarded Honorary Fellowship of the Institute for Research in the Humanities (IRH), UW-Madison. The Inter-University Centre of Indian Institute of Advanced Study, Shimla awarded him Associateship during 2004 to 2006. His areas of interest include Contemporary Literary and Cultural Theory, Indian Writing in English, Mystical Poetry of Diverse Poetic Traditions (Sufi in Particular), Postcolonial and Diasporic Literature, Indian Literary Aesthetics, and Communication Studies.

Hem Raj Bansal is Assistant Professor of English in the Department of English and European Languages, Central University of Himachal Pradesh, Dharamshala. Recipient of the Best Research Paper Award in a Seminar on Shakespeare in 2015, Bansal has also been awarded Library Fellowship by the Forum on Contemporary Theory, Baroda during June-July 2009. He has more than a dozen research articles to his credit and has presented papers in several national and international conferences. His co-edited book entitled *Homogeneity in Heterogeneity: Memory, Culture, and Resistance in Aboriginal Literatures from Around the World* is in the process of publication. As a poet, Bansal writes both in English as well as Hindi, and his debut collections shall be soon out. His areas of interest include Diaspora Writings, Australian Aboriginal Literature, Indo-Canadian Drama, Postcolonial Literature, World Literature and Communication Skills.

Anamika

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Mapping Diaspora Identities India and Beyond *Edited by :* **Dr. Roshan Lal Sharma Dr. Hem Raj Bansal**

Anamika

Mapping Diaspora Identities India and Beyond

Edited by

Roshan Lal Sharma
Hem Raj Bansal

This book charts out the trajectory of Indian diaspora from pre-colonial to the contemporary times. Grappling with pressing issues of conflict between home and host culture, female sensibility, pain of rootlessness resulting from exile, narratives of partition, and those concerning forced dispersal of people from Kashmir, the anthology adds to the critical corpus on Indian diaspora literature. Besides discussing the works of writers such as Salman Rushdie, Amitav Ghosh, Jhumhpa Lahiri, Manju Kapoor, Aga Shahid Ali, M.G. Vassan Ji, Jamila Hashmi, Bharati Mukherjee, Anita Desai, Henrique de Senna Fernandes, and Samuel Selvon, the anthology also examines the nature and scope of diaspora alongside issues pertaining to its literary representation in literature from India and beyond.

Chapter

Microbial Depolymerization

January 2018
DOI:10.1007/978-981-10-7413-4_4
In book: Waste Bioremediation (pp.61-103)

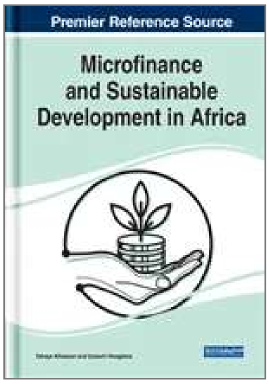
Authors:

Anvita Sheel

Deepak Pant
Central University of Himachal Pradesh

Abstract

Depolymerization is a tertiary recycling technique which represents the transformation of polymer chain into monomer units along with oligomers. Depolymerization of polymers with heterofunctional groups can be targeted by microorganisms, especially by fungi. Polyethylene terephthalate (PET), polyurethane (PU), polyester are among such polymers. Hydrolysis is the main cause behind microbial degradation, and hydrophobic surface binding of involving enzyme is responsible for microscopic description. Microbial participation techniques can be modified by using a chemical-assisted hybrid technique, and even halogenated polymer is dechlorinated by this technique.



Microfinance for achieving Sustainable Development Goals: Pondering Over Indian Experiences for the Preservation of Magnificent African Natural Resources

Manpreet Arora (School of Commerce and Management Studies, Central University of Himachal Pradesh, Dharamshala, India) and Swati Singh (Maharaja Agrasen University, India)

Source Title: Microfinance and Sustainable Development in Africa (/book/microfinance-sustainable-development-africa/264266)

Pages: 21
DOI: 10.4018/978-1-7998-7499-7.ch004

Abstract

This chapter focuses on the possibilities of exploring the areas where credit intervention can be done by the government in the form of schemes which are dependent on sustainable business practices. Nature has provided us abundance of raw material which if used wisely can help to remove poverty across the globe; on the same hand we can preserve the natural resources also if we use sustainable practices. In the current scenario where the world is facing pandemic and natural calamities, the time has to come to focus on sustainable rural micro financing activities which can not only solve the problem of linking the deprived sections of society with the mainstream, but it can also help them to improve their standard of living, and simultaneously, it can take care of various environmental issues too.

Chapter Preview

Top

Background

Various studies and researches provide evidence that the formal financial institutions used to fail to reach to the poorer sections of the society (**Ledgerwood, 1998**; Ananth & Öncü, 2013) especially to the rural people because there is information asymmetry and there are certain issues relating to enforcement of various programmes initiated by the governments in order to reach to the poorest of the poor residing in the rural areas therefore there is a dire need of having certain alternatives for the rural credit systems which can help us to solve the various problems relating with the rural credits which can serve the marginalized and vulnerable sections of the society. Financial institutions are regarded as the saviors who can help to solve various problems relating to the issues which generally the formal financial institutions or banking structures face while reaching to the deprived sections of the society. The concept of group lending phenomenon, joint liability system and mentored credit help us to solve various problems relating with the issues which generally are pertaining to the formal banking structure (**Bangoura, 2012**).

Key Terms in this Chapter

Microfinance (/dictionary/microfinance/49064): Microfinance refers to the financial services provided to low-income individuals or groups who are typically excluded from traditional banking.

Sustainability (/dictionary/sustainability/28851): Sustainability means meeting our own needs without compromising the ability of future generations to meet their own needs.

Sustainable Development Goals (/dictionary/sustainable-development-goals/62674): The Sustainable Development Goals are the blueprint established by United Nations to achieve a better and more sustainable future for all.

Sustainable Practices (/dictionary/sustainable-practices/78504): Sustainable practices are the processes services employ to maintain the qualities that are valued in the physical environment. Living sustainably is about living within the means of natural systems (environment) and ensuring that our lifestyle doesn't harm other people.

Complete Chapter List

Search this Book: Full text search terms

Reset

[Estd: 1954]

Raja Bahadur Venkata Rama Reddy Women's College (Autonomous)

Affiliated to Osmania University, Accredited with 'B++' Grade by NAAC
Narayanguda, Hyderabad

INTERNAL QUALITY ASSURANCE CELL (IQAC)

NAAC Sponsored Two Day National Seminar (Online Mode)
on **"Promoting Quality Research & Innovation in Higher Educational Institutions"**
[4th & 5th March 2022]

SUMMARY OF THE PROCEEDINGS

With the encouragement and support from the NAAC, Bengaluru, the IQAC of RBVRR Women's College organized a Two Day Virtual Seminar on "**Promoting Quality Research & Innovation in Higher Educational Institutions**" on 4th& 5th March 2022. Around 300 participants, comprising of Principals, Professors, Associate Professors, Assistant Professors IQAC coordinators and Research Scholars from various Colleges and Universities allover India participated in this Seminar.

The objectives of the seminarwere:

1. To create awareness on Quality Research among faculty and students
2. To inculcate research culture in HEI's
3. To know about contribution of Research and Innovation in the Nation/Society development
4. To create awareness on ecosystem for fostering innovation
5. To generate measures to promote research and innovation HEI's

The Inaugural began with the tradition of Lighting the Lamp and Prayer Song. The Program was presided by Prof. G. Sudarshan Reddy, Secretary cum Correspondent. Dr. J. Achyutha Devi, Principal welcomed the guests and shared a few details about the college. Dr. B. Anupama, Program Convenor and Coordinator, IQAC has welcomed Guests and spoke about the Seminar.

INAUGURATION FUNCTION:

Chief Guest: **Prof. R. Limbadri, Chairman, Telangana State Council of Higher Education** has released the proceedings of the Seminar. He discussed about quality research and innovation and its need in contemporary situations.

Guest of Honour and Keynote Speaker: **Dr. M. S. Shyamasundar, Adviser, NAAC, Southern Region** has given Keynote Lecture on Innovation and Research Culture. He has made discussion on two dimensions - one on digital transformation and other on Green renewable energy.

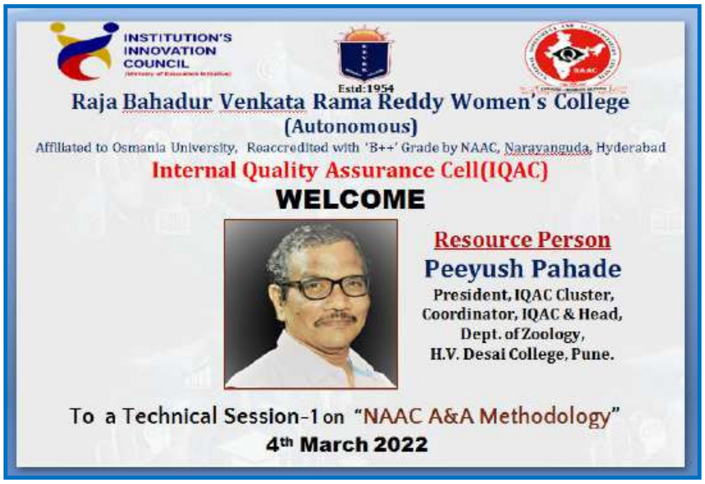
Chief Guest: Prof. R. Limbadri, Chairman, TSCHE - Seminar Proceeding Release

Guest of Honour & Keynote Speaker: Dr. M. S. Shyamasundar, Adviser, NAAC, Southern Region

THE TECHNICAL SESSIONS ON VARIOUS THEMES BEGAN WITH THE FOLLOWING EMINENT SPEAKERS:

Session 1: "NAAC A&A Methodology"

Pahade Peeyush Manohar, Coordinator, IQAC & Head, Dept. of Zoology, H V. Desai College, Pune, has highlighted the changes in SSR preparation such as decreased time in preparation, Fragmented authority, NAAC SSR framework, 7 criteria's , key indicators and metrics.



Session 2: "Research Methodology & Goals of Research"

Prof. Veena Soraganvi, Dean (R &D) Basaveshwar Engineering College (Autonomous), Bagalkot has enlightened on meaning of research, its significance, goals, types and role of research in science and technology. She emphasized on research problem formulation, identification of a problem, selection, statement of a problem, steps involved in literature review, formulate working hypothesis, research design, interpretation and report writing.

Session 3: "Contribution of research & Innovation in the development of Society/ Nation"

Shri. Manoj Kumar Bada Ghar Wala, Founder & Director, Badgharwala Technologies Pvt Ltd, Hyderabad has elaborated on impact of research and innovation in day-to-day life, changing technologies, different areas like Agriculture, irrigation, health and wellbeing, education, environmental studies and waste management. He highlighted the Grass root innovation which helps to solve local community problems and achieve sustainability.

Session 4: "Innovation in ways of Linking Research with Teaching"

Shri. Ravi Pandey, Research Establishment Officer (IP & Tech Transfer)Start-up Incubation and Innovation Center, IIT Kanpur, UP , has discussed about concepts linking innovations and research with teaching such as entrepreneurship, government initiatives for entrepreneurship, illustrations of innovation promotion in an organization. He has interacted with participants on innovations in organizations such as PRFAQ, Alexa, converting non smart TV into smart TV, and how to connect teaching with corporate. He has briefed about the different government initiatives for Incubation, patents, funding agencies for Incubation and IP in Q & A session.

Session 5: "Research Ethics"

Prof. Joseph Dorairaj, Professor of English, School of English and Foreign Languages, Gandhigram Rural Institute, Gandhigram, Tamilnadu, has elaborated on schemes of research, characteristics of good research work, what is ethics, academic ethics, research ethics, academic integrity. He has highlighted the 25 crucial issues in research practices and illustrated the unethical practices that are followed by researchers, academicians and students in their thesis, research articles and projects. He has enlightened on research misconduct in India, and measures taken to overcome these issues.

Session 6: "Inculcating Research culture through Curriculum Design"

Prof. Suresh H. Jangamshetti, Head, Dept. of EEE , Basaveshwar Engineering College(Autonomous), Bagalkot , has discussed about research activities and stages of research process. He explained about three attributes of quality research i.e. research infrastructure, research idea (guide) and research scholar. He has elaborated about research culture, quality research and outcomes of research and also briefed about how research is inculcated through curriculum design and development by including case studies.

TECHNICAL PRESENTATIONS SESSION:

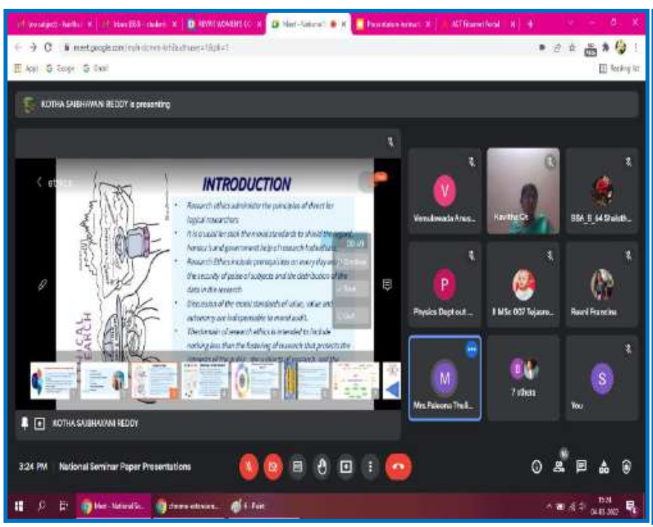
Abstracts/Full Papers are invited for Presentation on the following Sub-Themes:

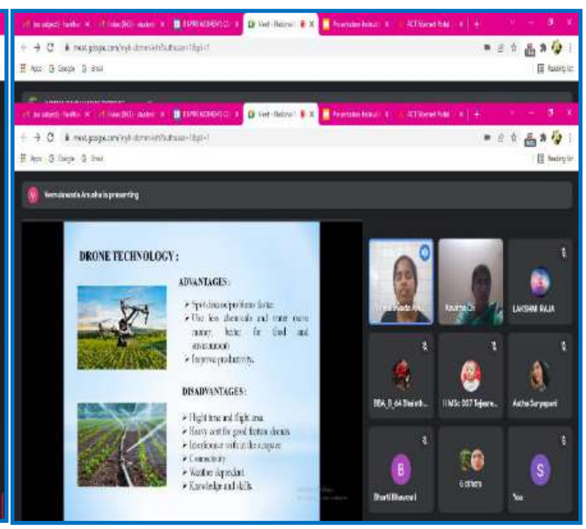
1. Strategies to promote quality Research & Innovation in Education.
2. Role of Research and Innovation in Nation building
3. Technology in Research & Innovation
4. Ecosystem for Fostering Innovation
5. Research Ethics
6. Innovation in HEI’s

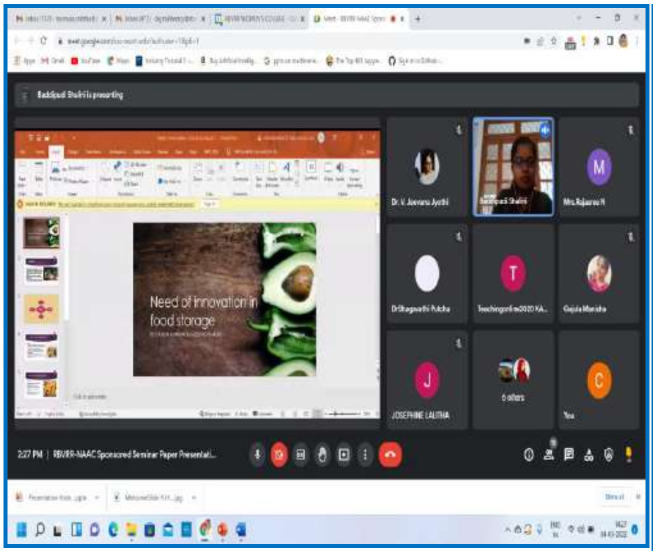
Hundred(100) abstracts are received for paper presentations on various themes such as “Role Of Research and Innovations in Nation Building”, “National Education Policy (2020): A Boost For Innovation In Education”,“Strategies to Promote Quality Research and Innovation in Education” “A Study on Ethical Practices in Research” and “Research Promotion Ideas For Higher Education” from all over India.

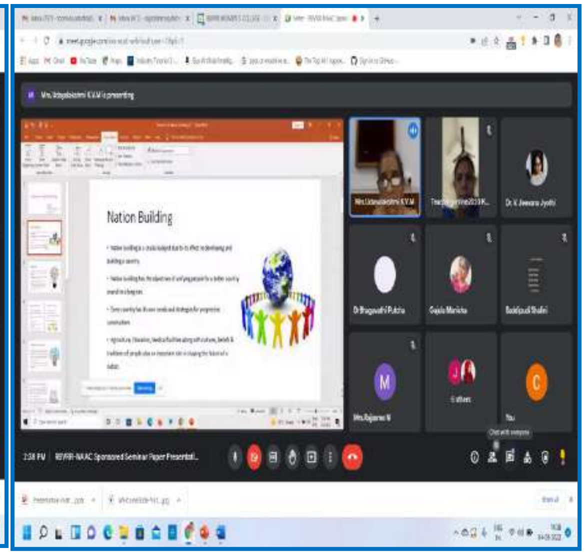
Sixty Three(63) articles were presented from various states across the India.

PAPER PRESENTATIONS







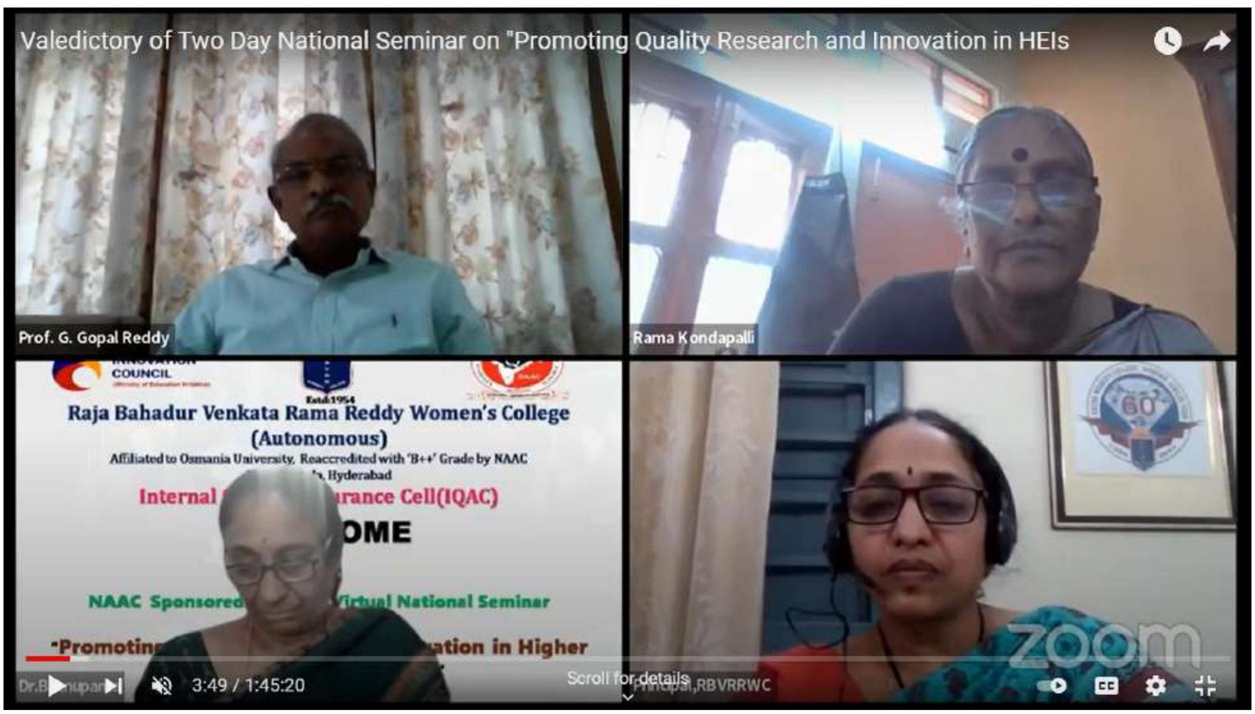


VALEDICTORY FUNCTION:

The Valedictory session began with the valedictory address by Dr. J. Achyutha Devi, Principal, RBVRR Women's College. The session was presided by Prof. G. Sudarshan Reddy, Secretary cum Correspondent, RBVRR Women's College. Prof K Muthyam Reddy, Secretary, HMVS addressed the gathering during the valedictory session. Seminar Report was presented by Dr. B. Anupama, Convener of the event and IQAC Coordinator, RBVRR Women's College. The Chief Guest of the session was chaired by Prof. G. Gopal Reddy, Pro Vice Chancellor, Mahatma Gandhi Central University,Bihar, and the Guest of Honour was Dr Rama Kondapalli, Adviser, Higher Education Councils and Central Universities. Announced Best Paper Presentations and collected Feedback from the participation. The session ended with a vote of thanks by Dr. A. Lalitha Praveena, member, IQAC and Assistant Professor, Department of Business Management, RBVRR women's college.

Chief Guest- Prof. G.Gopal reddy, Pro. Vice Chancellor,
Mahatma Gandhi Central University, Bihar

Guest of Honour: Dr. Rama Kondapally, Advisor,
Higher Education Council of States and Central Universities.



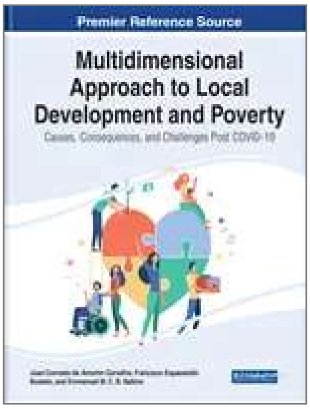
Valedictory Session

Dr. B Anupama

Dr. B. Anupama
Program Convenor & Coordinator,IQAC,RBVRRWC

J. Achyutha Devi

Dr. J. Achyutha Devi
Principal, RBVRRWC



Ruminating on Microfinance as a Contrivance of Economic Development and Women Upliftment: A Case of India ⊗

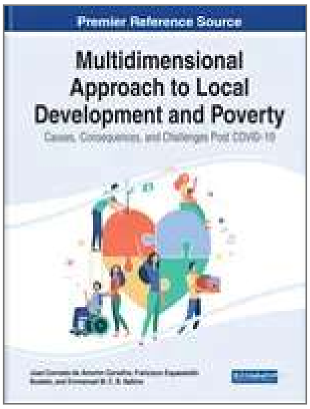
Manpreet Arora (Central University of Himachal Pradesh, Dharmashala, India)

Source Title: Multidimensional Approach to Local Development and Poverty: Causes, Consequences, and Challenges Post COVID-19 (/gateway/book/274537)

Pages: 20
ISBN13: 9781799889250ISBN10: 1799889254ISBN13 Softcover: 9781799889267EISBN13: 9781799889274
DOI: 10.4018/978-1-7998-8925-0.ch008

Abstract

This chapter focuses on the success of microfinance as a tool to create opportunities for women and other deprived sections in the formal financial structure of India. It also aims at focusing on the role of various institutions which play a significant position in providing rural credit through various channels. The author contends that in this pandemic where economies at global level require resilience strategies, microfinance once again can prove to be an effective strategy to bounce back in terms of improving the economic conditions of poor and vulnerable who suffered the most due to lockdowns, especially in developing nations like India. Localization and promotion of local product can help boost entrepreneurial activities at various levels, which can improve the distorted global supply chain slowly and gradually but effectively.



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XXII DAE High Energy Physics Symposium pp 59–64

Search for Sterile Neutrino Signal in the ^{7}Be Solar Neutrino Measurement with KamLAND

Ashish Sharma, Govind Singh, Gazal Sharma, Shankita Bhardwaj, Surender Verma & B. C. Chauhan

Conference paper | First Online: 24 May 2018

Part of the Springer Proceedings in Physics book series (SPPHY,volume 203)

Abstract

The present data from KamLAND detector experiment for Solar and Geo-neutrinos detection has assured its magnificent performance in the low energy region upto 2–3 MeV energy scale. KamLAND confirms that Large Mixing Angle (LMA) is the leading solution to the Solar Neutrino Problem (SNP), but there is possibility of time modulation of the low energy solar neutrino flux, which can also be related to solar magnetic activity. In continuation to our previous work, we re-investigate our model predictions done for ^{7}Be neutrino signal variation in KamLAND. We examine the recent ^{7}Be solar neutrino data of KamLAND solar mode and search for sterile neutrino signal, as a consequence of solar neutrino flux modulation due to solar magnetic field variation.

Keywords

Solar neutrinos **Sterile neutrinos** **Time modulation** **RSFP**

References

Chapter

The Role of Employer Branding in the Creation of Powerful Corporate Brands

By Harsh Mishra, Aditi Sharma

Book [Employer Branding for Competitive Advantage (https://www.taylorfrancis.com/books/mono/10.1201/9781003127826/employer-branding-competitive-advantage?refId=e9b80f6a-0de8-4ee1-9ec8-b78d7d03ff54&context=ubx)]

Edition	1st Edition
First Published	2021
Imprint	CRC Press
Pages	18
eBook ISBN	9781003127826

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ABSTRACT

Previous Chapter (chapters/edit/10.1201/9781003127826-2/assessing-validity-employer-branding-predicting-talent-oriented-outcomes-jeevan-jyoti-roomi-rani?context=ubx)
Next Chapter (chapters/edit/10.1201/9781003127826-4/toward-integration-collaborator-experience-digital-management-employer-brand-zakaria-lissaneddine-mostapha-el-idrissi-youn%C3%A8s-el-manzani?context=ubx)

Chapter

Synthesis, Characterization, and Application of Metal Oxide Nanoparticles

By Ashutosh Dixit, Renu Bala, Vivek Sheel Jaswal

Book [Nanotherapeutics in Cancer (https://www.taylorfrancis.com/books/mono/10.1201/9781003334538/nanotherapeutics-cancer?refId=e72b2058-66eb-4d62-9079-bb6405cadb0c&context=ubx)]

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Imprint	Jenny Stanford Publishing
Pages	21
eBook ISBN	9781003334538

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ABSTRACT

< Previous Chapter (chapters/edit/10.1201/9781003334538-1/introduction-nanotherapeutics-synthetic-preview-nahid-rehman-anjana-pandey?context=ubx)
Next Chapter > (chapters/edit/10.1201/9781003334538-3/current-scenario-nanomaterials-cancer-diagnostics-jagjit-kaur?context=ubx)

Women Empowerment
A New Perspective

Dr. Bhawana Bhardwaj
Dr. Vinod Negi

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Women Empowerment
Dr. Bhawana Bhardwaj
Dr. Vinod Negi

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A New Perspective

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WOMEN EMPOWERMENT
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Foreword

Women Empowerment has been a subject of debate since long. It is during last 50 years this subject became the central point of discussion among academicians, politicians and administrators. The policy planners started focusing on the issue and started launching various schemes for the upliftment of weaker sex and bring this issue to the centre of debate. All started talking of action plans which enhances the women share and give them a respectable place in the society. The present book is endeavour on the part of the editors to present the issue of women empowerment more seriously in a systematic manner. India attained independence on 15th August 1947 and the issue of women conditions and their empowerment still remains a matter of concern for all of us. No doubt, the government has taken a number of policy measures to uplift the status of the women in the society and bring that at par with their male counterpart. They constitute nearly 50 percent of the total population hence the subject of women empowerment becomes much more important.

The issue of empowerment of women is drawing attention all over the world including India since last few decades. United Nations in their reports and through their agencies have accentuated that gender issue is vital and needs priority. It has been realized that women cannot be expected to wait for any more impartiality.

The status of women all across the world has risen in the twentieth century. In India women have mostly lived in the four walls of their household and they were treated like objects. They have shown their dependence on males for most of the issues

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Women Empowerment and Gender Equality in India

A Myth or Reality?

ARCHNA KATOCH

ABSTRACT

The purpose of this paper is to determine whether women's empowerment is becoming a reality or not in India by focusing on gender disparities and current social status of women. Education is the keystone of women's empowerment because it enables them to change their lives by increasing their self-confidence and facilitating them to make informed decisions about their lives. The situations of women's life have certainly improved to some extent but they still face all types of violence and gender injustice. They cannot be empowered by just enacting laws without any public awareness and support to empower women. Constitutional and legal reforms, proper policy measures for women's empowerment in the social, educational, economic and political fields, and changing the mind-set of orthodox, patriarchal, stereotypical society are significant ways to achieve women's empowerment in India.

Keywords: Disparities, gender injustice, patriarchal, public opinion.

INTRODUCTION

Former United Nations Secretary-General Kofi Annan once said, "There is no tool for development more effective than the empowerment of women," (UNNC, 2005). Without empowered women, a nation cannot see its distant dream, as women are the mothers who create the future of a generation. The term empowerment is a multidimensional social process that supports people in attaining control over their own lives and fully participating in the processes and decisions that shape their lives. It refers to increasing the economic, political, social, educational, gender, or spiritual strength of individuals and communities.

Now the question, which arises, is that what it means by the term women's empowerment. In simple words, it is the creation of a social environment, where women can make independent decisions on their personal development as well as shine as equals in society. Women empowerment is essential to guarantee such human dignity. "Women's empowerment is a process of personal and social change through which they gain power, meaningful choices and control over their lives" (O'Neil, Domingo & Valters, 2014). However, most unfortunate thing is that women today are not empowered in this way. Despite the principles of gender equality enshrined in the constitution, women of India continue to withstand the most horrible societal discrimination, and even today nation require schemes like 'Beti Bachao Beti Padhao'. Base of all democracies is equal opportunities to all but, men and women differ in their access to honour, stature, and power.

Women make almost half of the world natives but society too often muzzles, marginalizes, or circumscribes the voice of this half of the population (Mayor, 1999). Traditionally, men have been always first in queue when it arises to who gets what, when, and how. Social-conflict analysis elucidates contemporary sex roles in terms of authority, subordination, and sexism. Women of India are in very pathetic state of affairs, whether it is their well-being, economic status, education, political situations, mortality rate or any other development parameter. Starting from the birth to death, females are under the grasps of copious evils acts such as violence, discriminations, cruelties, etc.

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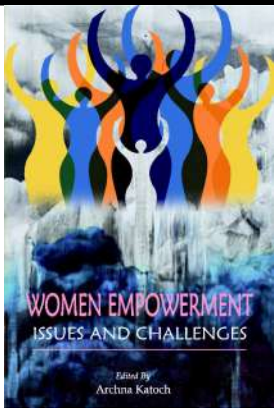

WOMEN EMPOWERMENT

Issues and Challenges

Edited By

Archna Katoch

Women empowerment and gender justice result in the development of a nation, reinforcement of its economy, invigoration of society, and eventually the establishment of peace every-where. An effort has been made in this book to raise the voice of women to achieve gender equality and women empowerment, which is the United Nations' fifth Sustainable Development Goal (SDG) by defying patriarchy, discrimination, exploitation, poverty, and violence against them. It endeavours to explore actual conditions of women in the society, different aspects of women's liberation, various issues related to imparting quality education, apposite health care, various constitutional and legal provisions, and to improve their social, economic and political status. The book comprises research articles on different issues, which advocate how women's empowerment can be achieved by raising their living levels, creating conducive conditions to their growth, increasing their freedom, and removing sexism, gender bias, and stereotypes. Despite various constitutional and legal provisions in India to protect the rights of women, gender inequality remains a primary barrier to meet the UN Sustainable Development Goals because just endorsing the laws without public support and opinion is nothing but a bundle of papers. This book attempts to create a strong public opinion by disseminating relevant information, knowledge, illumination, and creating public awareness to change the mindset of patriarchal, orthodox and tradition-oriented society. When women become aware of their rights and break the glass ceiling to access all the opportunities available to them without any prejudice, only then development, prosperity, and harmony will take place.

Archna Katoch is an Assistant Professor in the Department of Journalism and Mass Communication at Central University of Himachal Pradesh, Dharamshala, India.

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